1 Equivalence average cost and discounted cost

For a machine with lifetime $L \sim F(.)$, preventive cost c and corrective cost $c + a$, the average cost $J(u)$ of planning preventive maintenance after a time u has passed equals

$$J(u) = \frac{(c+a)\mathbb{P}(L < u) + c\mathbb{P}(L \geq u)}{\mathbb{E}[\min\{L, u\}]} = \frac{c + aF(u)}{\mathbb{E}[\min\{L, u\}]}.$$

For continuous discount $e^{-\alpha t}$, the discounted cost equals

$$J_\alpha(u) = (c + a + J_\alpha(u))\mathbb{P}(L < u)\mathbb{E}[e^{-\alpha L}|L < u] + (c + J_\alpha(u))e^{-\alpha u}\mathbb{P}(L \geq u)$$

$$\Rightarrow J_\alpha(u)(1 - \mathbb{P}(L < u)\mathbb{E}[e^{-\alpha L}|L < u] - e^{-\alpha u}\mathbb{P}(L \geq u))$$

$$= c(\mathbb{P}(L < u)\mathbb{E}[e^{-\alpha L}|L < u] + e^{-\alpha u}\mathbb{P}(L \geq u)) + a\mathbb{P}(L < u)\mathbb{E}[e^{-\alpha L}|L < u]$$

$$\Rightarrow J_\alpha(u) = \frac{c(\mathbb{P}(L < u)\mathbb{E}[e^{-\alpha L}|L < u] + e^{-\alpha u}\mathbb{P}(L \geq u)) + a\mathbb{P}(L < u)\mathbb{E}[e^{-\alpha L}|L < u]}{1 - \mathbb{P}(L < u)\mathbb{E}[e^{-\alpha L}|L < u] - e^{-\alpha u}\mathbb{P}(L \geq u)}$$

$$= \frac{c\mathbb{E}[e^{-\alpha \min\{L,u\}}] + aF(u)\mathbb{E}[e^{-\alpha L}|L < u]}{\mathbb{E}[1 - e^{-\alpha \min\{L,u\}}]}$$

Then

$$\lim_{\alpha \to 0} \alpha J_\alpha(u) = \lim_{\alpha \to 0} \alpha \frac{c\mathbb{E}[e^{-\alpha \min\{L,u\}}] + aF(u)\mathbb{E}[e^{-\alpha L}|L < u]}{\mathbb{E}[\alpha \min\{L, u\} + o(\alpha^2)]}$$

$$= \lim_{\alpha \to 0} \frac{c\mathbb{E}[e^{-\alpha \min\{L,u\}}] + aF(u)\mathbb{E}[e^{-\alpha L}|L < u]}{\mathbb{E}[\min\{L, u\} + o(\alpha)]} = \frac{c + aF(u)}{\mathbb{E}[\min\{L, u\}]} = J(u).$$