SCHOOL OF MATH DEPARTMENT

HodgeRank on Ranking Human Age

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ABSTRACT

We use hodgerank method ranking the age of 30 people. We both gain great results when using two different statistical model. Then we employ hodge decomposition theorem to analyze the inconsistency of our graph. We get interesting result that the local inconsistency in our ranking result is really related with the local inconsistency analyzed in curl flow part of hodge deomposition.

1 INTRODUCTION

Human ageing is a complicated problem of human physiology that has been studied in various disciplines such as computer vision , image processing and so on. The task of estimating the age of human age from a facial image is a challenging since several factors that influence it. As complicated as human ageing is, humans seem to be able to predict the ages of other humans relatively accurately.

data introduction The data we used in this paper contains three parts. First part are 30 images from human age dataset . The second part is the groundtruth age of each individual. The first column of this data is the individual id, and the second column is the groundtruth age of each individual. The third and most important data is the pairwise comparison data of human age in those 30 images. The annotator is presented with two images and given his choice of which one is older (or difficult to judge). Totally, the data obtain 14,011 pairwise comparisons from 94 annotators. There are four columns. The first column is annotator id, the second and third columns are individual id, the last column is shows the annotator's choice :

1 indicates the second column is older than the third one,

-1 indicateds the second column is younger than the third one, 0 indicates the second and third are difficult to judge.

Paired compariosn refers to any process comparing those image of different human in pairs by raters to judge which human is more younger. Different people maybe have different judgement about the age of these image. The method of paired comparison has been widely used in various fields.

In this report we use hodgerank decomposition of the paired comparison dataset to make the ranking and analyze the inconsistency. Firstly we give the basic model on how to rank the age based on the paired comparison data. Then we present our ranking result of two different satistical model. Compared the ranking result with the real age of each human, we find both two models perform well. More precisely, we have global consistency and some local inconsistency. Then we introduce the Hodge decomposition theorem, based on which we do some analysis on the inconsistency of our graph and ranking result. We get interesting result that the local inconsistency showed in our ranking result really related with the loacl inconsistency of decomposition of graph.

The report is organized as followed. In section 2, we give the basic model and section 3 gives the ranking result. Then we make some evaluation of the inconsistency in section4 and finally gives summary in section 5.

2 HodgeRank on Graph

Let $\Lambda = \{1, \ldots, 94\}$ be a set of annotators and $V = \{1, \ldots, 30\}$ be the set of human's image to be ranked.Paired comparison data is as a function on $\Lambda \times V \times V$, which is skew-symmetric for each annotators α, i.e., $Y^{\alpha}_{ij} = -Y^{\alpha}_{ji}$ representing the degree that α compare the individual i and individual j. The simplest setting is the binary choice, where

$$Y^{\alpha}_{ij} = \begin{cases} 1 & \text{if } \alpha \text{ provides individual i is older than individual j} \\ -1 & \text{otherwise} \end{cases}$$

A nonnegative weight fuction $\omega : \Lambda \times V \times V \to [0, \infty)$ is defined as,

$$\omega^{\alpha}_{ij} = \begin{cases} 1 & \text{if } \alpha \text{ makes a comparison for \{ i,j\}j} \\ 0 & \text{otherwise} \end{cases}$$

Our statistical rank aggregation problem is to look for some global ranking score $s : V \to R$

such that

$$\min_{s\in R^{\|V\|}} \sum_{i,j,\alpha} \omega_{ij}^{\alpha}(s_i - s_j - Y_{ij}^{\alpha})^2$$

which is equivalent to the following weigthed least square problem

$$\min_{s\in R^{\|V\|}} \sum_{i,j} \omega_{ij}^{\alpha}(s_i - s_j - \hat{Y}_{ij})^2$$

where $\hat{Y}_{ij} = (\sum_\alpha \omega_{ij}^{\alpha} Y_{ij}^{\alpha})/(\sum_\alpha \omega_{ij}^{\alpha})$ and $\omega_{ij} = \sum_\alpha \omega_{ij}^{\alpha})$.

A graph structure arises naturally from ranking data as follows. Let $G = (V, E)$ be a paired ranking graph whose vertex set is V , the set of videos to be ranked, and whose edge set is E, the set of video pairs which receive some comparisons, i.e.,

$$E = \left\{\{i, j\} \in \binom{V}{2} | \sum_\alpha \omega_{ij}^{\alpha} > 0\right\}$$

A pairwise ranking is called complete if each annotators α in Λ gives a total judgment of all images in V ; otherwise it is called incomplete. It is called balanced if the paired comparison graph is k-regular with equal weights $\omega_{ij} = \sum_\alpha \omega_{ij}^{\alpha}) \equiv c$ for all $\{i, j\} \in E$; otherwise it is called imbalanced. A complete and balanced ranking induces a complete graph with equal weights on all edges. The existing paired comparison methods in VQA often assume complete and balanced data.However this is an unrealistic assumption for real data.The HodgeRank approach adopted in this paper enables us a unified scheme which can deal with incomplete and imbalanced data emerged from random sampling in paired comparisons.

The minimization of the least square problem can be generalized to a family of linear models in paired comparison methods. To solve this problem , we firstly rewrite the least square problem as a much simpler form. We assume that for each image pair i,j, the number of comparisons is n_{ij}, among which a_{ij} annotators think individual i is younger than individual j(a_{ji}has the opposite meanings).Hence for each pair of those images, we have probability estimated by $\hat{\pi}_{ij} = a_{ij}/n_{ij}$. Accoding to this definition, the least square problem can be rewrited as

$$\min_{s\in R^{\|V\|}} \sum_{i,j} n_{ij}(s_i - s_j - (2\hat{\pi}_{ij} - 1))^2$$

since $\hat{Y}_{ij} = (a_{ij} - a_{ji})/n_{ij} = 2\hat{\pi}_{ij} - 1$.

General linear models assume that the true preference probability can be fully decided by a linear scaling function

$$\pi_{ij} = Prob\{i\ is\ younger\ than j\} = F(s_i^* - s_j^*)$$

for some $s^* \in R^{|V|}$. Here the function F can be chosen as any symmetric cumulated distribution function. When we only have an empirical preference probability $\hat{\pi}_{ij}$, we can map it to a skew-symmetric function by the inverse of F

$$\hat{Y}_{ij} = F^{-1}(\hat{\pi}_{ij})$$

where $\hat{Y}_{ij} = -\hat{Y}_{ji}$.However , in this case one can only expect that

$$\hat{Y}_{ij} = s_i^* - s_j^* + \varepsilon_{ij}$$

where ε_{ij} denote the noise. The general and simple case takes a linear F and is oftern called a uniform model. And also the function F has many different types as follows.

1.Uniform Model :

$$\hat{Y}_{ij} = 2\hat{\pi}_{ij} - 1$$

2.Bradley-Terry Model:

$$\hat{Y}_{ij} = \log \frac{\hat{\pi}_{ij}}{1 - \hat{\pi}_{ij}}$$

3 Ranking result

Based on the minimization problem and some statistical model intrduced above, we get the ranking result of thirty humans' age.

3.1 uniformly model

By applying the uniformly model we get the followed result.

ranking data	2.1599	1.9753	1.9509	1.8872	1.8224	1.3593	1.3560	1.3538
human number	26	10	19	18	1	9	25	16
real age	2	7	5	11	10	22	19	17

Table 3.1: uniformly model ranking result 1

ranking data	1.2874	1.2454	1.2422	1.1072	1.0480	1.0340	0.9672	0.9467
human number	15	2	13	30	23	12	20	8
real age	15	15	18	23	25	32	36	18

Table 3.2: uniformly model ranking result 2

ranking data	0.9404	0.9078	0.8874	0.8373	0.8083	0.7595	0.6969	0.6793
human number	22	24	17	6	4	27	21	29
real age	30	18	32	24	26	30	20	33

Table 3.3: uniformly model ranking result 3

ranking data	0.6547	0.6330	0.6200	0.5623	0.5054	0.4754
human number	5	11	7	14	3	28
real age	31	40	39	36	51	46

Table 3.4: uniformly model ranking result 4

Explicitly analyzing the result, we find that ranking of the age is almost consist with the real age. Especially we can see that the ages in each table are almost increaing which indicate local consisting of the result to some degree. Besides, we can see that when two human's age are in great difference, then they will difinitely be ranked correctly. It indicates the global consistency of our ranking result.

ranking data	9.8069	9.3920	7.2574	6.2872	6.1843	2.2687	2.1662	2.0041
human number	26	19	18	10	1	9	25	16
real age	2	5	11	7	10	22	19	17

Table 3.5: Bradley-Terry modelranking result 1

ranking data	1.8714	1.7284	1.7023	1.2948	1.1675	1.0004	0.9351	0.8820
human number	15	2	13	30	23	12	20	24
real age	15	15	18	23	25	32	36	18

Table 3.6: Bradley-Terry model ranking result 2

ranking data	0.8492	0.7516	0.7330	0.6363	0.5437	0.4290	0.3847	0.3251
human number	22	17	8	6	4	27	21	5
real age	30	32	18	24	26	30	20	31

Table 3.7: Bradley-Terry model ranking result 3

ranking data	0.3170	0.2928	0.2295	0.2174	0.1306	0.1083
human number	29	11	7	14	3	28
real age	33	40	39	36	51	46

Table 3.8: Bradley-Terry model ranking result 4

As we can see directly the result is similar with the uniform model. They both have global consistency but may not have local consistency. This will be discussed in our final section, the evaluation of the local inconsistency.

4 Evaluation of the inconsistency

4.1 Hodge Decomposition

Let $\hat{Y}_{ij}$ be a paired comparison flow on graph $G = (V, E)$, $\hat{Y}_{ij} = -\hat{Y}_{ji}$ for$\{i, j\} \in E$,and $\hat{Y}_{ij} =$ 0otherwise. There is a unique decomposition of $\hat{Y}$satisfying

$$\hat{Y} = \hat{Y}^g + \hat{Y}^h + \hat{Y}^c$$

where

$$\hat{Y}^g_{ij} = \hat{s}_i - \hat{s}_j,\ for\ some\ \hat{s} \in R^V$$

$$\hat{Y}^h_{ij} + \hat{Y}^h_{jk} + \hat{Y}^h_{ki} = 0,\ for\ each\ \{i, j, k\} \in T$$

$$\sum_{j \sim i} \omega_{ij} \hat{Y}^h_{ij} = 0,\ for\ each\ i \in V$$

The decomposition above is orthogonal under the following inner product on $R^{|E|}$,$\langle u, v\rangle_\omega = \sum_{\{i,j\} \in E} \omega_{ij} u_{ij} v_{ij}$.

The following provides some remarks on the decomposition based on which we will give some method to evaluate the inconsistency of the graph. Of course this evaluation will finally give information of the inconsistency of our final ranking result.

- $\hat{Y}^g$ is a rank two skew-symmetric matrix and gives a linear score function $\hat{s} \in R^V$ up to translations. This score function gives us the ranking result.
- $\hat{Y} - \hat{Y}^g = \hat{Y}^h + \hat{Y}^c$ which indicate the total inconsistency of the graph. This will be used to estimate our total inconsistency in next susection.
- The residue $\hat{Y}^c$ measures the amount of intrinsic (local) inconsistancy in characterized by the triangular trace. So the formula

  $$\frac{||\hat{Y}^c||^2_\omega}{||\hat{Y}||^2_\omega}$$

  can give us information of the local inconsistency. Besides, the following relative curl,

  $$\text{curl}^r_{ijk} = \frac{|\hat{Y}^c_{ij} + \hat{Y}^c_{jk} + \hat{Y}^c_{ki}|}{|\hat{Y}_{ij}| + |\hat{Y}_{jk}| + |\hat{Y}_{ki}|} = \frac{|\hat{Y}_{ij} + \hat{Y}_{jk} + \hat{Y}_{ki}|}{|\hat{Y}_{ij}| + |\hat{Y}_{jk}| + |\hat{Y}_{ki}|}$$

  can be used to characterize triangular intransitivity; $\text{curl}^r_{ijk} = 1$ iff i,j,k contains an intransitive triangle of $\hat{Y}$.

4.2 Evaluation of the inconsistency

In this section, we make some evalution of the inconsistency of the ranking result based on the above subsection.

4.2.1 Total inconsistency

Firstly, we derive the total inconsistency evalution formula.

$$\text{Inc.Total}(\hat{Y}) = \frac{||\hat{Y} - \hat{Y}^g||_\omega^2}{||\hat{Y}||_\omega^2} = \frac{\sum_{ij} \omega_{ij} (\hat{s}_i - \hat{s}_j - \hat{Y}_{ij})^2}{\sum_{ij} \omega_{ij} \hat{Y}_{ij}^2}$$

Besides, we will see later that the total inconsistency is actually the local inconsistency which we will discuss in next subsection.

4.2.2 Local inconsistency

Cause the graph is complete, we can see from the reference[Statistical ranking and combinatorial hodge theory] that there is no global inconsistency. So we only need to consider the local inconsistency. Which characterized by the following formula in our problem.

$$\text{Inc.Curl}(\hat{Y}) = \frac{||\hat{Y}^c||_\omega^2}{||\hat{Y}||_\omega^2} = \frac{||\hat{Y} - \hat{Y}^g||_\omega^2}{||\hat{Y}||_\omega^2}$$

Then we give the measure of local inconsistency of our ranking result. For the uniform model, $\text{Inc.Curl}(\hat{Y}) = 0.4368$ and for the Bradley-Terry model $\text{Inc.Curl}(\hat{Y}) = 0.4740$. They indicate that there may be many local inconsistency in the ranking result. Actually, when we compare the tables in section 3, we will see in table 3.2 and 3.3, there are some local misarrangement of the human number compared with their age. But we can see the global arrangement is correct. And the unform model performs better than Bradley-Terry model which we can see from the table.

4.2.3 Transitivity measurement

As we have mentioned,

$$\text{curl}_{ijk}^r = \frac{|\hat{Y}_{ij}^c + \hat{Y}_{jk}^c + \hat{Y}_{ki}^c|}{|\hat{Y}_{ij}| + |\hat{Y}_{jk}| + |\hat{Y}_{ki}|} = \frac{|\hat{Y}_{ij} + \hat{Y}_{jk} + \hat{Y}_{ki}|}{|\hat{Y}_{ij}| + |\hat{Y}_{jk}| + |\hat{Y}_{ki}|}$$

can be used to characterize trangular intransitivity. By this method, we can find which trangular circle have great inconsistency. Firstly we find if there is any 3-cliques have $\text{curl}_{ijk}^r = 1$. As we have mentioned before, this means the triangle is intransitive. And there are 45 3-cliques that is intransitive compared with total 4060 3-cliques. Compare with these 3-cliques we get an interesting and exciting result. For example, the first 3-clique that have $\text{curl}_{ijk}^r = 1$ is 2,9,15. Compared it with the table in section three

ranking data	1.3593	1.2874	1.2454
human number	9	15	2
real age	22	15	15

Table 4.1: 3-clique2,9,15

We can directly see that the ranking order is not consist. Obviously number 9 human is older than the other two humans. This example verify that the curl folw can really indicate the local inconsistency of the ranking result.

5 SUMMARY

We use Hodgerank method to rank the age of 30 people. Our ranking result performs well. Then we use hodge decomposition to analyze the inconsistency. We find that the local inconsistency in our ranking result is really related with the local inconsistency showed in one part of our hodge decomposition. That is really interesting and shows hodgerank is useful.

Appendices

```
install("R.matlab")
library("R.matlab")
path<-('/Users/xuxiangcan1/Downloads/age')
##pathname<-file.path(path,s.mat)
pathname<-file.path(path,'Agedata.mat')
testdata<-readMat(pathname)
print(testdata)
dataage=testdata$Pair.Compar
unique(dataage[,1])
c=dataage[which(l==comb1[1]),]


l=complex(real=dataage[,2]+dataage[,3],imaginary=dataage[,2]*dataage[,3]);
comb1=complex(real=b[,1]+b[,2],imaginary=b[,1]*b[,2])
result1=matrix(0,nrow=435,ncol=2);

for (i in 1:435){
subset1=dataage[which(l==comb1[i]),];
result1[i,1]=nrow(subset1);
# subset2=subset1[which((subset1[,2]-subset1[,3])*subset1[,4]>0),];
# result1[i,2]=nrow(subset2);
result1[i,2]= length(which((subset1[,2]-subset1[,3])*subset1[,4]>0));
}
```

```
result2=cbind(b,result1);
pi_ij= result1[,2]/result1[,1];
y_ij_1=2*pi_ij-1;
y_ij_2=log(pi_ij/(1-pi_ij));

datareuslt=data.frame(i=result2[,1],j=result2[,2],n_ij=result2[,3],y_ij_1=y_ij_1,y_ij_2=y_
index=datareuslt[,1:2];
result3=matrix(0,nrow=30,ncol=30);
for (i in 1:435){
result3[index[i,1],index[i,2]]=y_ij_1[i];
result3[index[i,2],index[i,1]]=-y_ij_1[i];
}

result4=matrix(0,nrow=30,ncol=30);
for (i in 1:435){
result4[index[i,1],index[i,2]]=y_ij_2[i];
result4[index[i,2],index[i,1]]=-y_ij_2[i];
}

result5=matrix(0,nrow=30,ncol=30);
for (i in 1:435){
result5[index[i,1],index[i,2]]=result2[i,3];
result5[index[i,2],index[i,1]]=result2[i,3];
}

write.csv(result5,file="resultn_ij.csv")
write.csv(result4,file="resulty_ij2.csv")
write.csv(result3,file="resulty_ij1.csv")



clear all
Y=csvread('datay2.csv');
GG=csvread('datan.csv');


% Compute the 0-order coboundary operator dd0
dd0 = [];

% Compute the 1-dimensional simplices, i.e. edges

ss1 = [];
for i=1:length(GG),
```

```
for j=(i+1):length(GG),
ss1 = [ss1; [i,j]];
dd_row = zeros(1,length(GG));
dd_row(i) = 1;
dd_row(j) = -1;
dd0 = [dd0; dd_row];
end
end
ss1 = ss1'; % edge matrix of 2-by-(# edges), each column is a node pair

dd1 = [];
ss2 = [];
for i=1:length(GG),
for j=(i+1):length(GG),
for k=(j+1):length(GG),
if ((GG(i,j)>0)&(GG(j,k)>0)&(GG(k,i)>0)),
ss2 = [ss2; [i,j,k]];
dd_row = zeros(1,size(dd0,1));
dd_row(find(ss1(1,:)==i&ss1(2,:)==j))=1;
dd_row(find(ss1(1,:)==j&ss1(2,:)==k))=1;
dd_row(find(ss1(1,:)==i&ss1(2,:)==k))=-1;
dd1 = [dd1; dd_row];
end
end
end
end

ss2 = ss2';

dd2 = [];
ss3 = [];
for i=1:length(GG),
for j=(i+1):length(GG),
for k=(j+1):length(GG),
if ((GG(i,j)>0)&(GG(j,k)>0)&(GG(k,i)>0)),
ss3 = [ss3; [i,j,k]];
dd_row = zeros(1,size(dd0,1));
dd_row(find(ss1(1,:)==i&ss1(2,:)==j))=1;
dd_row(find(ss1(1,:)==j&ss1(2,:)==k))=1;
dd_row(find(ss1(1,:)==i&ss1(2,:)==k))=1;
dd2 = [dd2; dd_row];
end
end
end
```

```
end


ss3 = ss3';
w=squareform(GG);
y=squareform(Y)';

Q=diag(w);
L0 =dd0'*Q*dd0;


div =dd0'*Q*y;

x= lsqr(L0,-div);
B=exp(x);
yg=dd0*x;
r=norm(Q*(y-yg))/norm(Q*y);
L1=Q*dd1';
curl=Q*y;
z=lsqr(L1,curl);
yc=dd1'*z;

yh=y-yg-yc;

curl=abs(dd1*y);
absy=abs(y);
abscurl=dd2*absy;
ratio=curl./abscurl;

[a]=find(ratio==1)
```