Classical Cryptography

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History

- Egyptians 4000 years ago
- The scytale transposition cipher was used by the Spartan military

A Scytale, an early device for encryption.

Enigma machine

- World war 1&2
- As a tool to protect national secrets and strategies

Ceasar cipher (no key)

Shift by 3

a → d
b → e
c → f
…
z → c

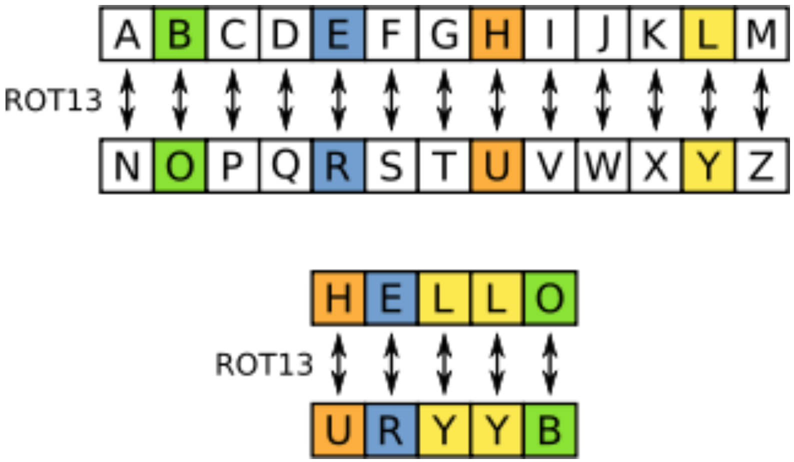
$$c = E_k(\text{"abc"})$$

$$= \text{"def"}$$

This system is completely broken when the algorithm description is known

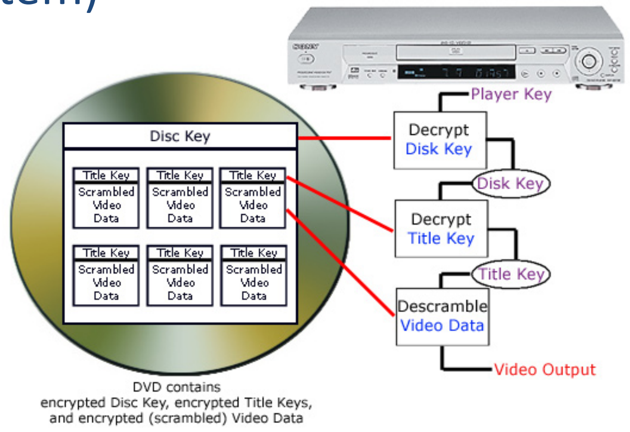
ROT 13

- Ceasar cipher with key of 13
- 13 chosen since encryption and decryption are same operation



Basic assumptions

- The system is completely known to the attacker
- Only the key is secret; that is, crypto algorithms are not secret
- This is known as **Kerckhoffs' principle**
- Why do we make this assumption?
  - Experience has shown that secret algorithms are weak when exposed (e.g., DVD content scrambling system)
  - Secret algorithms never remain secret
  - Better to find weaknesses beforehand



Substitution cipher with shift

Shift by k

a → c
b → d
c → e
…
z → b

Key k

$$c = E_k(\text{“apple”}),\ k = 2$$

$$= \text{“crrmg”}$$

How to break this cipher?

- A simple substitution (shift by n) is used
  - But the key is unknown
- Given ciphertext: crrmg
- How to find the key?
- Only 26 possible keys — try them all!
- **Exhaustive key search**
- Solution: key is $k = 2$
  - Q. How can we check the validity of the key?

Substitution cipher with permutation

Use any permutation of letters

a → e
b → c
c → q
…
z → a

Key permutation

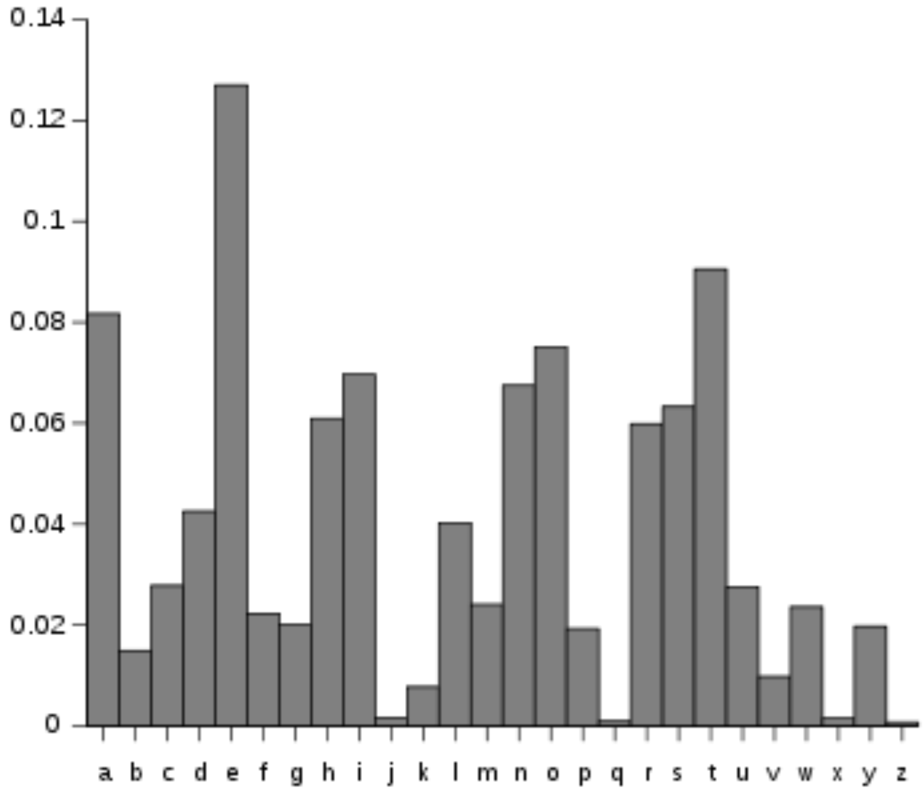
$c = E_k(\text{“abc”})$

$= \text{“ecq”}$

Then 26! > 2^{88} possible keys!

How to break this cipher?

- Use letter frequencies; most common letters in English are e, t, a, I, o, n, s, h, r, d, l, u



Additional frequency features

- Digraph (groups of 2 letters) frequencies
  - Common digraphs: EN, RE, ER, NT
- Vowels other than E rarely followed by another vowel
- The letter Q is followed only by U
- Etc.

Example of cryptanalysis

- Ciphertext:

PBFPVYFBQXZTYFPBFEQJHDXXQVAPTPQJKTOYQWIPBVWLXTOXBTFXQWAXBVCXQWAXFQJVWLE
QNTOZQGGQLFXQWAKVWLXQWAEBIPBFXFQVXGTVJVWLBTPQWAEBFPBFHCVLXBQUFEVWLXG
DPEQVPQGVPPBFTIXPFHXZHVFAGFOTHFEFBQUFTDHZBQPOTHXTYFTODXQHFTDPTOGHFQPBQ
WAQJJTODXQHFOQPWTBDHHIXQVAPBFZQHCFWPFHPBFIPBQWKFABVYYDZBOTHPBQPQJTQOT
OGHFQAPBFEQJHDXXQVAVXEBQPEFZBVFOJIWFFACFCCFHQWAUVWFLQHGFXVAFXQHFUFHILTT
AVWAFFAWTEVOITDHFHFQAITIXPFHXAFQHEFZQWGFLVWPTOFFA

Analyze this message using statistics below

Ciphertext frequency counts:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
21	26	6	10	12	51	10	25	10	9	3	10	0	1	15	28	42	0	0	27	4	24	22	28	6	8

This might be ‘e’.

Vigenère Cipher

- 16th century – the Vigenère

plaintext	tobeornottobethatistheques ···
key	runrunrunrunrunrunrunrunru ···
ciphertext	KIOVIEEIGKIOVNURNVJNUVKHVM ···

- What is the size of key space?
  - If keys are 14-character strings; then key space has size $26^{14} \approx 2^{66}$
- How can we break this?

Variant Vigenère cipher

- Easier to work with ASCII plaintext and hex ciphertext
  - Easier to implement
  - Easier to use (plaintext not limited to lowercase characters)

- Easier to work with byte-wise XOR rather than modular addition

Variant Vigenère cipher

- The key is a string of bytes
- The plaintext is a string of ASCII characters
- To encrypt, XOR each character in the plaintext with the next character of the key
- Decryption just reverses the process

Example

- Say plaintext is “Hello!” and key is 0xA1 2F
- “Hello!” = 0x48 65 6C 6C 6F 21
- XOR with 0xA1 2F A1 2F A1 2F
- 0x48 $\oplus$ 0xA1
  - 0100 1000 $\oplus$ 1010 0001 = 1110 1001 = 0xE9

- Ciphertext: 0xE9 4A CD 43 CE 0E

Attacking the Vigenère cipher

- Two steps:
  - Determine the key length
  - Determine each character of the key

Determining the key length

- Let p_i (for $0 \le i \le 255$) be the frequency of byte i in plaintext (assuming English text)
  - I.e., p_{97} = frequency of 'a'
  - The distribution is far from uniform
- If the key length is N, then every N^{th} character of the plaintext is encrypted using the same "xor"
  - If we take every N^{th} character and calculate frequencies, we should get the p_i's in permuted order
  - If we take every M^{th} character (M not a multiple of N) and calculate frequencies, we should get something close to uniform

Determining the key length

- How to distinguish these two?
- For some candidate distribution $q_0, \ldots, q_{255}$, compute $\Sigma\, q_i^2$
  - If close to uniform, $\Sigma\, q_i^2 \approx 256 \cdot (1/256)^2 = 1/256$
  - If a permutation of p_i, then $\Sigma\, q_i^2 \approx \Sigma\, p_i^2$
    - Could compute $\Sigma\, p_i^2$ (but somewhat difficult)
    - Key point: will be much larger than 1/256
- Try all possibilities for the key length, compute $\Sigma\, q_i^2$, and look for maximum value

Index of Coincidence - Plaintext

Letter	a	b	c	d	e	f	g	h	i	j	k	l	m
Frequency	.082	.015	.028	.043	.127	.022	.020	.061	.070	.002	.008	.040	.024
Letter	n	o	p	q	r	s	t	u	v	w	x	y	z
Frequency	.067	.075	.019	.001	.060	.063	.091	.028	.010	.023	.001	.020	.001

Beker and Piper, *Cipher Systems: The Protection of Communications*, Wiley.

$$\begin{array}{ccccccccc} \text{aa} & \text{or} & \text{bb} & \text{or} & \text{cc} & \text{or} & \ldots & \text{or} & \text{zz} \\ .082\times.082 & + & .015\times.015 & + & .028\times.028 & + & \ldots & + & .001\times.001 \end{array}$$

$$I \approx 0.0656010$$

Index of Coincidence - Uniform

$$I \approx \underbrace{\left(\frac{1}{26}\times\frac{1}{26}\right)+\left(\frac{1}{26}\times\frac{1}{26}\right)+\left(\frac{1}{26}\times\frac{1}{26}\right)+\ldots+\left(\frac{1}{26}\times\frac{1}{26}\right)}_{\text{26 terms}}=\frac{1}{26}\approx 0.038$$

Determining the i^{th} byte of the key

- Assume the key length N is known
- Look at every N^{th} character of the ciphertext, starting with the i^{th} character
  - Call this the i^{th} ciphertext “stream”
  - Note that all bytes in this stream were generated by XORing plaintext with the same byte of the key
- Try decrypting the stream using every possible byte value B
  - Get a candidate plaintext stream for each value

When the guessed key length is 3,

KIOVIEEIGKIOVNURNVJNUVKHVM ···

KVEKVRJVKHVM ···

K = ⟨Space⟩ ?

Determining the i^{th} byte of the key

- When the guess B is correct:
  - Frequencies of lowercase letters (as a fraction of all lowercase letters) should be close to known English-letter frequencies
    - Tabulate $q_a, \ldots, q_z$
    - Should find $\Sigma\, q_i\, p_i \approx \Sigma\, p_i^2 \approx 0.065$
    - In practice, take B that maximizes $\Sigma\, q_i\, p_i$, subject to caveat above (and possibly others)

Attack time?

- The key length is between 1 and L
- Determining the key length: $\approx$ 256 L
- Determining all bytes of the key:
  - Guessing B at ith character: 256
  - Calculating $\Sigma\ q_i\ p_i$ at ith character: 256
  - Total: 256^2 L

- Brute-force key search: $\approx 256^L$

The attack in practice

- Attacks get more reliable as the ciphertext length grows larger

- Attacks still work for short(er) ciphertexts, but more “tweaking” and manual involvement is needed

One Time Pad (OTP)

First example of a "secure" cipher

Choose key k as random bit string as long the message!

$\mathcal{M} = \mathcal{C} = \{0, 1\}^n$

$\mathcal{K} = \{0, 1\}^n$

Encryption: $c = m \oplus k$

Very fast enc/dec !!

… but long keys (as long as plaintext)

Quiz

You are given a message (m) and its OTP encryption (c).

Q. Can you compute the OTP key from m and c ?

Yes, the key is $k = m \oplus c$.

OTP has perfect secrecy

Q. What is the perfect secrecy?

Information Theoretic Security
(Shannon 1949)

Def: A cipher (***E***, ***D***) over $(\mathcal{K}, \mathcal{M}, \mathcal{C})$ has **perfect secrecy** if

$\forall m_0, m_1 \in \mathcal{M} \quad (|m_0| = |m_1|) \quad$ and $\quad \forall c \in \mathcal{C}$

$$Pr[\, \boldsymbol{E}(\boldsymbol{k}, \boldsymbol{m_0}) = \boldsymbol{c} \,] \;=\; Pr[\, \boldsymbol{E}(\boldsymbol{k}, \boldsymbol{m_1}) = \boldsymbol{c} \,] \quad \text{where} \quad k \xleftarrow{R} \mathcal{K}$$

- Given c, we can't tell if m is m_0 or m_1
- Any adversary can't learn any information about m from c
- In other words, m and c are independent
- No ciphertext only attack!! (but other attacks might be possible)

OTP has perfect secrecy
(Shannon 1949)

Proof:

$\forall m, c$:

$$\Pr_k[E(k, m) = c] = |\{ k \in \mathcal{K} : E(k, m) = c \}| \; / \; |\mathcal{K}|$$

$$= 1 \; / \; |\mathcal{K}|$$

Remember $\boldsymbol{k = m \oplus c}$**.**

Unfortunately …

(Shannon’s theorem)

<u>Thm</u>: If a shared-key encryption has perfect secrecy

$\Rightarrow \quad |\mathcal{K}| \geq |\mathcal{M}|$

- That is, key length should be greater than message length to achieve perfect secrecy.
- It is hard to use in practice!!!
- Computational adversary is needed
  - E.g., The encryption scheme cannot be broken with probability better than 2^{-80} in 200 yrs with the fastest supercomputer

Using the same key twice?

- $c_1 = k \oplus m_1$

  $c_2 = k \oplus m_2$

- Attacker can compute

  $c_1 \oplus c_2 = (k \oplus m_1) \oplus (k \oplus m_2) = m_1 \oplus m_2$

- This leaks information about m_1, m_2!
  - No longer perfectly secret! (e.g., $m_1 \oplus m_2$ reveals whether m1 is different from m2)
  - Frequency analysis

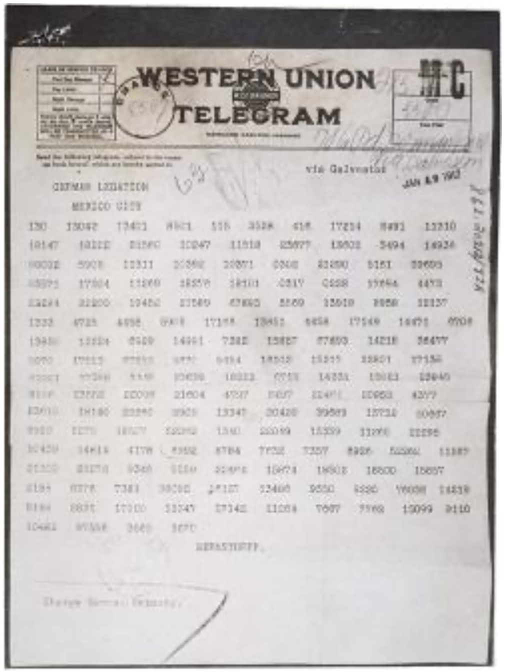
OTP in practice: The Moscow-Washington hotline

- A device called *Electronic Teleprinter Cryptographic Regenerative Repeater Mixer II* (ETCRRM II) encrypted the teletype messages.
- ETCRRM II used OTP.
- Each country delivered keying tapes used to encode its messages via its **embassy** abroad.

Codebook Cipher

- Literally, a book filled with “codewords”
- Zimmerman Telegram encrypted via codebook

Februar	13605
fest	13732
finanzielle	13850
folgender	13918
Frieden	17142
Friedenschluss	17149
:	:



- Modern block ciphers are codebooks!
- More about this later…

One more thing!

- Fundamental concepts
  - **Confusion** —— obscure relationship between plaintext and ciphertext
  - **Diffusion** —— spread plaintext statistics through the ciphertext
- One-time pad is confusion-only

Double transposition

Plaintext: attackxatxdawn

	col 1	col 2	col 3
row 1	a	t	t
row 2	a	c	k
row 3	x	a	t
row 4	x	d	a
row 5	w	n	x

Permute rows and columns

$\Rightarrow$

	col 1	col 3	col 2
row 3	x	t	a
row 5	w	x	n
row 1	a	t	t
row 4	x	a	d
row 2	a	k	c

- Ciphertext: xtawxnattxadakc
- What is the key?
- Key is matrix size and permutations: (3,5,1,4,2) and (1,3,2)
- Double transposition is diffusion-only

Questions?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