王松年 第5次作业

2.40 解:

由题得:

$$(P\vdots E)=\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\2&-1&0&0&1&0\\2&1&1&0&0&1\end{array}\right]\xrightarrow[r_3-2r_1]{r_2-2r_1}\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&-1&0&-2&1&0\\0&1&1&-2&0&1\end{array}\right]\xrightarrow{r_3+r_2}\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&-1&0&-2&1&0\\0&0&1&-4&1&1\end{array}\right]$$

$$\xrightarrow{r_2\times(-1)}\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&0&2&-1&0\\0&0&1&-4&1&1\end{array}\right]$$ ，∴ P可逆，且 $P^{-1}=\begin{bmatrix}1&0&0\\2&-1&0\\-4&1&1\end{bmatrix}$

∵ $AP=PB$ ∴在等式两边同时右乘 P^{-1} 得 $A=PBP^{-1}$

∴ $A=PBP^{-1}$

$$=\begin{bmatrix}1&0&0\\2&-1&0\\2&1&1\end{bmatrix}\cdot\begin{bmatrix}1&0&0\\0&0&0\\0&0&-1\end{bmatrix}\cdot\begin{bmatrix}1&0&0\\2&-1&0\\-4&1&1\end{bmatrix}=\begin{bmatrix}1&0&0\\2&0&0\\2&0&-1\end{bmatrix}\cdot\begin{bmatrix}1&0&0\\2&-1&0\\-4&1&1\end{bmatrix}=\begin{bmatrix}1&0&0\\2&0&0\\6&-1&-1\end{bmatrix}$$

$A^2=(PB\cdot P^{-1})\cdot(PB\cdot P^{-1})=P\cdot B\cdot(P\cdot P^{-1})\cdot B\cdot P^{-1}=PB^2P^{-1}$

…… 同理可推出 $A^5=PB^5P^{-1}$

∵ B为对角矩阵

∴ $B^5=\begin{bmatrix}1^5&0&0\\0&0&0\\0&0&(-1)^5\end{bmatrix}=\begin{bmatrix}1&0&0\\0&0&0\\0&0&-1\end{bmatrix}=B$ ∴ $A^5=PB^5P^{-1}=PBP^{-1}=A$

2.41 证明:

① 若矩阵M可逆，则有 $M^{-1}=\begin{bmatrix}X&Y\\Z&W\end{bmatrix}$ 使得 $M\cdot M^{-1}=E$

即: $\begin{bmatrix}A&0\\C&B\end{bmatrix}\cdot\begin{bmatrix}X&Y\\Z&W\end{bmatrix}=\begin{bmatrix}E&0\\0&E\end{bmatrix}$

$$\begin{bmatrix}A&0\\C&B\end{bmatrix}\cdot\begin{bmatrix}X&Y\\Z&W\end{bmatrix}=\begin{bmatrix}AX&AY\\CX+BZ&CY+BW\end{bmatrix}=\begin{bmatrix}E&0\\0&E\end{bmatrix}$$

即 $AX=E$，所以A可逆，且 $X=A^{-1}$； $AY=0,\Rightarrow A^{-1}AY=A^{-1}\cdot 0\Rightarrow Y=0$

∴ $CY+BW=BW=E$，∴ B可逆且 $W=B^{-1}$

$CX+BZ=0$，即 $BZ=-CX=-CA^{-1}\Rightarrow Z=-B^{-1}\cdot C\cdot A^{-1}$

∴ $M^{-1}=\begin{bmatrix}A^{-1}&0\\-B^{-1}CA^{-1}&B^{-1}\end{bmatrix}$

②.

由①得：$M^{-1}=\begin{bmatrix} A^{-1} & 0 \\ -B^{-1}CA^{-1} & B^{-1} \end{bmatrix}$

若A和B都可逆，则

$$M\cdot M^{-1}=\begin{bmatrix} A & 0 \\ C & B \end{bmatrix}\cdot\begin{bmatrix} A^{-1} & 0 \\ -B^{-1}CA^{-1} & B^{-1} \end{bmatrix}$$

$$=\begin{bmatrix} A\cdot A^{-1} & 0 \\ CA^{-1}-B(B^{-1}CA^{-1}) & B\cdot B^{-1} \end{bmatrix}$$

∵ A, B可逆 ∴ $A\cdot A^{-1}=E \quad B\cdot B^{-1}=E$

$CA^{-1}-B(B^{-1}CA^{-1})=CA^{-1}-CA^{-1}=0$

即 $M\cdot M^{-1}=\begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}$. 证毕。

例2.42 解：

$$(A\,\vdots\,E)=\left(\begin{array}{ccc|ccc} 0 & 2 & -1 & 1 & 0 & 0 \\ 1 & -3 & 2 & 0 & 1 & 0 \\ 1 & -1 & 2 & 0 & 0 & 1 \end{array}\right)\xrightarrow{r_1\leftrightarrow r_2}\begin{bmatrix} 1 & -3 & 2 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 0 & 1 \end{bmatrix}\xrightarrow{r_3-r_1}$$

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 1 \end{array}\right)\xrightarrow{r_3-r_2}\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right)\xrightarrow[r_1-2r_3]{r_2+r_3}\left(\begin{array}{ccc|ccc} 1 & -3 & 0 & 2 & 3 & -2 \\ 0 & 2 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{r_2\times\frac{1}{2}}\left(\begin{array}{ccc|ccc} 1 & -3 & 0 & 2 & 3 & -2 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right)\xrightarrow{r_1+3r_2}\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & \frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right)$$

∴ A可逆，$A^{-1}=\begin{pmatrix} 2 & \frac{3}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ -1 & -1 & 1 \end{pmatrix}$

例2.43 证明： $(a^3-b^3)=(a-b)(a^2+ab+b^2)$

∵ $A^3=0$ ∴ $A^3-8I_n=A^3-(2I_n)^3=8I_n$

$A^3-(2I_n)^3=(A-2I_n)\cdot(A^2+2A+4I_n)=8I_n$

即：$(A-2I_n)\cdot\dfrac{(A^2+2A+4I_n)}{8}=I_n$

∴ $A-2I_n$ 可逆，且 $(A-2I_n)^{-1}=\frac{1}{8}(A^2+2A+4I_n)$

例2.44 课本 P105 10.

例2.45

正确，

理由：若A,B可交换，则 $AB=BA$，两边同时取逆：$(AB)^{-1}=(BA)^{-1}$

即 $B^{-1}A^{-1}=A^{-1}B^{-1}$，所以 A^{-1}, B^{-1} 可交换。

同理，若 A^{-1}, B^{-1} 可交换，则 $A^{-1}B^{-1}=B^{-1}A^{-1}$，两边同时取逆：

$(A^{-1}B^{-1})^{-1}=(B^{-1}A^{-1})^{-1}$，即 $(B^{-1})^{-1}\cdot(A^{-1})^{-1}=(A^{-1})^{-1}(B^{-1})^{-1} \Rightarrow BA=AB$.

所以A,B可交换。

例2.46. 解：

A为对角阵，$\therefore A^{-1}=\begin{bmatrix}3&0&0\\0&4&0\\0&0&7\end{bmatrix}$，由题得：$A^{-1}BA-BA=6A$

即 $(A^{-1}-E)BA=6A$，同时右乘 A^{-1}，$(A^{-1}-E)B=6E$

同时左乘 $(A^{-1}-E)^{-1}$，$B=6(A^{-1}-E)^{-1}$

$A^{-1}-E=\begin{bmatrix}2&0&0\\0&3&0\\0&0&6\end{bmatrix}$，对角阵 $(A^{-1}-E)^{-1}=\begin{bmatrix}1/2&0&0\\0&1/3&0\\0&0&1/6\end{bmatrix}$

$\therefore B=6(A^{-1}-E)^{-1}=\begin{bmatrix}3&0&0\\0&2&0\\0&0&1\end{bmatrix}$

例2.47. 解：

由题得：$\alpha\cdot\alpha^T=\begin{bmatrix}a\\0\\\vdots\\a\end{bmatrix}[a\ 0\cdots0\ a]=\begin{bmatrix}a^2&0&\cdots&0&a^2\\0&0&\cdots&0&0\\\vdots&&\ddots&&\vdots\\0&\cdots&\cdots&0&0\\a^2&0&\cdots&0&a^2\end{bmatrix}$

$\therefore A=E-\alpha\alpha^T=\begin{bmatrix}1-a^2&0&\cdots&0&-a^2\\0&1&\cdots&0&0\\\vdots&&\ddots&&\vdots\\0&&&1&0\\-a^2&0&\cdots&0&1-a^2\end{bmatrix}$ $B=E+\frac{1}{a}\alpha\alpha^2=\begin{bmatrix}1+a&0&\cdots&0&a\\0&1&\cdots&0&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&1&0\\a&0&\cdots&0&1+a\end{bmatrix}$

.A的逆矩阵为B，即 $A\cdot B=B\cdot A=E$

$$AB=\begin{bmatrix}(1-a^2)(1+a)-a^3&0&\cdots&0&(1-a^2)\cdot a-a^2(1+a)\\0&1&\cdots&0&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&1&0\\-a^2(1+a)+a(1-a^2)&0&\cdots&0&-a^3+(1-a^2)(1+a)\end{bmatrix}=\begin{bmatrix}1&&&&\\&1&&&\\&&1&&\\&&&1&\\&&&&\ddots&\\&&&&&1\end{bmatrix}$$

所以有：$(1+a^2)(1+a)-\cancel{a^3}=1$ ①

$(1+a^2)a-a^2(1+a)=0$ ②

由题目已知：$a<0$ ③

联立①②③解得 $a=-1$.

例 2.48 解：

由题得：$A=\begin{pmatrix} A_{11} & 0 \\ 0 & A_{22} \end{pmatrix}$ 所以 $A^{-1}=\begin{pmatrix} A_{11}^{-1} & 0 \\ 0 & A_{22}^{-1} \end{pmatrix}$

其中：$A_{11}=\begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$ $A_{22}=\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$

$$(A_{11}\vdots E)=\left(\begin{array}{cc:cc} 5 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right)\xrightarrow[r_2/2]{r_1/5}\left(\begin{array}{cc:cc} 1 & \frac{2}{5} & \frac{1}{5} & 0 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} \end{array}\right)\xrightarrow{r_2-r_1}\left(\begin{array}{cc:cc} 1 & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & \frac{1}{10} & -\frac{1}{5} & \frac{1}{2} \end{array}\right)$$

$$\xrightarrow{r_1-4r_2}\left(\begin{array}{cc:cc} 1 & 0 & 1 & -2 \\ 0 & \frac{1}{10} & -\frac{1}{5} & \frac{1}{2} \end{array}\right)\xrightarrow{r_2\times 10}\left(\begin{array}{cc:cc} 1 & 0 & 1 & -2 \\ 0 & 1 & -2 & 5 \end{array}\right)\quad \therefore A_{11}^{-1}=\begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$$

$$(A_{22}\vdots E)=\left(\begin{array}{cc:cc} 1 & -2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right)\xrightarrow{r_2-r_1}\left(\begin{array}{cc:cc} 1 & -2 & 1 & 0 \\ 0 & 3 & -1 & 1 \end{array}\right)\xrightarrow{r_2\times\frac{1}{3}}\left(\begin{array}{cc:cc} 1 & -2 & 0 & 1 \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} \end{array}\right)$$

$$\xrightarrow{r_1+2r_2}\left(\begin{array}{cc:cc} 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} \end{array}\right)\quad \therefore A_{22}^{-1}=\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

$$\therefore A^{-1}=\begin{pmatrix} 1 & -2 & 0 & 0 \\ -2 & 5 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & -\frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

例 2.49 解：

由题得：

$$E+A=C=\begin{bmatrix} 2 & 0 & 0 & 0 \\ -2 & 4 & 0 & 0 \\ 0 & -4 & 6 & 0 \\ 0 & 0 & -6 & 8 \end{bmatrix}\quad E-A=D=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & -2 & 0 & 0 \\ 0 & 4 & -4 & 0 \\ 0 & 0 & 6 & -6 \end{bmatrix}$$

$C=\begin{bmatrix} C_{11} & 0 \\ C_{21} & C_{22} \end{bmatrix}$，其中 $C_{11}=\begin{bmatrix} 2 & 0 \\ -2 & 4 \end{bmatrix}$ $C_{21}=\begin{bmatrix} 0 & -4 \\ 0 & 0 \end{bmatrix}$ $C_{22}=\begin{bmatrix} 6 & 0 \\ -6 & 8 \end{bmatrix}$

形如 $A=\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$（其中 A_{11}, A_{22} 为方阵）的逆为 $A^{-1}=\begin{bmatrix} A_{11}^{-1} & 0 \\ -A_{22}^{-1}A_{21}A_{11}^{-1} & A_{22}^{-1} \end{bmatrix}$ …Ⅰ式

可以看出，方阵 C, C_{11}, C_{22} 都满足Ⅰ式。

所以

$$C_{11}^{-1}=\begin{bmatrix}\frac{1}{2} & 0\\ -\frac{1}{4}\times(-2)\times\frac{1}{2} & \frac{1}{4}\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0\\ \frac{1}{4} & \frac{1}{4}\end{bmatrix}$$

$$C_{22}^{-1}=\begin{bmatrix}\frac{1}{6} & 0\\ -\frac{1}{8}\times(-6)\times\frac{1}{6} & \frac{1}{8}\end{bmatrix}=\begin{bmatrix}\frac{1}{6} & 0\\ \frac{1}{8} & \frac{1}{8}\end{bmatrix}$$

$$-C_{22}^{-1}\cdot C_{21}\cdot C_{11}^{-1}=-\begin{bmatrix}\frac{1}{6} & 0\\ \frac{1}{8} & \frac{1}{8}\end{bmatrix}\cdot\begin{bmatrix}0 & -4\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}\frac{1}{2} & 0\\ \frac{1}{4} & \frac{1}{4}\end{bmatrix}=-\begin{bmatrix}0 & -\frac{2}{3}\\ 0 & -\frac{1}{2}\end{bmatrix}\begin{bmatrix}\frac{1}{2} & 0\\ \frac{1}{4} & \frac{1}{4}\end{bmatrix}$$

$$=-\begin{bmatrix}-\frac{1}{6} & -\frac{1}{6}\\ -\frac{1}{8} & -\frac{1}{8}\end{bmatrix}=\begin{bmatrix}\frac{1}{6} & \frac{1}{6}\\ \frac{1}{8} & \frac{1}{8}\end{bmatrix}$$

所以 $(E+A)^{-1}=C^{-1}=\begin{bmatrix}C_{11}^{-1} & 0\\ -C_{22}^{-1}C_{21}C_{11}^{-1} & C_{22}^{-1}\end{bmatrix}$

$$=\begin{bmatrix}\frac{1}{2} & 0 & 0 & 0\\ \frac{1}{4} & \frac{1}{4} & 0 & 0\\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0\\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8}\end{bmatrix}$$

所以 $B=(E+A)^{-1}\cdot(E-A)=\begin{bmatrix}\frac{1}{2} & 0 & 0 & 0\\ \frac{1}{4} & \frac{1}{4} & 0 & 0\\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0\\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8}\end{bmatrix}\cdot\begin{bmatrix}0 & 0 & 0 & 0\\ 2 & -2 & 0 & 0\\ 0 & 4 & -4 & 0\\ 0 & 0 & 6 & -6\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 0\\ \frac{1}{2} & -\frac{1}{2} & 0 & 0\\ \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} & 0\\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4}\end{bmatrix}$

$E+B=\begin{bmatrix}1 & 0 & 0 & 0\\ \frac{1}{2} & \frac{1}{2} & 0 & 0\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0\\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}\end{bmatrix}=F=\begin{bmatrix}F_{11} & 0\\ F_{21} & F_{22}\end{bmatrix}$，其中 $F_{11}=\begin{bmatrix}1 & 0\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$ $F_{21}=\begin{bmatrix}\frac{1}{3} & \frac{1}{3}\\ \frac{1}{4} & \frac{1}{4}\end{bmatrix}$

$F_{22}=\begin{bmatrix}\frac{1}{3} & 0\\ \frac{1}{4} & \frac{1}{4}\end{bmatrix}$

所以 $F_{11}^{-1}=\begin{bmatrix}1^{-1} & 0\\ -(\frac{1}{2})^{-1}\times\frac{1}{2}\times(1)^{-1} & (\frac{1}{2})^{-1}\end{bmatrix}=\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}$

$$F_{22}^{-1}=\begin{bmatrix}(\frac{1}{3})^{-1} & 0\\ -(\frac{1}{4})^{-1}\times\frac{1}{4}\times(\frac{1}{3})^{-1} & (\frac{1}{4})^{-1}\end{bmatrix}=\begin{bmatrix}3 & 0\\ -3 & 4\end{bmatrix}$$

$$-F_{22}^{-1}\cdot F_{21}\cdot F_{11}^{-1}=-\begin{bmatrix}3 & 0\\ -3 & 4\end{bmatrix}\begin{bmatrix}\frac{1}{3} & \frac{1}{3}\\ \frac{1}{4} & \frac{1}{4}\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}=-\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}=\begin{bmatrix}0 & -2\\ 0 & 0\end{bmatrix}$$

所以 $(E+B)^{-1}=F^{-1}=\begin{bmatrix}F_{11}^{-1} & 0\\ -F_{22}^{-1}F_{21}F_{11}^{-1} & F_{22}^{-1}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ -1 & 2 & 0 & 0\\ 0 & -2 & 3 & 0\\ 0 & 0 & -3 & 4\end{bmatrix}$

例2.50 解:

(1) 证明: 等式两边左乘 A: $2B = AB - 4A$

$\therefore\ AB - 2B = 4A$

$(A-2E)B = 4A$

同时右乘 $\frac{1}{4}A^{-1}$: $(A-2E)(\frac{1}{4}BA^{-1}) = E$

$\therefore$ $A-2E$可逆. 其逆矩阵为 $\frac{1}{4}BA^{-1}$.

(2) $\because\ 2A^{-1}B = B - 4E$ 同时右乘 B^{-1}:

$2A^{-1} = E - 4B^{-1}$ $\quad\therefore\ A^{-1} = \frac{1}{2}(E - 4B^{-1})$

$\because\ B = \begin{pmatrix} 1 & -2 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ $\therefore\ B = \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix}$, 其中 $B_{11} = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}$ $B_{22} = [2]$

$$(B_{11} \vdots E) = \left(\begin{array}{cc:cc} 1 & -2 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \xrightarrow{r_2 - r_1} \left(\begin{array}{cc:cc} 1 & -2 & 1 & 0 \\ 0 & 4 & -1 & 1 \end{array}\right) \xrightarrow{r_2 \times \frac{1}{4}} \left(\begin{array}{cc:cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{4} \end{array}\right) \xrightarrow{r_1 + 2r_2} \left(\begin{array}{cc:cc} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{4} & \frac{1}{4} \end{array}\right)$$

$\therefore\ B_{11}^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$

$\therefore\ B^{-1} = \begin{bmatrix} B_{11}^{-1} & 0 \\ 0 & B_{22}^{-1} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$ $\quad E - 4B^{-1} = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\therefore\ A = (A^{-1})^{-1} = [\frac{1}{2}(E - 4B^{-1})]^{-1} = 2(E - 4B^{-1})^{-1}$

$(E - 4B^{-1}) = C = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix}$, 其中 $C_{11} = \begin{bmatrix} -1 & -2 \\ 1 & 0 \end{bmatrix}$ $C_{22} = [-1]$

$$(C_{11} \vdots E) = \left(\begin{array}{cc:cc} -1 & -2 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{r_2 + r_1} \left(\begin{array}{cc:cc} -1 & -2 & 1 & 0 \\ 0 & -2 & 1 & 1 \end{array}\right) \xrightarrow[r_2 \times (-\frac{1}{2})]{r_1 \times (-1)} \left(\begin{array}{cc:cc} 1 & 2 & -1 & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \end{array}\right)$$

$$\xrightarrow{r_1 - 2r_2} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 1 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \end{array}\right) \quad \therefore\ C_{11}^{-1} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

$\therefore\ (E - 4B^{-1})^{-1} = C^{-1} = \begin{bmatrix} C_{11}^{-1} & 0 \\ 0 & C_{22}^{-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\therefore\ A = 2(E - 4B^{-1})^{-1} = \begin{bmatrix} 0 & 2 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$