Exercise 2.1

1. Assume v and w are two vectors of unit length, i.e $\|\boldsymbol{v}\| = \|\boldsymbol{w}\| = 1$. Using the formula above, what angle between v and w maximizes the dot product? Using the formula above, what angle between v and w minimizes the dot product?

   Solution.
   The formula for a dot product is:

   $$\boldsymbol{v} \cdot \boldsymbol{w} = \|\boldsymbol{v}\|\|\boldsymbol{w}\|cos(\theta)$$

   Given that v and w are unit vectors, this can be simplified to:

   $$\boldsymbol{v} \cdot \boldsymbol{w} = cos(\theta)$$

   Therefore, to maximize $\boldsymbol{v}\cdot\boldsymbol{w}$, θ is 0 or π radians. To minimize $\boldsymbol{v}\cdot\boldsymbol{w}$, θ is $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ radians.

   ■

2. Compute $\boldsymbol{v} \cdot \boldsymbol{w}$ where

   $$\boldsymbol{v} = \begin{bmatrix} 1 \\ 3 \\ -4 \\ 6 \end{bmatrix}, \text{ and } \boldsymbol{w} = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 3 \end{bmatrix}$$

   Solution.

   $$\boldsymbol{v} \cdot \boldsymbol{w} = \|\boldsymbol{v}\|\|\boldsymbol{w}\|cos(\theta) = 12$$

   ■

Exercise 2.2

1. Find a row vector $\boldsymbol{t}$ so that the product $\boldsymbol{tf}$ tells you the number of fruits in your refrigerator.

   Solution.

   $$\boldsymbol{t} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

   ■

2. Find a row vector $\boldsymbol{c}$ such that the product $\boldsymbol{cf}$ tells you the total number of *citrus* fruits in your refrigerator.

Solution.

$$\boldsymbol{c} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

∎

3. Suppose that in the genetically engineered future, all apples weigh 100 g, all grapefruits weigh 250 g and all oranges weigh 120 g. Find a row vector $\boldsymbol{w}$, such that the product $\boldsymbol{wf}$ tells you the total weight of fruits in your refrigerator.

   Solution.

$$\boldsymbol{w} = \begin{bmatrix} 100 & 250 & 120 \end{bmatrix}$$

∎

Exercise 2.3

1. Recall the matrix $\boldsymbol{G}$ and the vector $\boldsymbol{f}$, which kept track of the number of fruit of different types. What does the vector $\boldsymbol{Gf}$ represent?

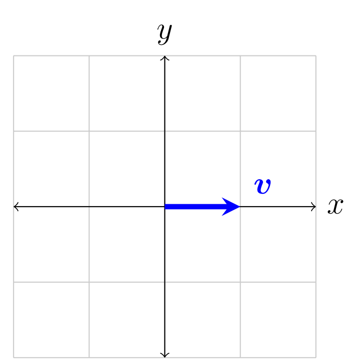
   Solution. $\boldsymbol{Gf}$ is a 2x1 vector that specifies the total number of fruit and the number of citrus fruit. ∎

Exercise 2.4

1. Please draw the spacial vector

$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

   Solution.



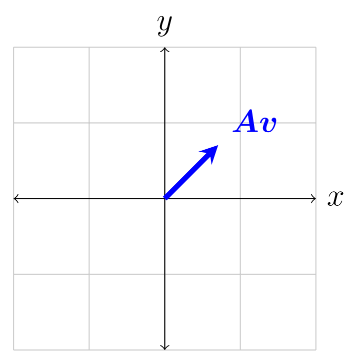
∎

2. Please draw the vector $\boldsymbol{w} = \boldsymbol{A}\boldsymbol{v}$ where $\boldsymbol{A}$ is

$$\boldsymbol{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Solution.

$$\boldsymbol{w} = \boldsymbol{A}\boldsymbol{v} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$



■

3. What happened to $\boldsymbol{v}$ when you multiplied by $\boldsymbol{A}$?

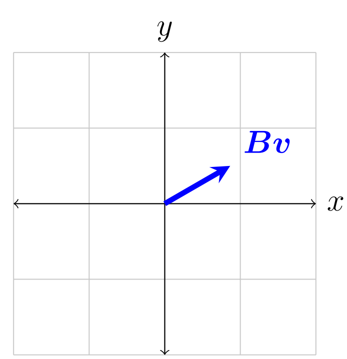
Solution. $\boldsymbol{A}\boldsymbol{v}$ is a 45 degree rotation of $\boldsymbol{v}$. ■

4. Please draw the vector $u = \boldsymbol{B}v$, where $\boldsymbol{B}$ is

$$\boldsymbol{B} = \begin{bmatrix} cos(30°) & -sin(30°) \\ sin(30°) & cos(30°) \end{bmatrix}$$

Solution.

$$\boldsymbol{B}\boldsymbol{v} = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$$



∎

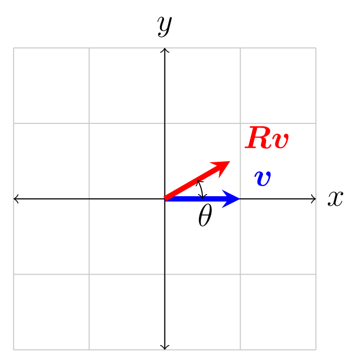
5. What happened to $\boldsymbol{v}$ when you multiplied by $\boldsymbol{B}$?

   Solution. $\boldsymbol{Bv}$ is a 30 degree rotation of $\boldsymbol{v}$. ∎

6. Please draw the vector $\boldsymbol{t} = \boldsymbol{Rv}$, where $\boldsymbol{R}$ is

$$R = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

   Solution.



∎

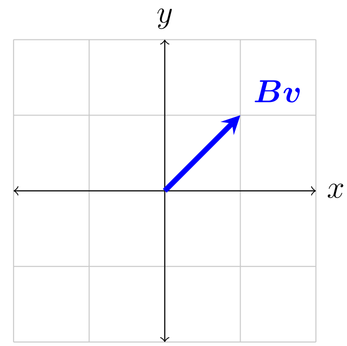
7. What happened to v when you multiplied by R?

   Solution. $\boldsymbol{Rv}$ is the rotation of $\boldsymbol{v}$ by θ. ∎

8. Please draw a new spacial vector

$$\boldsymbol{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

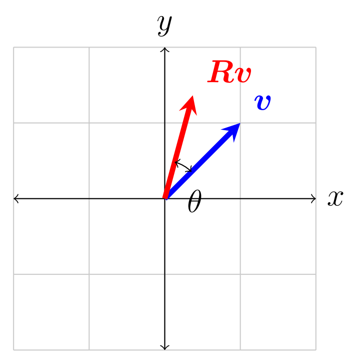
   Solution.



■

9. Please draw the vector $\boldsymbol{x} = \boldsymbol{R}\boldsymbol{w}$

   Solution.



■

10. What does multiplying *any* vector by $\boldsymbol{R}$ do?

    Solution. Multiplying by $\boldsymbol{R}$ rotates the vector by θ. ■

Exercise 2.6

1. Using the definitions for A, v, and u from above, please predict the output of the following commands and then solve them using MATLAB.

   Solution.

```
A = [2 1; 3 -1; 0 4];
v = [-2; 1];
u = [2 -3 1];

A * v
% ans =
```

```
% -3
% -7
% -4

u * A
% ans =
% -5 -9

A(1:2, :) * v
% ans =
% -3
% -7

u * A(:, 2)
% ans = 9
```

■

Exercise 2.7

1. Write a for loop that creates the following matrix:

```
M_squares = [1 1; 2 4; 3 9; 4 16]
```

Solution.

```
for i=1:4
M_squares(i, :) = [i i^2]
end
```

■

Exercise 2.8

1.
```
A * v
```

Solution.

$$\boldsymbol{A} * \boldsymbol{v} = \begin{bmatrix} -3 \\ -7 \\ 4 \end{bmatrix}$$

■

2.
```
u * A
```

Solution.

$$\boldsymbol{u} * \boldsymbol{A} = \begin{bmatrix} -5 & 9 \end{bmatrix}$$

■

3.
```
A * u
```

Solution. Undefined (incorrect dimensions) ■

4.
```
v * A
```

Solution. Undefined (incorrect dimensions) ■

5.
```
A(1:2, :) * v
```

Solution.

$$\boldsymbol{A}_1 * \boldsymbol{v} = \begin{bmatrix} -3 \\ -7 \end{bmatrix}$$

■

6.
```
u * A(:, 2)
```

Solution.

$$\boldsymbol{u} * \boldsymbol{A}_2 = 9$$

■

7.
```
A(:, 2:4) * v
```

Solution. Undefined (out of bounds error) ■

8.
```
u * A(1, :)
```

Solution. Undefined (incorrect dimensions). ■

Exercise 2.9

1. Evaluate $4\boldsymbol{A} - 5\boldsymbol{B}$

$$\boldsymbol{A} = \begin{bmatrix} 3 & 4 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$
$$\boldsymbol{B} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \end{bmatrix}$$

Solution.

$$4\boldsymbol{A} - 5\boldsymbol{B} = \begin{bmatrix} 7 & 6 & -3 \\ 2 & -6 & -1 \end{bmatrix}$$

■

Exercise 2.10

1. If the matrix $\boldsymbol{A}$ has dimensions of 4×5, what are the dimensions of $\boldsymbol{A}^{\intercal}$?

   Solution. 5×4 ■

Exercise 2.11

1. If the matrix $\boldsymbol{A}$ is 4×5 and the vector $\boldsymbol{v}$ is 5×1, what are the dimensions of $\boldsymbol{Av}$ and $(\boldsymbol{Av})^{\intercal}$

   Solution. $\boldsymbol{Av}$ is 4×1 and $(\boldsymbol{Av})^{\intercal}$ is 1×4 ■

Exercise 2.12

1. How do you find the transpose of a vector or matrix in MATLAB?

   Solution. Appending ' to the matrix returns the transpose ■

Exercise 2.13

1.

$$\boldsymbol{A} = \begin{bmatrix} -2 & 4 \\ 0 & 3 \end{bmatrix}$$
$$\boldsymbol{B} = \begin{bmatrix} 5 & -3 \\ -1 & 1 \end{bmatrix}$$

Find the matrix product $\boldsymbol{AB}$.

Solution.

$$\boldsymbol{AB} = \begin{bmatrix} -14 & 10 \\ -3 & 3 \end{bmatrix}$$

■

Exercise 2.14

1. Find the matrix product $\boldsymbol{BA}$.

   Solution.

$$\boldsymbol{BA} = \begin{bmatrix} -10 & 11 \\ 2 & -1 \end{bmatrix}$$

■

Exercise 2.15

1.

$$\boldsymbol{C} = \begin{bmatrix} -5 & -1 \\ -3 & 2 \end{bmatrix}$$

   Calculate $\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C})$

   Solution.

$$\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C}) = \begin{bmatrix} -16 & 20 \\ -12 & 9 \end{bmatrix}$$

■

Exercise 2.16

1. Calculate $\boldsymbol{AB}+\boldsymbol{AC}$. Is it equal to your previous answer?

   Solution.

$$\boldsymbol{AB}+\boldsymbol{AC} = \begin{bmatrix} -16 & 20 \\ -12 & 9 \end{bmatrix}$$

   This is equal to my previous answer. ■

Exercise 2.17

1. $\boldsymbol{A}^2$

Solution.

$$\boldsymbol{A}^2 = \begin{bmatrix} 4 & 4 \\ 0 & 9 \end{bmatrix}$$

■

2. $\boldsymbol{B}^2$

Solution.

$$\boldsymbol{B}^2 = \begin{bmatrix} 28 & -18 \\ -6 & 4 \end{bmatrix}$$

■

Exercise 2.18

1. Square Matrix

Solution. A square matrix is a matrix with the same number of rows and columns.
ex:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

■

2. Rectangular Matrix

Solution. A rectangular matrix is a matrix where the elements are arranged into a number of rows and a number of columns.
ex:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

■

3. Diagonal Matrix

Solution. A diagonal matrix is a square matrix where the elements outside of the main diagonal are 0.

ex:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

■

4. Identity Matrix

Solution. An identity matrix is a diagonal matrix where the numbers on the main diagonal are all 1.

ex:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

■

5. Symmetric Matrix

Solution. A symmetric matrix is a square matrix that is equal to its transpose.

ex:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

■

Exercise 2.19

1.

$$\boldsymbol{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Find a 3×3 matrix $\boldsymbol{M}$ such that $\boldsymbol{M}\boldsymbol{v} = 3\boldsymbol{v}$ for any vector v.

Solution.

$$\boldsymbol{M} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

■

2. Find the 3×3 matrix which scales the x component by 3 and the y component by 5 and leaves the z component the same.

   Solution.

$$\boldsymbol{M} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

■

3. Find the 3×3 matrix which scales the x component by a and the y component by b and the z component by c.

   Solution.

$$\boldsymbol{M} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

■

Exercise 2.20

1. Are the temperatures for each city contained in the rows or columns of this matrix?

   Solution. The temperature for each city is stored in each row. ■

2.
   (a) Define a matrix of the same shape as T with all its entries equalling 32, and call this matrix B

   (b) Define a square, diagonal matrix of the appropriate dimensions which when multiplying another matrix scales all its entries by $\frac{5}{9}$.

   (c) Provide 1 line of MATLAB code which generates a new matrix Y which contains the temperature data in Celsius.

   Solution.

```
loads('temps.mat') % load T
offset = 32 * ones(size(T))
scale = 5 / 9 * eye(4)

% convert farenheit to celsius
T_celsius = scale * (T - offset)
```

■

3. Extract the temperatures for each city into 4 different vectors ***t*1**, ***t*2**, ***t*3**, ***t*4**, and check that the dimensions of these vectors are as expected.

   Solution.

```
t1 = T(1, :)
size(t1)
% ans = 1 7670

t2 = T(2, :)
size(t2)
% ans = 1 7670

t3 = T(3, :)
size(t3)
% ans = 1 7670

t4 = T(5, :)
size(t5)
% ans = 1 7670
```

■

4. Which of the vectors corresponds to which city?

   Solution.

```
mean(t1)
% 51.7667

mean(t2)
% 51.914

mean(t3)
```

```
% 58.4365

mean(t4)
% 55.9451
```

(a) ***t1***: Boston
(b) ***t2***: Providence
(c) ***t3***: Washington D.C.
(d) ***t4***: New York

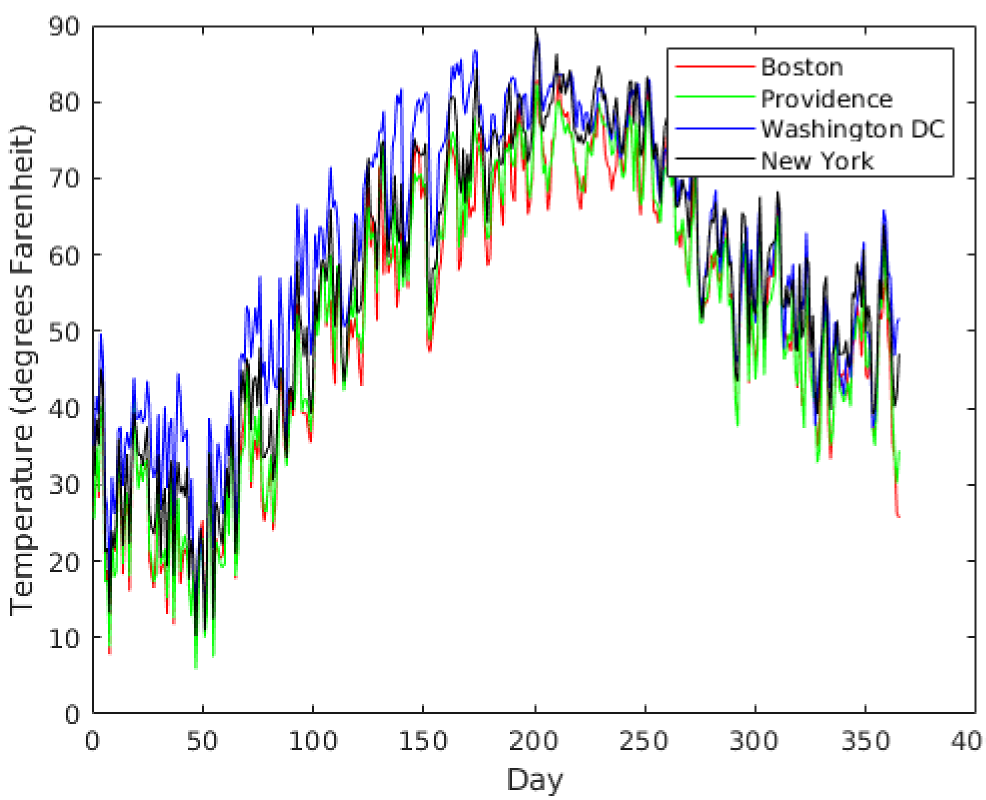
■

5. What are the maximum and minimum temperatures for Boston?

   Solution.
   Maximum Temperature: 90.7°F
   Minimum Temperature: 0.7°F ■

6. Plot graphs for the daily temperatures for the four cities for the last year you have data.

   Solution.



■

7. From the matrix T, extract a 3×365 matrix of daily temperatures for the last year in Boston, Providence and Washington DC.

Solution.

```
t_predict = T(1:3, end-364:end)
```

■

8. A good approximation for the temperature in New York on a given day is given by:

$$T_n = 0.2235T_b + 0.4193T_p + 0.3856T_w$$

Formulate a matrix equation which uses the matrix from the previous part and the formula from the genie to guess the daily temperature in New York for the last year. Apply this equation in MATLAB.

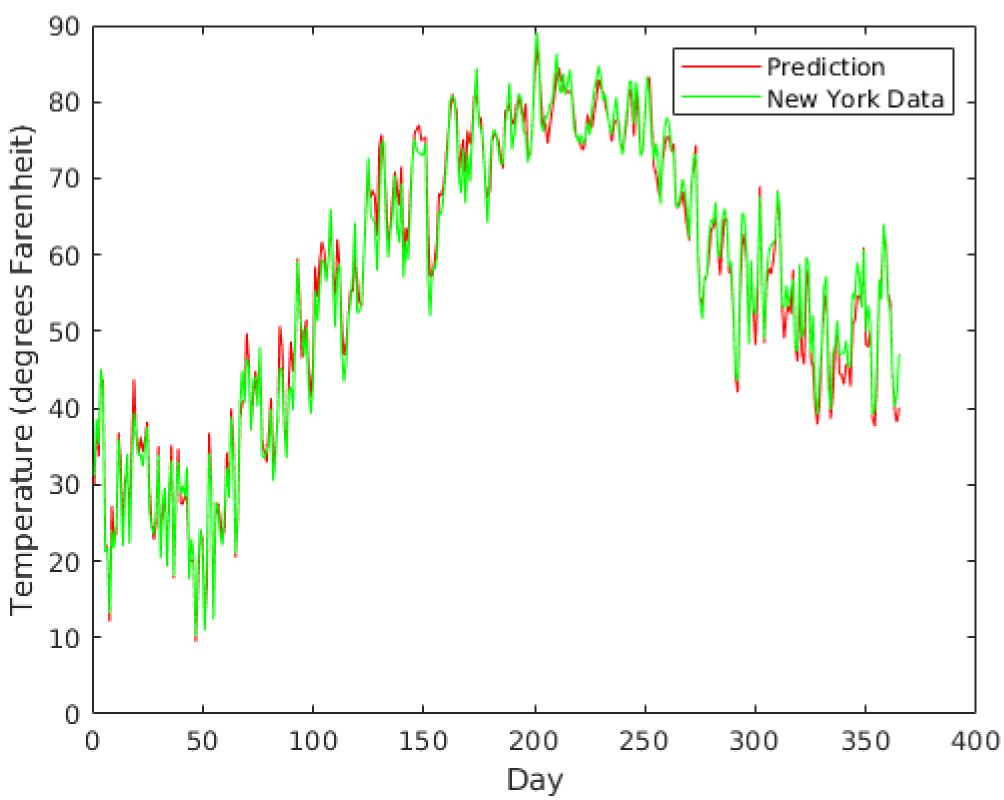
Solution.

```
t_n = 0.2235 * T_predict(1, :) + 0.4193 * T_predict(2, :) + 0.3856 *
    ↪ T_predict(3, :)
```

■

9. Plot your prediction for the temperature in New York with the actual temperature data. Is the prediction close?

Solution.



The prediction looks significantly accurate compared to the recorded data. ■