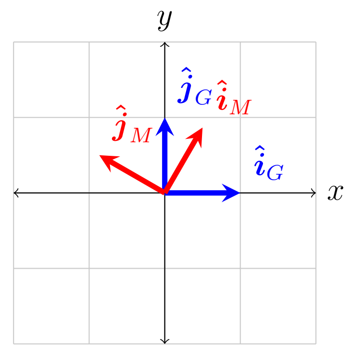
Exercise 42.1

1. The frame M is a counterclockwise rotation of the global frame G by $\frac{\pi}{3}$ radians. Draw the basis vectors for frame G and frame M.

Solution.



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2. Plot the frame G coordinates $(2, -1)_G$ Now express the frame G coordinates $(2, -1)_G$ in the frame M, and confirm that this is the same point by plotting it using the frame M.

Solution.

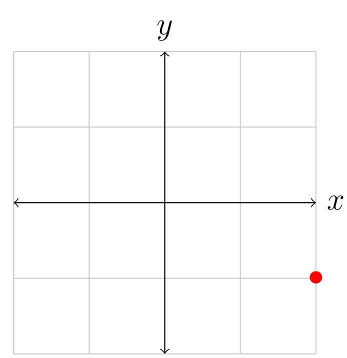
$$R_{MG} = \begin{bmatrix} \hat{\boldsymbol{i}}_G \hat{\boldsymbol{i}}_M & \hat{\boldsymbol{j}}_G \hat{\boldsymbol{i}}_M \\ \hat{\boldsymbol{i}}_G \hat{\boldsymbol{j}}_M & \hat{\boldsymbol{j}}_G \hat{\boldsymbol{j}}_M \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$r_M = R_{MG} r_G$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.134 \\ -2.232 \end{bmatrix}$$

$$r_M = 0.134 \hat{\boldsymbol{i}}_M - 2.232 \hat{\boldsymbol{j}}_M$$



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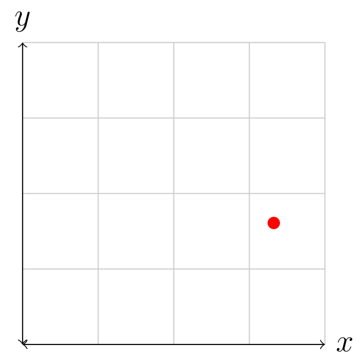
3. Plot the frame M coordinates $(3, -2)_M$ Now express the frame M coordinates $(3, -2)_M$ in the frame G, and confirm that this is the same point by plotting it using the frame G.

Solution.

$$\begin{aligned}
R_{GM} &= \begin{bmatrix} \hat{\boldsymbol{\imath}}_M \hat{\boldsymbol{\imath}}_G & \hat{\boldsymbol{\jmath}}_M \hat{\boldsymbol{\imath}}_G \\ \hat{\boldsymbol{\imath}}_M \hat{\boldsymbol{\jmath}}_G & \hat{\boldsymbol{\jmath}}_M \hat{\boldsymbol{\jmath}}_G \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \\
r_G &= R_{GM} r_M \\
&= \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 3.323 \\ 1.598 \end{bmatrix} \\
r_G &= 3.323 \hat{\boldsymbol{\imath}}_G + 1.598 \hat{\boldsymbol{\jmath}}_G
\end{aligned}$$



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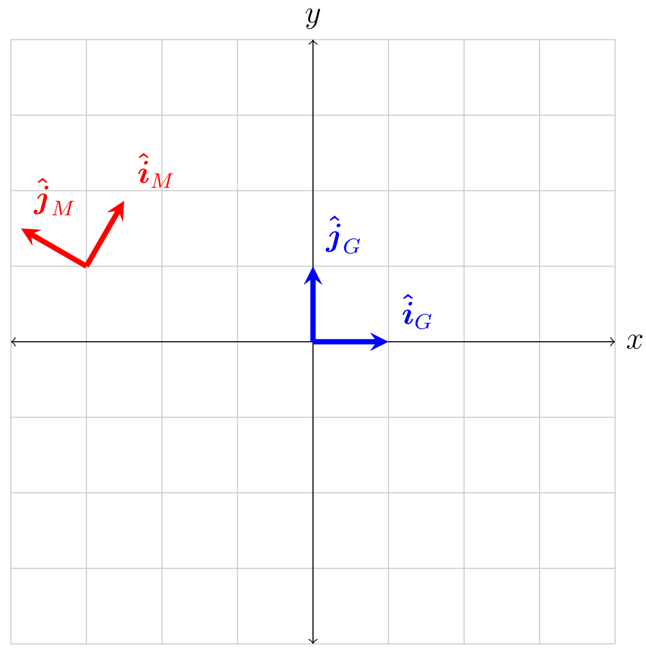
Exercise 42.2

1. The frame M is a counterclockwise rotation of the global frame G by $\frac{\pi}{3}$ radians, and has its origin at $(-3, 1)_G$

   Draw the origin and basis vectors for frame G and frame M.

   Solution.



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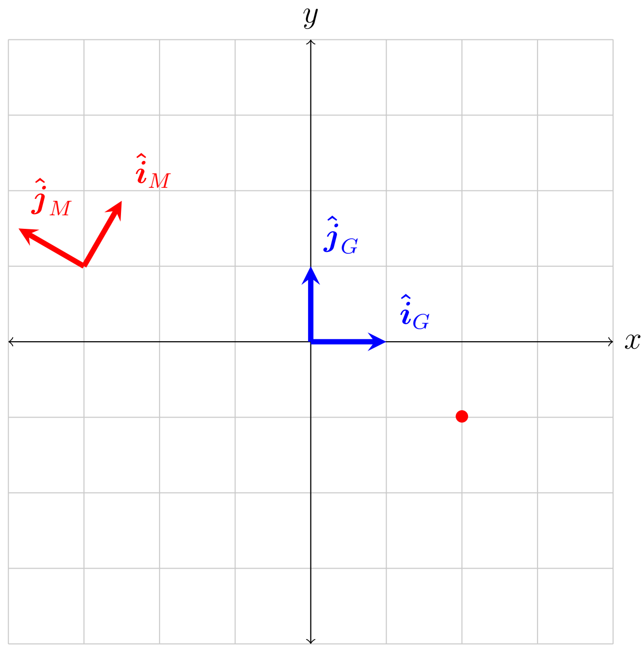
2. Plot the frame G coordinates $(2, -1)_G$ Now express the frame G coordinates $(2, -1)_G$ in the frame M, and confirm that this is the same point by plotting it using the frame M.

Solution.

$$
\begin{aligned}
R_{MG}T_{MG} &= \begin{bmatrix} \cos\frac{\pi}{3} & \sin\frac{\pi}{3} & 0 \\ -\sin\frac{\pi}{3} & \cos\frac{\pi}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0.5 & 0.866 & 0.634 \\ -0.866 & 0.5 & -3.098 \\ 0 & 0 & 1 \end{bmatrix} \\
r_M &= R_{MG}T_{MG}r_G \\
&= \begin{bmatrix} 0.7679 \\ -5.3301 \\ 1 \end{bmatrix} \\
r_M &= 0.7679\hat{\boldsymbol{\imath}}_M - 5.3301\hat{\boldsymbol{\jmath}}_M
\end{aligned}
$$



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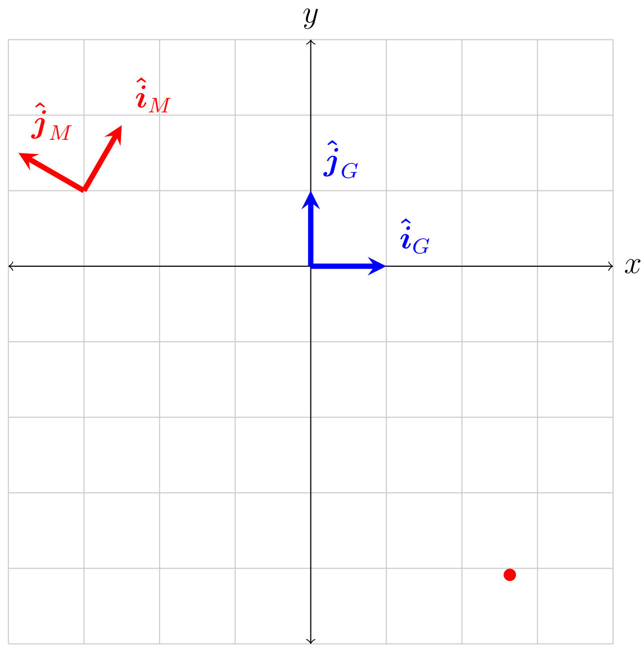
3. Plot the frame M coordinates $(3, -2)_M$ Now express the frame M coordinates $(3, -2)_M$ in the frame G, and confirm that this is the same point by plotting it using the frame G.

Solution.

$$\begin{aligned}
R_{GM}T_{GM} &= \begin{bmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} & 0 \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0.5 & -0.866 & -2.366 \\ 0.866 & 0.5 & -2.098 \\ 0 & 0 & 1 \end{bmatrix} \\
r_M &= R_{MG}T_{MG}r_G \\
&= \begin{bmatrix} 2.634 \\ -4.098 \\ 1 \end{bmatrix} \\
r_G &= 2.634\hat{\boldsymbol{i}}_G - 4.098\hat{\boldsymbol{j}}_G
\end{aligned}$$



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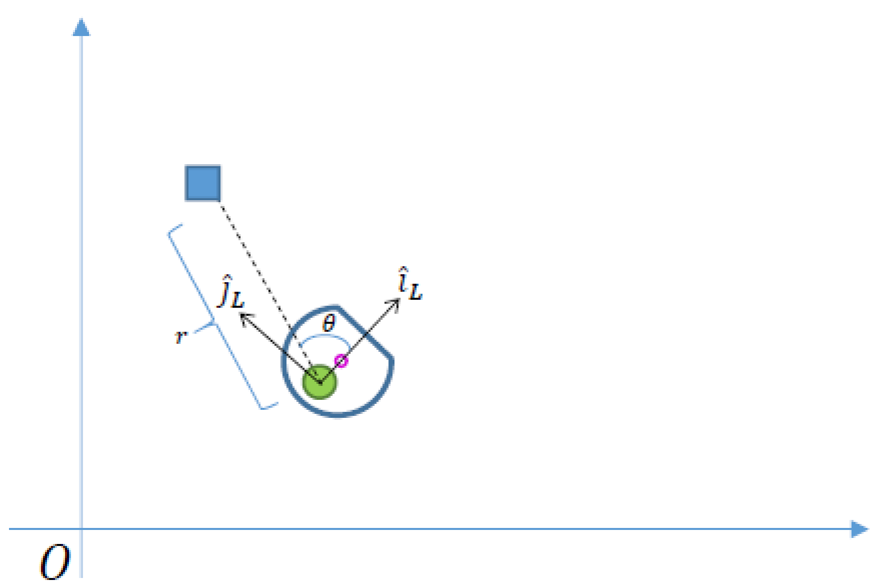
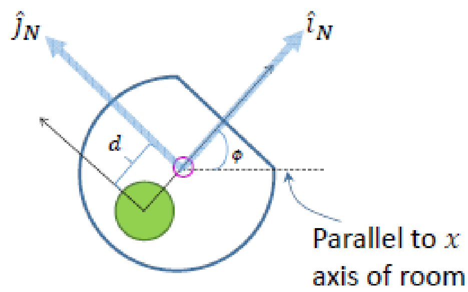
Exercise 42.3

1. Suppose that the LIDAR returns a value of (r, θ) when scanning an object. With reference to Figure 42.5, please express the location of the object with respect to the LIDAR frame L.





Solution.

$$r_L = r\cos\theta\hat{\boldsymbol{\imath}}_L + r\sin\theta\hat{\boldsymbol{\jmath}}_L$$
$$r_L = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}$$

■

2. With reference to Figures 42.5 and 42.6, express the location of the object with respect to the NEATO frame N.

Solution.

$$r_N = r_{NL} r_L$$
$$r_{NL} = \begin{bmatrix} 1 & 0 & -d \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$r_N = \begin{bmatrix} r\cos\theta - d \\ r\sin\theta \end{bmatrix}$$

■

3. Express the location of the square object in the global frame G. Assume that the center of rotation of the NEATO is located at $(x_N, y_N)_G$.

Solution.

$$T_{GN} = \begin{bmatrix} 1 & 0 & x_n \\ 0 & 1 & y_n \\ 0 & 0 & 1 \end{bmatrix}$$
$$R_{GN} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\begin{aligned} r_G &= R_G N T_G N T_N L r_L \\ &= \begin{bmatrix} r\cos(\theta+\phi) - d\cos\phi + x_N \\ r\sin(\theta+\phi) - d\sin\phi + y_N \end{bmatrix} \end{aligned}$$

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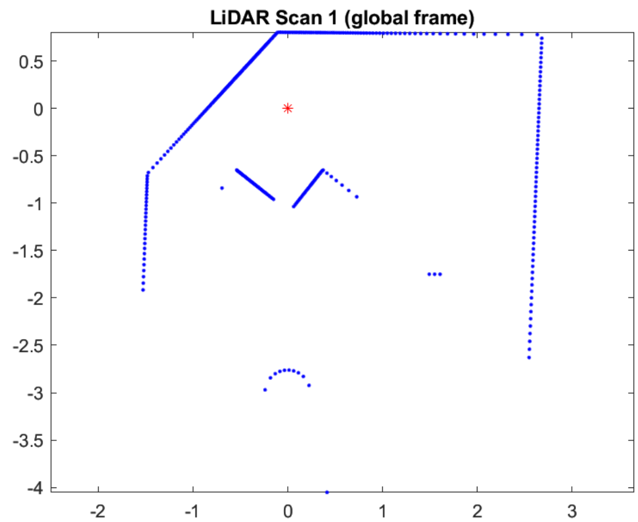
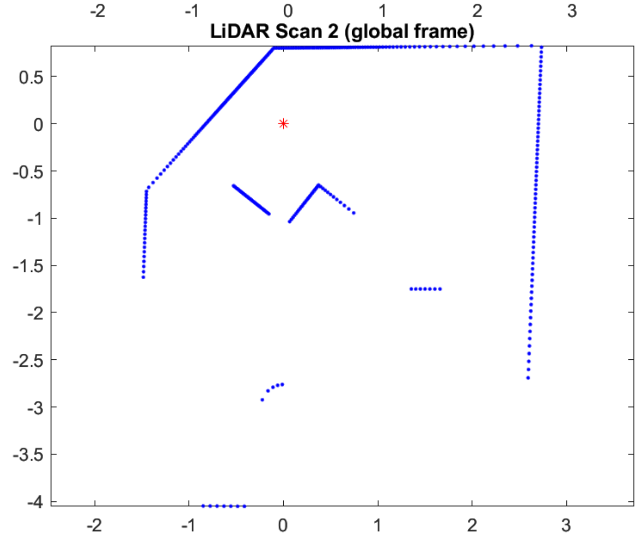
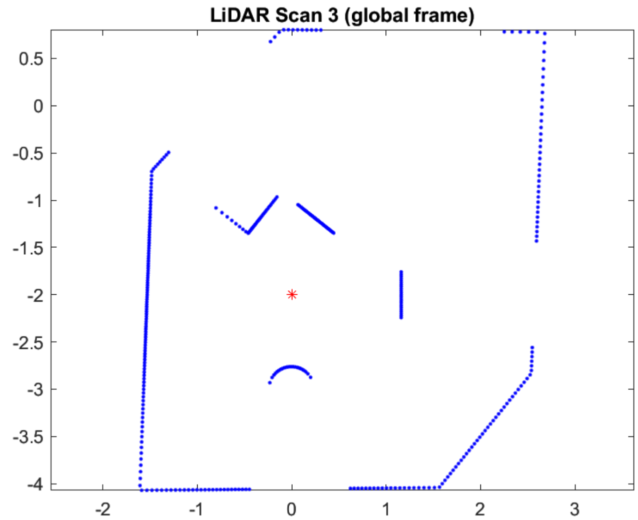
Exercise 42.4

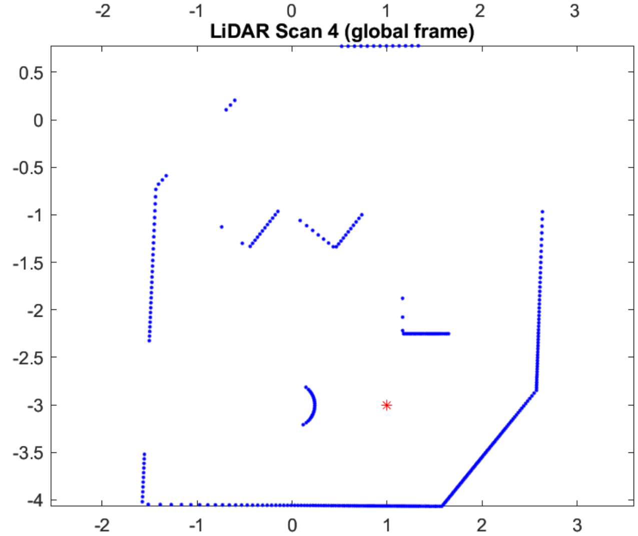
1. Caputer 4 simulated LiDAR scans in different points and orientations in the gauntlet. Plot them in the global frames and overlay them.

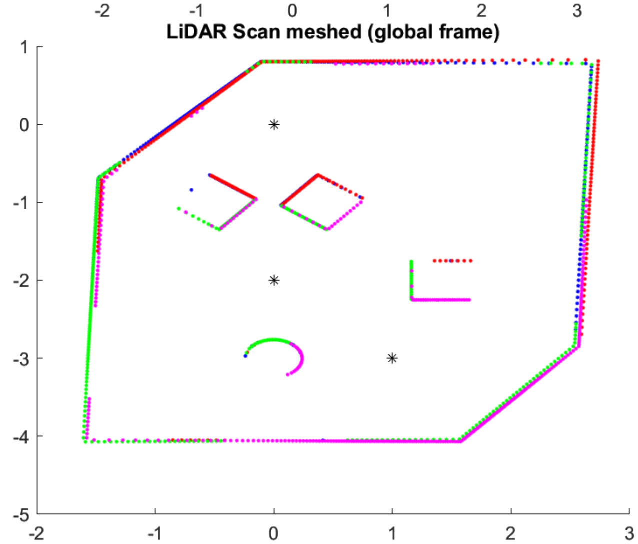
   Solution. The script used to generate the plots can be found here.






LiDAR Scan 3 (global frame)
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-2
-1
0
1
2
3


LiDAR Scan 4 (global frame)
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-2
-1
0
1
2
3


LiDAR Scan meshed (global frame)
1
0
-1
-2
-3
-4
-5
-2
-1
0
1
2
3