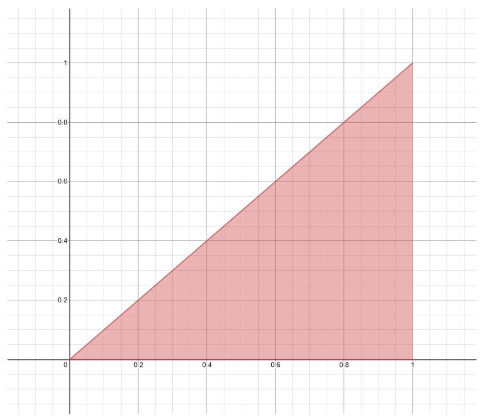
Exercise 27.1

1. Sketch the regions of integration and compute the area of the enclosed region by evaluating the double integral.

$$\int_0^1 \int_0^x dydx$$

Solution.



$$\begin{aligned}\int_0^1 \int_0^x dydx &= \int_0^1 xdx \\ &= \frac{1}{2}x^2\Big|_0^1 \\ &= \frac{1}{2}\end{aligned}$$

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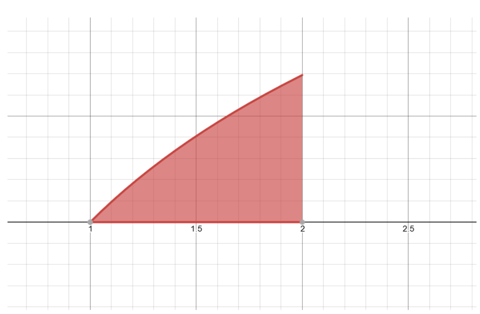
2.

$$\int_1^2 \int_0^{\ln x} dydx$$

Solution.



$$
\begin{aligned}
\int_1^2 \int_0^{\ln x} dy dx &= \int_1^2 \ln x dx \\
&= (x \ln x - x) \Big|_0^1 \\
&= 2 \ln 2 - 1
\end{aligned}
$$

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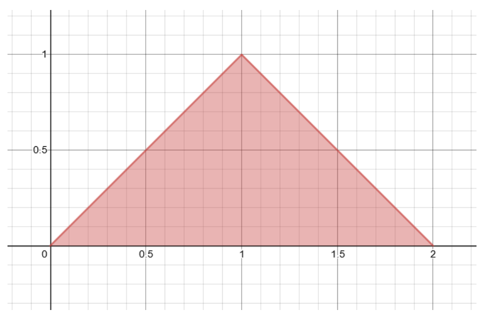
Exercise 27.2

1.

$$\int_0^1 \int_y^{2-y} dx dy$$

Solution.



$$
\begin{aligned}
\int_0^1 \int_y^{2-y} dx dy &= \int_0^1 (2 - 2y) dy \\
&= \left(2y - y^2\right) \Big|_0^1 \\
&= 1
\end{aligned}
$$

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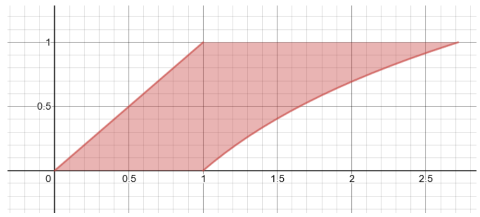
2.

$$\int_0^1 \int_y^{\exp y} dx dy$$

Solution.



$$\begin{aligned}\int_0^1 \int_y^{\exp y} dxdy &= \int_0^1 (\exp y - y)dy \\ &= \exp y\Big|_0^1 - \frac{y^2}{2}\Big|_0^1 \\ &= e - \frac{3}{2}\end{aligned}$$

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Exercise 27.3

1. Repeat the previous two questions, but change the order of integration.

$$\int_0^1 \int_y^{2-y} dxdy$$

Solution.

$$\begin{aligned}\int_0^1 \int_y^{2-y} dxdy &= \int_0^1 \int_0^x dydx + \int_0^1 \int_0^{2-x} dydx \\ &= \int_0^1 xdx + \int_1^2 (2-x)\,dx \\ &= \frac{x^2}{2}\Big|_0^1 + \left(2x - \frac{x^2}{2}\right)\Big|_1^2 \\ &= 1\end{aligned}$$

■

2.

$$\int_0^1 \int_y^{\exp y} dxdy$$

Solution.

$$\begin{aligned}\int_0^1 \int_y^{\exp y} dxdy &= \int_0^1 \int_0^x dydx + \int_1^e \int_{\ln x}^1 dydx \\ &= \int_0^1 xdx + \int_1^e (1-\ln x)\, dx \\ &= \frac{x^2}{2}\Big|_0^1 + x\Big|_1^e + (x\ln x - x)\Big|_1^e \\ &= e - \frac{3}{2}\end{aligned}$$

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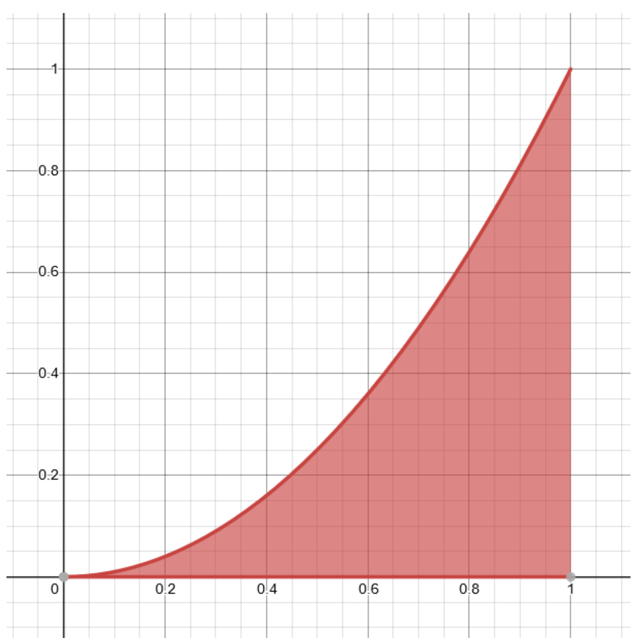
Exercise 27.4

1. Sketch the following region of integration in the plane, and evaluate the integral using WolframAlpha.

$$\int_D xcosydA$$

Solution.



$$\int_0^1 \int_0^{x^2} x\cos(y)dydx = \sin^2\left(\frac{1}{2}\right) \approx 0.229849$$

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Exercise 27.5

1. Find the total mass and center of mass of the 1cm thin aluminum plate bounded by the parabola $y = x^2$, $y = 10$, and $x = 0$. Assume x and y are measured in centimeters.

   Solution.

$$M = H \int_0^{10} \int_0^{\sqrt{y}} \rho dx dy$$
$$M = \rho \int_0^{10} \sqrt{y} dy$$
$$M = \rho \frac{2}{3} y^{\frac{3}{2}} \Big|_0^{10}$$
$$M = \rho \frac{2}{3} 10^{\frac{3}{2}}$$
$$M \approx 56.9$$

$$x_{COM} = \frac{H}{M} \int_0^{10} \int_0^{\sqrt{y}} x \rho dx dy$$
$$x_{COM} = \frac{\rho}{\rho \frac{2}{3} 10^{\frac{3}{2}}} \int_0^{10} \frac{y}{2} dy$$
$$x_{COM} = \frac{3}{2\left(10^{\frac{3}{2}}\right)} \left(\frac{y^2}{4} \Big|_0^{10} \right)$$
$$x_{COM} = \frac{3}{2\left(10^{\frac{3}{2}}\right)} \left(\frac{10^2}{4} \right)$$
$$x_{COM} = \frac{3}{8} 10^{1/2}$$
$$x_{COM} \approx 1.186$$

$$y_{COM} = \frac{H}{M} \int_0^{10} \int_0^{\sqrt{y}} y \rho dx dy$$
$$y_{COM} = \frac{\rho}{\rho \frac{2}{3} 10^{\frac{3}{2}}} \int_0^{10} y^{\frac{3}{2}} dy$$
$$y_{COM} = \frac{3}{2\left(10^{\frac{3}{2}}\right)} \left(\frac{y^{\frac{5}{2}}}{\frac{5}{2}} \Big|_0^{10} \right)$$
$$y_{COM} = 6$$

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