

## Homework 5

Try to be precise and to the point. Your answers should be short.

1. Run the *Dijkstra*( $G, s$ )–algorithm on graph  $G$  in Figure 1 of Lecture Note 9, starting at  $v$ . You need to draw a table (similar to the table in LN9.pdf).
2. Let  $T$  be a heap tree (representing an array  $H$ ) of height  $n$ .
  - (a) What is the number of internal nodes of  $T$  with exactly one child? Explain your answer.
  - (b) Explain why every leaf of  $T$  is at distance  $n$  or  $n - 1$  from the root of  $T$ .
3. Let  $G$  be a graph. A  $k$ –colouring of  $G$  is a colouring of vertices with at most  $k$  colours such that no two adjacent vertices have the same colour.
  - (a) Describe 1-colourable graphs.
  - (b) What is the minimal number of colors needed to colour a tree with more than 1 node. Explain your answer.
  - (c) Explain that the bipartite graph  $K_{n,m}$  can be 2-colourable.
4. Let  $k$  be the maximal among the degrees of all vertices of graph  $G$ . Write down a greedy and linear time algorithm that colours  $G$  with  $k + 1$  colours.
5. Suppose  $S = \{a, b, c, d, e, f\}$ . Give two examples of prefix codes for these letters. Present the prefix codes as binary trees.