

Master's Thesis Defense

The Prize-Collecting Steiner Tree Problem and Related Problems

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The fun we are having today:

- 1 Introduction
- 2 The Prize-Collecting Steiner Tree Problem
- 3 Other Ways of Collecting Prize
- 4 The Median Tree Problem
- 5 Summary / Reflections

Outline

- 1 Introduction
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Overview

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What happened in the last six months?

- I read a stack of research papers about the PCSTP.
- I read a smaller stack of research papers about related problems.
- *I was indecisive.*
- I worked on a solver for the Median Tree Problem.

Motivation

Why the PCSTP?

- It's a hard problem. Hard problems are worth solving.
- It relates to the Steiner Tree Problem, and shares some of its characteristics.

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Why the PCSTP?

- It's a hard problem. Hard problems are worth solving.
- It relates to the Steiner Tree Problem, and shares some of its characteristics.
- Finally: *It is an interesting problem.*

Motivation

The Survey

What makes a survey necessary?

Motivation

The Survey

What makes a survey necessary?

- A lot is written about the PCSTP, but it is unstructured and disjoint.
- Some of these papers touch on very complex subjects and are sometimes short and unintuitive.
- The PCSTP is a good “case study” for an ILP problem.

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The Survey

What makes a survey necessary?

- A lot is written about the PCSTP, but it is unstructured and disjoint.
 - Some of these papers touch on very complex subjects and are sometimes short and unintuitive.
 - The PCSTP is a good “case study” for an ILP problem.
- **There is a lot to learn.**

Motivation

Why the Median Tree Problem?

Motivation

Why the Median Tree Problem?

We'll get back to that later.

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The Prize-Collecting Steiner Tree Problem

Short History

- Defined by Egon Balas in 1988 as a side effect.
- Subject to steady focus in the late 90's and early 00's.
- One of the subjects of the 11th DIMACs Implementation Challenge in 2014.

The Prize-Collecting Steiner Tree Problem

Problem Definition

Given an undirected graph

$$G = (V, E, c, p)$$

where $c : E \rightarrow \mathbb{R}^+$ defines edge weights, and $p : V \rightarrow \mathbb{R}^+$ defines vertex *prizes*, then the solution to the *PCSTP* is a tree

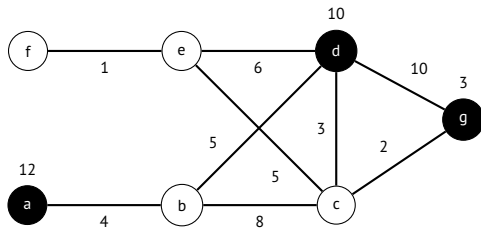
$$T = (V_T, E_T, c, p) \subseteq G$$

which minimizes

$$c(T) = \min_T \sum_{(i,j) \in E_T} c_{ij} + \sum_{v \in (V \setminus V_T)} p_v.$$

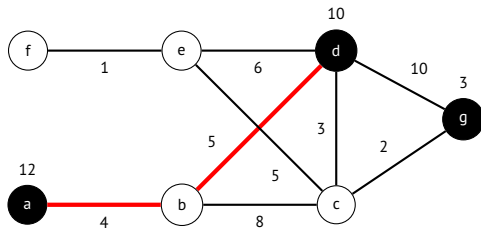
The Prize-Collecting Steiner Tree Problem

Example



The Prize-Collecting Steiner Tree Problem

Example



The Survey

Quick Summary

Contents of the Survey:

- ① The history of solving the PCSTP.
- ② Preprocessing routines.
- ③ Two heuristic algorithms. LP-based and search based.
- ④ An approximation algorithm: the GW Algorithm.
- ⑤ How to separate GSECs.
- ⑥ The DHEA and SCIP-Jack solvers.

The Survey

Main Points

What have we learned?

The Survey

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- 1 The PCSTP is well covered all things considered.

The Survey

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- ② Preprocessing is *very* good for the PCSTP.

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- ① The PCSTP is well covered all things considered.
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- ③ Directed formulations of the problem are preferable for branch and bound.

The Survey

Main Points

What have we learned?

- ① The PCSTP is well covered all things considered.
- ② Preprocessing is *very* good for the PCSTP.
- ③ Directed formulations of the problem are preferable for branch and bound.
- ④ Heuristics are aplenty.

The Survey

Main Points

Curiosities

- Not a lot of focus on applications.
- Linear progress?
- Somewhat general methods besides preprocessing.

The Survey

Main Points

Curiosities

- Not a lot of focus on applications.
- Linear progress?
- Somewhat general methods besides preprocessing.

On to other problems.

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Prize-Collecting Tours

Three main variants:

- ① The Prize-Collecting Travelling Salesman Problem
- ② The Orienteering Problem
- ③ The Profitable Tour Problem

Prize-Collecting Tours

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Some Notes:

- Main point of the Balas paper in 88.
- Shares approximation algorithms.
- Apart from the OP, not well covered.

Median Subgraphs

- Assignment Problem.
- Different shapes of facility.
- Median Trees.

Summary

- Two axis' of similarity: structure and prize function.
- PCSTP is the most well researched problem in the family.

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What now?

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Motivation

Options

- Stay with the PCSTP.
- Look at the Profitable Tour Problem.
- Look at the Median Tree Problem.

Motivation

Median Trees over Prize-Collecting Tours

- A feasible solution to the PCSTP is a feasible solution to the MTP.
- Collect Prize vs. Assignment: similar – although not the exact same – trade offs.

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Median Trees over Prize-Collecting Tours

- A feasible solution to the PCSTP is a feasible solution to the MTP.
- Collect Prize vs. Assignment: similar – although not the exact same – trade offs.

And a splash of subjectivity.

Median Tree Problem

Problem Definition

Let $G = (V, E, c, d)$ be an undirected graph. Denote $c : E \rightarrow \mathbb{R}^+$ as an *edge cost* function and $d : V \times V \rightarrow \mathbb{R}^+$ be an *assignment cost* function where we have

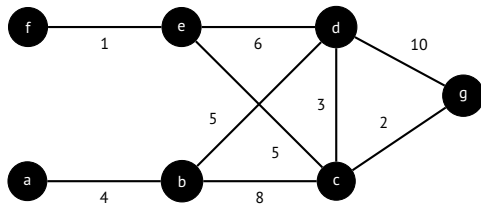
$$d_{ii} = 0.$$

Then the *Median Tree Problem* is defined as finding a *connected subgraph* $T = (V_T, E_T)$ of G where $V_T \subseteq V$ and $E_T \subseteq E$ which minimises the cost function,

$$c(T) = \sum_{ij \in E_T} c_{ij} + \sum_{i \in V} \min_{j \in V_T} d_{ij}.$$

Median Tree Problem

Example



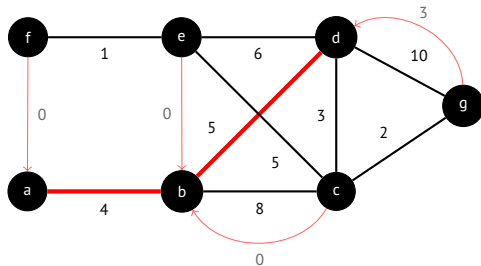
Median Tree Problem

Example

d_{ij}	a	b	c	d	e	f	g
a	0	12	12	12	12	12	12
b	0	0	0	0	0	0	0
c	0	0	0	0	0	0	0
d	10	10	10	0	10	10	10
e	0	0	0	0	0	0	0
f	0	0	0	0	0	0	0
g	3	3	3	3	3	3	0

Median Tree Problem

Example



Median Tree Problem

ILP Formulation

$$\begin{array}{ll} \underset{\mathbf{x}, \mathbf{y}}{\text{minimize}} & \sum_{ij \in E} c_{ij} x_{ij} + \sum_{i, j \in V} d_{ij} y_{ij} \end{array} \quad (1a)$$

$$\begin{array}{ll} \text{subject to} & \sum_{ij \in E} x_{ij} = \sum_{i \in V} y_{ii} - 1 \end{array} \quad (1b)$$

$$x(E(S)) \leq \sum_{i \in S \setminus \{s\}} y_{ii} \quad \forall S \subseteq V, s \in S \quad (1c)$$

$$\sum_{j \in V} y_{kj} = 1 \quad \forall k \in V \quad (1d)$$

$$y_{ik} \leq y_{kk} \quad \forall i, k \in V \quad (1e)$$

$$y_{kk} \leq \sum_{i \in \delta(k)} x_{ik} \quad \forall k \in V \quad (1f)$$

$$\mathbf{x} \in \mathbb{B}^{|E|} \quad (1g)$$

$$\mathbf{y} \in \mathbb{B}^{|V \times V|} \quad (1h)$$

Median Tree Problem

Valid Inequalities

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Forced self-assignment:

$$y_{ii} \geq x_{ji} \quad \forall i \in V, \forall j \in \delta(i).$$

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Degree of Nonterminals:

$$\sum_{j \in \delta(i)} x_{ij} \geq 2x_{ik} \quad \forall i \in N, \forall k \in \delta(i)$$

The Solver

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- Based on the Gurobi MIP solver.
- Callbacks written in Python 3.
- Applies methods from the PCSTP survey:
 - Primal heuristic from the DHEA solver.
 - User cuts based on GSEC separation from an article by Lucena and Resende for the PCSTP.

Results

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What did I manage to do?

haha

Improvements

The Survey

Improvements

The Solver

Further Work

Solve a real problem

haha

Lessons Learned

Postface

Questions?