Learning under Noisy Supervision

Part II: Statistical Learning under Noisy Supervision

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Structure

Basics

Consistent algorithms

Transition matrix

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Learning without noisy labels

Problem setup:

Data: $S = \{(\boldsymbol{x}_1, y_1), \ldots, (\boldsymbol{x}_n, y_n)\} \sim D^n$.

Aim: Learn a classifier $f \in F$, such that $\forall(\boldsymbol{x}, y) \sim D$, $f(\boldsymbol{x})$ is a good prediction for y.

What is the best classifier we can obtain?

w.r.t. accuracy

To measure the accuracy, we define loss function

$$\ell(\boldsymbol{x}, y), f \mapsto \ell\big(y, f(\boldsymbol{x})\big) \in \mathbb{R}.$$

For example, 0-1 loss: $\mathbf{1}(y \neq \operatorname{sign}(f(\boldsymbol{x})))$.

The best classifier should be the one that has the minimum loss on all the possible data from the domain.

Expected risk, Bayes classifier

Theoretically,

The expected risk:

$$R_{D,0-1}(f) = \mathbb{E}_{(X,Y)\sim D}[\mathbf{1}(Y \neq \operatorname{sign}(f(X)))].$$

Bayes classifier:

$$f_\rho(\boldsymbol{x}) = \operatorname{argmin}_f R_{D,0-1}(f) = \arg\max_y P(Y = y | X = \boldsymbol{x}).$$

Restricted Bayes classifier:

$$f^* = \operatorname*{argmin}_{f\in\mathcal{F}} R_{D,0-1}(f).$$

Empirically,

In reality, we can only observe a sample of data $S = \{(\boldsymbol{x}_1, y_1), \dots, (\boldsymbol{x}_n, y_n)\} \sim D^n$.

We approximate the expected risk $R(f)$ via the empirical risk: $\widehat{R}_{D,\ell}(f) = \frac{1}{n}\sum_{i=1}^{n} \ell\big(y_i, f(\boldsymbol{x}_i)\big)$.

We minimize the empirical risk to find a predictor:

$$f_n = \arg\min_{f \in \boldsymbol{\mathcal{F}}} \widehat{R}_{D,\ell}(f).$$

Statistically consistent classifier [1,2]:

With high probability, as $n \longrightarrow \infty$,
we have: $R_{D,\ell}(f_n) \longrightarrow R_{D,\ell}(f^*)$.

[1] Mohri et al. *Foundations of machine learning*. MIT press, 2018.
[2] Devroye, et al. *A probabilistic theory of pattern recognition*. Vol. 31. Springer Science & Business Media, 2013.

Aim:

Designing algorithms whose outputs will approach
$f_\rho(\boldsymbol{x}) = \arg\max_y P(Y = y|X = \boldsymbol{x})$.

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Noisy sample: $\tilde{S} = \{(\boldsymbol{x}_1, \tilde{y}_1), \ldots, (\boldsymbol{x}_n, \tilde{y}_n)\} \sim \widetilde{D}^n$,
where $\tilde{y}$ stands for noisy labels and $\widetilde{D}$ the noisy distribution.

What is the best classifier we can learn?

Can we approach $f_\rho(\boldsymbol{x}) = \arg\max_y P(Y = y | X = \boldsymbol{x})$?

Learning with noisy labels

One category: extracting confident examples.

SOTA, e.g., Co-teaching [3]; Joint Optim [4].

Another category: label-noise learning [5].

Methodology, i.e., statistically consistent algorithms.

[3] Han, Bo, et al. "Co-teaching: Robust training of deep neural networks with extremely noisy labels." *NeurIPS* 2018.
[4] Tanaka, Daiki, et al. "Joint optimization framework for learning with noisy labels." *CVPR* 2018.
[5] Xia, Xiaobo, et al. “Are anchor points really indispensable in label-noise learning?.” *NeurIPS* 2019.

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Model label noise

Transition matrix:

$$\begin{bmatrix} P(\tilde{Y} = 1 | Y = 1, \boldsymbol{x}) & \cdots & P(\tilde{Y} = 1 | Y = C, \boldsymbol{x}) \\ \vdots & \ddots & \vdots \\ P(\tilde{Y} = C | Y = 1, \boldsymbol{x}) & \cdots & P(\tilde{Y} = C | Y = C, \boldsymbol{x}) \end{bmatrix}.$$

Transition matrix

$$\begin{bmatrix} P(\tilde{Y} = 1|\boldsymbol{x}) \\ \vdots \\ P(\tilde{Y} = C|\boldsymbol{x}) \end{bmatrix} = \underbrace{\boxed{\begin{bmatrix} P(\tilde{Y} = 1|Y = 1, \boldsymbol{x}) & \cdots & P(\tilde{Y} = 1|Y = C, \boldsymbol{x}) \\ \vdots & \ddots & \vdots \\ P(\tilde{Y} = C|Y = 1, \boldsymbol{x}) & \cdots & P(\tilde{Y} = C|Y = C, \boldsymbol{x}) \end{bmatrix}}}_{T^\top(\boldsymbol{x})} \begin{bmatrix} P(Y = 1|\boldsymbol{x}) \\ \vdots \\ P(Y = C|\boldsymbol{x}) \end{bmatrix}$$

Why called "label-noise learning"?

- Label-noise learning [5]
- Learning with noisy labels [6]

[5] Xia, Xiaobo, et al. "Are anchor points really indispensable in label-noise learning?." *NeurIPS* 2019.
[6] Natarajan, Nagarajan, et al. "Learning with noisy labels." *NeurIPS* 2013.

Model Label Noise

(1) Random Classification Noise (RCN) [7]:

$$\rho_{\tilde{Y},Y}(X) = P\left(\tilde{Y}\middle|Y,X\right)=P\left(\tilde{Y}\middle|Y\right) = \rho, \forall\, Y \neq \tilde{Y}.$$

(2) Class-conditional Noise (CCN) [6]:

$$\rho_{\tilde{Y},Y}(X) = P\left(\tilde{Y}\middle|Y,X\right)=P\left(\tilde{Y}\middle|Y\right).$$

(3) Instance-dependent Noise (IDN) [8,9]:

$$\rho_{\tilde{Y},Y}(X) = P\left(\tilde{Y}\middle|Y,X\right).$$

[7] Angluin, Dana, and Philip Laird. “Learning from noisy examples.” *Machine Learning* 2.4: 343-370, 1988.
[8] Cheng, Jiacheng, et al. “Learning with bounded instance and label-dependent label noise.” *ICML* 2020.
[9] Berthon, Antonin, et al. "Confidence scores make instance-dependent label-noise learning possible." *ICML*, 2021

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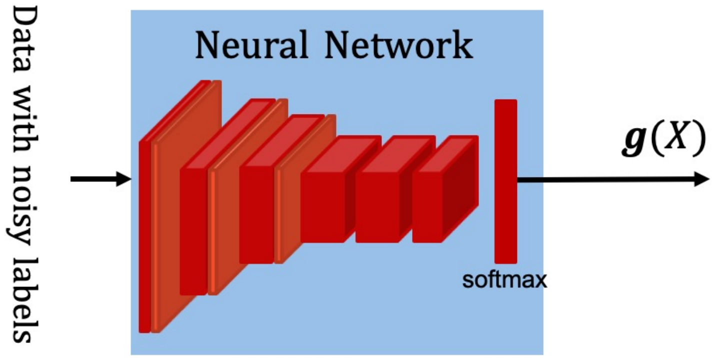
Class-conditional Noise (CCN)

The loss correction method:
Modify ℓ to be $\tilde{\ell}$ such that

$$\mathbb{E}_{(X,\tilde{Y})\sim\tilde{D}}\left[\tilde{\ell}\left(f(X),\tilde{Y}\right)\right] = \mathbb{E}_{(X,Y)\sim Y}\left[\ell\left(f(X),Y\right)\right]$$

By exploiting the model of label noise:

$$\begin{bmatrix} P(\tilde{Y}=1|\boldsymbol{x}) \\ \vdots \\ P(\tilde{Y}=C|\boldsymbol{x}) \end{bmatrix} = \begin{bmatrix} P(\tilde{Y}=1|Y=1) & \cdots & P(\tilde{Y}=1|Y=C) \\ \vdots & \ddots & \vdots \\ P(\tilde{Y}=C|Y=1) & \cdots & P(\tilde{Y}=C|Y=C) \end{bmatrix} \begin{bmatrix} P(Y=1|\boldsymbol{x}) \\ \vdots \\ P(Y=C|\boldsymbol{x}) \end{bmatrix}$$



$$\begin{bmatrix} P(\tilde{Y}=1|\boldsymbol{x}) \\ \vdots \\ P(\tilde{Y}=C|\boldsymbol{x}) \end{bmatrix} = \begin{bmatrix} P(\tilde{Y}=1|Y=1) & \cdots & P(\tilde{Y}=1|Y=C) \\ \vdots & \ddots & \vdots \\ P(\tilde{Y}=C|Y=1) & \cdots & P(\tilde{Y}=C|Y=C) \end{bmatrix} \begin{bmatrix} P(Y=1|\boldsymbol{x}) \\ \vdots \\ P(Y=C|\boldsymbol{x}) \end{bmatrix}$$

Importance reweighting [11]:

$$\tilde{\ell}_{ir}(f(\boldsymbol{x}), y) = \frac{P(\boldsymbol{x}, y)}{\tilde{P}(\boldsymbol{x}, y)} \ell(f(\boldsymbol{x}), y) = \frac{\boldsymbol{g}_y(\boldsymbol{x})}{(T^\top \boldsymbol{g})_y(\boldsymbol{x})} \ell(f(\boldsymbol{x}), y),$$

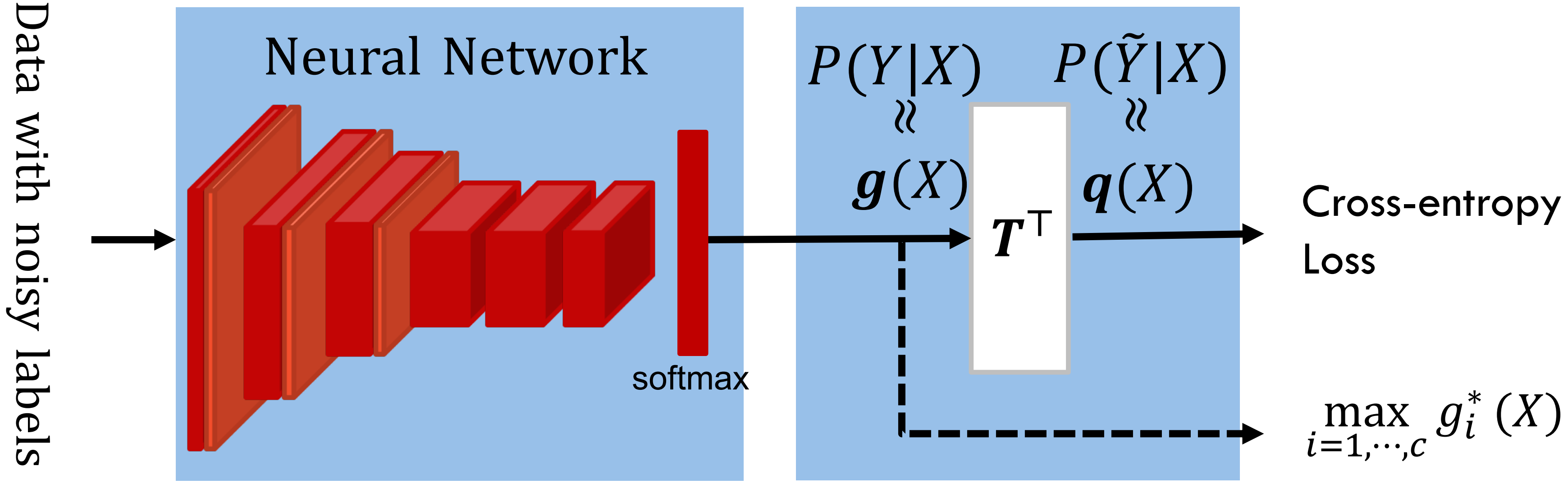
where $f(\boldsymbol{x}) = \arg \max_{j \in \{1,\ldots,C\}} \boldsymbol{g}_j(\boldsymbol{x})$.

Thus, $\mathbb{E}_{(X,\tilde{Y})\sim\tilde{D}}\left[\tilde{\ell}_{ir}\left(f(X), \tilde{Y}\right)\right] = \mathbb{E}_{(X,Y)\sim D}[\ell(f(X), Y)]$

[11] Liu, Tongliang, and Dacheng Tao. "Classification with noisy labels by importance reweighting." *IEEE Transactions on pattern analysis and machine intelligence* 38.3 (2015): 447-461..

$$\begin{bmatrix} P(\tilde{Y}=1|\boldsymbol{x}) \\ \vdots \\ P(\tilde{Y}=C|\boldsymbol{x})\end{bmatrix} = \begin{bmatrix} P(\tilde{Y}=1|Y=1) & \cdots & P(\tilde{Y}=1|Y=C) \\ \vdots & \ddots & \vdots \\ P(\tilde{Y}=C|Y=1) & \cdots & P(\tilde{Y}=C|Y=C)\end{bmatrix}\begin{bmatrix}P(Y=1|\boldsymbol{x})\\ \vdots \\ P(Y=C|\boldsymbol{x})\end{bmatrix}$$

Forward correction [12]:



[12] Patrini, Giorgio, et al. "Making deep neural networks robust to label noise: A loss correction approach." *CVPR* 2017.

A summary of consistent algorithms

- Many methods for dealing with noisy labels
  Loss correction, Sample selection, label correction, …

- Model label noise
  Random Classification Noise (RCN)
  Class-conditional Noise (CCN)
  Instance-dependent Noise (IDN)

- Three loss correction methods
  Importance reweighting (risk-consistent), forward correction (classifier-consistent)

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Transition matrix

How to estimate the transition matrix

Given the noisy data
$\tilde{S} = \{(\boldsymbol{x}_1, \tilde{y}_1), \ldots, (\boldsymbol{x}_n, \tilde{y}_n)\} \sim \widetilde{D}$.

How to estimate the transition matrix T?

Definition

If $P\left(Y=i\middle|\boldsymbol{x}^i\right)=1$, then $\boldsymbol{x}^i$ is called the anchor point for the i-th class.

[11] Liu, Tongliang, and Dacheng Tao. "Classification with noisy labels by importance reweighting." *IEEE Transactions on pattern analysis and machine intelligence* 38.3 (2015): 447-461..

Anchor point assumption

$$\begin{bmatrix} P(\tilde{Y}=1|X) \\ \vdots \\ P(\tilde{Y}=C|X) \end{bmatrix} = \underset{T}{\boxed{\begin{bmatrix} P(\tilde{Y}=1|Y=1) & \cdots & P(\tilde{Y}=1|Y=C) \\ \vdots & \ddots & \vdots \\ P(\tilde{Y}=C|Y=1) & \cdots & P(\tilde{Y}=C|Y=C) \end{bmatrix}}} \begin{bmatrix} P(Y=1|X) \\ \vdots \\ P(Y=C|X) \end{bmatrix}$$

$$\begin{bmatrix} P(\tilde{Y}=1|X=\boldsymbol{x}^1) \\ \vdots \\ P(\tilde{Y}=C|X=\boldsymbol{x}^1) \end{bmatrix} = \begin{bmatrix} P(\tilde{Y}=1|Y=1) & \cdots & P(\tilde{Y}=1|Y=C) \\ \vdots & \ddots & \vdots \\ P(\tilde{Y}=C|Y=1) & \cdots & P(\tilde{Y}=C|Y=C) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P(\tilde{Y}=1|X=\boldsymbol{x}^i) \\ \vdots \\ P(\tilde{Y}=C|X=\boldsymbol{x}^i) \end{bmatrix} = \begin{bmatrix} P(\tilde{Y}=1|Y=i) \\ \vdots \\ P(\tilde{Y}=C|Y=i) \end{bmatrix}$$

[11] Liu, Tongliang, and Dacheng Tao. "Classification with noisy labels by importance reweighting." *IEEE Transactions on pattern analysis and machine intelligence* 38.3 (2015): 447-461..

How to find anchor points

Binary classification, find the anchor points:

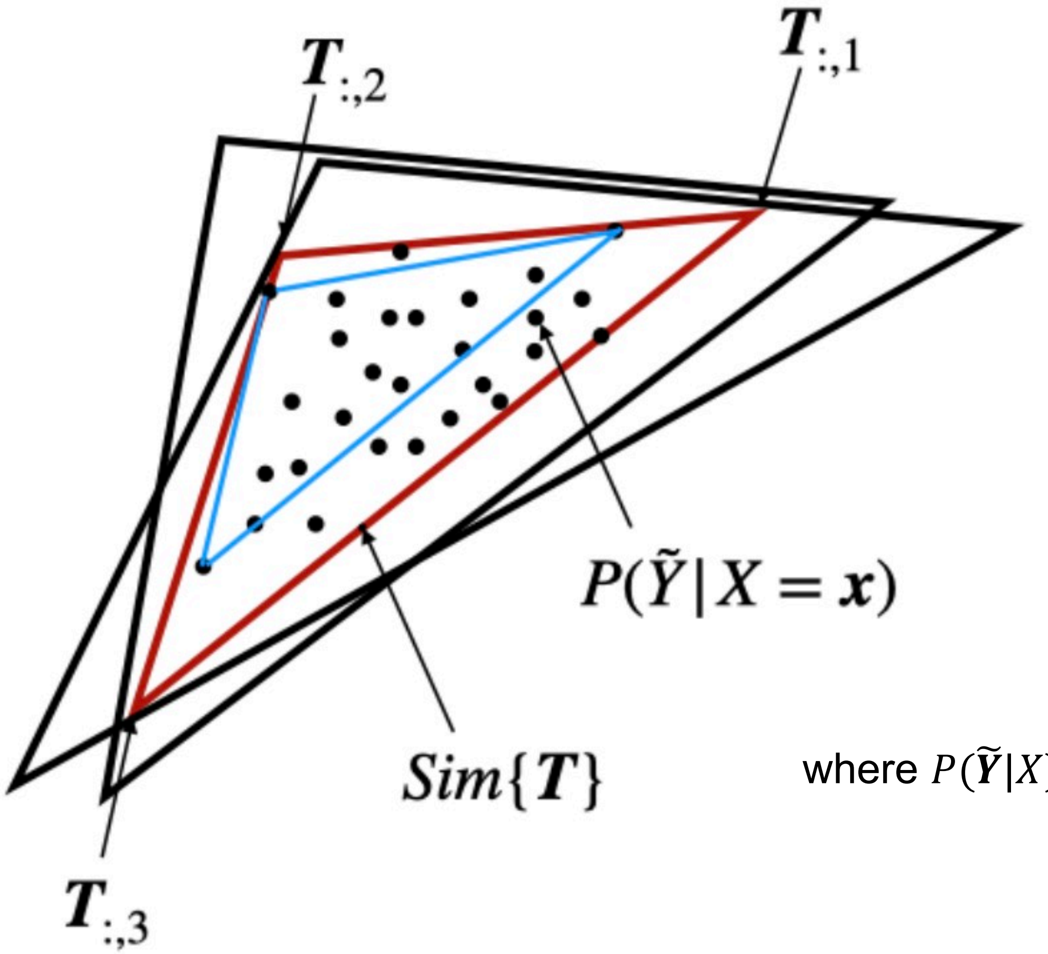
$$\boldsymbol{x}^y = \underset{\boldsymbol{x} \in X}{\operatorname{argm}ax}\, P(\tilde{Y} = y | \boldsymbol{x}).$$

Multi-classification, approximate the anchor points for multi-class learning:

$$\boldsymbol{x}^y \approx \underset{\boldsymbol{x} \in X}{\operatorname{argm}ax}\, P(\tilde{Y} = y | \boldsymbol{x}).$$

[11] Liu, Tongliang, and Dacheng Tao. "Classification with noisy labels by importance reweighting." *IEEE Transactions on pattern analysis and machine intelligence* 38.3 (2015): 447-461..

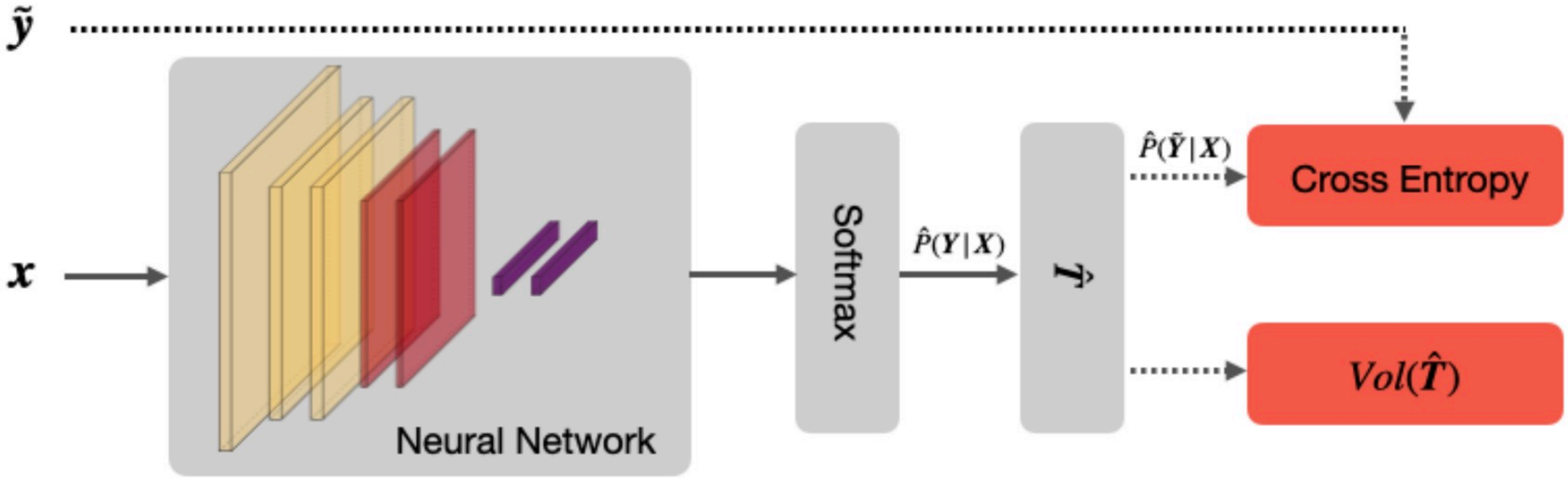
Sufficiently scattered assumption vs anchor point assumption



where $P(\tilde{\boldsymbol{Y}}|X) = \begin{bmatrix} P(\tilde{Y}=1|X) \\ \vdots \\ P(\tilde{Y}=C|X) \end{bmatrix} = \underbrace{\boxed{\begin{bmatrix} P(\tilde{Y}=1|Y=1) & \cdots & P(\tilde{Y}=1|Y=C) \\ \vdots & \ddots & \vdots \\ P(\tilde{Y}=C|Y=1) & \cdots & P(\tilde{Y}=C|Y=C) \end{bmatrix}}}_{T} \begin{bmatrix} P(Y=1|X) \\ \vdots \\ P(Y=C|X) \end{bmatrix}$

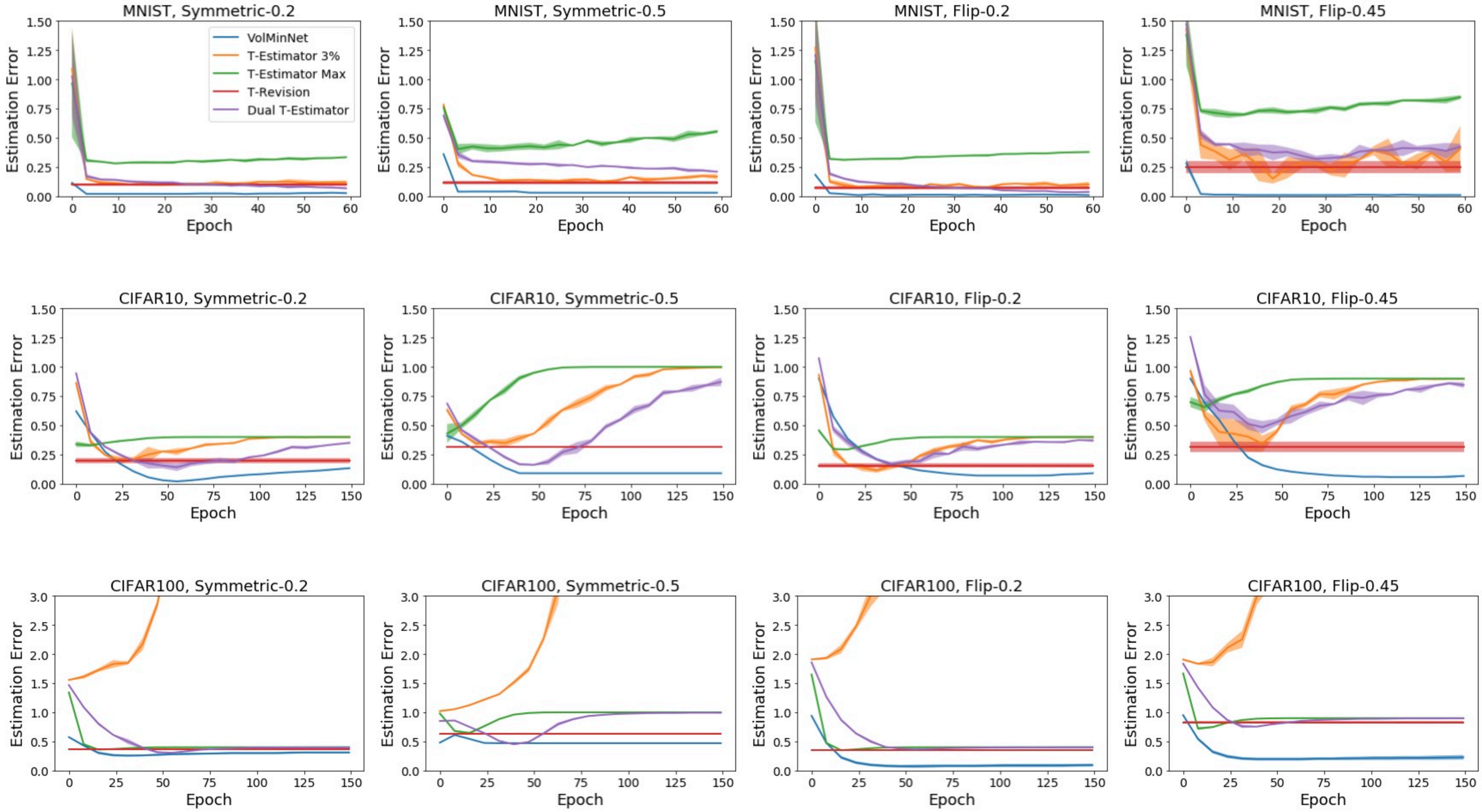
[14] Li, Xuefeng, et al. "Provably end-to-end label-noise learning without anchor points." *ICML* 2021.

VolMinNet [14]



$$\min_{\hat{T}\in\mathbb{T}} \mathrm{vol}\,(\hat{T})$$
$$\text{s.t.}\, \hat{T}h_\theta = P(\widetilde{\boldsymbol{Y}}|X)$$

[14] Li, Xuefeng, et al. "Provably end-to-end label-noise learning without anchor points." *ICML* 2021.



[14] Li, Xuefeng, et al. "Provably end-to-end label-noise learning without anchor points." *ICML* 2021.

T revision [15]

$$T = \begin{bmatrix} 0.5 & 0.3 & 0.2 & & \\ & 0.5 & .25 & .25 & \\ & & 0.5 & 0.3 & 0.2 \\ 0.2 & & & 0.5 & 0.3 \\ 0.3 & 0.2 & & & 0.5 \end{bmatrix} \qquad \tilde{T} = \begin{bmatrix} 0.5 & 0.3 & 0.2 & & \\ & 0.5 & 0.3 & 0.2 & \\ & & 0.5 & 0.3 & 0.2 \\ 0.2 & & & 0.5 & 0.3 \\ 0.3 & 0.2 & & & 0.5 \end{bmatrix}$$

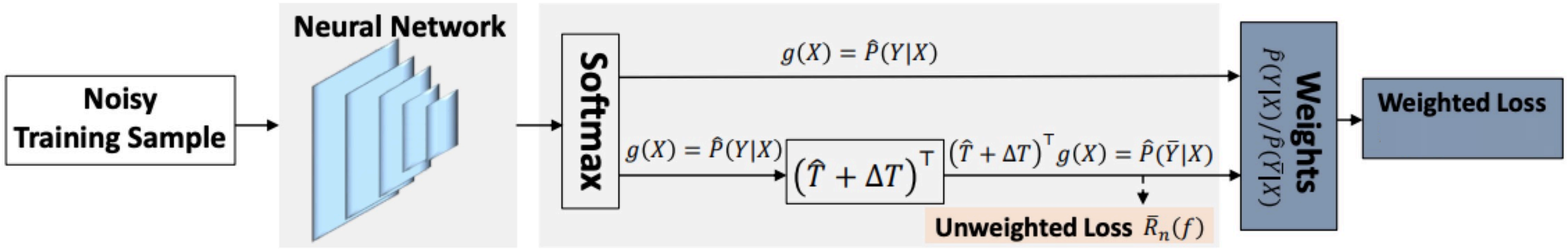
If $P(\widetilde{\boldsymbol{Y}}|X = \boldsymbol{x}) = [0.141; 0.189; 0.239; 0.281; 0.15]$,

then, $P(\boldsymbol{Y}|X = \boldsymbol{x}) = (T^\top)^{-1} P(\widetilde{\boldsymbol{Y}}|X = \boldsymbol{x}) =$
$[0.15; 0.28; 0.25; \boxed{0.3;}\, 0.02]$.

$$P(\boldsymbol{Y}|X = \boldsymbol{x}) = (\tilde{T}^\top)^{-1} P(\widetilde{\boldsymbol{Y}}|X = \boldsymbol{x})$$
$$= [0.1587; 0.2697; \boxed{0.2796;}\, 0.2593; 0.0325].$$

[15] Xia X, et al. Are Anchor Points Really Indispensable in Label-Noise Learning? NeurIPS. 2019

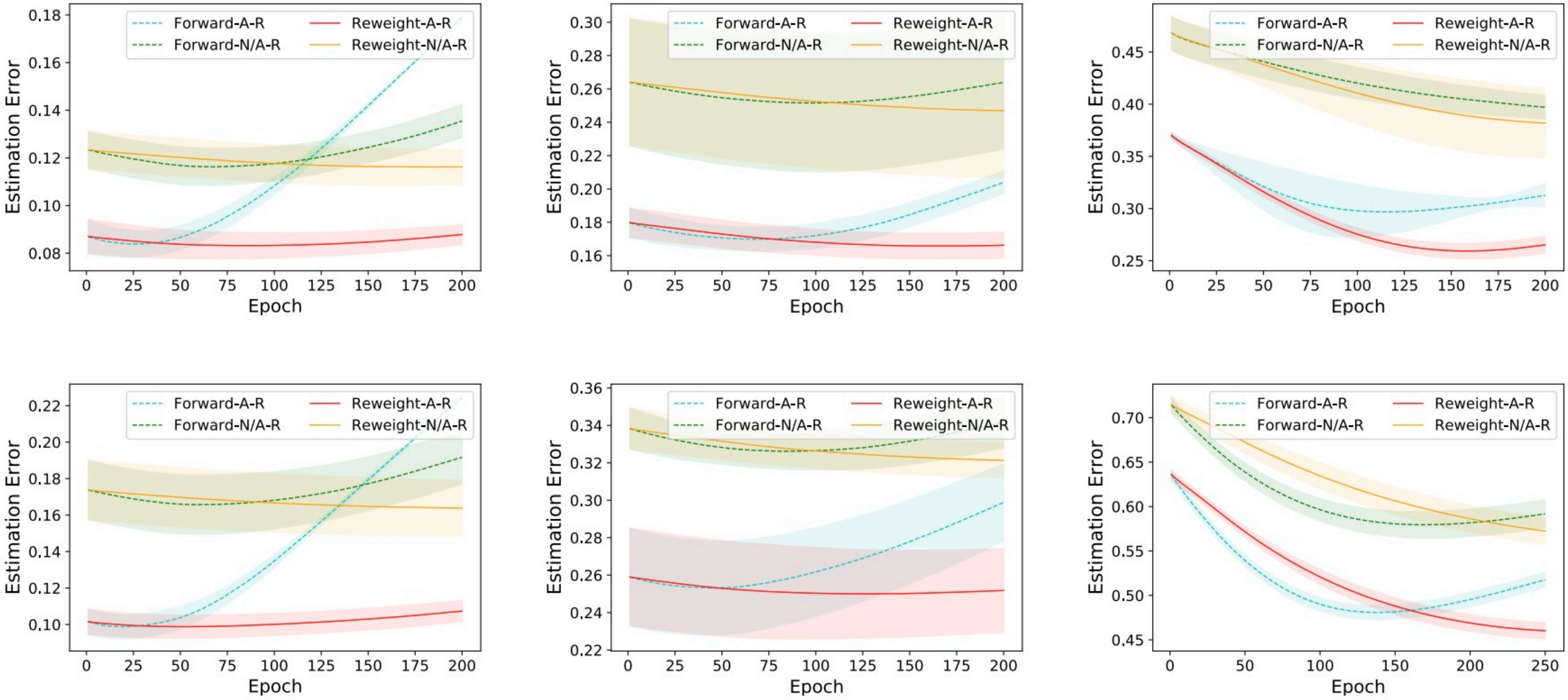
T revision [15]



Weighted loss $= \frac{1}{\mathrm{n}} \sum_{i=1}^{n} \frac{\boldsymbol{g}_{\tilde{y}_i}(\boldsymbol{x}_i)}{(\boldsymbol{T}^{\top}\boldsymbol{g})_{\tilde{y}_i}(\boldsymbol{x}_i)} L(f(\boldsymbol{x}_i), \tilde{y}_i)$,

where $f(\boldsymbol{x}) = \operatorname{argmax}_{i \in \{1,\ldots,C\}} \boldsymbol{g}_i(\boldsymbol{x})$.

[15] Xia X, et al. Are Anchor Points Really Indispensable in Label-Noise Learning? NeurIPS. 2019



(a) *MNIST* (b) *CIFAR-10* (c) *CIFAR-100*

[15] Xia X, et al. Are Anchor Points Really Indispensable in Label-Noise Learning? NeurIPS. 2019

A summary of estimating transition matrix

- How to estimate the transition matrix given only noisy data?
  Method: T estimator (by exploiting anchor points)

- How about if there is no anchor points?
  Method: VolMinNet (using the sufficiently scattered assumption)

- How to deal with poorly estimated transition matrix
  Method: T revision (revising the matrix by using a slack variable)

Conclusion and future directions

- Conclusion
  - Statistically consistent algorithms: the classifier learned by using noisy data will converge to the optimal one defined by using clean data
  - Statistically consistent algorithms are robust to the data distribution and label noise type
  - Modelling the label noise and estimating the transition matrix are cores in label-noise learning
- Future directions
  - Design effectively loss correction methods for deep learning
  - How to address the finite/small sample problem
  - How to use a small set of clean data to better estimate the transition matrix
  - How to model and estimate the instance-dependent label noise (IDN)