Final Exam (ICE2003-46)

Student ID:__________________

Name:______________________

1. **(20 pts)** Explain given terms.

(a) **(5 pts)** <u>Write conditions (in equation)</u> that RP is stationary.

(b) **(10 pts)** Consider random sinusoids. <u>Answer questions with appropriate proof</u>.

Random amplitude sinusoid: $X(t) = A\cos(2\pi t)$, where A is some RV. Is this RP wide-sense-stationary (5 pts)?

Random phase sinusoid: $X(t) = \cos(\omega t + \Theta)$, where Θ is some RV. Is this RP wide-sense-stationary (5 pts)?

(c) **(5 pts)** A component has a constant failure rate function $r(t) = -R'(t)/R(t) = \lambda$, where $R(t) = P[T > t]$ is the reliability at time t and T is lifetime of the component. <u>Find mean time to failure (MTTF)</u>.

2. (20 pts) Assume that we have input RV X and output RV Y of a system. The joint pdf is as below:

$$f_{X,Y}(x,y) = \begin{cases} ce^{-x}e^{-y} & 0 \le y \le x < \infty \\ 0 & elsewhere \end{cases}$$

(a) **(4 pts)** Find the coefficient c.

(b) **(4 pts)** Find marginal pdf's of X and Y.

(c) **(2 pts)** Are X and Y independent?

(d) **(2 pts)** $E[X] = 3/2$, $VAR[X] = 5/4$, $E[Y] = 1/2$, $VAR[Y] = 1/4$. **Find the correlation coefficient** $\rho_{X,Y}$ and **tell if X and Y are correlated**.

(e) **(8 pts)** Find the MAP and ML estimators for X in terms of Y.

ML:

MAP:

3. **(20 pts)** The input X to a communication channel assumes the values +1 or -1 with probabilities 1/3 and 2/3. The output Y of the channel is given by $Y = X + N$, where N is a zero-mean, unit-variance Gaussian RV.

Hint: pdf of Gaussian RV is: $f_X(x) = \dfrac{e^{-(x-m)^2/2\sigma^2}}{\sqrt{2\pi}\sigma}$ $\quad E[X] = m \quad \text{VAR}[X] = \sigma^2$

(a) **(10 pts)** Find the conditional pdf of Y given $X = +1$, and given $X = -1$.

(b) **(10 pts)** Find $P[X = +1 \mid Y > 0]$. Hint: Q(1) = 0.159

4. **(20 pts)** A radio transmitter sends a signal $s > 0$ to a receiver using three paths. The signals that arrive at the receiver along each path are:

$X_1 = s + N_1$, $X_2 = s + N_2$, and $X_3 = s + N_3$

where N_1, N_2, and N_3 are independent Gaussian RVs with zero mean and unit variance. (pdf of Gaussian RV is shown in Problem 3)

(a) **(6 pts)** Find the joint pdf of $\mathbf{X} = (X_1, X_2, X_3)$.

(b) **(7 pts)** Find the probability that a majority of the signals are positive. (Write the result in terms of Gaussian cdf F)

(c) **(7 pts)** Find the mean vector and covariance matrix for $\mathbf{X} = (X_1, X_2, X_3)$.

5. **(20 pts)** A student uses pens whose lifetime is an exponential RV with mean 3 weeks. Use the central limit theorem to determine the minimum number of pens he should buy at the beginning of a 15-week semester, so that with probability 0.999 he does not run out of pens during the semester.

TABLE 4.3 $Q(x) = 10^{-k}$

k	$x = Q^{-1}(10^{-k})$
1	1.2815
2	2.3263
3	3.0902
4	3.7190
5	4.2649
6	4.7535
7	5.1993
8	5.6120
9	5.9978
10	6.3613