Zero-Jitter Task Chains via Algebraic Rings

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Outline

1. Motivation
2. Model of tasks
3. Chains of LET tasks
4. Zero-jitter chains of LET tasks

Motivation

- Communication through **shared memory** is very popular in automotive applications
  - very efficient (just a `MOV`/`LOAD` instruction)
- When the communicated data is samples of a physical process, it is normally **unbuffered**
  - fresher values just overwrites old values
- The read/write instants are affected by the task schedule
  - *Logical Execution Time (LET)* mitigates this aspect: read/writes instants of each job are at pre-determined instants
- Still **chains** of LET tasks may lose the regular timing of LET tasks
- The presented research is about guaranteeing a regular timing (**zero jitter**) of chains of LET tasks

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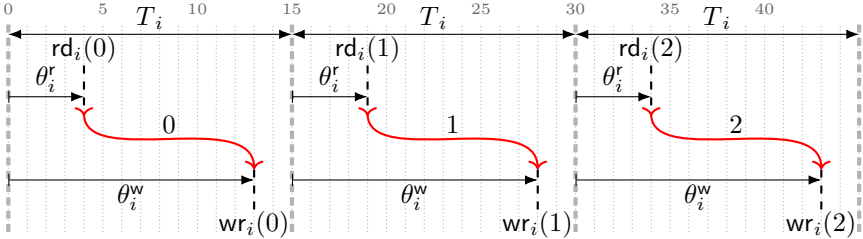
Model of a LET task τ_i



- A LET *task* τ_i is composed by *periodic jobs* (curvy arrows) indexed by $0, 1, 2, \ldots$
  - in LET, we only need the read and write instants

θ_i^{r}	*read phasing* of τ_i, relative to the *period* T_i (aka the *offset*)
θ_i^{w}	*write phasing* of τ_i, rel. to *period* T_i ($\theta_i^{\mathsf{w}} - \theta_i^{\mathsf{r}}$ is aka the *deadline*)
$\mathsf{rd}_i(j) = j\,T_i + \theta_i^{\mathsf{r}}$	*read instant* of job j of τ_i
$\mathsf{wr}_i(j) = j\,T_i + \theta_i^{\mathsf{w}}$	*write instant* of job j of τ_i

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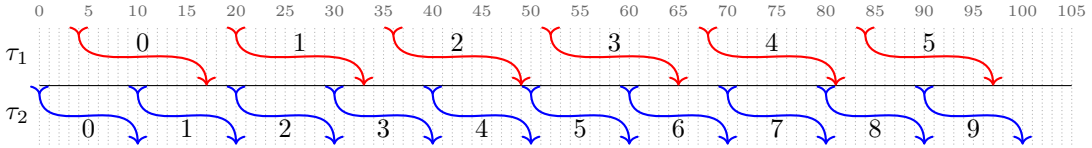
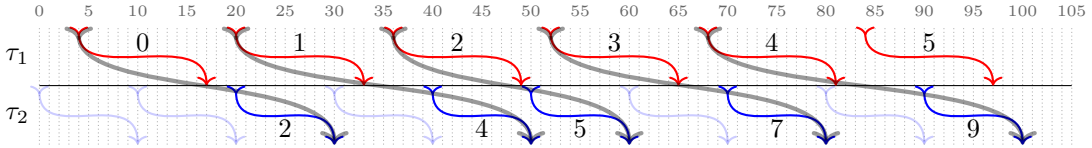
Chain of 2 LET tasks



- Chain of LET tasks
  - each task reads data written by the previous one
  - data is on **shared memory**
    - τ_1 may over-write data before τ_2 reads (if $T_1 < T_2$)
    - τ_2 may read again the same data (if $T_1 > T_2$)

Chain of 2 LET tasks



- Chain of LET tasks
  - each task reads data written by the previous one
  - data is on **shared memory**
    - ⋆ τ_1 may over-write data before τ_2 reads (if $T_1 < T_2$)
    - ⋆ τ_2 may read again the same data (if $T_1 > T_2$)
- Chains of jobs have **variable** read-to-write delay

			rd→wr delay
(j_1, j_2)	$\mathsf{rd}_1(j_1)$	$\mathsf{wr}_2(j_2)$	$\mathsf{wr}_2(j_2) - \mathsf{rd}_1(j_1)$
$(0, 2)$	4	30	$26 = 13 + \mathbf{3} + 10$
$(1, 4)$	20	50	$30 = 13 + \mathbf{7} + 10$
$(2, 5)$	36	60	$24 = 13 + \mathbf{1} + 10$
$(3, 7)$	52	80	$28 = 13 + \mathbf{5} + 10$
$(4, 9)$	68	100	$32 = 13 + \mathbf{9} + 10$
...	...	...	...

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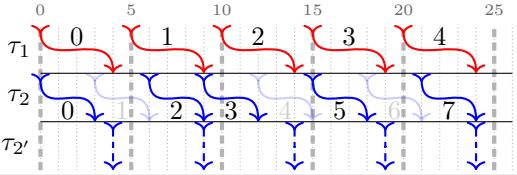
Making a zero jitter chain of 2 LET tasks: add a copier task

- if $T_1 \geqslant T_2$ then we add $\tau_{2'}$ after τ_2

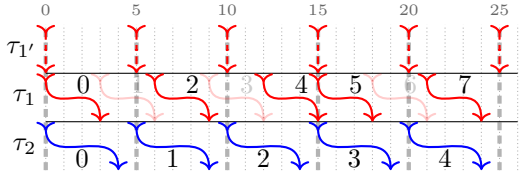
	T_i	θ_i^{r}	θ_i^{w}
τ_1	5	0	4
τ_2	3	0	3
$\tau_{2'}$	5	9	9



(j_1, j_2)	$\mathsf{wr}_2(j_2) - \mathsf{rd}_1(j_1)$	$\mathsf{wr}_{2'}(j_{2'}) - \mathsf{rd}_1(j_1)$
$(0, 2)$	9	9
$(1, 3)$	7	9
$(2, 5)$	8	9
$(3, 7)$	9	9

- if $T_1 \leqslant T_2$ then we add $\tau_{1'}$ before τ_1

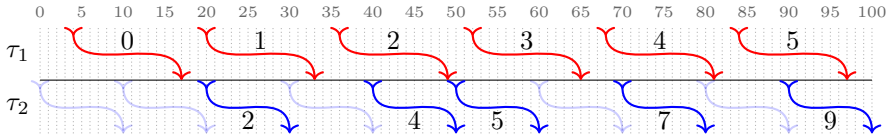
	T_i	θ_i^{r}	θ_i^{w}
$\tau_{1'}$	5	-5	-5
τ_1	3	0	3
τ_2	5	0	4



(j_1, j_2)	$\mathsf{wr}_2(j_2) - \mathsf{rd}_1(j_1)$	$\mathsf{wr}_2(j_2) - \mathsf{rd}_{1'}(j'_1)$
$(0, 1)$	9	9
$(2, 2)$	8	9
$(4, 3)$	7	9
$(5, 4)$	9	9

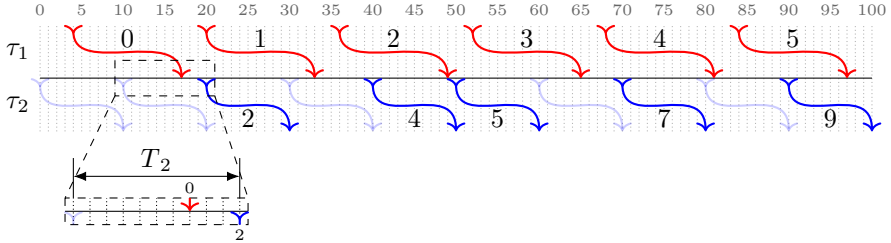
Variability of the input-output delay

- The source of variability is $\mathsf{rd}_2(j_2) - \mathsf{wr}_1(j_1)$. In the example: $3, 7, 1, 5, 9, 3, 7, \ldots$



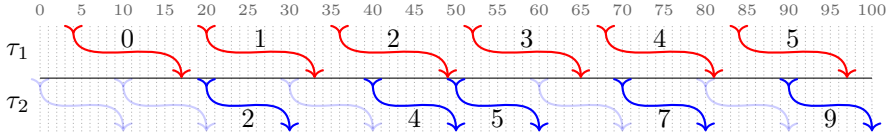
Variability of the input-output delay

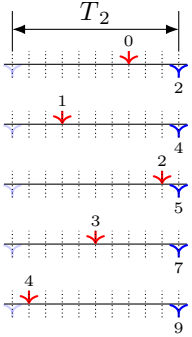
- $\mathsf{rd}_2(j_2) - \mathsf{wr}_1(j_1) < T_2$ ($= 10$ in the example), always. Let's zoom in ...



Variability of the input-output delay

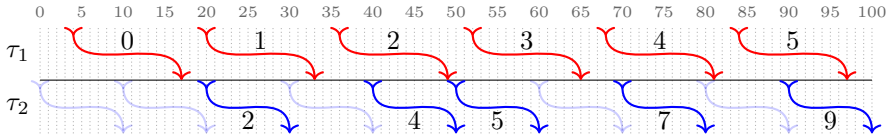
- ... and take $\mathrm{rd}_2(j_2)$ as reference for all $\mathrm{wr}_1(j_1) \to \mathrm{rd}_2(j_2)$ delays.

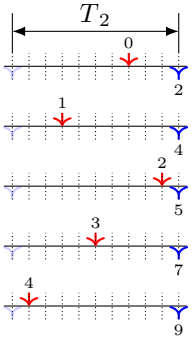


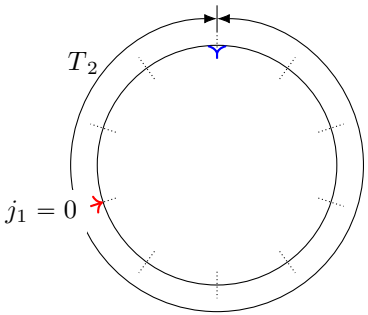


Variability of the input-output delay

- A repetitive requence: let's wrap all the T_2-long segments over the (algebraic) ring $\mathbb{Z}/T_2\mathbb{Z}$.

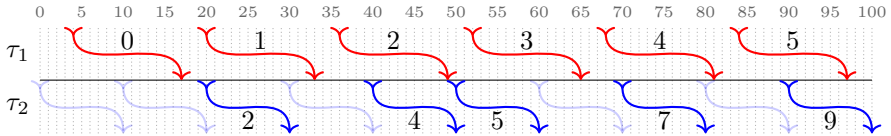


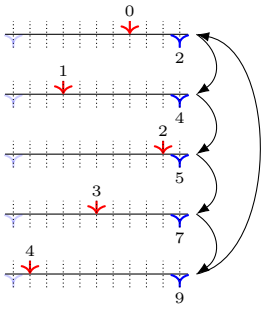


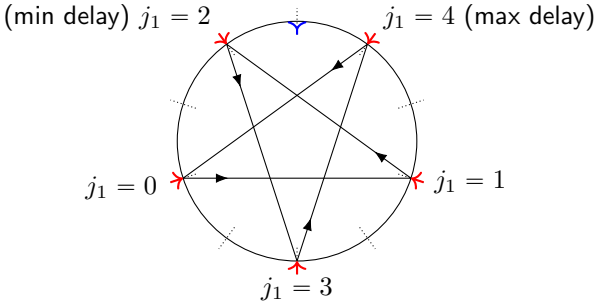


Variability of the input-output delay

- By inverting over the ring $\mathbb{Z}/T_2\mathbb{Z}$, max/min delay is found **without** unrolling the schedule.

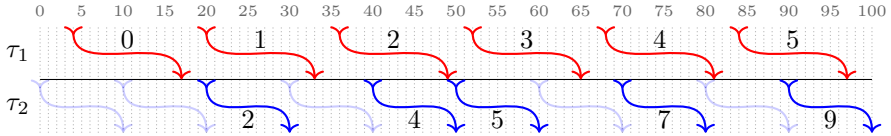


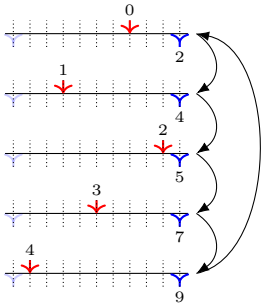


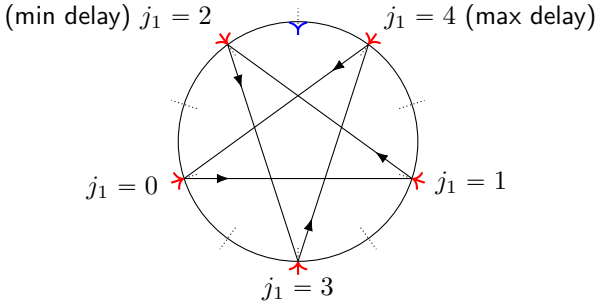


Variability of the input-output delay

- The parameters of the copier task follows from the maximum delay.







Conclusions

- The LET model allows a zero-jitter timing **for the tasks**
- Chains of LET tasks do **have non-zero jitter**
- By adding a copier, we can **eliminate the jitter**
- The parameters of the copier can be found with operations over an **algebraic ring**
- Published on IEEE Transactions of Computers, 2023
- Some Python code implements the whole machinery