Database Systems
Lecture17 – Chapter 16: Query Optimization

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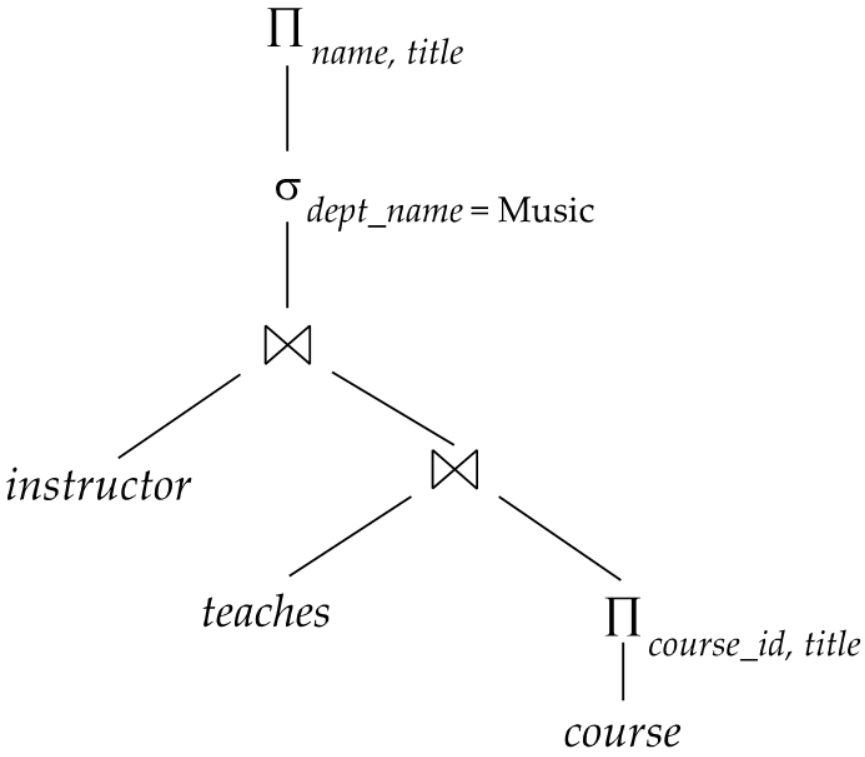
Outline

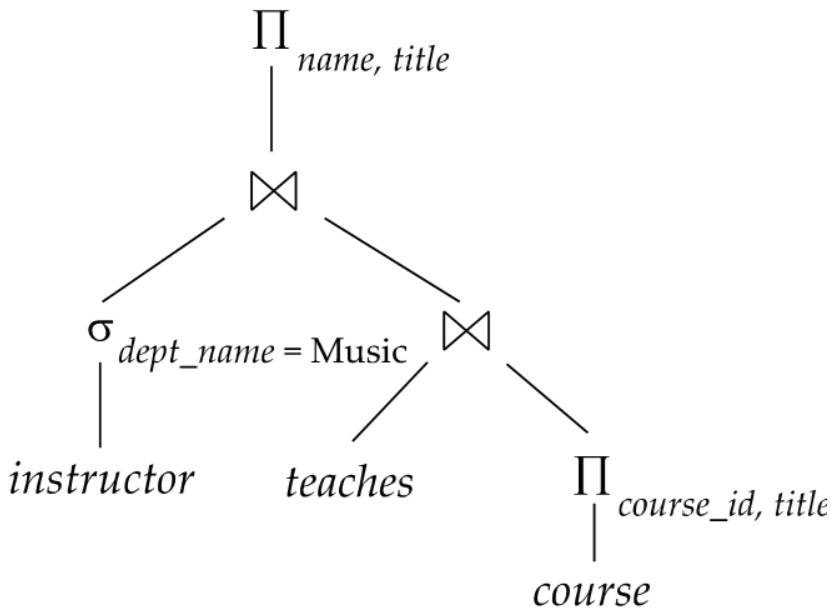
- Introduction
- Transformation of Relational Expressions
- Catalog Information for Cost Estimation
- Statistical Information for Cost Estimation
- Cost-based optimization
- Dynamic Programming for Choosing Evaluation Plans

Introduction

- Alternative ways of evaluating a given query
  - Equivalent expressions
  - Different algorithms for each operation



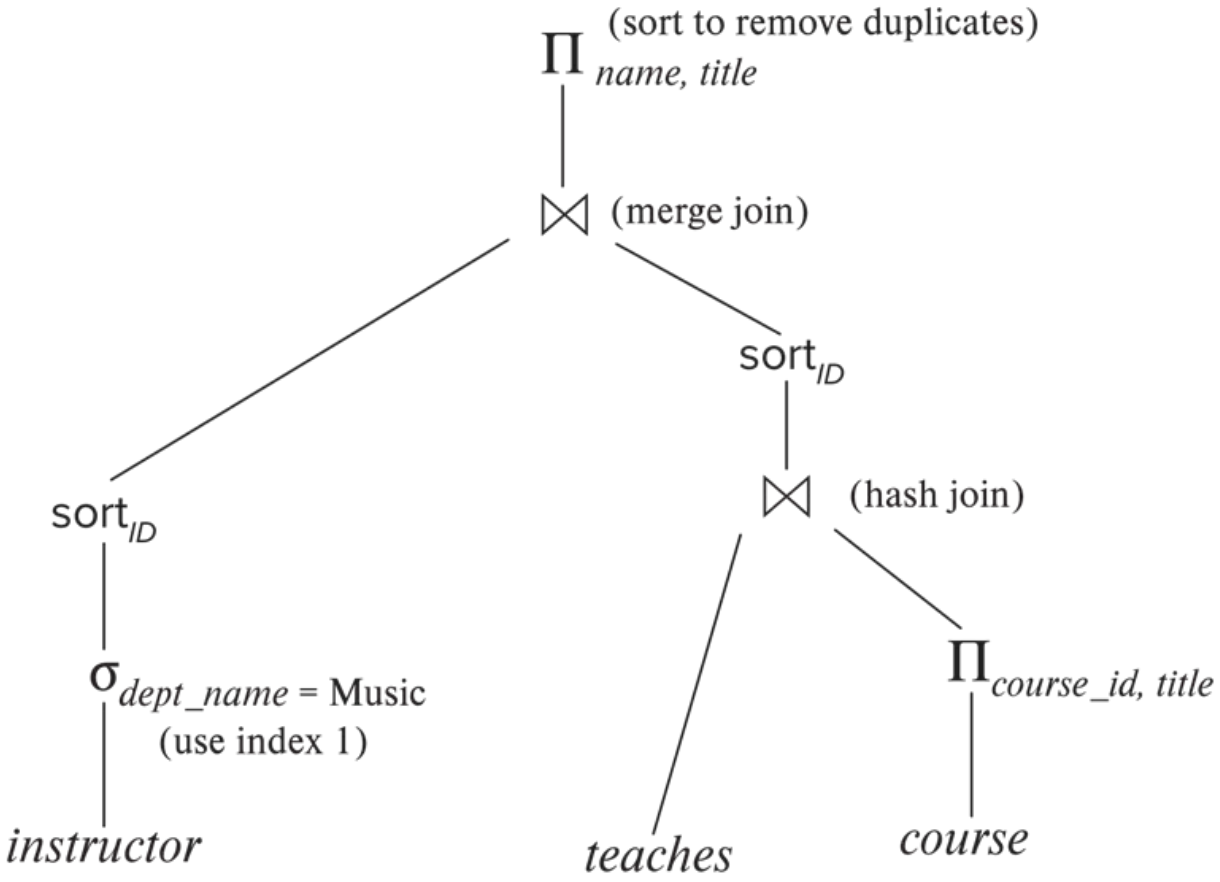
(a) Initial expression tree



(b) Transformed expression tree

Introduction (Cont.)

- An **evaluation plan** defines exactly what algorithm is used for each operation, and how the execution of the operations is coordinated.



Introduction (Cont.)

- Cost difference between evaluation plans can be enormous

- Steps in **cost-based query optimization**
  1. Generate logically equivalent expressions using **equivalence rules**
  2. Annotate resultant expressions to get alternative query plans
  3. Choose the cheapest plan based on **estimated cost**

- Estimation of plan cost based on:
  - Statistical information about relations.
    Examples:
    - number of tuples, number of distinct values for an attribute
  - Statistics estimation for intermediate results
    - to compute cost of complex expressions
  - Cost formulae for algorithms, computed using statistics

Viewing Query Evaluation Plans

- Most database support **explain** <query>
  - Displays plan chosen by query optimizer, along with cost estimates
  - Some syntax variations between databases
    - Oracle: **explain plan for** <query>
      e.g., **explain plan for select** * **from** table
    - SQL Server: **set showplan_text on**
    - MySQL: **explain select * from** table
  - Some databases (e.g. PostgreSQL) support
    explain analyse <query>
    - Shows actual runtime statistics running the query, in addition to showing the plan
- Some databases (e.g. PostgreSQL) show cost as *f..l*
  - *f* is the cost of delivering first tuple and *l* is cost of delivering all results

Generating Equivalent Expressions

Transformation of Relational Expressions

- Two relational algebra expressions are said to be **equivalent** if the two expressions generate the same set of tuples
  - Note: order of tuples is irrelevant

- **equivalence rule** →
  - replace expression of first form by second, or vice versa

Equivalence Rules

1. Conjunctive selection operations can be deconstructed into a sequence of individual selections.

$$\sigma_{\theta_1 \wedge \theta_2}(E) \equiv \sigma_{\theta_1}(\sigma_{\theta_2}(E))$$

2. Selection operations are commutative.

$$\sigma_{\theta_1}(\sigma_{\theta_2}(E)) \equiv \sigma_{\theta_2}(\sigma_{\theta_1}(E))$$

3. Only the last in a sequence of projection operations is needed, the others can be omitted.

$$\Pi_{L_1}(\Pi_{L_2}(\ldots(\Pi_{L_n}(E))\ldots)) \equiv \Pi_{L_1}(E)$$

where $L_1 \subseteq L_2 \ldots \subseteq L_n$

4. Selections can be combined with Cartesian products and theta joins.

   a. $\sigma_{\theta}(E_1 \times E_2) \equiv E_1 \bowtie_{\theta} E_2$

   b. $\sigma_{\theta_1}(E_1 \bowtie_{\theta_2} E_2) \equiv E_1 \bowtie_{\theta_1 \wedge \theta_2} E_2$

Equivalence Rules (Cont.)

5. Theta-join operations (and natural joins) are commutative.

$$E_1 \bowtie E_2 \equiv E_2 \bowtie E_1$$

6. (a) Natural join operations are associative:

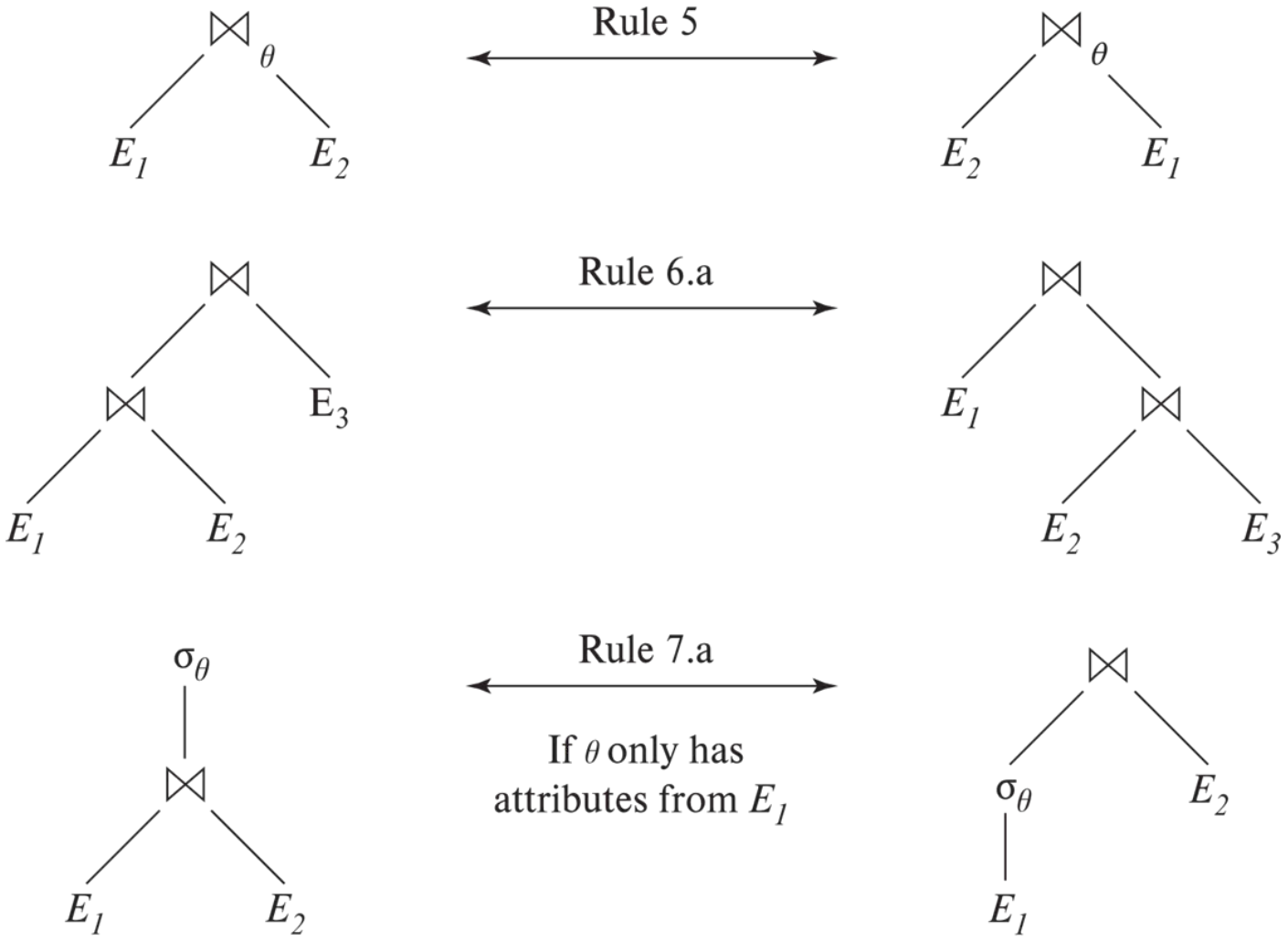
$$(E_1 \bowtie E_2) \bowtie E_3 \equiv E_1 \bowtie (E_2 \bowtie E_3)$$

(b) Theta joins are associative in the following manner:

$$(E_1 \bowtie_{\theta_1} E_2) \bowtie_{\theta_2 \wedge \theta_3} E_3 \equiv E_1 \bowtie_{\theta_1 \wedge \theta_3} (E_2 \bowtie_{\theta_2} E_3)$$

where θ_2 involves attributes from only E_2 and E_3.

Pictorial Depiction of Equivalence Rules



Equivalence Rules (Cont.)

7. The selection operation distributes over the theta join operation under the following two conditions:
   (a) When all the attributes in θ_0 involve only the attributes of one of the expressions (E_1) being joined.

$$\sigma_{\theta_0}(E_1 \bowtie_{\theta} E_2) \equiv (\sigma_{\theta_0}(E_1)) \bowtie_{\theta} E_2$$

   (b) When θ_1 involves only the attributes of E_1 and θ_2 involves only the attributes of E_2.

$$\sigma_{\theta_1 \wedge \theta_2}(E_1 \bowtie_{\theta} E_2) \equiv (\sigma_{\theta_1}(E_1)) \bowtie_{\theta} (\sigma_{\theta_2}(E_2))$$

Equivalence Rules (Cont.)

8. The projection operation distributes over the theta join operation as follows:

   (a) if θ involves only attributes from $L_1 \cup L_2$:

   $$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) \equiv \Pi_{L_1}(E_1) \bowtie_\theta \Pi_{L_2}(E_2)$$

   (b) In general, consider a join $E_1 \bowtie_\theta E_2$.

   - Let L_1 and L_2 be sets of attributes from E_1 and E_2, respectively.
   - Let L_3 be attributes of E_1 that are involved in join condition θ, but are not in $L_1 \cup L_2$, and
   - let L_4 be attributes of E_2 that are involved in join condition θ, but are not in $L_1 \cup L_2$.

   $$\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) \equiv \Pi_{L_1 \cup L_2}(\Pi_{L_1 \cup L_3}(E_1) \bowtie_\theta \Pi_{L_2 \cup L_4}(E_2))$$

   Similar equivalences hold for outerjoin operations: $\overset{\bowtie}{}$, $\overset{\bowtie}{}$, and $\overset{\bowtie}{}$

Equivalence Rules (Cont.)

9. The set operations union and intersection are commutative

$$E_1 \cup E_2 \equiv E_2 \cup E_1$$
$$E_1 \cap E_2 \equiv E_2 \cap E_1$$

(set difference is not commutative).

10. Set union and intersection are associative.

$$(E_1 \cup E_2) \cup E_3 \equiv E_1 \cup (E_2 \cup E_3)$$
$$(E_1 \cap E_2) \cap E_3 \equiv E_1 \cap (E_2 \cap E_3)$$

11. The selection operation distributes over $\cup$, $\cap$ and $-$.

a. $\sigma_\theta (E_1 \cup E_2) \equiv \sigma_\theta (E_1) \cup \sigma_\theta(E_2)$
b. $\sigma_\theta (E_1 \cap E_2) \equiv \sigma_\theta (E_1) \cap \sigma_\theta(E_2)$
c. $\sigma_\theta (E_1 - E_2) \equiv \sigma_\theta (E_1) - \sigma_\theta(E_2)$
d. $\sigma_\theta (E_1 \cap E_2) \equiv \sigma_\theta(E_1) \cap E_2$
e. $\sigma_\theta (E_1 - E_2) \equiv \sigma_\theta(E_1) - E_2$

preceding equivalence does not hold for $\cup$

12. The projection operation distributes over union

$$\Pi_L(E_1 \cup E_2) \equiv (\Pi_L(E_1)) \cup (\Pi_L(E_2))$$

Equivalence Rules (Cont.)

13. Selection distributes over aggregation as below

$\sigma_\theta({}_A g_f(E)) \equiv {}_A g_f(\sigma_\theta(E))$

provided θ only involves attributes in G

14. a. Full outerjoin is commutative:

$E_1 \mathbin{⟗} E_2 \equiv E_2 \mathbin{⟗} E_1$

b. Left and right outerjoin are not commutative, but:

$E_1 \mathbin{⟕} E_2 \equiv E_2 \mathbin{⟖} E_1$

c. Left and right outerjoins are not associative

$(r \mathbin{⟕} s) \mathbin{⟕} t \not\equiv r \mathbin{⟕} (s \mathbin{⟕} t)$

15. Selection distributes over left and right outerjoins as below, provided θ_1 only involves attributes of E_1

a. $\sigma_{\theta_1}(E_1 \mathbin{⋈_\theta} E_2) \equiv (\sigma_{\theta_1}(E_1)) \mathbin{⋈_\theta} E_2$

b. $\sigma_{\theta_1}(E_1 \mathbin{⟕_\theta} E_2) \equiv E_2 \mathbin{⟖_\theta} (\sigma_{\theta_1}(E_1))$

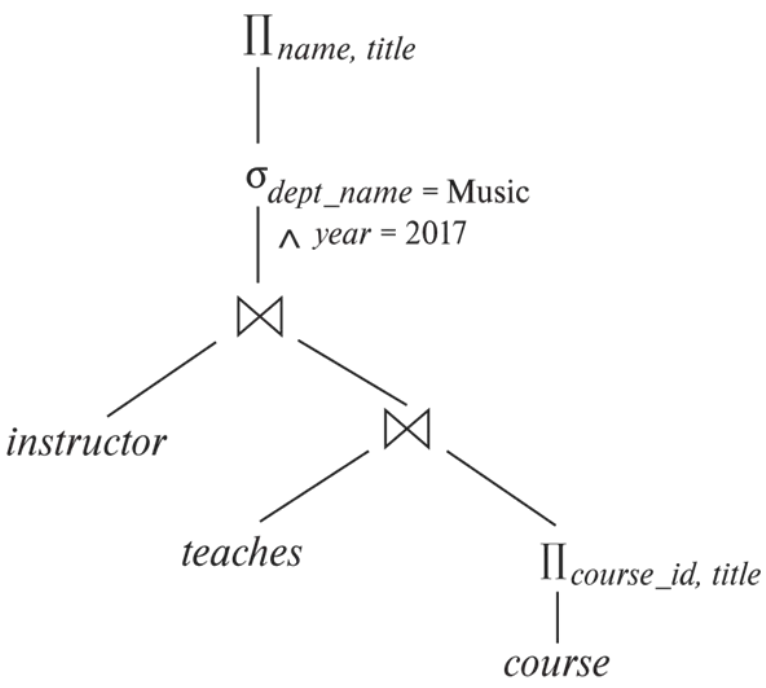
Transformation Example: Pushing Selections

- Query: Find the names of all instructors in the Music department, along with the titles of the courses that they teach
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\text{'Music'}}(instructor \bowtie (teaches \bowtie \Pi_{course\_id,\ title}(course))))$

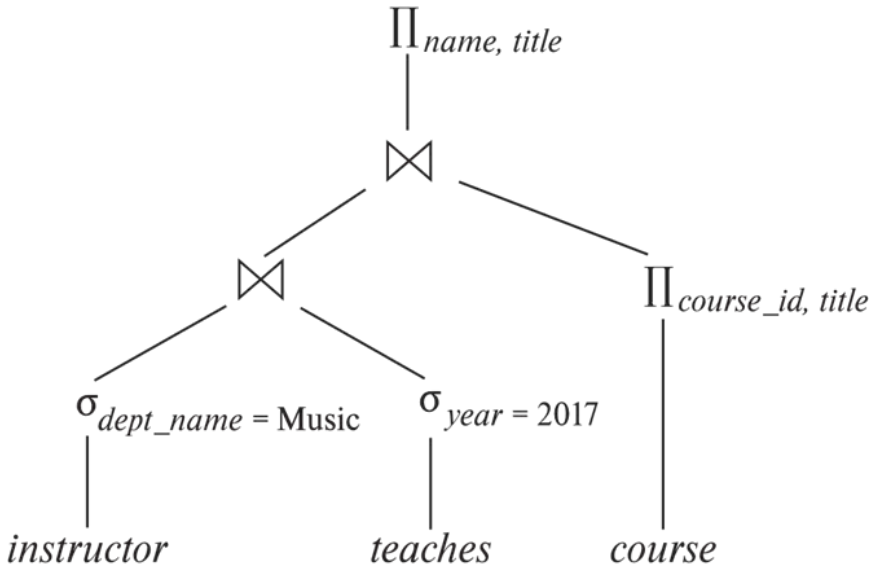
- Transformation using rule 7a.
  - $\Pi_{name,\ title}((\sigma_{dept\_name=\text{'Music'}}(instructor)) \bowtie (teaches \bowtie \Pi_{course\_id,\ title}(course)))$

- Performing the selection as early as possible reduces the size of the relation to be joined.

Multiple Transformations (Cont.)



(a) Initial expression tree



(b) Tree after multiple transformations

Join Ordering Example

- (Join Associativity) $\bowtie$

  For all relations r_1, r_2, and r_3,

  $$(r_1 \bowtie r_2) \bowtie r_3 = r_1 \bowtie (r_2 \bowtie r_3)$$

- If $r_2 \bowtie r_3$ is quite large and $r_1 \bowtie r_2$ is small, we choose

  $$(r_1 \bowtie r_2) \bowtie r_3$$

  so that we compute and store a smaller temporary relation.

Join Ordering Example (Cont.)

- Consider the expression

  $\Pi_{name,\ title}(\sigma_{dept\_name=\text{“Music”}}(instructor) \bowtie teaches \bowtie \Pi_{course\_id,\ title}(course))$

- Could compute $teaches \bowtie \Pi_{course\_id,\ title}(course)$ first, and join with

  $\sigma_{dept\_name=\text{“Music”}}(instructor)$

  but the result of the first join is likely to be a large relation.

- Only a small fraction of the university's instructors are likely to be from the Music department
  - it is better to compute

    $\sigma_{dept\_name=\text{“Music”}}(instructor) \bowtie teaches$

    first.

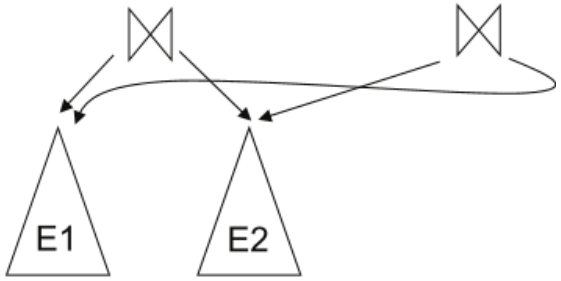
Enumeration of Equivalent Expressions

- Query optimizers use equivalence rules to **systematically** generate expressions equivalent to the given expression
- Can generate all equivalent expressions as follows:
  - Repeat
    - apply all applicable equivalence rules on every subexpression of every equivalent expression found so far
    - add newly generated expressions to the set of equivalent expressions

    Until no new equivalent expressions are generated above
- The above approach is very expensive in space and time
  - Two approaches
    - Optimized plan generation based on transformation rules
    - Special case approach for queries with only selections, projections and joins

Implementing Transformation Based Optimization

- Space requirements reduced by sharing common sub-expressions:
  - when E1 is generated from E2 by an equivalence rule, usually only the top level of the two are different, subtrees below are the same and can be shared using pointers
    - E.g., when applying join commutativity



  - Same sub-expression may get generated multiple times
    - Detect duplicate sub-expressions and share one copy
- Time requirements are reduced by not generating all expressions
  - Dynamic programming
    - We will study only the special case of dynamic programming for join order optimization

Cost Estimation

- Cost of each operator computed as described in Chapter 15
  - Need statistics of input relations
    - E.g., number of tuples, sizes of tuples
- Inputs can be results of sub-expressions
  - Need to estimate statistics of expression results
  - To do so, we require additional statistics
    - E.g., number of distinct values for an attribute
- More on cost estimation later

Choice of Evaluation Plans

- Must consider the interaction of evaluation techniques when choosing evaluation plans
  - choosing the cheapest algorithm for each operation may not yield best overall algorithm. E.g.
    - merge-join may be costlier than hash-join, but may provide a sorted output which reduces the cost for an outer level aggregation.
    - nested-loop join may provide opportunity for pipelining

- Practical query optimizers incorporate the following approaches:
  1. Search all the plans and choose the best plan in a cost-based fashion.
  2. Uses heuristics to choose a plan.

Cost-Based Optimization

- Consider finding the best join-order for $r_1 \bowtie r_2 \bowtie \ldots \bowtie r_n$.
- There are $(2(n-1))!/(n-1)!$ different join orders for above expression. With $n = 7$, the number is 665280, with $n = 10$, the number is greater than 176 billion!
- No need to generate all the join orders.
- Using dynamic programming, the least-cost join order for any subset of
  $\{r_1, r_2, \ldots r_n\}$ is computed only once and stored for future use.

Dynamic Programming in Optimization

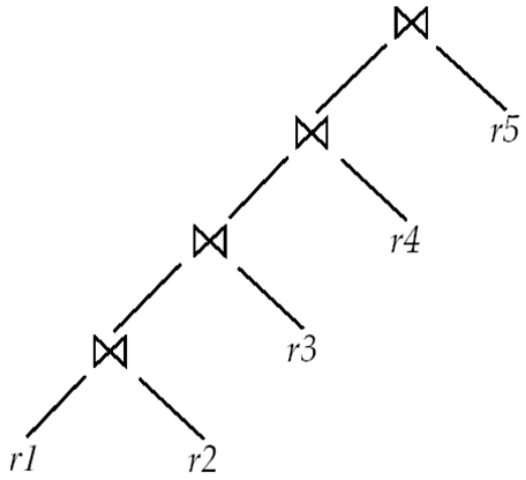
- To find best join tree for a set of *n* relations:
  - Consider all possible plans of the form: $S_1 \bowtie (S - S_1)$ where S_1 is any non-empty subset of S.
  - Recursively compute costs for joining subsets of S to find the cost of each plan.
  - Choose the cheapest plan.
  - Base case for recursion: single relation access plan
    - Apply all selections on R_i using best choice of indices on R_i
  - When plan for any subset is computed, store it and reuse it when it is required again, instead of recomputing it
    - Dynamic programming
    - **Memoization**

Join Order Optimization Algorithm

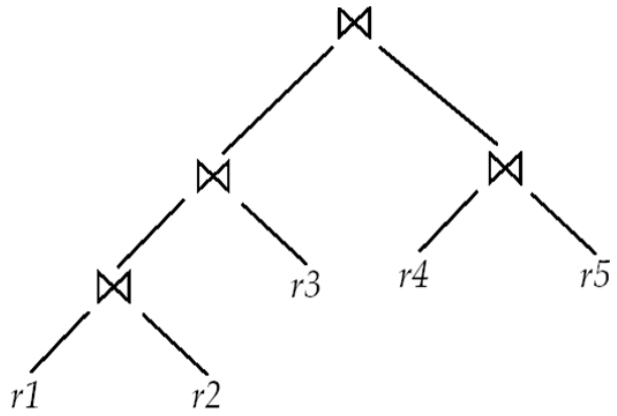
```
procedure findbestplan(S)
  if (bestplan[S].cost ≠ ∞)
        return bestplan[S]
  // else bestplan[S] has not been computed earlier, compute it now
  if (S contains only 1 relation)
     set bestplan[S].plan and bestplan[S].cost based on the best way
     of accessing S  using selections on S and indices (if any) on S else
  for each non-empty subset S1 of S such that S1 ≠ S
        P1= findbestplan(S1)
        P2= findbestplan(S - S1)
        for each algorithm A for joining results of P1 and P2
           … compute plan and cost of using A ..
            if cost < bestplan[S].cost
                  bestplan[S].cost = cost
                  bestplan[S].plan = plan;
  return bestplan[S]
```

Left Deep Join Trees

- In **left-deep join trees,** the right-hand-side input for each join is a relation, and the left-hand-side input is the result of an intermediate join.



(a) Left-deep join tree



(b) Non-left-deep join tree

- If only left-deep trees are considered, time complexity of finding best join order is decreased from $O(3^n)$ to $O(n\ 2^n)$

Interesting Sort Orders

- Consider the expression $(r_1 \bowtie r_2) \bowtie r_3$ (with A as common attribute)
- An **interesting sort order** is a particular sort order of tuples that could make a later operation (join/group by/order by) cheaper
  - An **interesting sort order** allows a query optimizer to consider plans that could be locally sub-optimal, but produce ordered output beneficial for other operators, and thus be part of a globally optimal plan.
  - Using merge-join to compute $r_1 \bowtie r_2$ may be costlier than hash join but generates result sorted on A
  - Which in turn may make merge-join with r_3 cheaper, which may reduce cost of join with r_3 and minimizing overall cost

Heuristic Optimization

- Cost-based optimization is expensive, even with dynamic programming.
- Systems may use ***heuristics*** to reduce the number of choices that must be made in a cost-based fashion.
  - **Heuristic** technique is a practical method that is not guaranteed to be optimal, perfect or rational, but which is nevertheless sufficient for reaching an immediate, short-term goal.
- Heuristic optimization transforms the query-tree by using a set of rules that typically (but not always) improve execution performance:
  - Perform selection early (reduces the number of tuples)
  - Perform projection early (reduces the number of attributes)
  - Perform most restrictive selection and join operations (i.e., with smallest result size) before other similar operations.
  - Some systems use only heuristics, others combine heuristics with partial cost-based optimization.

Structure of Query Optimizers

- Many optimizers considers only left-deep join orders.
  - Plus heuristics to push selections and projections down the query tree
  - Reduces optimization complexity and generates plans amenable to pipelined evaluation.
- Heuristic optimization used in some versions of Oracle:
  - Repeatedly pick “best” relation to join next
    - Starting from each of n starting points. Pick best among these
- Intricacies of SQL complicate query optimization
  - E.g., nested subqueries

Statistics for Cost Estimation

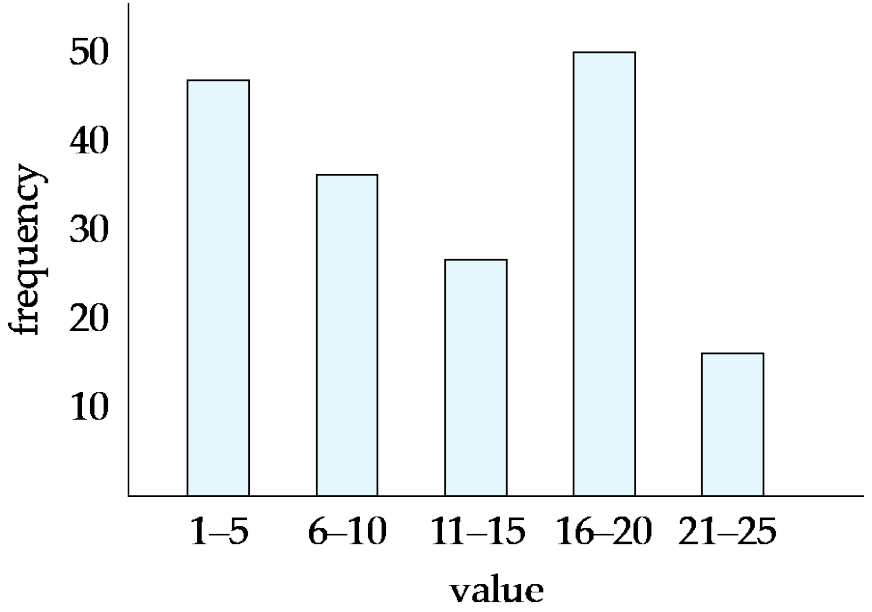
Statistical Information for Cost Estimation

- n_r: number of tuples in a relation *r.*
- b_r: number of blocks containing tuples of *r.*
- l_r: size of a tuple of *r.*
- f_r: blocking factor of *r* — i.e., the number of tuples of *r* that fit into one block.
- *V(A, r):* number of distinct values that appear in *r* for attribute *A;* same as the size of $\prod_A(r)$.
- If tuples of *r* are stored together physically in a file, then:

$$b_r = \left\lceil \frac{n_r}{f_r} \right\rceil$$

Histograms

- Histogram on attribute *age* of relation *person*



- **Equi-width** histograms
- **Equi-depth** histograms break up range such that each range has (approximately) the same number of tuples
  - E.g. (4, 8, 14, 19)
- Many databases also store *n* **most-frequent values** and their counts
  - Histogram is built on remaining values only

Histograms (cont.)

- Histograms and other statistics usually computed based on a **random sample**
- Statistics may be out of date
  - Some database require a **analyze** command to update statistics
  - Others automatically recompute statistics
    - e.g., when number of tuples in a relation changes by some percentage

Selection Size Estimation - Skipped

- $\sigma_{A=V}(r)$
  - n_r / *V(A,r)* : number of records that will satisfy the selection
  - Equality condition on a key attribute: *size estimate* = 1
- $\sigma_{A \leq V}(r)$ (case of $\sigma_{A \geq V}(r)$ is symmetric)
  - Let c denote the estimated number of tuples satisfying the condition.
  - If min(A,r) and max(A,r) are available in catalog
    - c = 0 if v < min(A,r)
    - c = $n_r . \frac{v - \min(A,r)}{\max(A,r) - \min(A,r)}$
  - If histograms available, can refine above estimate
  - In absence of statistical information *c* is assumed to be $n_r / 2$.

Size Estimation of Complex Selections - Skipped

- The **selectivity** of a condition θ_i is the probability that a tuple in the relation *r* satisfies θ_i .
  - If s_i is the number of satisfying tuples in *r*, the selectivity of θ_i is given by s_i / n_r.
- **Conjunction:** $\sigma_{\theta 1 \wedge \theta 2 \wedge \ldots \wedge \theta n}(r)$. *Assuming independence,* estimate of tuples in the result is: $$n_r * \frac{s_1 * s_2 * \ldots * s_n}{n_r^n}$$
- **Disjunction:** $\sigma_{\theta 1 \vee \theta 2 \vee \ldots \vee \theta n}(r)$. Estimated number of tuples: $$n_r * \left(1 - (1 - \frac{s_1}{n_r}) * (1 - \frac{s_2}{n_r}) * \ldots * (1 - \frac{s_n}{n_r})\right)$$
- **Negation:** $\sigma_{\neg\theta}(r)$. Estimated number of tuples: $n_r - size(\sigma_\theta(r))$

Join Operation: Running Example - Skipped

Running example:

student ⋈ *takes*

Catalog information for join examples:

- $n_{student}$ = *5*,000.
- $f_{student}$ = *50,* which implies that
  $b_{student}$ =5000/50 = 100.
- n_{takes} = 10000.
- f_{takes} = 25, which implies that
  b_{takes} = 10000/25 = 400.
- *V(ID, takes)* = 2500, which implies that on average, each student who has taken a course has taken 4 courses.
  - Attribute *ID* in *takes* is a foreign key referencing *student.*
  - *V*(*ID, student*) = 5000 (*primary key!*)

Estimation of the Size of Joins - Skipped

- The Cartesian product r x s contains $n_r . n_s$ tuples; each tuple occupies $s_r + s_s$ bytes.
- If $R \cap S = \varnothing$, then $r \bowtie s$ is the same as r x s.
- If $R \cap S$ is a key for R, then a tuple of s will join with at most one tuple from r
  - therefore, the number of tuples in $r \bowtie s$ is no greater than the number of tuples in s.
- If $R \cap S$ *in* S is a foreign key in S referencing R, then the number of tuples in $r \bowtie s$ is exactly the same as the number of tuples in s.
  - The case for $R \cap S$ being a foreign key referencing S is symmetric.
- In the example query *student* ⋈ *takes, ID* in *takes* is a foreign key referencing *student*
  - hence, the result has exactly n_{takes} tuples, which is 10000

Estimation of the Size of Joins (Cont.) - Skipped

- If $R \cap S = \{A\}$ is not a key for R or S.
  If we assume that every tuple t in R produces tuples in $R \bowtie S$, the number of tuples in $R \bowtie S$ is estimated to be:

$$\frac{n_r * n_s}{V(A,s)}$$

  If the reverse is true, the estimate obtained will be:

$$\frac{n_r * n_s}{V(A,r)}$$

  The lower of these two estimates is probably the more accurate one.
- Can improve on above if histograms are available
  - Use formula similar to above, for each cell of histograms on the two relations

Estimation of the Size of Joins (Cont.) - Skipped

- Compute the size estimates for *depositor* ⋈ *customer* without using information about foreign keys:
  - *V(ID, takes)* = 2500, and
    V(ID, student) = 5000
  - The two estimates are 5000 * 10000/2500 = 20,000 and 5000 * 10000/5000 = 10000
  - We choose the lower estimate, which in this case, is the same as our earlier computation using foreign keys.

Size Estimation for Other Operations - Skipped

- Projection: estimated size of $\Pi_A(r)$ = $V(A,r)$
- Aggregation : estimated size of ${}_G\gamma_A(r)$ = $V(G,r)$
- Set operations
  - For unions/intersections of selections on the same relation: rewrite and use size estimate for selections
    - E.g., $\sigma_{\theta 1}(r) \cup \sigma_{\theta 2}(r)$ can be rewritten as $\sigma_{\theta 1 \text{ or } \theta 2}(r)$
  - For operations on different relations:
    - estimated size of $r \cup s$ = size of r + size of s.
    - estimated size of $r \cap s$ = minimum size of r and size of s.
    - estimated size of $r - s$ = r.
    - All the three estimates may be quite inaccurate, but provide upper bounds on the sizes.

Size Estimation (Cont.) - Skipped

- Outer join:
  - Estimated size of $r \bowtie s$ = *size of* $r \bowtie s$ + *size of* r
    - Case of right outer join is symmetric
  - Estimated size of $r ⟕ s$ = *size of* $r \bowtie s$ + size of r + size of s

Estimation of Number of Distinct Values - Skipped

Selections: $\sigma_\theta(r)$

- If θ forces A to take a specified value: $V(A,\sigma_\theta(r)) = 1$.
  - e.g., $A = 3$
- If θ forces A to take on one of a specified set of values:
  $V(A,\sigma_\theta(r))$ = number of specified values.
  - (e.g., ($A = 1$ V $A = 3$ V $A = 4$)),
- If the selection condition θ is of the form A *op* r
  estimated $V(A,\sigma_\theta(r)) = V(A.r) * s$
  - where s is the selectivity of the selection.
- In all the other cases: use approximate estimate of
  $\min(V(A,r), n_{\sigma\theta(r)})$
  - More accurate estimate can be got using probability theory, but this one works fine generally

Estimation of Distinct Values (Cont.) - Skipped

Joins: $r \bowtie s$

- If all attributes in A are from r
  estimated $V(A, r \bowtie s) = \min(V(A,r), n_{r \bowtie s})$
- If A contains attributes $A1$ from r and $A2$ from s, then estimated $V(A, r \bowtie s)$ =

$$\min(V(A1,r)*V(A2 - A1,s), V(A1 - A2,r)*V(A2,s), n_{r \bowtie s})$$

  - More accurate estimate can be got using probability theory, but this one works fine generally

Estimation of Distinct Values (Cont.) - Skipped

- Estimation of distinct values are straightforward for projections.
  - They are the same in $\Pi_{A\,(r)}$ as in *r*.
- The same holds for grouping attributes of aggregation.
- For aggregated values
  - For min(*A*) and max(*A*), the number of distinct values can be estimated as min(V(*A,r*), *V*(*G,r*)) where G denotes grouping attributes
  - For other aggregates, assume all values are distinct, and use *V*(*G,r*)