Exercise 24.3

1. What do the two conditions for static equilibrium say about the linear and angular acceleration of the system?

   Solution. The conditions state that the linear and angular acceleration of the system is 0. ■

2. For a system to be in static equilibrium, does it need to be at rest? Why or why not?

   Solution. No. If the system is moving at constant velocity it is at static equilibrium. ■

3. For a two-dimensional (x,y) system, how many equations are needed to prove static equilibrium? Please write them. How about for a three-dimensional (x,y,z) system?

   Solution. For a two-dimensional system, three equations are needed to prove static equilibrium.

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum \tau_z = 0$$

   For a three-dimensional system, six equations are needed to prove static equilibrium.

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$
$$\sum \tau_x = 0$$
$$\sum \tau_y = 0$$
$$\sum \tau_z = 0$$

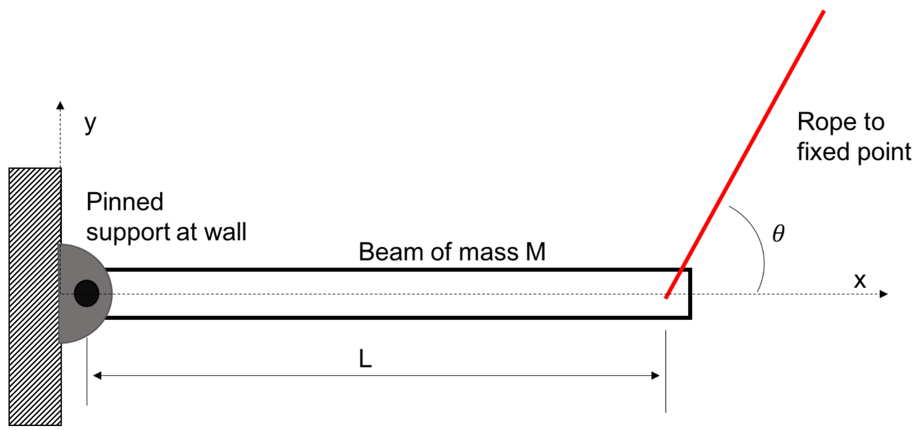
   ■

4. For a two-dimensional (x,y) system in static equilibrium, what is the maximum number of unknown forces/torques that can be solved for?

   Solution. There are a maximum of three unknowns that can be solved for. ■

Exercise 24.4

1. Consider the pinned beam with constant cross section and mass distribution shown below. The beam is supported at one end by a pinned joint. For this exercise, you can assume the beam is stationary ($\boldsymbol{a} = \boldsymbol{\alpha} = 0, \boldsymbol{v} = \dot{\boldsymbol{\theta}} = 0$)

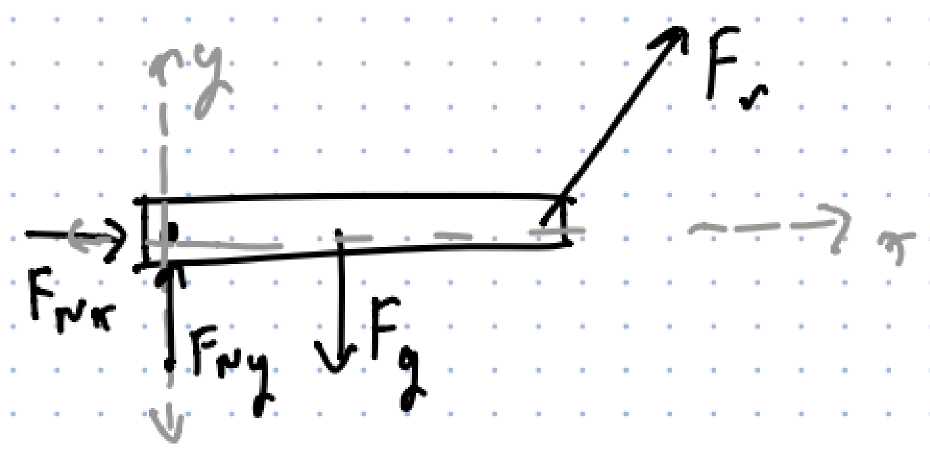


When drawing a free body diagram for the pinned beam, which body should you isolate?

Solution. I would isolate the beam. ■

2. Draw the FBD for the body you have isolated.

Solution.



■

3. Is the beam in static equilibrium? How do you know?

Solution. Yes. This problem states that $\boldsymbol{a} = \boldsymbol{\alpha} = 0$ ■

4. Write the appropriate equations for find the unknown forces and torques acting on the beam.

Solution.

$$\sum F_x = 0 = F_{Nx} + F_r \cos\theta$$
$$\sum F_y = 0 = -F_{Ny} + F_g - F_r \sin\theta$$
$$\sum \tau_z = 0 = F_g \frac{L}{2} - F_r \sin\theta L$$

■

5. How many unknowns do you have in your system of equations?

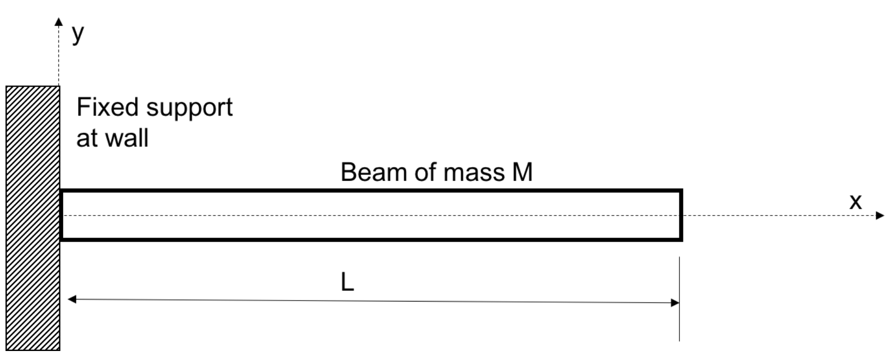
Solution. 3: F_{Nx}, F_{Ny}, F_r ■

6. Is the pinned beam system statically determinant? Why or why not?

Solution. Yes. The number of equations is equal to the number of unknowns. ■

Exercise 24.5

1. Consider the cantilevered beam with constant cross section and mass distribution shown below. The beam has mass M, and is supported at one end by a fixed joint. For this exercise, you can assume the beam is stationary ($\boldsymbol{a} = \boldsymbol{\alpha} = 0, \boldsymbol{v} = \dot{\boldsymbol{\theta}} = 0$)

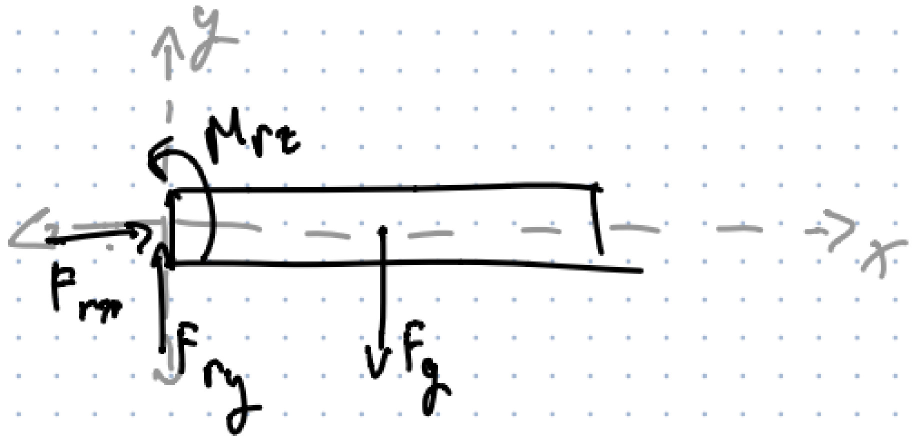


When drawing a free body diagram for the cantilevered beam, which body should you isolate?

Solution. I would isolate the beam. ■

2. Draw the FBD for the body you have isolated.

Solution.



∎

3. Is the beam in static equilibrium? How do you know?

   Solution. Yes. This problem states that $\boldsymbol{a} = \boldsymbol{\alpha} = 0$ ∎

4. Write the appropriate equations for find the unknown forces and torques acting on the beam.

   Solution.

$$\sum F_x = 0 = F_{rx}$$
$$\sum F_y = 0 = -F_{ry} + F_g$$
$$\sum \tau_z = 0 = F_g \frac{L}{2} - M_{rz}$$

∎

5. How many unknowns do you have in your system of equations?

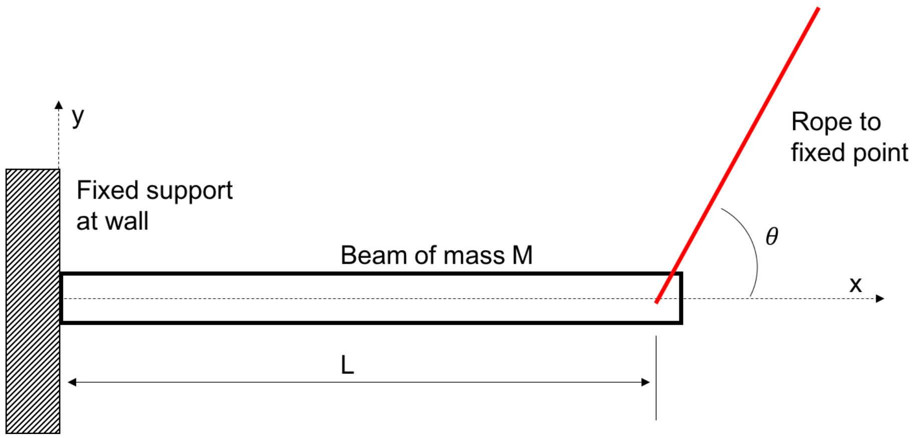
   Solution. 3: F_{Nx}, F_{Ny}, F_r ∎

6. Is the cantilevered beam system statically determinant? Why or why not?

   Solution. Yes. The number of equations is equal to the number of unknowns. ∎

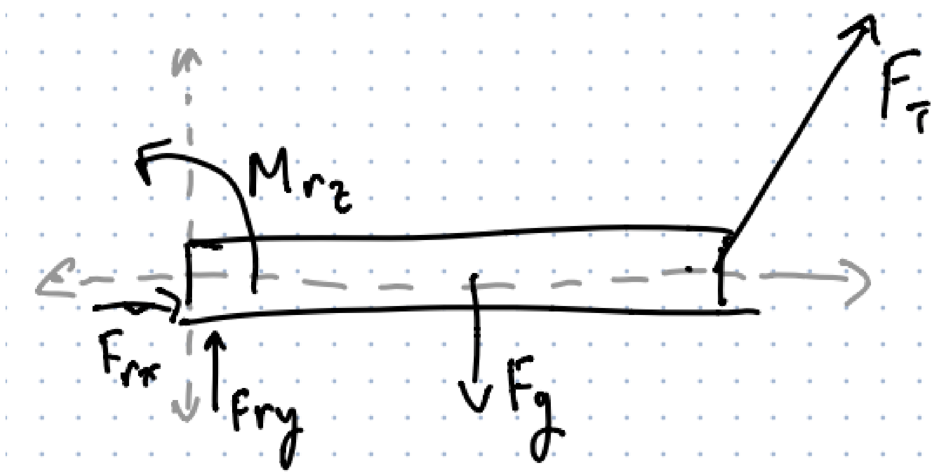
Exercise 24.6

1. Consider the cantilevered beam, of constant cross section and mass distribution, with a rope, shown below.



Draw the FBD.

Solution.



■

2. Write the appropriate equations for find the unknown forces and torques acting on the beam. How many equations are there?

Solution.

$$\sum F_x = 0 = F_{rx} + F_T \cos\theta$$
$$\sum F_y = 0 = -F_{ry} + F_g - F_T \sin\theta$$
$$\sum \tau_z = 0 = F_g \frac{L}{2} - F_T \sin\theta L - M_{rz}$$

■

3. How many unknowns do you have in your system of equations?

Solution. Four unknowns: M_{rz}, F_{rx}, F_{ry}, and F_T. ■

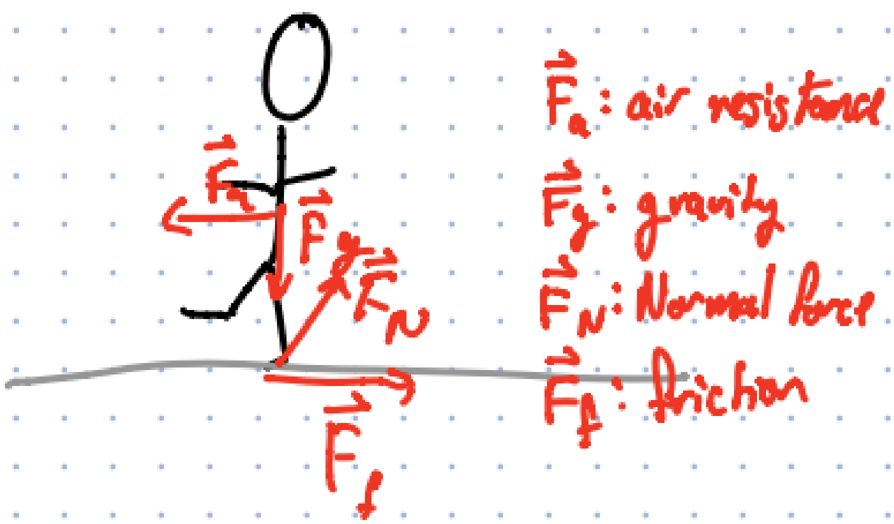
4. Is the cantilevered beam system statically determinant? Why or why not?

   Solution. No. There are more unknowns than equations. ■

Exercise 24.7

1. Consider a sprinter who is running (and accelerating) over a level surface. Draw a free body diagram for the person. What force is propelling the runner forward?

   Solution.



   The friction and normal forces are propelling the runner forward. ■

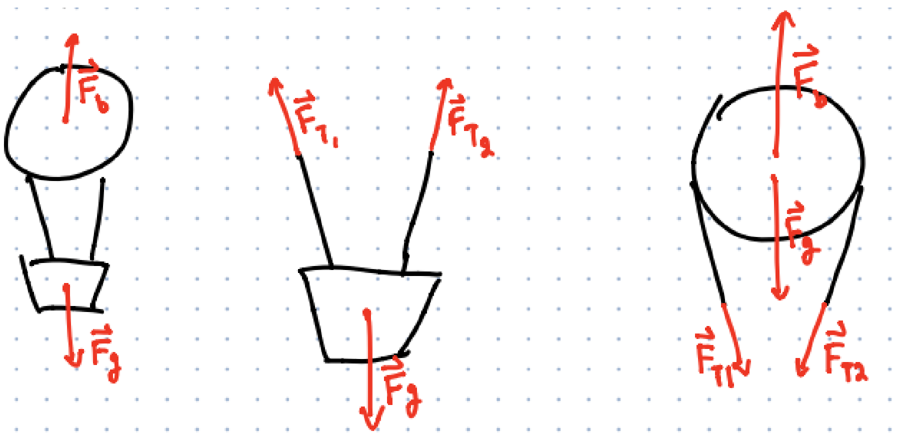
2. Consider a hot air balloon carrying a basket with two riders. It is floating at a given altitude. Draw free body diagrams for the following:

   (a) Draw a FBD for the whole system (balloon + basket) using equivalent forces to represent all distributed (body and contact) forces.

   (b) Draw an FBD for the basket using equivalent forces to represent all distributed forces.

   (c) Draw a FBD for the balloon (not including the basket) using distributed forces.

   Solution.



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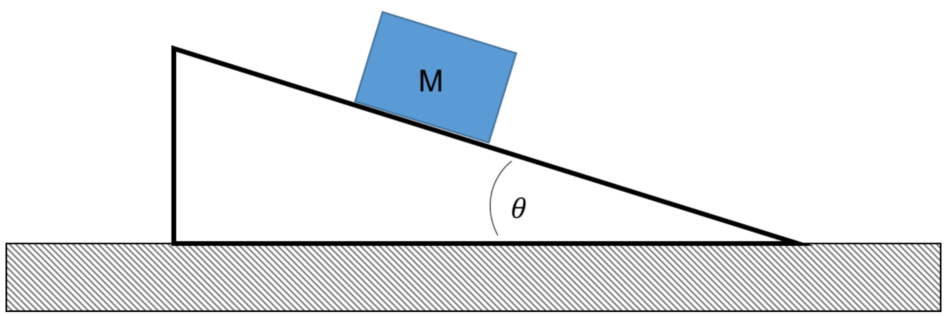
Exercise 24.9

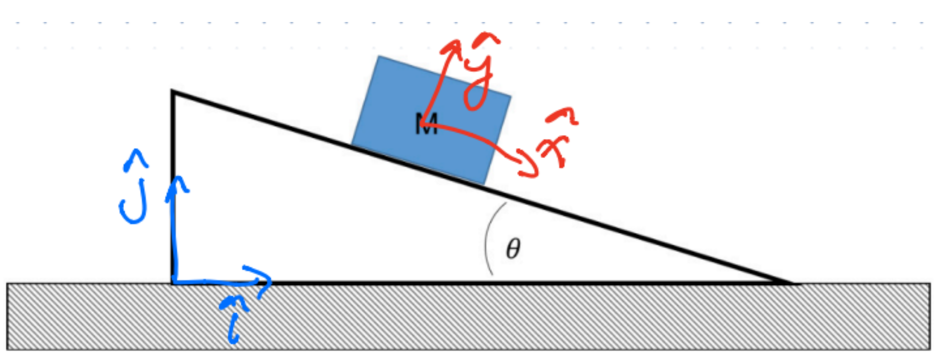
1. For the system below, define two reference frames for the system using orthogonal unit vectors



   (a) One reference frame will be known as the "global frame" and defined by the unit vectors $\left[\hat{i}, \hat{j}\right]$. The $\hat{i}$ vector should be parallel to the floor, with the $\hat{j}$ vector perpendicular to the floor.

   (b) The second reference frame will be known as the "ramp frame" and defined by the unit vectors $[\hat{x}, \hat{y}]$. The $\hat{x}$ vector should be parallel to the ramp, with the $\hat{y}$ vector perpendicular to the ramp.

Solution.



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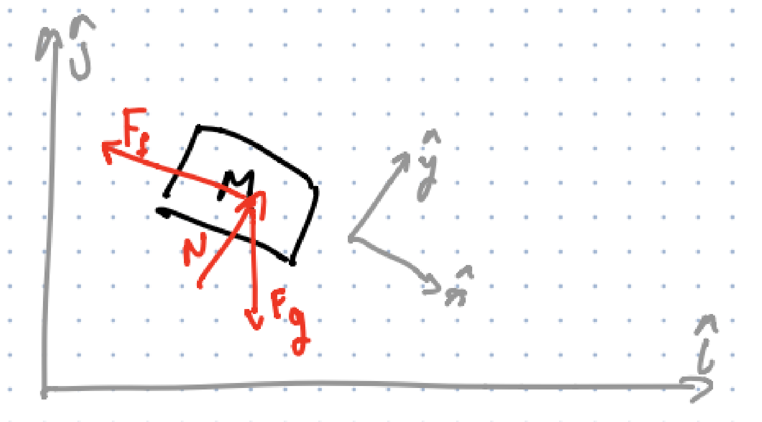
Exercise 24.10

1. Draw a FBD for the box by isolating it from the ramp. Draw and label appropriate vectors for the forces acting on the box.

Solution.



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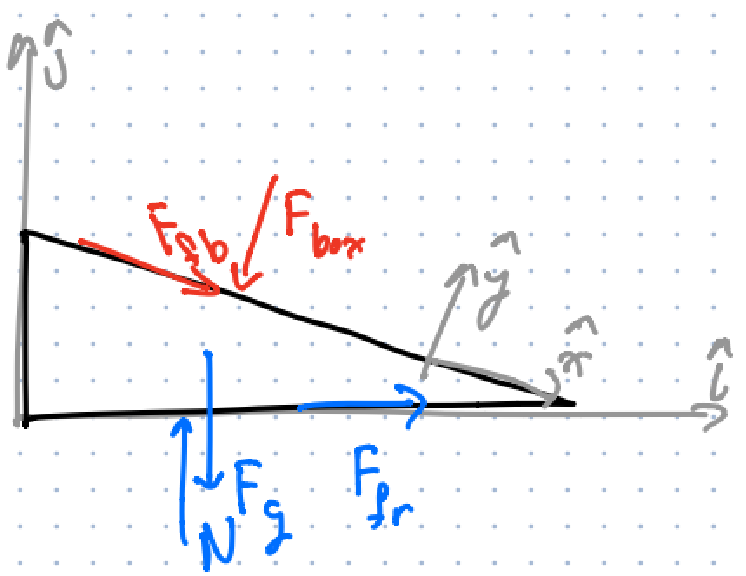
2. Draw a FBD for the ramp. Draw and label appropriate vectors for the forces.

Solution.



■

3. Look carefully at the force vectors you have drawn for the box and ramp FBDs. What reference frame is each force defined in?

Solution.

Force	Acting on	Reference Frame	Direction
F_f	Box	Box	$-\hat{x}$
F_g	Box	Box	$-\hat{j}$
N	Box	Box	$\hat{y}$
F_{fb}	Ramp	Ramp	$\hat{x}$
F_{box}	Ramp	Ramp	$-\hat{y}$
F_{fr}	Ramp	Ramp	$\hat{i}$
F_g	Ramp	Ramp	$-\hat{j}$
N	Ramp	Ramp	$\hat{j}$

■

Exercise 24.11

1. Write down an equation for the summation of all of the forces acting on the box.

Solution.

$$\sum F_{box} = -F_g\hat{j} - F_f\hat{x} + N\hat{y}$$
$$\sum F_{ramp} = -F_g\hat{j} + N\hat{j} + F_{fr}\hat{i} + F_{fb}\hat{x} - F_{box}\hat{y}$$

■

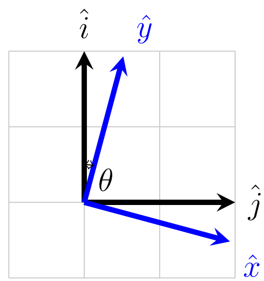
2. Typically, for a 2D problem, we would split the force equation into two equations, $\sum F_{\hat{i}}$ and $\sum F_{\hat{j}}$. Can we do that here? What about the forces that are defined in the $[\hat{x}, \hat{y}]$ frame? Can forces in different frames be added?

Solution. Yes, we can split these equations into two separate equations, but then not all reference frames would be represented. Forces in different reference frames do not add. ■

Exercise 24.12

1. Draw the unit vectors that define the global and ramp frame co-located at the same origin. Specify the angle θ that defines the rotation between the two coordinate systems.

Solution.



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2. Write equations for $\hat{x}$ and $\hat{y}$ in terms of $\hat{i}$ and $\hat{j}$.

   Solution.

$$\hat{x} = \cos\theta\hat{i} - \sin\theta\hat{j}$$
$$\hat{y} = \sin\theta\hat{i} + \cos\theta\hat{j}$$

■

3. Rewrite the equations above in the form of a rotation matrix.

   Solution.

$$\begin{bmatrix}\hat{x}\\ \hat{y}\end{bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\hat{i}\\ \hat{j}\end{bmatrix}$$

■

Exercise 24.14

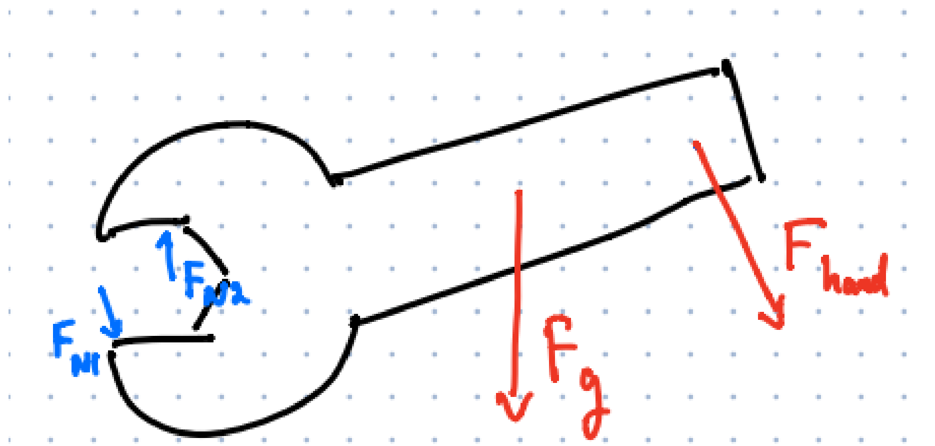
1. The picture below shows a person trying to tighten a nut with a wrench. The point of view is that of standing, facing the end of the bolt as shown and turning clockwise. Draw a free body diagram for

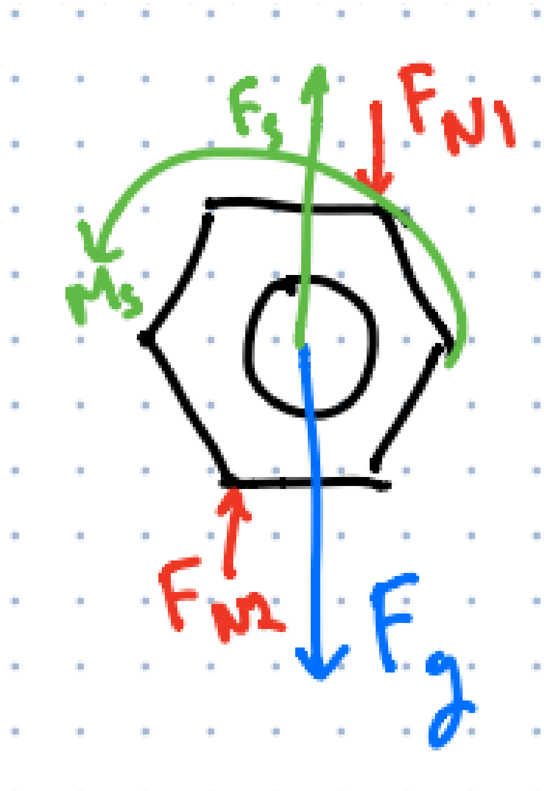
   (a) The wrench only

   (b) The nut only

   Then, using your FBDs, explain how the wrench is used to tighten the nut.

   Solution.





The wrench transfers torque to the nuts by the normal force applied to the flat faces. If the normal force exceeds the reaction moment ■