1 Fluid models

Given a CTMC $X(t)$ with states s_i for $i = 1, ..., n$ and infinitesimal generator matrix Q, we want to predict some event using a Fluid Model with fluid level $L(t)$ with initial distribution $F_L(q)$ and density $f_L(q)$, we want to find the rates $r_i \leq 0$ for each state s_i.
Let X_k denote the k-th state that the CTMC visits, let T_k denote the time the CTMC spends in this state and let F_k denote the occurrence of the event while the CTMC is in its k-th state. We assume that the event occurs when the fluid level reaches zero.
We then have

$$\mathbb{P}(F_k | X_k = s_i) = \int_0^\infty \mathbb{P}(T_k > \frac{l}{-r_i}) f_{L_i}(l) dl = \int_0^\infty e^{-q_{ii}/r_i}) f_{L_i}(l) dl$$

Where f_{L_i} denotes the density of the fluid level when the CTMC arrives in s_i. Given some density function f_{L_i}, we can use this to make a maximum likelihood estimator for r_i. Also another relation holds between the fluid levels of different states[1]:

$$\frac{\partial}{\partial t} p_i(t, l) + \frac{\partial}{\partial l} r_i p_i(t, l) = \sum_{k=1}^n q_{ki} p_k(t, l)$$

References

[1] Marco Gribaudo and Miklós Telek. Fluid models in performance analysis. In *International School on Formal Methods for the Design of Computer, Communication and Software Systems*, pages 271–317. Springer, 2007.