RSA

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The RSA problem

- Here: group $\mathbb{Z}^*_N$ of order $\phi(N)$
- Choose e with $\gcd(e, \phi(N)) = 1$
  - Raising to the e-th power is a permutation on $\mathbb{Z}^*_N$!
- If $ed = 1 \bmod \phi(N)$, raising to the d-th power is the *inverse* of raising to the e-th power
  - i.e., $(x^e)^d = x \bmod N$, $(x^d)^e = x \bmod N$
  - x^d is the *e-th root of x modulo N*

The RSA problem

- If p, q are known:
  - $\Rightarrow \phi(N)$ can be computed
  - $\Rightarrow d = e^{-1} \bmod \phi(N)$ can be computed
  - $\Rightarrow$ possible to compute e-th roots modulo N
- If p, q are *not* known:
  - $\Rightarrow$ computing $\phi(N)$ is as hard as factoring N
  - $\Rightarrow$ co

Very useful for public-key cryptography!

The RSA assumption (informal)

- Informally: given N and e, hard to compute the e-th root of a uniform element $y \in \mathbb{Z}^*_N$

Implementing RSA

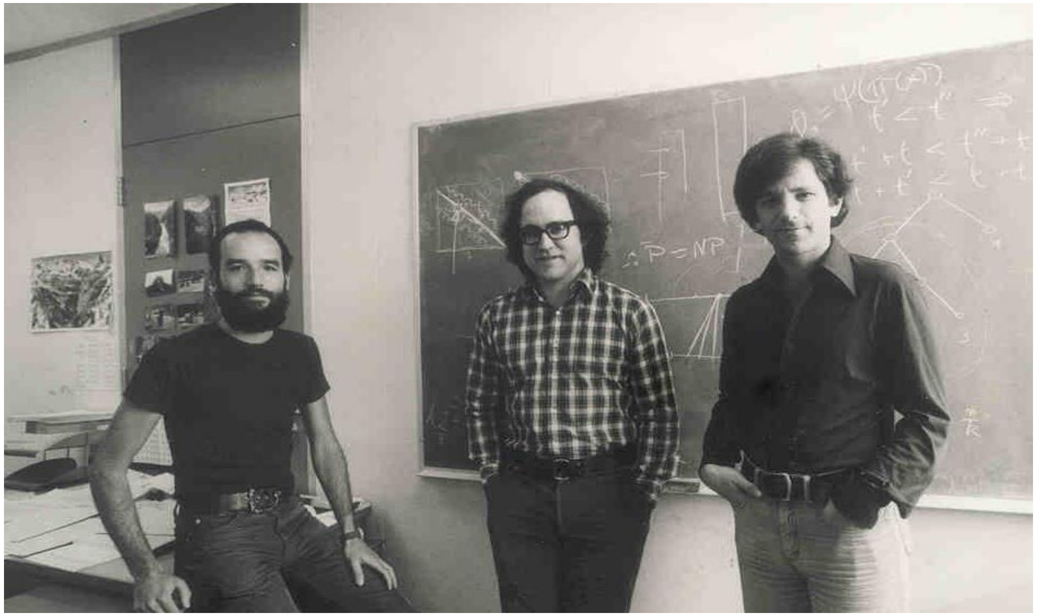
- One way to implement RSA:
  - Generate uniform n-bit primes p, q
  - Set $N := pq$
  - Choose arbitrary e with $\gcd(e, \phi(N))=1$
  - Compute $d := [e^{-1} \bmod \phi(N)]$
  - Output (N, e, d)

Implementing RSA

- Choice of e?
  - Does not seem to affect hardness of the RSA problem
  - e = 3, $2^{16} + 1$ (1 0000 0000 0000 0001; i.e., efficient for square and multiply; prime number) for efficient exponentiation

RSA public key crypto

- Proposed in 1977 by Ron Rivest, Adi Shamir and Leonard Adleman at MIT
- It is believed to be secure and still widely used



Shamir *Rivest* *Adleman*

(Plain) RSA crypto system

- Private key is two large primes p, q or d
- Public key is $n = pq$ and public exponent e
  - e is a relatively prime to $\phi(N)$ (= $(p-1)(q-1)$)
  - find d where $de = 1 \pmod{\phi(N)}$
- Encryption: $c = m^e \pmod{n}$
- Decryption: $m = c^d \pmod{n}$
  - $m^{ed} = m^{(1+k(p-1)(q-1)) \bmod (p-1)(q-1)} = m$

Simple RSA example (1)

- Select large(?) primes $p = 11, q = 3$
- Then $N = pq = 33$ and $(p-1)(q-1) = 20$
- Choose $e = 3$ (relatively prime to 20)
- Find d such that $ed = 1 \bmod 20$
  - We find that $d = 7$ works
- **Public key:** $(N, e) = (33, 3)$
- **Private key:** $d = 7$

Simple RSA example (2)

- **Public key:** $(N, e) = (33, 3)$
- **Private key:** $d = 7$
- Suppose message $M = 8$
- Ciphertext C is computed as

$$C = M^e \bmod N = 8^3 = 512 = 17 \bmod 33$$

- Decrypt C to recover the message M by

$$\begin{aligned} M = C^d \bmod N &= 17^7 = 410{,}338{,}673 \\ &= 12{,}434{,}505 * 33 + 8 = 8 \bmod 33 \end{aligned}$$

Security of (plain) RSA crypto

- This scheme is deterministic
  - Cannot be CPA-secure! (i.e., CPA-secure crypto must be randomized)
- RSA assumption only refers to hardness of computing the e^{th} roots of uniform C
  - C is not uniform unless M is
  - Partial information about the e^{th} root may be leaked

Plain RSA should never be used!

Questions?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