$$\hat{\mathbf{R}}_{um} = \frac{1}{\mathbf{Z}_{um}} \sum_{k=1}^{K} \mathbf{L}_{um}^{(k)} \mathbf{R}_{um}^{(k)}$$

(1)

??
$\mathbf{R}^h \mathcal{U}^h$
$\mathbf{R}_{um}^h \mathbf{R} h \mathbf{R}^h um^\top$
N, M
$\mathbf{R}$
$\in$
$R^{N \times M}$
K
$\ll$
$\min(N, M)$
D $\ll$
$\min(N, M)$
$i = \langle u, m \rangle \mathbf{R} i$
$\mathbf{P}_u^{(k)}$ $uk \in$
R^D
$\mathbf{Q}_m^{(k)}$ $mk \in$
R^D
$z_i z_{u,m}$ $ii =$
$\langle u, m \rangle$
θ_u $u \in$
R^K
ϕ_k $k \in$
R^M
α
β
Ψ_0
? $\phi_k k \phi_{k,m} k m K K \theta_u u \theta_{u,k} u k Dir(\alpha) Dir(\beta) \theta \phi \alpha \beta$

$$P(\{\langle u, m \rangle\}) \propto \prod_{\langle u,m \rangle} \left(\sum_k \theta_{u,k} \phi_{k,m} \right),$$

(2)

$\langle u, m \rangle um$ **??** $\langle u, m \rangle$
$\mathbf{P}_u^{(k)} \in$
R^D
$\mathbf{Q}_m^{(k)} \in$
$R^D um k$**?**
?
$\mathbf{P}^{(k)} \mathbf{Q}^{(k)} \Psi_{\mathbf{P}}^{(k)} =$
$\{\mu_{\mathbf{P}}^{(k)}, \Lambda_{\mathbf{P}}^{(k)}\} \Psi_{\mathbf{Q}}^{(k)} =$
$\{\mu_{\mathbf{Q}}^{(k)}, \Lambda_{\mathbf{Q}}^{(k)}\}$

$$P(\Psi^{(k)} | \Psi_0^{(k)}) = \mathcal{N}(\mu^{(k)} | \mu_0^{(k)}, (\xi_0^{(k)} \Lambda^{(k)})^{-1}) \mathcal{W}(\Lambda^{(k)} | \mathbf{W}_0^{(k)}, \nu_0^{(k)})$$

(3)

$\nu_0^{(k)} \mathcal{W}^{(k)} \mathbf{W}_0^{(k)} \mathbf{W}_0^{(k)} \in$
$R^{N \times N} \mathbf{W}_0^{(k)} \in$
$R^{M \times M}$
$\Psi_0^{(k)} =$
$\{\mu_0^{(k)}, \nu_0^{(k)}, \mathbf{W}_0^{(k)}\} k$**?**
$\mu_0^{(k)*} =$
$\frac{\beta_0 \mu_0 + N^{(k)} \mathbf{P}^{(k)}}{\beta_0 + N^{(k)}}, \beta_0^{(k)*} =$
$\beta_0 +$
$N^{(k)}, \nu_0^{(k)*} =$
$\nu_0 +$
$N^{(k)}$
$[\mathbf{W}_0^{(k)*}]^{-1} =$
$\mathbf{W}_0^{-1} +$
$N^{(k)} \bar{S}^{(k)} +$
$\frac{\beta_0 N^{(k)}}{\beta_0 + N^{(k)}} (\mu_0 -$
$\bar{\mathbf{P}}^{(k)}) (\mu_0 -$
$\bar{\mathbf{P}}^{(k)})^{\mathrm{T}}$
$\bar{\mathbf{P}}^{(k)} =$
$\frac{1}{N^{(k)}} \sum_{u \in U^{(k)}} \mathbf{P}_u^{(k)}, \bar{S} =$
$\frac{1}{N^{(k)}} (\mathbf{P}_u^{(k)} -$