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Rocking spectrum intensity measures for seismic assessment of rocking rigid blocks

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ABSTRACT

In this study two novel spectral ground motion intensity measures (IMs) are presented. The proposed IMs are calculated throughout the rocking rotational and the angular velocity spectra which are illustrated as 3D surface graphs, with the slenderness α and the period parameter T, the two horizontal axes. Thus, the IMs are defined as the volume under the spectra, evaluated throughout integration. The proposed IMs are intended to have a strong correlation with the rocking response of free standing blocks. Several rocking rigid blocks subjected to multiple ground motion records are assumed in order to identify the capability of the IMs to predict the rocking response concerning the variability of the block dimensions. The selection of the dimensions was made in order to correspond to slender rocking structures such as electrical equipment, ancient monolithic columns and bridges piers. The evaluation of the IMs was also made by examining characteristics such as efficiency, proficiency and sufficiency. Furthermore, the performance of the proposed IMs are compared with the performance of other well-known ground motion parameters to highlight the adequacy of the former in predicting the rocking seismic response.

1. Introduction

Probabilistic seismic assessment has emerged in recent years as a useful tool for both the design of new structures and the assessment of existing ones. Vulnerability assessment is based on the probabilistic seismic demand model (PSDM) that links the structural response with the intensity of the ground motion. Consequently, these two parameters are described in the literature as Engineering Demand Parameter (EDP) and Intensity Measure (IM). The demand uncertainties introduced in the model depend on the IM used. Thus, the selection of the IM that describes the response of the examined structural system with better accuracy is of significant importance. Several criteria have been proposed in the literature to propose an IM as an optimal one [1–4].

Over the years several researchers have developed ground motion parameters to represent seismic excitation intensity. These IMs can be classified into time history, energy, spectral, and frequency content parameters [5]. Among the others, the spectral parameters are the only ones that contain structural information. Therefore, they aim to have a strong correlation with the structural damage [6]. Nevertheless, response spectra of SDOF oscillators are not adequate to describe the rocking response [7]. Thus, defining spectral intensity measures that could enclose information about the rocking structures is expected to be a better descriptor of the rocking response.

The seismic response of structural systems that exhibit pure rocking behavior is very sensitive to even slight alteration on the excitation characteristics or on the blocks' dimensions [8]. Spanos and Koh [9] taking that fact into account, have studied the rocking response under a probabilistic framework. Recently, Acikgoz and DeJong [10] examined large flexible rocking structures subjected to pulse type excitations in order to lead to a predictive framework. Seismic vulnerability assessment of rocking structural systems are presented in the literature. Psycharis et al. [11] have assessed the seismic reliability of ancient multidrum columns, while Dimitrakopoulos and Paraskeva [12] have evaluated the efficiency of dimensionless IMs in predicting rocking response. Kavvadias et al. [13] have examined several IMs in order to conclude to these that describe the rocking response of rigid blocks with better accuracy.

Taking all the above into consideration, two scalar IMs, intended to have strong correlation with the seismic rocking performance, are proposed in the present study. These IMs, based on rotation and rotational velocity spectra, form an attempt to quantify the information that the shape of the spectra provide. These ground motion parameters are

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defined by the volume under the normalized rocking spectra. The Rocking Rotation Spectral Intensity (RRSI) is evaluated by integration of the rocking rotation spectrum while the Rotational Velocity Spectral Intensity (RSVI) by integration of the rotational velocity spectrum. The integration limits of the period parameter T and the slenderness α was adopted in order to construct IMs that could take into account structural response information for the majority of the slender rocking blocks. Using these IMs, a reduction of the uncertainty associated with the median demand of rocking response is sought.

To assess the adequacy of the IMs, 12 rigid blocks subjected to a set of 35 ground motion records are examined. Correlation coefficients between the proposed ground motion parameters and the developed damage, as well as criteria such as efficiency, proficiency and sufficiency are adopted in order to evaluate the IMs. Moreover, velocity based and frequency content IMs are also examined. These IMs are known to have a strong correlation with the rocking response [12,13]. Thus, the comparison of their performance with the performance of the two proposed IMs could assure their suitability for usage in vulnerability analysis of rocking structures.

2. Seismic rocking response of rigid block

A rigid block standing free on a rigid base, with slenderness α, semi-diagonal R and frequency parameter p, oscillates about the centers of rotation O and O′ when rocking motion initiates (Fig. 1). The minimum acceleration of a ground excitation that enables rocking can be computed from static analysis, and yields to $\ddot{u}/g \geq \tan(\alpha)$. The problem of a rigid rocking block motion under a seismic excitation can be described by the following equation [14]:

$$\ddot{\theta} = -p^2 \cdot \left\{ \sin\left[\alpha \cdot sgn(\theta) - \theta\right] + \frac{\ddot{u}_g}{g} \cdot \cos\left[\alpha \cdot sgn(\theta) - \theta\right] \right\} \tag{1}$$

where sgn() is the sign function and $p = \sqrt{3g/4R}$ is the frequency parameter of the rigid block.

During the rocking motion, energy is lost only during impact (when the rotation changes sign at θ = 0) which causes a reduction of the rotational velocity after it:

$$\dot{\theta}_{n+1}{}^2 = r \cdot \dot{\theta}_n{}^2 \tag{2}$$

where r is the restitution coefficient, $\dot{\theta}_n$ is the velocity before the impact and $\dot{\theta}_{n+1}$ is the velocity after the impact.

Considering that the angular momentum remains constant about point O exactly before the impact and right after it, the coefficient of restitution for a rigid rectangular block is given by the following equation [15]:

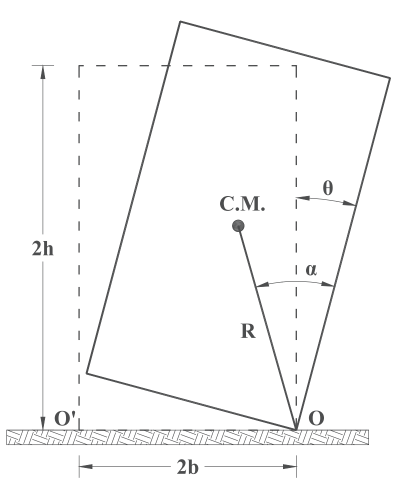

Fig. 1. Characteristics of a rigid rocking block.

$$r = [1 - \frac{3}{2}(\sin\alpha)^2]^2 \tag{3}$$

The restitution coefficient depends only on the slenderness and expresses the energy dissipation during rocking motion considering the inelastic impact. Since the actual restitution coefficient depends on parameters such as the slenderness, the material, the impact velocity and the imperfections, several experiments were performed to determine it accurately [16,17] and improved formulas have been proposed [18]. In the current study, the restitution coefficient of Eq. (3) is used for the analyses. The seismic rocking response is calculated by solving the equations listed above, in Matlab R2016 [19], using the Runge-Kutta integration technique named ode45.

3. Rocking spectra

In correspondence with response spectra for single-degree-of-freedom (SDOF) models, rotation and velocity rocking spectra are firstly presented by Makris and Konstantinidis [7]. Despite the fact that the elastic SDOF oscillator can be characterized by the period T and the damping ζ, the rocking block has two representative parameters, the frequency parameter p and slenderness α. Thus, the rocking spectra are usually depicted with diagrams which correspond to blocks with individual slenderness.

In this study rocking spectra are pictured as 3D surface graphs in which the two horizontal axes are the period parameter T and the slenderness α. In such a way, the outcome of the ground motions to the rocking response is easily observed. The surface plot compared with the classic 2D one, provides the rocking response of blocks with a wide range of geometric sizes more clearly. Moreover, taking into account features of the rocking seismic response, characteristics of the ground motion excitations, such as maximum absolute acceleration and frequency contain, can be estimated by rocking spectra. The values of the rotational rocking spectra are normalized to the slenderness of the blocks α, while the angular velocity spectra are normalized to the frequency parameter p. It is known that, in rare cases, a rocking block could develop rotation higher than its slenderness without overturning, while when it overturns the EDP takes infinite value. Assuming that the rocking block develop infinite rotation, cannot be suitable for the definition of the proposed IMs. Thus, the limit of the normalized rotation when the collapse occurs is considered $\theta_{max}/\alpha = 1$. As the numerical integration of the equation of motion is completed when θ = α, the maximum rotational velocity is captured until that time step. The rocking spectra are created for a range of slenderness α = 0.1–0.3 rad and T = 1–8 rad/s.

Rocking spectra obtained by two ground motions are presented in Fig. 2. In the above figure, the surface spectra are depicted together with the acceleration time histories. The excitations have PGA values adequate to uplift the total range of the blocks. However, due to their frequency content, differences in the spectra are observed. As seen in spectrum created from Imperial Valley, rocking is initiated for most blocks, but due to its low mean period T_m [20], it can barely induce uplift to a block with slenderness higher than α = 0.2 rad. On the contrary, the Northridge excitation, which has the same PGA but higher T_m, proves to have a more detrimental effect on rocking. In respect to the previous remarks the angular velocities values are also presented in Fig. 2.

4. Proposed intensity measures

A proper selection of IMs is crucial to exhibit high correlation with the structural damage. By representing ground motion intensity through appropriate IMs, it is possible to obtain an optimized response prediction. It is known that rocking response is affected by the frequency content of the excitation [15]. The strong dependence of the velocity characteristics of the ground motion with rocking response is

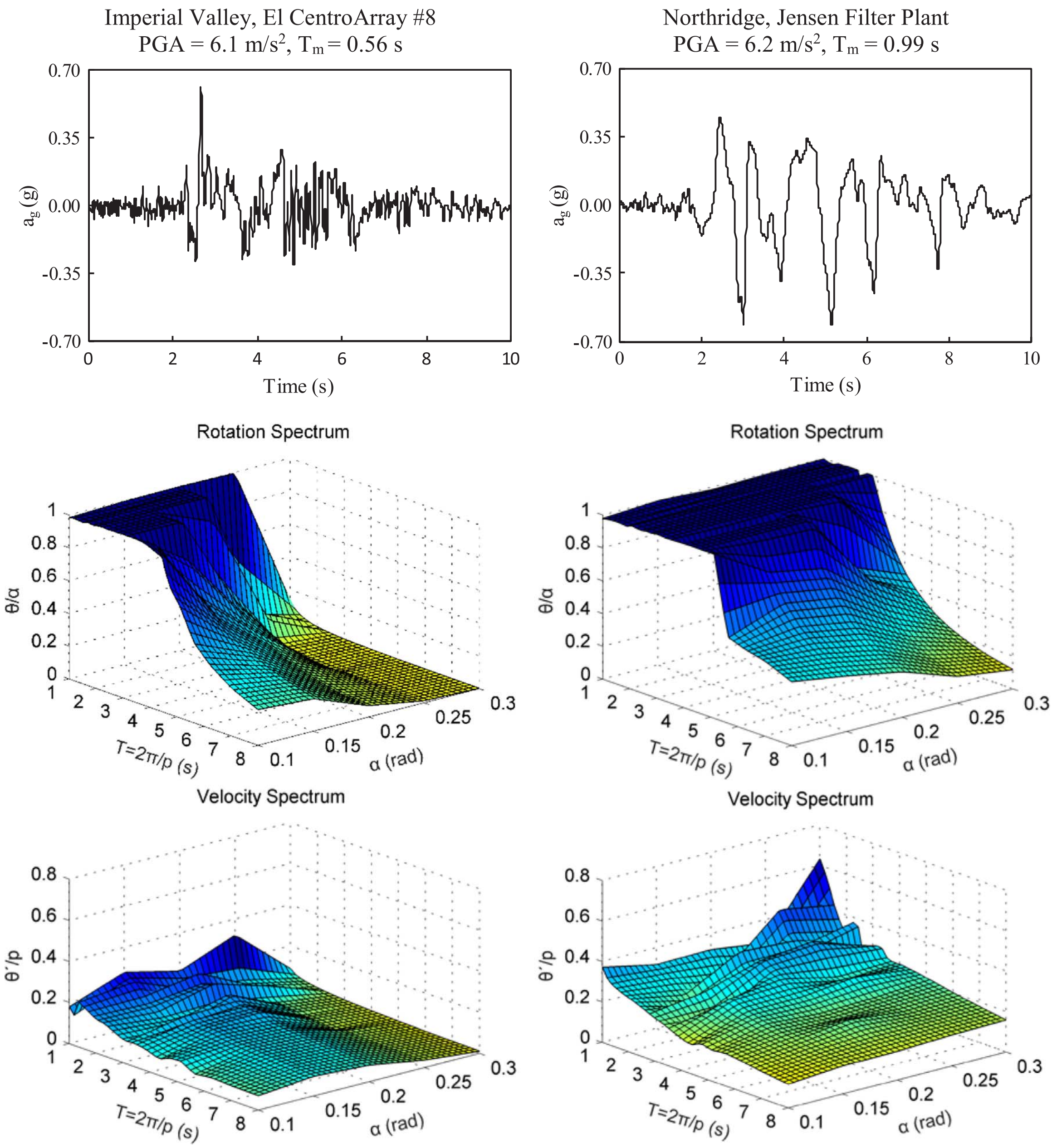

Fig. 2. Acceleration time histories and the corresponding normalized rocking and rotational velocity spectra.

verified in recent studies [12]. Also, intensity measures such as Housner's Spectral Intensity (SI_H) [21], containing information about the elastic response of SDOF oscillators, significantly affects the response of free standing rigid blocks [13].

The IMs that contain not only ground motion information, but also information about the structural response are expected to be more efficient ones regarding the engineering demand prediction. Spectral parameters constitute IMs that contain information of the structural features. Although, there are numerous IMs calculated by the elastic SDOF response spectra, none is proposed based on the rocking one. Thus, a rational approach is to select IMs defined by the rocking spectra, which could form a strongly correlated parameter with the rocking response.

In the present study the proposed IMs are determined in order to contain information about the earthquake record as well as information about the structural response of free standing rigid blocks, aiming for a better description of the rocking performance. The proposed novel ground motion parameters named Rocking Rotation Spectral Intensity (RRSI) and Rotational Velocity Spectral Intensity (RVSI) are defined as the volumes under the spectra calculated via integration of the rotation and angular velocity spectrum respectively. The definition of the IMs are the following:

$$RRSI = \int_{a_{\min}}^{a_{\max}} \int_{T_{\min}}^{T_{\max}} S_r(T, a)\, dTda \tag{4}$$

$$RVSI = \int_{a_{\min}}^{a_{\max}} \int_{T_{\min}}^{T_{\max}} S_v(T, a) dTda \tag{5}$$

where S_r is the rocking rotation spectrum surface and S_v is the rotational velocity spectrum surface.

The values of the rocking spectra are dimensionless due to the fact that the rocking rotations and the rotational velocities are normalized. As such the physical dimensions of the proposed IMs are rad·s. The definition of the IMs depends on the assumption that the overturning rocking rotation takes values of $|\theta|/\alpha = 1$. Although another threshold for the overturning rocking rotation may adopted, the relation of the values of the proposed IMs for each ground motion, comparatively, should be the same. As such, they could describe the potential destructiveness of a ground motion on rocking structural systems. The integration limits of period parameter T and slenderness α are chosen in order to reflect a wide variety of slender rocking structures. For the evaluation of the frequency parameter p examples from previous studies are considered ranging from $p \approx 3.5$ rad/s for a typical tombstone to $p < 1$ rad/s for relatively tall building frame structures [10]. As a result, the integration limits for the period parameter T are adopted from 1 to 8 s. Concerning slenderness, the adopted integration limits are $\alpha = 0.1–0.3$ rad moving from a slender structure to stocky ones. In these values rocking structures from Electrical transformers ($\alpha \approx 0.30$ rad) and ancient columns ($\alpha \approx 0.15$ rad) to bridge piers ($\alpha \approx 0.10$ rad) are included.

5. Optimal selection of intensity measure used for vulnerability analysis

In recent years, probabilistic analysis has become widely used for structural engineering purposes and Probabilistic Seismic Demand Models (PSDM) have been developed aiming to predict the response of a structure under earthquakes of certain intensities. Specifically, the fragility analysis expresses the probability (P) that the capacity (C) of a specifically measured engineering demand parameter (EDP) of a structure will exceed a certain level of demand (D), for a specific ground motion intensity measure (IM), as defined by the following equation [1]:

$$P = P\,[D \geq C|\,IM] \tag{6}$$

Regarding the rocking response, the results of the time history analyses have to be treated with two different methods. In order to calculate the probability of exceeding a certain capacity value, using only the non-collapse data, the following expression is used [4]:

$$P\,[D \geq C|\,IM] = \Phi\left(\frac{\ln(S_d/S_c)}{\sqrt{\beta_{D|IM}{}^2}}\right) \tag{7}$$

where Φ is the standard normal cumulative distribution function, S_c is the median value of the capacity which is estimated through the adopted limit states and $\beta_{D|IM}$ is the dispersion or logarithmic standard deviation for the demand conditioned the IM.

The median seismic demand S_d is related to an examined IM with the following expression:

$$S_d = a\,(IM)^b \tag{8}$$

where α and b are the linear regression coefficients for the logarithmic expression of the assumed scale law.

The linear regression analysis is performed between the engineering demand parameter calculated via the time history analyses and the corresponding IM values. The logarithmic standard deviation (Eq. (8)) of the linear regression analysis indicates the dispersion of the median demand. This parameter constitutes the demand uncertainty introduced in the probabilistic model.

$$\beta_{D|IM} \cong \sqrt{\frac{\sum\,(\ln(d_i) - \ln(a\,(IM)^b))^2}{N-2}} \tag{9}$$

where N is the total number of the data.

The above methodology (Eqs. (7)–(9)) is performed by using the data of the restricted rocking responses, excluding the rocking collapse cases. To calculate the fragility of the structural system by taking into account the rocking overturn data the Eq. (7) is altered to:

$$P\,[D \geq C|\,IM] = P_o + (1 - P_o)\cdot\Phi\left(\frac{\ln(S_d/S_c)}{\sqrt{\beta_{D|IM}{}^2}}\right) \tag{10}$$

where P_o is the probability of rocking collapse.

When rocking overturn occurs the rotation values reach infinite values. Assuming this fact, the problem should be considered as a categorical one, by grouping the data into non-collapse and collapse ones in order to estimate the probability of overturning. So, to estimate the parameters of the fragility function (mean μ and standard deviation β) that provide the probability of collapse, the maximum likelihood approach is adopted [22,23]. The maximum likelihood function L is defined as follows:

$$L = \prod_{j=1}^{m}\binom{n_j}{z_j}p_j^{\,z_j}(1 - p_j)^{n_j - z_j} \tag{11}$$

where the probability of z_j collapses out of n_j ground motions of a certain value of IM is given by the binomial distribution, p_j is the probability that a ground motion of a particular IM value, will cause the collapse of the structure, m is the number of IM levels and Π denotes a product over all IM levels.

Assuming lognormal cumulative distribution for the overturning probability, Eq. (11) converts to the following:

$$L = \prod_{j=1}^{m}\binom{n_j}{z_j}\Phi\left(\frac{\ln x_j - \mu}{\beta}\right)^{z_j}\left(1 - \Phi\left(\frac{\ln x_j - \mu}{\beta}\right)\right)^{n_j - z_j} \tag{12}$$

The maximization of L gives the statistical moments $\hat{\mu}_{MLE}$ and $\hat{\beta}_{MLE}$ via an optimization process.

$$\{\hat{\mu}_{MLE},\ \hat{\beta}_{MLE}\} = \max_{\mu,\,\beta}\prod_{j=1}^{m}\binom{n_j}{z_j}\Phi\left(\frac{\ln x_j - \mu}{\beta}\right)^{z_j}\left(1 - \Phi\left(\frac{\ln x_j - \mu}{\beta}\right)\right)^{n_j - z_j} \tag{13}$$

The appropriate selection of an IM plays an important role in the accuracy of a probabilistic seismic demand analysis (PSDA) for structures. As such, the choice must be made based on criteria, presented in the literature, which help to distinguish the accuracy of the seismic assessment. An optimal IM is defined by primary factors such as efficiency, practicality, proficiency and sufficiency [1–4].

Efficient IMs are the ones that eliminate the dispersion of the results about the median, which means a decrease in the uncertainties introduced to the PSDM, resulting in superior fragility curves during the vulnerability assessment. A distinguished IM according to its efficiency is represented by a lower logarithmic standard deviation $\beta_{D|IM}$ (Eq. (9)).

Proficiency is a complex measure assessing the effect of both practicality and efficiency. Practicality refers to whether or not there is any direct correlation between an IM and the demand placed on the structure and is measured by the regression parameter b in the PSDM. Thus, the proficiency consists of parameters that combine the logarithmic standard deviation and the slope of the regression analysis and defined as [4]:

$$\zeta = \frac{\beta_{D|IM}}{b} \tag{14}$$

Sufficiency forms a criterion which indicates the dependency of an IM on seismic excitation parameters, such as magnitude (M) and epicentral distance (R) [2]. A regression analysis of the residuals obtained from the PSDM per each IM, ε|IM, relative to the former ground motion characteristics is carried out to evaluate their correlation.

$$\varepsilon|IM = a + bM \ \text{ and } \varepsilon|IM = a + b\ln(R) \tag{15}$$

The statistical significance of the regression is quantified by the p-value of the slope b, with smaller p-values indicating evidence of an insufficient IM. For the purposes of this study, no residual dependence on a 95% confidence interval is termed sufficient. In other words, a p-value smaller than 0.05 suggests significant dependency. In attenuation relationships, R is often treated in log form, therefore, ln(R) could be used [3].

From the aforementioned characteristics, efficiency and proficiency are most commonly used to distinguish whether an IM is optimal regarding its appropriateness in predicting the structural damage.

Another measure to evaluate the grade of interdependency between the examined IMs and the EDP is the correlation coefficient by Pearson' (Eq. (16)) [24] which shows how well the data fit a linear relationship and is derived from the regression analysis of each PSDM.

$$\rho_{Pearson} = \frac{\sum_{i=1}^{N}(X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{N}(X_i - \overline{X})^2 \sum_{i=1}^{N}(Y_i - \overline{Y})^2}} \tag{16}$$

where:

X_i is the value of the seismic intensity parameter of the i-th ground motion excitation, Y_i is the value of the seismic response parameter to the i-th excitation and finally $\overline{X}$, $\overline{Y}$ are the mean values of X_i and Y_i.

To assess the adequacy of an IM, the computability of the seismic hazard in terms of the examined IM is important to be considered. The hazard computability refers to the effort required to develop seismic hazard curves [1]. Seismic hazard information is commonly available in terms of PGA or spectral acceleration at a certain period $S_a(T)$.

6. IMs evaluation procedure

The adequacy of the proposed IMs in describing the rocking behavior of slender rigid blocks is thoroughly examined. In order to investigate the robustness of the IMs regarding the structural characteristics, 12 different rigid blocks are considered. The blocks have slenderness of 0.10, 0.15 and 0.20 rad and frequency parameter p equal to 2.73, 2.00, 1.35 and 0.86 rad/s, that correspond to blocks with semi-diagonal R values of 1.0, 1.8, 4.0 and 10.0 m respectively. The selection of the blocks dimensions and slenderness was made to be in accordance with typical rocking structures such as bridge piers, ancient columns, electrical transformers and laboratory or home equipment. The procedure could be extended also to rocking frames [25], rocking flexible oscillators [26], or even to more complex rocking systems [27].

A cloud analysis is conducted using a set of 35 ground motion time histories as input base acceleration for the analyses. The earthquake magnitudes (Ms) range from 5.3 to 7.6, including both near-fault and far-fault records. A further aspect which has been taken into consideration is the expected damage potential of the seismic excitation on the structure, a fact that is crucial for conducting a cloud analysis. Seismic excitations which provide a wide spectrum of structural damage, from negligible to severe, are taken into account. The complete list of the natural ground motion records used for the analyses is shown in Table 1 [28,29].

Table 1
List of the used ground motions [28,29].

Seismic events	Recording stations	Date	M_s	R (km)	RRSI (rad s)	RVSI (rad s)
Aigio	AIGA	25/06/1995	6.40	21.50	0.31	0.14
Athens	ATH4	07/09/1999	5.90	16.62	0.09	0.05
Bucharest	Bucharest	04/03/1977	7.50	115.0	0.41	0.08
Chi-Chi	CHY035	20/09/1999	7.62	12.56	0.28	0.08
Chi-Chi	CHY101	20/09/1999	7.62	9.94	0.89	0.21
Chi-Chi	TCU88	20/09/1999	7.62	18.16	0.19	0.08
Chuetsuoki	Nakanoshima Nagaoka	16/07/2007	6.80	19.89	0.39	0.11
Coalinga	Fault Zone 14	02/05/1983	6.36	29.48	0.58	0.23
Corinth	Corinth	24/02/1981	6.60	10.28	0.20	0.07
Darfield	Papanui High School	04/09/2010	7.00	26.76	0.42	0.07
Denali	TAPS Pump Station #10	03/11/2002	7.90	2.74	0.53	0.12
Duzce	Lamont 531	12/11/1999	7.14	8.03	0.04	0.07
Erzincan	Erzincan	19/03/1992	6.69	4.38	0.96	0.26
Friuli	Tolmezzo	06/05/1976	6.50	15.82	0.32	0.13
Gazli	Karakyr	17/05/1976	6.80	5.46	0.72	0.20
Imperial Valley	El Centro Array #9	19/05/1940	6.95	6.09	0.43	0.14
Imperial Valley	El Centro Array #7	15/02/1979	6.53	0.56	0.52	0.14
Imperial Valley	El Centro Array #8	15/02/1979	6.53	3.86	0.48	0.15
Irpinia	Sturno	23/11/1980	6.90	10.84	0.52	0.11
Iwake	Ichinoseki Maikawa	14/07/2008	6.90	23.02	0.02	0.03
Kalamata	Kalamata	13/09/1986	6.20	10.00	0.24	0.06
Kobe	Nishi-Akashi	16/01/1995	6.90	7.08	0.42	0.18
Kobe	Port Island	16/01/1995	6.90	3.31	0.74	0.17
Landers	Joshua Tree	28/06/1992	7.28	11.03	0.37	0.09
Loma Prieta	Los Gatos – Lexington Dam	17/11/1986	6.93	5.02	0.72	0.24
El Mayor Cucapah	Chihuahua	04/04/2010	7.20	19.47	0.85	0.22
Northridge	Jensen Filter Plant	17/01/1994	6.69	5.92	0.99	0.35
Northridge	Newhall – Fire Station	17/01/1994	6.69	5.43	0.56	0.16
Northridge	Paicoma Dam	17/01/1994	6.69	7.01	0.36	0.14
Parkfield	Cholame #2	28/06/1966	6.20	17.64	0.64	0.22
San Fernando	Paicoma Dam	09/02/1970	6.61	1.81	0.12	0.06
Taiwan SMAR-T1(40)	Smart1 M02	15/111986	6.32	60.89	0.14	0.04
Superstition Hill	Mtn Camera	24/11/1987	6.54	5.61	0.35	0.16
Tabas	Dayhook	16/09/1978	7.35	13.94	0.19	0.09
Whittier	LA – Obregon Park	01/10/1987	5.27	13.62	0.06	0.06

The engineering demand parameter (EDP) chosen in the present study provides an estimate of the structural damage state and collapse potential. Due to the nature of the rocking behavior, the most representative parameter to assess structural capacity has to be based on the rocking rotation. As a result, the absolute peak rocking rotation $|\theta_{max}|$ scaled with respect to the slenderness α is adopted:

$$EDP = \frac{|\theta_{max}|}{\alpha} \tag{17}$$

The correlation coefficients by Pearson are demonstrated, as the metric of evaluating the interdependency of the IMs with the EDP. Moreover, to highlight the efficiency of the proposed parameters the dispersion estimators β and ζ, calculated by Eq. (9) and Eq. (14) respectively are presented. Assuming the maximum likelihood estimator to calculate the overturning fragility, β_{MLE} values are presented (Eq. (13)), due to the fact that it could be considered as an additional efficiency parameter. Additionally, the sufficiency of the proposed IMs regarding their correlation with parameters such as magnitude M and distance R is examined.

Further, the performance of the PGV, SI_H [21], I_{FVF} [30], T_m [20] and L_m [31] in assessing the vulnerability of the examined structural systems is also presented. These ground motion parameters are strongly correlated with the rocking response of slender rigid blocks [13]. The definitions of the examined IMs are the followings:

$$SI_H = \int_{0.1}^{2.5} PSV(T, \quad \xi = 0.05)\, dT \tag{18}$$

$$I_{FVF} = PGV \cdot t_D^{0.25} \tag{19}$$

$$T_m = \frac{\sum_i C_i^2 (1/f_i)}{\sum_i C_i^2} \tag{20}$$

$$L_m = PGV \cdot T_m \tag{21}$$

where PSV is the pseudo-velocity response spectra of elastic SDOF oscillators, t_D is the strong motion duration as defined by Trifunac and Brady [32], the C_i are the Fourier amplitudes of the accelerogram and f_i the Fourier transform frequencies between 0.25 and 20 Hz.

The comparison between the results which are obtained by all the examined IMs is going to deduce the performance of the proposed, in this study, spectral IMs, regarding their appropriateness in describing the damage potential of an earthquake, on the rocking response.

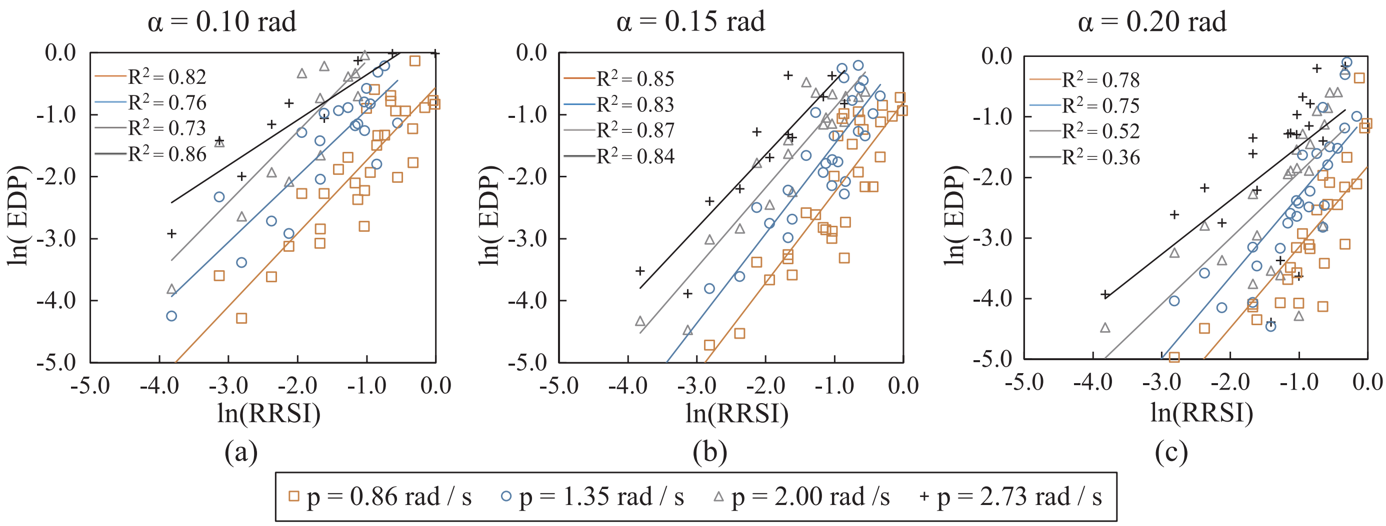

Fig. 3. Linear regression analysis between ln(RRSI) and ln(EDP).

7. Results

In Figs. 3, 4 the linear regression between the peak rocking rotation and the proposed IMs, RRSI and RVSI are depicted respectively. It can be seen that the linear regression is capable of describing the relationship between the IM and the EDP. Moreover, at first glance, the RRSI seems to be more suitable in describing the safe rocking response for the total of the rocking blocks, despite the RVSI which gives better performance in less slender blocks. From these diagrams, the size effect is also easily observed with larger blocks being more stable as they developed smaller values of rocking rotation.

In Fig. 5 the correlation coefficients between the proposed and the other examined IMs with the EDP are presented. Three subfigures are depicted, each one corresponding to blocks with a certain value of slenderness. The RRSI displays, on average, the highest correlation with the rocking response in contrast with the other IMs (Table 2). In more detail, it affects significantly the blocks with slenderness less than 0.15 rad regardless of their size. Its performance tends to decrease only according to less slender blocks ($\alpha = 0.20$ rad) of small size ($R = 1.0$ and 1.8 m). The RVSI exhibits low correlation with the response of more slender blocks ($\alpha = 0.10$ rad). On the other hand, it correlates strongly with the response of less slender blocks. In comparison also with the other IMs considered in this study, RVSI is the only one that performs at such a high rate in predicting the response of blocks with slenderness $\alpha = 0.20$ rad and especially with frequency parameter values $p = 2.00$ and 2.73 rad/s. From the other examined IMs, those that constitute velocity based ones demonstrate similar performance with the RRSI, while the T_m is the IM that depicts the lowest grade of interdependency with the rocking response, albeit without being negligible.

The logarithmic standard deviations β, which are calculated via the linear regression analysis, are depicted in Fig. 6. Among the two proposed IMs, the RRSI present considerable low values of β regardless the rigid block examined, while the RVSI demonstrates noted efficiency only when describing the response of less slender blocks. Regarding the other examined IMs, the SI_H and the I_{FVF} decrease notably the dispersion of the PSDM. Moreover, T_m and L_m perform well, especially for more slender blocks. Between the two proposed IMs and the other 5 examined, the RRSI presents the lowest logarithmic standard deviation on average (Table 2). Despite the propitious performance of the proposed IMs, regarding the description of the rocking response, it should not be overlooked the fact that for their definition important computational effort is needed.

Apart from the logarithmic standard deviations β, another more complex dispersion metric, the proficiency ζ, is calculated. Fig. 7 displays the ζ parameter per block, for each one of the IMs. According to

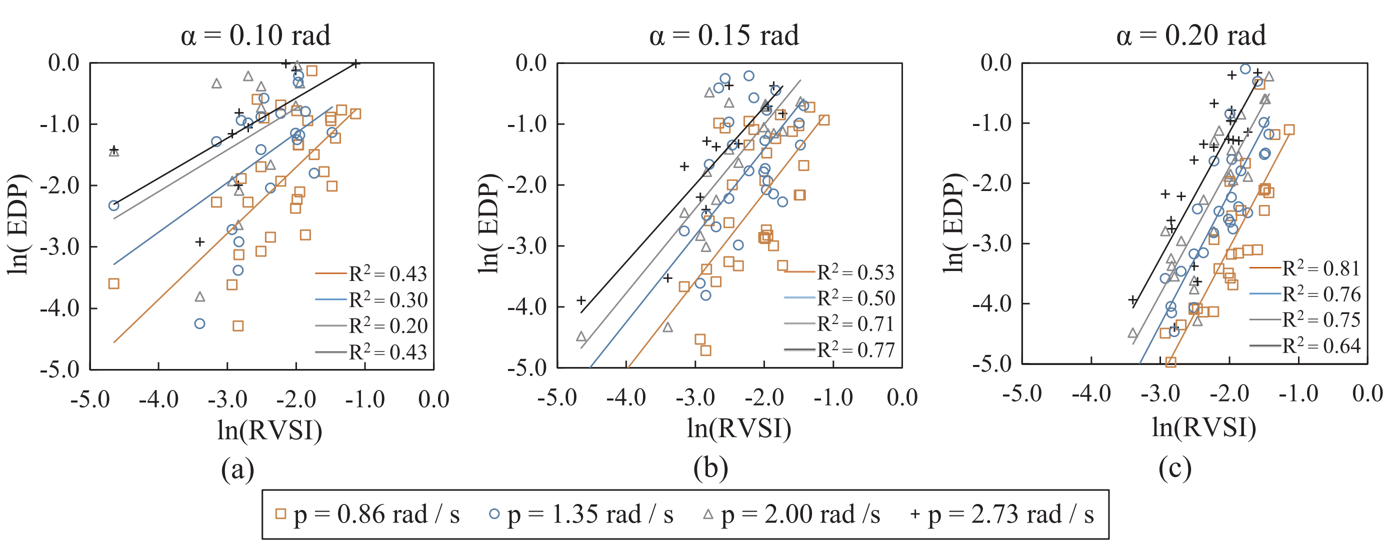

Fig. 4. Linear regression analysis between ln(RVSI) and ln(EDP).

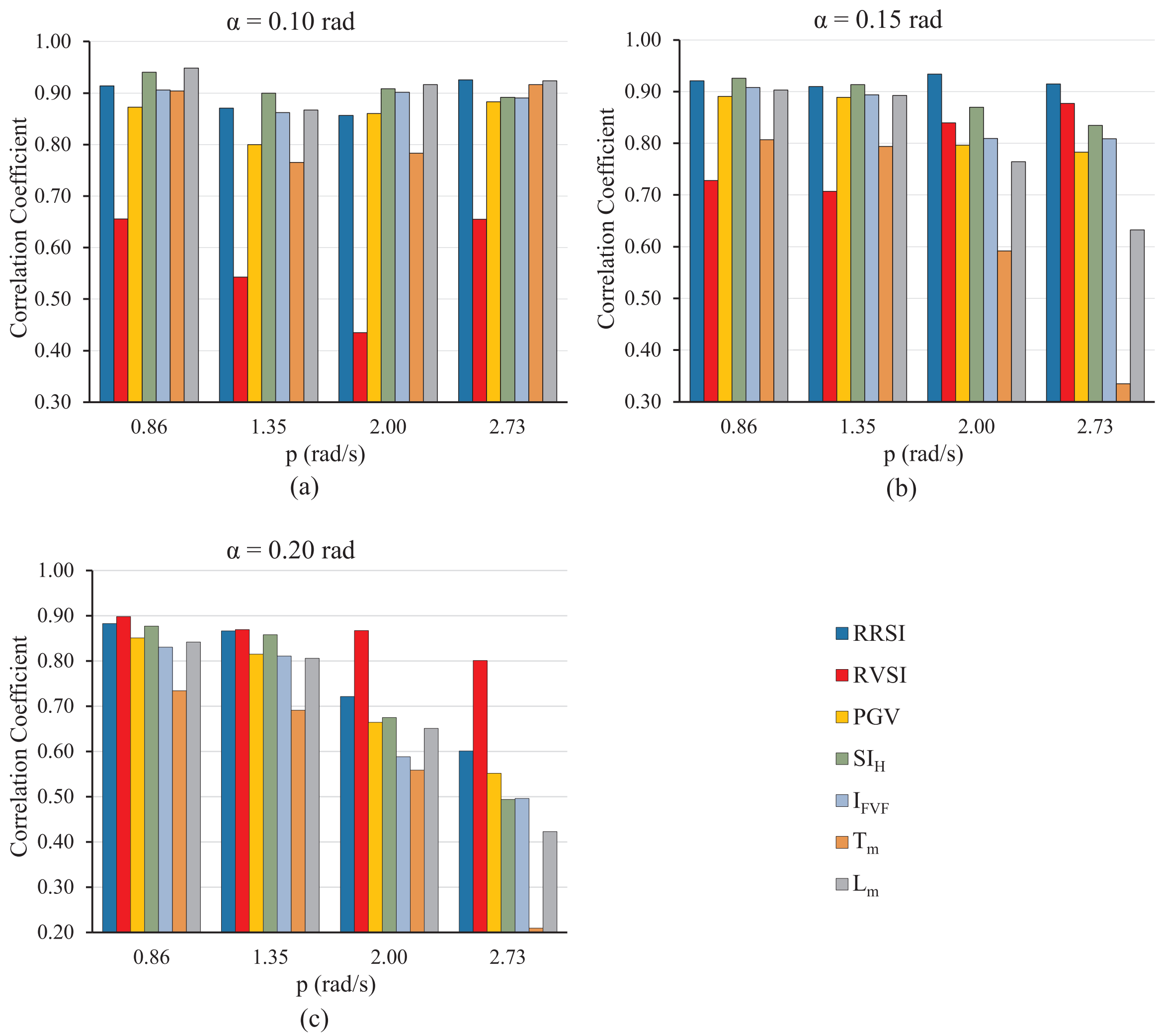

Fig. 5. Correlation coefficients by Pearson between the IMs and the EDP.

Table 2
Mean values of $\rho_{Pearson}$, β, ζ and β_{MLE}.

Statistic parameters	Intensity measures						
	RRSI	RVSI	PGV	SI_H	I_{FVF}	T_m	L_m
$\bar{\rho}_{Pearson}$	0.86	0.74	0.81	0.84	0.81	0.67	0.80
$\bar{\beta}$	0.62	0.86	0.73	0.65	0.71	0.88	0.72
$\bar{\zeta}$	0.55	0.74	0.48	0.51	0.46	0.83	0.92
$\bar{\beta}_{MLE}$	0.35	0.73	0.30	0.45	0.30	0.75	0.72

these results, the I_{FVF} displays the better performance on average. The lowest proficiency is presented by T_m and L_m, while the other parameters perform in the same way the efficiency parameter β with the RRSI being one of the best parameters. It is evident that the variability on the response in regard with the examined IMs, except of the RVSI, is increased as the slenderness decreases. That is observed due to the fact that the response of more stocky blocks is affected also by the acceleration amplitude of the ground motions.

The standard deviation β_{MLE} obtained by the maximum likelihood estimator approach could also form an efficiency parameter. In Fig. 8 the standard deviation β_{MLE}, obtained by the appliance of the maximum likelihood parameter, are pictured. The block with slenderness $\alpha = 0.20$ rad and frequency parameter $p = 0.86$ rad/s ($R = 10$ m) did not collapse subjected to any of the 35 ground motions. Thus, at the third subfigure in the row, there are results depicted from only three blocks. Between the two proposed IMs the RRSI developed lower values of dispersion. The T_m and the L_m consist the parameters that could predict the overturning of the blocks with the larger values of dispersion compared with the other IMs. Apart from the RRSI, the PGV and the I_{FVF} lead to minimum values of dispersion. The β_{MLE} mean values per IM are presented in Table 2.

The p-values obtained by the regression analyses of the residuals (ε|IM), with the M and ln(R) are presented in Fig. 9. The results are displayed for each block per IM. Therefore, in the each subfigure 12 bullet points are displayed for each IM. Every point corresponds to an individual examined rocking SDOF oscillator case. Independence of the RRSI on both the magnitude M and the distance R, for all the considered rigid blocks, is suggested. On the other hand, the RVSI seems to be, even in some cases, significantly dependent on both M and R. Among the other IMs, the I_{FVF} and L_m are insufficient as they are dependent on R and both on M and R respectively. Subsequently, the PGV, SI_H and the T_m are form sufficient IMs.

The proposed IMs consist structural response specific ones. As such, increased computational effort is needed for calculating the ground motion hazard. The definition of the IMs takes into account the structural response of a large number of rocking rigid blocks. However, the proposed IMs are independent on the characteristics of the examined structures. That is a favorable feature, as it is not required the definition of hazard curves for each specific structure.

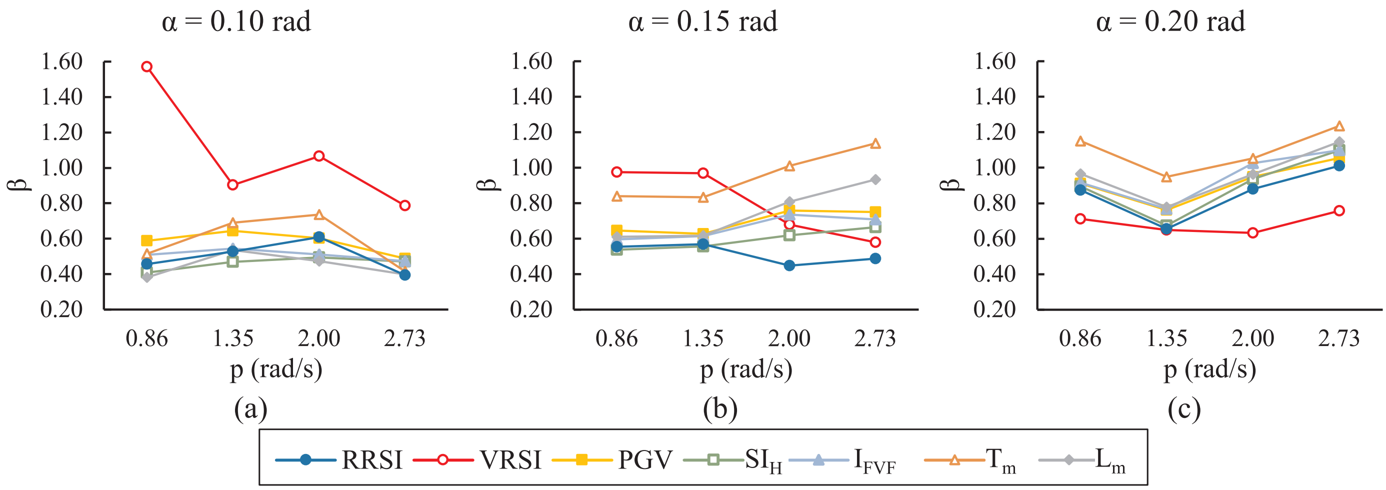

Fig. 6. Logarithmic standard deviations (β) of the examined IMs for the considered blocks.

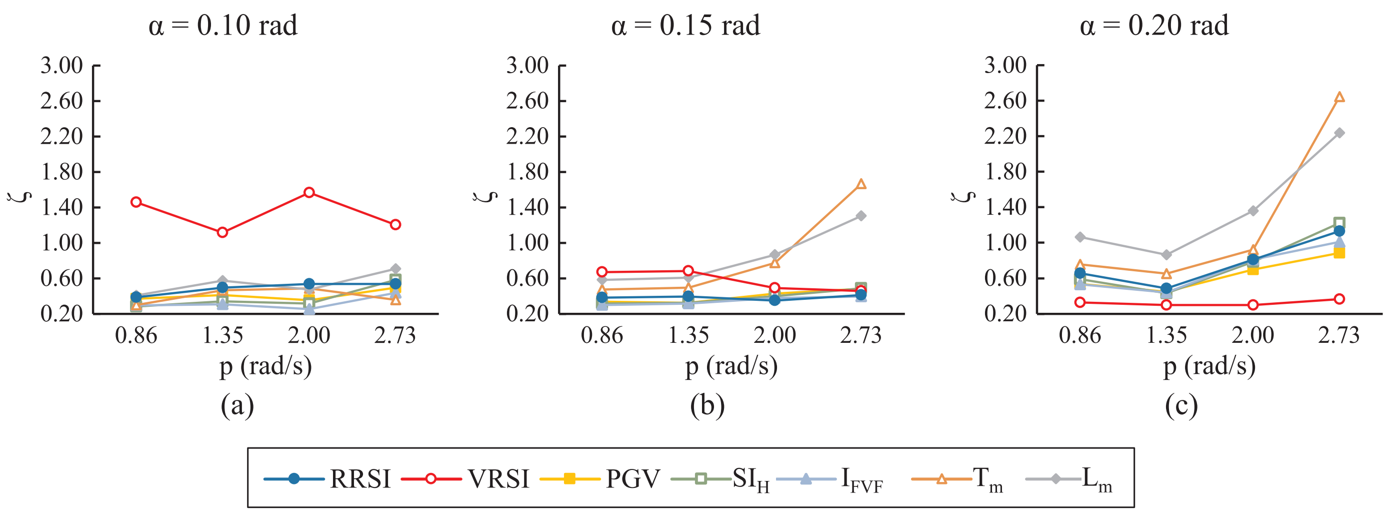

Fig. 7. Proficiency measure (ζ) of the examined IMs for the considered blocks.

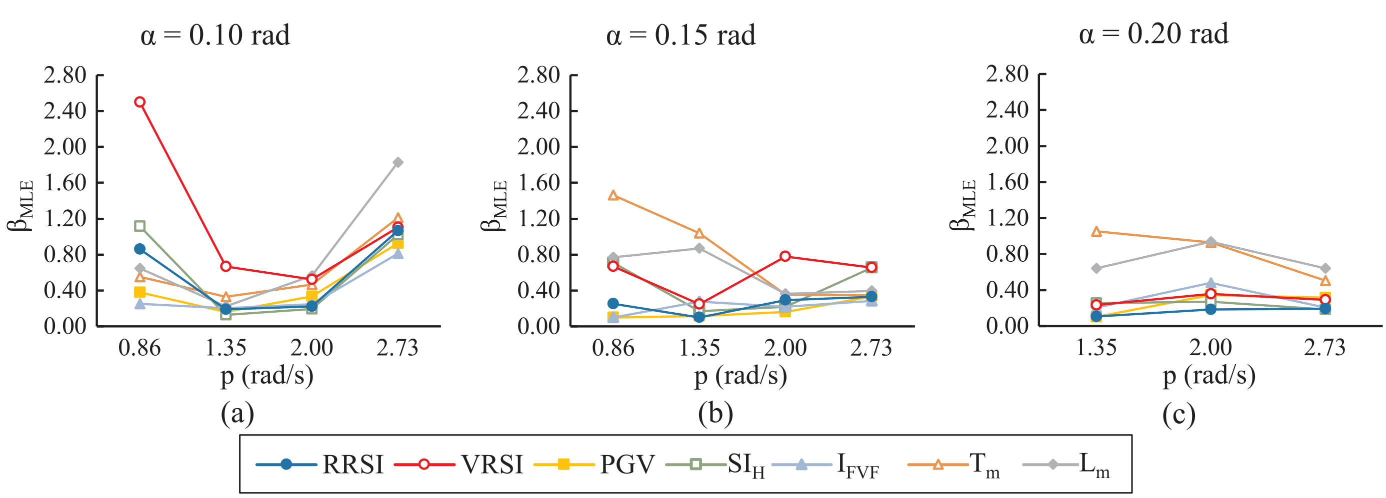

Fig. 8. Standard deviation (β_{MLE}) of overturning fragility curves for the examined rigid blocks.

8. Conclusions

Two novel scalar IMs named RRSI and RVSI are proposed in the present study. These parameters are defined by the volume under the rocking spectra pictured as surface graphs. The spectral volumes that constitute the RRSI and RVSI are evaluated by integration of the rocking rotation spectrum and the angular velocity spectrum respectively. The integration limits of the period parameter T and the

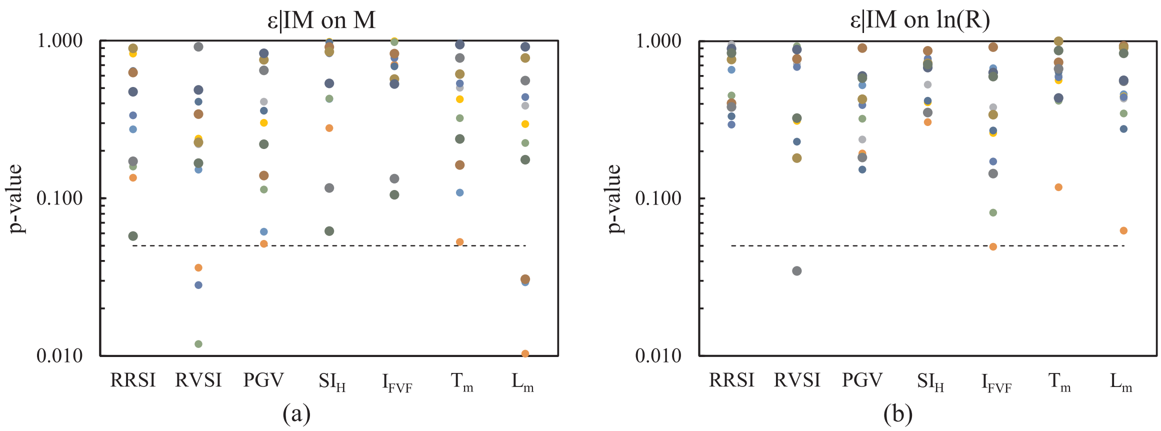

Fig. 9. p-values of coefficient b from the regression of the residuals on magnitude M and on source to site distance R.

slenderness α were adopted in order to construct IMs which take into account structural response information for the majority of the slender rocking blocks. The proposed IMs are determined with the intention of having strong correlation with the seismic rocking performance.

To assess the adequacy of the proposed IMs, 12 rigid blocks subjected to a set of 35 ground motion record are examined. Using only the non-collapse data, linear regression analyses are performed between the proposed IMs and the developed rocking rotation, from which the correlation coefficients are extracted. Further, criteria such as efficiency, proficiency and sufficiency are adopted in order to evaluate the IMs. Moreover, the standard deviations calculated by the maximum likelihood method are presented, as an efficiency parameter. In order to highlight the performance of the IMs, their results are compared with those obtained by 5 well-known IMs which affect the rocking response.

- Regarding the correlation coefficients, the RRSI developed the higher values on average ($\bar{\rho}_{Pearson} = 0.86$), while it ranged from 0.6 to 0.94. The RVSI presents moderate correlation on average, however, it outperforms the less slender blocks ($\alpha = 0.20$ rad).
- The RRSI proved to be an efficient IM on average ($\beta = 0.62$) having similar performance with the SI_H and the I_{FVF}, whilst the RVSI being more efficient in describing the response of stocky blocks.
- Based on the proficiency criteria, the I_{FVF} presents better performance. However, RRSI demonstrates low values of the parameters ζ too. Concerning the less slender blocks, the RVSI presents an exceptional performance, compared with the other examined IMs.
- Similar outcomes arise from the standard deviation which is calculated throughout the maximum likelihood estimator process, with the RRSI being the parameter that decreased the variation of the response in the definition of the overturn collapse.
- The dependency of the proposed IM with the magnitude M and distance R is also examined, with the RRSI being independent of these two parameters and the RVSI being insufficient.

References

[1] Giovenale P, Cornell AC, Esteva L. Comparing the adequacy of alternative ground motion intensity measures for the estimation of structural responses. Earthq Eng Struct Dyn 2004;33:951–79.
[2] Luco N, Cornell C. Structure-specific scalar intensity measures for near-source and ordinary earthquake motions. Earthq Spectra 2007;23:357–92.
[3] Mackie KR, Stojadinovi¢ B. Fragility basis for California highway overpass bridge seismic decision making, PEER report. Berkeley, CA: University of California; 2005.
[4] Padgett JE, Nielson BG, DesRoches R. Selection of optimal intensity measures in probabilistic seismic demand models of highway bridge portfolios. Earthq Eng Struct Dyn 2008;37:711–25.
[5] Elenas A. Seismic-parameter-based statistical procedures for the approximate assessment of structural damage. Math Probl Eng 2014:Article ID 916820http://dx.doi.org/10.1155/2014/916820. [22 pages].
[6] Kostinakis K, Fontara IK, Athanatopoulou AM. Scalar structure-specific ground motion intensity measures for assessing the seismic performance of structures: a review. J Earthq Eng 2016. http://dx.doi.org/10.1080/13632469.2016.1264323.
[7] Makris N, Konstantinidis D. The rocking spectrum and the limitations of practical design methodologies. Earthq Eng Struct Dyn 2003;32:265–89.
[8] Yim CS, Chopra AK, Penzien J. Rocking response of rigid blocks to earthquakes. Earthq Eng Struct Dyn 1980;8:565–87.
[9] Spanos PD, Koh AS. Analysis of block random rocking. Soil Dyn Earthq Eng 1986;5:178–83.
[10] Acikgoz MS, DeJong MJ. The rocking response of large flexible structures to earthquakes. Bull Earthq Eng 2014;12:875–908.
[11] Psycharis IN, Fragiadakis M, Stefanou I. Seismic reliability assessment of classical columns subjected to near-fault ground motions. Earthq Eng Struct Dyn 2013;42:2061–79.
[12] Dimitrakopoulos EG, Paraskeva TS. Dimensionless fragility curves for rocking response to near-fault excitations. Earthq Eng Struct Dyn 2015;44:2015–33.
[13] Kavvadias IE, Vasiliadis LK, Elenas E. Seismic response parametric study of ancient rocking columns. Int J Archit Herit 2017. http://dx.doi.org/10.1080/15583058.2017.1298009.
[14] Zhang J, Makris N. Rocking response of free-standing blocks under cycloidal pulses. J Eng Mech ASCE 2001;127:473–83.
[15] Housner GW. The behavior of inverted pendulum structures during earthquakes. Bull Seismol Soc Am 1963;53:403–17.
[16] Peña F, Lourenço PB, Campos Costa A. Experimental dynamic behavior of free-standing multi-block structures under seismic loadings. J Earthq Eng 2008;12:953–79.
[17] El Gawady MA, Ma Q, Butterworth JW, Ingham J. Effects of interface material on the performance of free rocking blocks. Earthq Eng Struct Dyn 2010;40:375–92.
[18] Kalliontzis D, Sritharan S, Schultz A. Improved coefficient of restitution estimation for free rocking members. J Struct Eng 2016;142.
[19] Natick MA. MATLAB, mathworks; 2016.
[20] Rathje EM, Abrahamson NA, Bray JD. Simplified frequency content estimates of earthquake ground motions. J Geotech Geoenviron Eng ASCE 1998;124:150–9.
[21] Housner GV. Spectrum intensities of strong motion earthquakes. In: Proceedings of the symposium on earthquake and blast effects on structures. Oakland, California: EERI; 1952, p. 20–36.
[22] Baker JW. Efficient analytical fragility function fitting using dynamic structural analysis. Earthq Spectra 2015;31:579–99.
[23] Shinozuka M, Feng MQ, Lee J, Naganuma T. Statistical analysis of fragility curves. J Eng Mech ASCE 2000;126:1224–31.
[24] Hartung J. Statistik. München: Oldenburg Verlag; 1995.
[25] Makris N, Vassiliou MF. Planar rocking response and stability analysis of an array of free-standing columns capped with a freely supported rigid beam. Earthq Eng Struct Dyn 2013;2013(42):431–49.
[26] Vassiliou MF, Mackie KR, Stojadinović B. Dynamic response analysis of solitary flexible rocking bodies: modeling and behavior under pulse-like ground excitation. Earthq Eng Struct Dyn 2014;43:1463–81.
[27] Lagomarsino S. Seismic assessment of rocking masonry structures. Bull Earthq Eng 2015;13:97–128.
[28] Pacific Earthquake Engineering Research Centre (PEER). Strong motion database; 2016. 〈http://ngawest2.berkeley.edu/〉.
[29] European strong-motion database; 2016. 〈http://www.isesd.hi.is/ESD_Local/frameset.htm〉.
[30] Fajfar P, Vidic T, Fischinger M. A measure of earthquake motion capacity to damage medium-period structures. Soil Dyn Earthq Eng 1990;9:236–42.
[31] Dimitrakopoulos E, Kappos AJ, Makris N. Dimensional analysis of yielding and pounding structures for records without distinct pulses. Soil Dyn Earthq Eng 2009;29:1170–80.
[32] Trifunac MD, Brady AG. A study on the duration of strong earthquake ground motion. Bull Seismol Soc Am 1975;65:581–626.