COMP 3311
DATABASE MANAGEMENT SYSTEMS

LECTURE 19
QUERY OPTIMIZATION

CATALOG INFORMATION FOR COST ESTIMATION

- The DBMS system catalog (data dictionary) stores, for each relation r, the following statistics:

  n_r the number of tuples in r.

  B_r the number of pages containing tuples of r.

  l_r the size of a tuple of r in bytes.

  bf_r the blocking factor of r (i.e., the number of tuples of r that fit into one disk page).

  $V(A, r)$ the number of distinct values that appear in r for attribute A.
  - Equivalent to $\pi_A(r)$.

☞ If tuples of r are stored together physically in a file, then $B_r = \lceil n_r / bf_r \rceil$.

CATALOG INFORMATION ABOUT INDEXES

- For indexes the system catalog stores the following information.
  - HT_i the number of levels in the index i (i.e., the height of i).
    - For a balanced tree index (such as a B^+-tree) on attribute A of relation r, $HT_i = \lceil \log_{f_i}(V(A, r)) \rceil$.
      - where f_i is the average fan-out of the internal nodes of the index.
      - min $f_i = \lceil n/2 \rceil$ and max $f_i = n$ where n is the maximum fan-out of an index node.
    - For a hash index, HT_i is 1 (or 1.2 if there are overflow buckets).
  - LB_i the number of lowest-level index pages in i (i.e., the number of pages at the leaf level of the index).

STATISTICS ESTIMATION

The output size of an operation determines (i) the cost of the operation and (ii) the cost of subsequent operations.

☞ **Its accurate estimation is important for optimization.**

- Therefore, the statistics should be updated whenever a relation is updated.

  ☞ **Can incur substantial overhead.**

- Statistics are updated only periodically.

  ☞ **They are likely to not be completely accurate.**

$V(A, r)$ - number of distinct values of A in r
f_i - average fan-out of index internal nodes

SELECTION CARDINALITY AND SELECTIVITY

$SC(\theta, r)$, the selection cardinality of predicate θ for relation r, is the *average number of tuples that satisfy the predicate* θ.

Selectivity(θ, r) = $SC(\theta, r)$ / n_r (i.e., the fraction of tuples that satisfy θ)

☞ **Selectivity is a number between 0 and 1.**

☞ **The smaller the selectivity, the fewer tuples are selected.**

$V(A, r)$ - number of distinct values of A in r
f_i - average fan-out of index internal nodes

SIZE ESTIMATION: EQUALITY SELECTION

Equality selection: $\sigma_{A=v}(r)$

$SC(A=v, r) = n_r / V(A, r) \Longrightarrow$ number of tuples / number of distinct values

Selectivity$(A=v, r) = 1 / V(A, r) \Longrightarrow$ 1 / number of distinct values

☞ **Assumes uniform distribution of values of *A* over *r*.**

- $\lceil SC(A=v, r) / bf_r \rceil \Longrightarrow$ the number of pages that these tuples will occupy if they are ordered on attribute A.
- If the tuples are not ordered on A, then each tuple may reside in a different page.

☞ **For equality selection on a key attribute:**

$SC(A=v, r) = 1$ (only 1 tuple will qualify)

Selectivity$(A=v, r) = 1 / n_r$

$V(A, r)$ - number of distinct values of A in r
f_i - average fan-out of index internal nodes

SIZE ESTIMATION: EQUALITY SELECTION EXAMPLE

Given Sailor with n_{Sailor} = 40,000 tuples in B_{Sailor} = 500 pages

```
select sailorId
from Sailor
where rating=8;
```

- If V(rating, Sailor) = 10 distinct ratings, then the condition rating=8 is expected to retrieve SC(rating=8, Sailor) = n_{Sailor}/V(rating, Sailor) = 40,000/10 = 4,000 tuples (i.e., 1/10 of the tuples *assuming uniform distribution of rating values*).

- The estimated cost (I/Os) to execute the query using
  - a clustering index on rating is 1/10*500 = 50 page I/Os.
  - a non-clustering index on rating is 4000 page I/Os since *each tuple* is assumed to require a page I/O.
    - In both cases we do not consider the cost of accessing the index because it is negligible. The index contains only 10 entries since there are only 10 rating values.
  - a file scan is 500 page I/Os.
  - a binary search (provided that the file is sorted on rating) is

binary search: $\lceil \log_2(B_r) \rceil$ + #pages of Sailor that qualify: $\lceil SC(A, r) / bf_r \rceil$ - 1 (1st page) = $\lceil \log_2(500) \rceil + \lceil 4000/80 \rceil - 1 = 9 + 49$
= 58 page I/Os.

n_{Sailor}: 40,000
B_{Sailor}: 500
V(rating, Sailor) = 10

SIZE ESTIMATION: NONEQUALITY SELECTIONS

Range selection: $\sigma_{A \le v}(r)$
(The case for $\sigma_{A \ge v}(r)$ is symmetric.)

- Assume that **min**(A, r) and **max**(A, r) are available in the catalog.

$SC(A \le v, r) = 0$ if $v < \min(A, r)$ (since no tuples will qualify)

$SC(A \le v, r) = n_r$ if $v \ge \max(A, r)$ (since all tuples will qualify)

$SC(A \le v, r) = n_r * \frac{v - \min(A,r)}{\max(A,r) - \min(A,r)}$ (assumes uniform distribution of tuples over the values)

Example:

Suppose min(rating)=1 and max(rating)=10.
For each rating there are 40,000/10=4,000 tuples.
Then SC(rating≤2, Sailor) = n_{Sailor} * (2 - 1) / (10 - 1)
= 40,000 * 1/9 = 4445 tuples (actual value 8000).

```
select sailorId
from Sailor
where rating<=2;
```

☞ **Can get more accurate estimates using histograms.**

HISTOGRAM

- The previous estimates assume that each value of *A* has the same probability (i.e., that the values of *A* are uniformly distributed over the tuples).

  ☞ **The uniformity assumption rarely holds in practice.**

- A histogram assumes local uniformity within each partition (but not global uniformity).

- **Equiwidth** - divide *A* values into subranges of equal size (e.g., equal rating ranges).

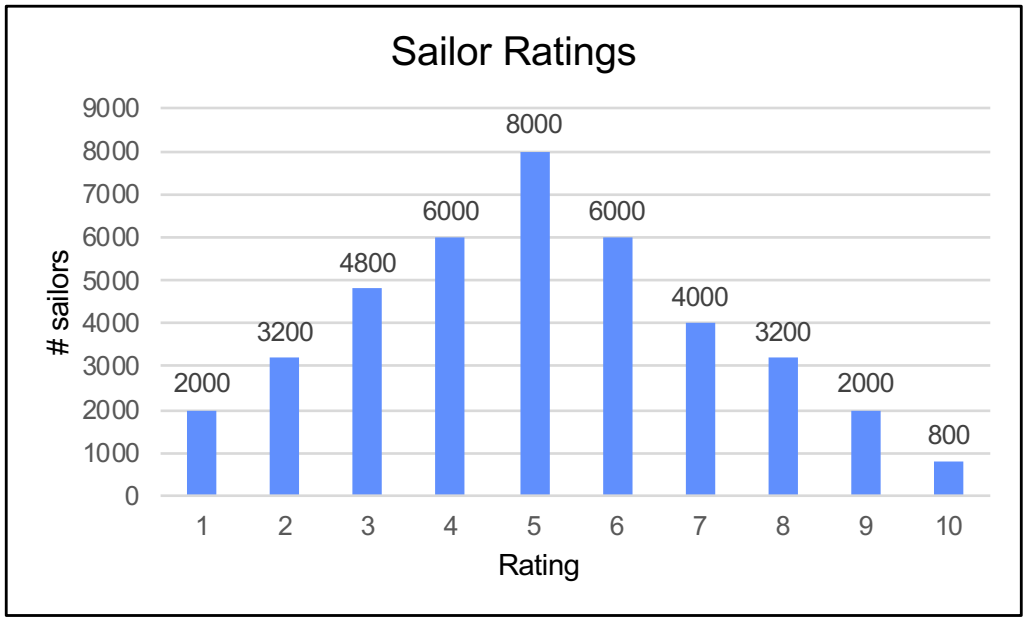
- **Equidepth** - divide *A* values into subranges such that the number of tuples within each subrange is equal.

n_{Sailor}: 40,000
B_{Sailor}: 500
V(rating, Sailor) = 10

HISTOGRAM (CONT'D)

Example: Given the histogram below on sailor ratings, then
SC(rating≤2, Sailor) = 2000 + 3200 = 5200.
(Compared to 4445 under the uniform distribution assumption.)

```
select sailorId
from Sailor
where rating<=2;
```



ATTRIBUTE INDEPENDENCE

- The previous estimates assume that values of attributes are independent.

☞ **The attribute independence assumption rarely holds in practice.**

Example: Consider the Sailor relation and assume that the rating of a sailor increases with her experience. We have histograms on both rating and age. Furthermore, the number of sailors with age 20 and 50 are equal.

- The two queries below are estimated to have the same *SC*.

```
select *
from Sailor
where rating=10 and age=20;
```

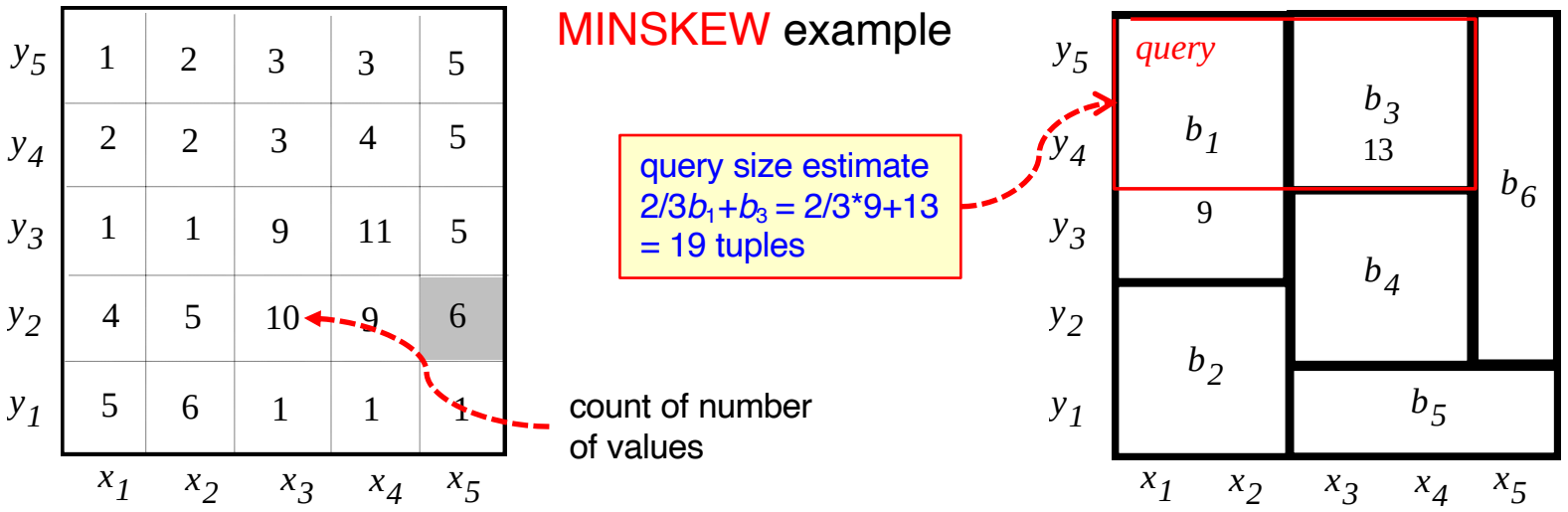
```
select *
from Sailor
where rating=10 and age=50;
```

- Which query is expected to retrieve more tuples?

Solution: Multidimensional histograms

MULTI-DIMENSIONAL HISTOGRAM

- Divide the space (e.g., age-rating) into partitions, so that data values in each partition have an *almost uniform distribution of values*.
- Keep in memory the partition extents (i.e., the ranges of values included in the partition) and the number of tuples per partition.
- Use this information to estimate the number of tuples in the query window.



SIZE ESTIMATION: COMPLEX SELECTIONS

Conjunction: $\sigma_{\theta 1 \wedge \theta 2 \wedge \ldots \wedge \theta n}(r)$

The probability that a tuple satisfies the conjunctive condition is the product of the selectivities of the predicates. If the selectivity of θ_1 is $\frac{s_1}{n_r}$, θ_2 is $\frac{s_2}{n_r}$ … θ_i is $\frac{s_n}{n_r}$, then

$$SC(\sigma_{\theta 1 \wedge \theta 2 \wedge \ldots \wedge \theta n}, r) = n_r * \frac{s_1}{n_r} * \frac{s_2}{n_r} * \cdots * \frac{s_n}{n_r} = n_r * \frac{s_1 * s_2 * \cdots * s_n}{n_r^n}$$

Disjunction: $\sigma_{\theta 1 \vee \theta 2 \vee \ldots \vee \theta n}(r)$

The probability that a tuple satisfies the disjunctive condition is 1 minus the probability that it will satisfy *none* of the conditions. If the selectivity of θ_i is $\frac{s_i}{n_r}$, then $\overline{\theta}_i$ is $(1 - \frac{s_i}{n_r})$ and

$$SC(\sigma_{\theta 1 \vee \theta 2 \vee \ldots \vee \theta n}, r) = n_r * (1 - (1 - \frac{s_1}{n_r}) * (1 - \frac{s_2}{n_r}) * \ldots * (1 - \frac{s_n}{n_r}))$$

based on the logical equivalence: $\theta_1 \vee \theta_2 \vee \cdots \vee \theta_n = \overline{\overline{\theta_1} \wedge \overline{\theta_2} \wedge \cdots \wedge \overline{\theta_n}}$

SIZE ESTIMATION: JOIN

The Cartesian product r x s contains $n_r * n_s$ tuples.
Each tuple occupies $s_r + s_s$ bytes.

- If $r \cap s = \varnothing$
  - $r \bowtie s$ is the same as r x s (no common attributes).
- If $r \cap s = A$ is a key for r
  - Each tuple of s joins with *at most one tuple* from r (since the join attributes are a key of r).

☞ The number of tuples in $r \bowtie s$ is no greater than number of tuples in s.

Example: In Sailor ⋈ Reserves, sailorId in Sailor is a key of Sailor.

☞ The result has exactly $n_{Reserves}$ tuples.

☞ If sailorId in Reserves can be null, then the number of tuples in the result will be less than the number in Reserves.

$V(A, r)$ - number of distinct values of A in r

SIZE ESTIMATION: JOIN (CONT'D)

The Cartesian product $r \times s$ contains $n_r * n_s$ tuples.
Each tuple occupies $s_r + s_s$ bytes.

- If $r \cap s = A$ is a (*not null*) foreign key *in* s referencing r
  - The number of tuples in $r \bowtie s$ is exactly the same as the number of tuples in s (since the join attribute is a foreign key in s of r).

☞ The case where $r \cap s$ is a foreign key *in* r referencing s is symmetric.

Example: Estimate the size result for Sailor ⋈ Reserves where the join attribute, sailorId, in Reserves is a foreign key of Sailor.

Catalog information: $n_{Sailor} = 10{,}000$ $n_{Reserves} = 5{,}000$

- Since sailorId is a foreign key in Reserves, the size of the result is the same as $n_{Reserves} = 5{,}000$.

$V(A, r)$ - number of distinct values of A in r

SIZE ESTIMATION: JOIN (CONT'D)

The Cartesian product r x s contains $n_r * n_s$ tuples.
Each tuple occupies $s_r + s_s$ bytes.

- If $r \cap s = A$ is *not a key* for r or s
  - If we assume that every tuple in r produces $n_s / V(A, s)$ tuples in $r \bowtie s$, then the number of tuples in $r \bowtie s$ is estimated to be:

$$\frac{n_r * n_s}{V(A, s)}$$

  - If instead we assume that every tuple in s produces $n_r / V(A, r)$ tuples in $r \bowtie s$, then the number of tuples in $r \bowtie s$ is estimated to be:

$$\frac{n_r * n_s}{V(A, r)}$$

  - The lower of these two estimates is probably the more accurate one.

$V(A, r)$ - number of distinct values of A in r

SIZE ESTIMATION: JOIN (CONT'D)

Example: Estimate the result size of Sailor ⋈ Reserves *without using information about foreign keys*.

Catalog information:

n_{Sailor} = 10,000 V(sailorId, Sailor) = 10,000 V(sailorId, Reserves) = 2,500

$n_{Reserves}$ = 5,000 ⟹ only 2,500 sailors have reservations

- The two estimates are:

$$\frac{n_{Sailor} * n_{Reserves}}{V(sailorId, Sailor)} = 10{,}000 * 5{,}000 / 10{,}000 = \underline{5{,}000}$$

$$\frac{n_{Sailor} * n_{Reserves}}{V(sailorId, Reserves)} = 10{,}000 * 5{,}000 / 2{,}500 = \underline{20{,}000}$$

- We choose the lower estimate, which, in this case, is the same as our earlier computation using foreign keys.

SIZE ESTIMATION: OTHER OPERATIONS

Projection: The estimated size of $\pi_A(r) = V(A, r)$

Aggregation: The estimated size of group-by $A = V(A, r)$

Set operations

- For union / intersection of selections on the same relation, rewrite the query and use size estimate for selections.
  - E.g. $\sigma_{\theta 1}(r) \cup \sigma_{\theta 2}(r)$ can be rewritten as $\sigma_{\theta 1 \vee \theta 2}(r)$.
- For operations on different relations:
  - Estimated size of $r \cup s$ = size of r + size of s.
  - Estimated size of $r \cap s$ = minimum (size of r , size of s).
  - estimated size of $r - s$ = r.

☞ **All three estimates may be quite inaccurate, but provide upper bounds on the sizes.**

SIZE ESTIMATION: NUMBER OF DISTINCT VALUES

Selection: $\sigma_\theta(r)$

- If the selection condition θ is on attribute A, estimate as $V(A, \sigma_\theta(r)) = V(A, r) * s$ where s is the selection *selectivity*.

  Example: $V(\text{rating}, \sigma_{\text{rating}=2}(\text{Sailor})) = 10/10 = 1$ (where s is 1/10).

- In all other cases (e.g., selection on another attribute) use the rough estimate $\min(V(A, r), n_{\sigma\theta(r)})$.

  Example: $V(\text{age}, \sigma_{\text{rating}=8}(\text{Sailor}))$ =
  min(# different age values, # tuples in $\sigma_{\text{rating}=8}(\text{Sailor})$.

- Join $r \bowtie s$ estimate as $V(A, r \bowtie s) = \min(V(A, r), n_{r \bowtie s}))$.

  Example: $V(\text{sName}, \text{Sailor} \bowtie \text{Reserves})$ =
  min(V(sName, Sailor), output size of join).

$V(A, r)$ - number of distinct values of A in r

SIZE ESTIMATION: NUMBER OF DISTINCT VALUES (CONT'D)

Projection: $\pi_A(r)$

- The number of distinct values is the same as $V(A, r)$. The same holds for the grouping attributes of an aggregation.

Aggregated values

- For **min**(A), **max**(A), the number of distinct values can be estimated as **min**($V(A, r)$, $V(G, r)$) where G denotes the grouping attributes.

Example:

```
select max(age)
from Sailor
group by rating;
```

V(max(age), group by rating) = min(#different age values, #different rating values) = #different rating values (i.e., for each rating we will get a different max(age) value).

- For other aggregates, assume all values are distinct and use $V(G, r)$.

Example:

```
select avg(age)
from Sailor
group by rating;
```

V(avg(age), group by rating) = #different rating values.

QUERY OPTIMIZATION: SUMMARY

- Since disk accesses slow query processing, it is worthwhile to allocate a considerable amount of processing to choose an execution strategy that will minimize disk accesses.

- The first step is to transform the user query into equivalent ones using relational algebra equivalence rules.

- Next, use statistics about relations to estimate the cost of each execution plan.

  ☞ **Histograms provide more accurate estimates.**

- Finally, use cost estimates and heuristics to choose an execution plan.