Homework 3

Try to be precise and to the point. Your answers should be short. Optional problems are not marked but they help to follow this course.

1. Solve Exercises 1-4 and Exercise 6 from LN4.pdf.
2. Draw three graphs G_1, G_2 and G_3 such that (1) each graph has exactly 5 vertices; (2) G_1 has exactly 1 component, G_2 has exactly 2 components, and G_3 has exactly 3 components.
3. Solve Exercises 1-5 in LN5.pdf
4. Fix $n > 0$ and $k > 0$. Draw a directed graph with $k \cdot n$ vertices such that the graph has exactly n strongly connected components.
5. Assume that you have an acyclic digraph G with exactly n vertices. What is the maximum number of topological orders G can have? What is the minimum number of topological orders G can have? For each give an example that meets your bound. (You can either draw the graphs or formally write down their descriptions).
6. What is the maximal number of edges a directed graph with n vertices can have? What is the maximal number of edges an undirected graph with n vertices can have? Explain your answer.
7. Give examples of $3n$ requests such that the solution based on "select the smallest interval" gives n requests, while an optimal solution consists of $2n$ intervals. You can explain your answer by drawing your solution or by explicitly writing the intervals down.
8. Suppose the selection rule for the interval scheduling problem is the following. Select a request that has the fewest possible requests overlapping it. Give an example where this rule does not provide an optimal solution.
9. For the interval scheduling problem consider the following rule to solve it. Always select the request, among available ones, with the latest starting time. Does this give an optimal solution to the problem? Explain your answer.