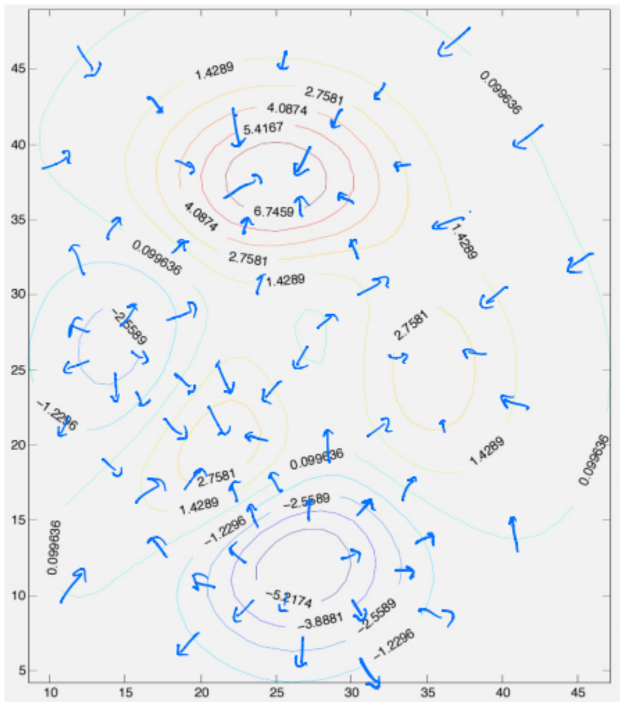
Exercise 40.3

1. Sketch the gradient field.

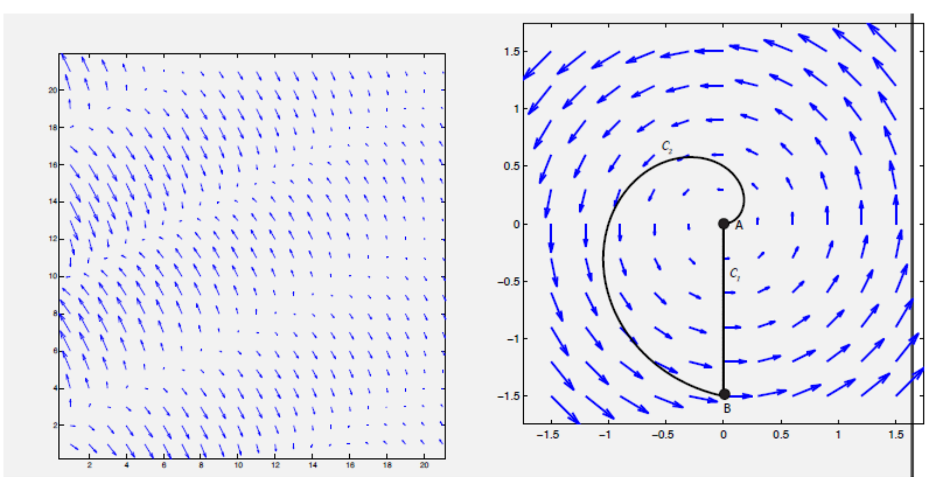
   Solution.



■

Exercise 40.4

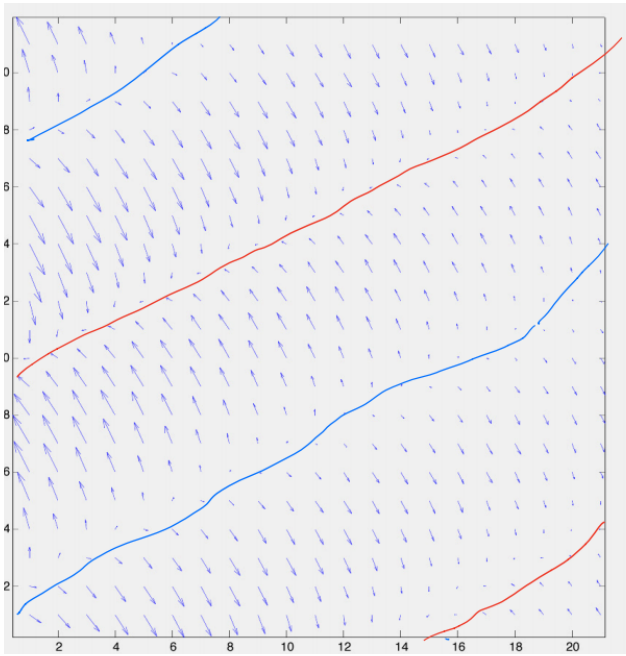
1. Which field can be a gradient?



Solution. The right field cannot be a gradient, because if so, it would not be continuous. This is because the arrows point in a circle (there is no minimum and maximum). ■

2. Sketch the contour.

   Solution.



■

Exercise 40.5

1. What are the signs of the partial derivatives $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$ at each of the points marked (R,S,T,U,V,W,X and Y)?

   Solution.

Point	Sign at $\frac{\partial g}{\partial x}$	Sign at $\frac{\partial g}{\partial y}$
R	-	+
S	-	+
T	+	0
U	0	-
V	+	0
W	-	-
X	0	-
Y	+	+

■

2. At which of the points marked does the gradient vector ∇g have the greatest magnitude?

Solution. The gradient vector ∇g is greatest at point Y because that is where it has the greatest change over distance. ∎

3. Let

$$\boldsymbol{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The directional derivative $D_{\hat{\boldsymbol{u}}} g$ is zero at exactly one of the points marked. Which is it?

Solution. $D_{\hat{\boldsymbol{u}}} g = 0$ at point S ∎

Exercise 40.7

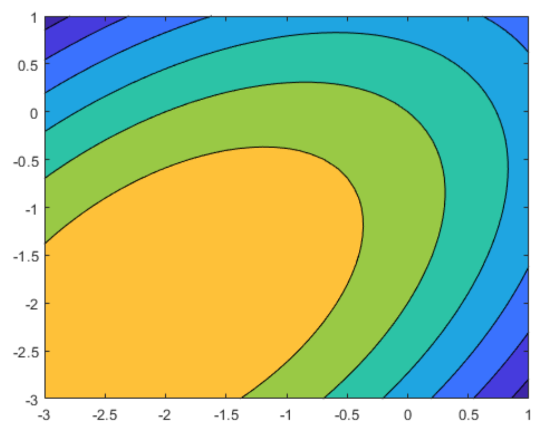
1. The mountain that will be climbed is defined as

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$$

where x, y, and f are measured in feet.

Visualize the contours of this function on the domain $(-3, 1) \times (-3, 1)$.

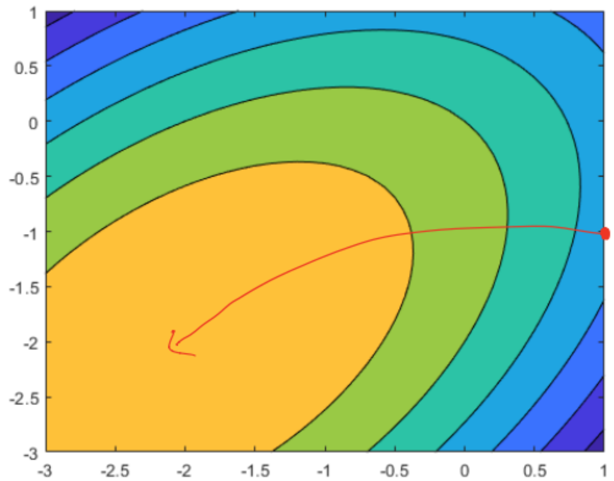
Solution.



∎

2. Draw the path of the steepest ascent if we were moving continuously from a starting point at $(1, -1)$.

Solution.



■

3. Find the gradient of this function.

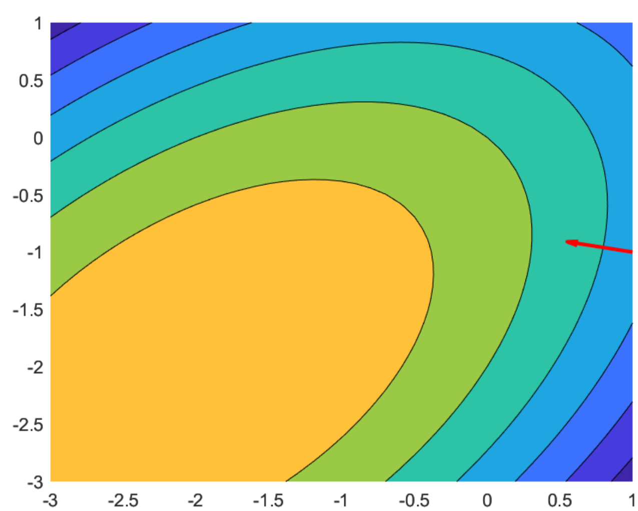
   Solution.
$$\nabla f = \begin{bmatrix} y - 2x - 2 \\ x - 2y - 2 \end{bmatrix}$$

■

4. Assuming $r_0 = (1, -1)$, what is the initial gradient at r_0? What would be a reasonable choice for λ_0 so that r_1 is not too far from the continuous path? Plot r_1 on your contour plot.

   Solution.
$$\nabla f(1, -1) = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$$

   A λ value of 0.1 keeps the r_1 value close to the continuous path.



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5. Assuming you place your NEATO at $(1, -1)$ pointing along the y-axis, how much do you have to rotate it in order to align it with the gradient at r_0? What would be a reasonable angular speed?

   Solution.

$$\theta = \arctan(\nabla f(1, -1)) = 1.37\text{rad}$$

■

6. Assuming that you are going to drive your NEATO at $0.1\frac{m}{s}$, how long would you drive in order to reach r_1?

   Solution.

$$|r_1| = 0.5099$$
$$t = \frac{|r_1|}{0.1} = 5.1$$

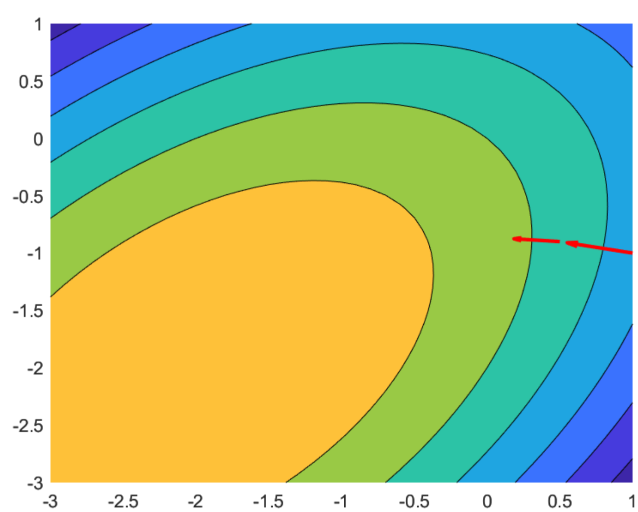
■

7. What is the gradient at r_1? What value of σ should you use so that λ_1 and r_2 are reasonable? Plot r_2 on your contour plot.

   Solution.

$$\nabla f(r_1) = \begin{bmatrix} -3.9 \\ 0.3 \end{bmatrix}$$

   With a σ value of 0.9, the λ_1 value is 0.09. This results in the following r_2 vector.



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Exercise 40.8

1. Write a script that determines the discrete points $r_1, r_2, ...$ given a discrete r_0.

   Solution. A script to generate the vectors $r_1, r_2, ...$ can be found here. ■