Master's Thesis Defense

The Prize-Collecting Steiner Tree Problem and Related Problems

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The fun we are having today:

1. Introduction
2. The Prize-Collecting Steiner Tree Problem
3. The Survey
4. The Median Tree Problem
5. Reflections

Outline

1. Introduction
2. The Prize-Collecting Steiner Tree Problem
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- I read a smaller stack of research papers about problems related to the PCSTP.
- *I was indecisive.*
- I worked on a solver for the Median Tree Problem.

Why the PCSTP?

- It is an NP-hard problem. Hard problems are worth solving.
- It is typically solved using ILP.
- It relates to the Steiner Tree Problem, and shares some of its characteristica.

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- It relates to the Steiner Tree Problem, and shares some of its characteristica.
- Finally: *I think it is an interesting problem.*

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– There is a lot to learn.

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We'll get back to that later.

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- One of the subjects of the 11th DIMACs Implementation Challenge in 2014.

The Prize-Collecting Steiner Tree Problem

Problem Definition

Given an undirected graph

$$G = (V, E, c, p)$$

where $c : E \to \mathbb{R}^+$ defines edge weights, and $p : V \to \mathbb{R}^+$ defines vertex *prizes*, then the solution to the *PCSTP* is a tree

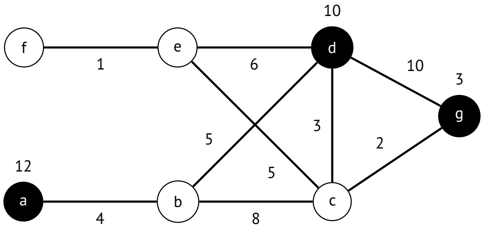
$$T = (V_T, E_T, c, p) \subseteq G$$

which minimizes

$$c(T) = \min_T \sum_{(i,j) \in E_T} c_{ij} + \sum_{v \in (V \setminus V_T)} p_v.$$

The Prize-Collecting Steiner Tree Problem

Example



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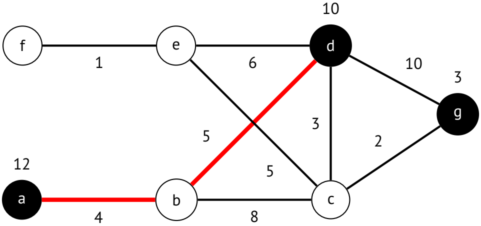

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The Survey

Quick Summary

Contents of the PCSTP Survey:

1. The history of solving the PCSTP.
2. Preprocessing routines.
3. Two heuristic algorithms. LP-based and search based.
4. An approximation algorithm: the GW Algorithm.
5. How to separate GSECs.
6. The DHEA and SCIP-Jack solvers.

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The Survey

Main Points

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The Survey

Main Points

What have we learned?

1. The PCSTP is well covered all things considered.
2. Preprocessing is *very* good for the PCSTP.
3. Directed formulations of the problem are preferable for branch and bound.
4. Heuristics are aplenty.

Curiosities

- Not a lot of focus on applications.
- Linear progress?
- Somewhat general methods besides preprocessing.

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On to other problems.

Three main variants:

1. The Prize-Collecting Travelling Salesman Problem
2. The Orienteering Problem
3. The Profitable Tour Problem

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1. The Prize-Collecting Travelling Salesman Problem
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Some Notes:

- Main point of the Balas paper in 88.
- Shares approximation algorithms.
- Apart from the OP, not well covered.

- Assignment Problem.
- Different shapes of facility.
- Median Trees.

Summary

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What now?

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I wanted to do some practical work.

The Median Tree Problem

Motivation

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Options

- Stay with the PCSTP.

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- Stay with the PCSTP.
- Look at the Profitable Tour Problem.

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Options

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- Look at the Median Tree Problem.

Motivation

Median Trees over Prize-Collecting Tours

- A feasible solution to the PCSTP is a feasible solution to the MTP.
- Collect Prize vs. Assignment: similar – although not the exact same – trade offs.

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Median Trees over Prize-Collecting Tours

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And a splash of subjectivity.

Let $G = (V, E, c, d)$ be an undirected graph. Denote $c : E \to \mathbb{R}^+$ as an *edge cost* function and $d : V \times V \to \mathbb{R}^+$ be an *assignment cost* function where we have

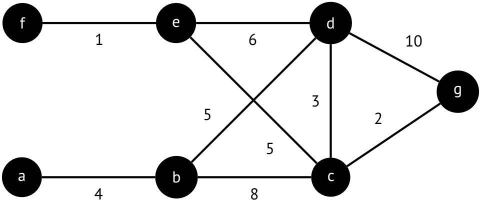
$$d_{ii} = 0.$$

Then the *Median Tree Problem* is defined as finding a *connected subgraph* $T = (V_T, E_T)$ of G where $V_T \subseteq V$ and $E_T \subseteq E$ which minimises the cost function,

$$c(T) = \sum_{ij \in E_T} c_{ij} + \sum_{i \in V} \min_{j \in V_T} d_{ij}.$$

Median Tree Problem

Example



Median Tree Problem

Example

d_{ij}	a	b	c	d	e	f	g
a	0	12	12	12	12	12	12
b	0	0	0	0	0	0	0
c	0	0	0	0	0	0	0
d	10	10	10	0	10	10	10
e	0	0	0	0	0	0	0
f	0	0	0	0	0	0	0
g	3	3	3	3	3	3	0

Median Tree Problem

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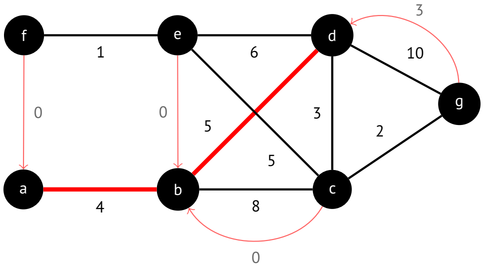

Median Tree Problem

ILP Formulation

$$\begin{aligned}
\underset{\mathbf{x},\mathbf{y}}{\text{minimize}} \quad & \sum_{ij \in E} c_{ij} x_{ij} + \sum_{i,j \in V} d_{ij} y_{ij} && \text{(1a)}\\
\text{subject to} \quad & \sum_{ij \in E} x_{ij} = \sum_{i \in V} y_{ii} - 1 && \text{(1b)}\\
& x(E(S)) \leq \sum_{i \in S \setminus \{s\}} y_{ii} \ \forall S \subseteq V, s \in S && \text{(1c)}\\
& \sum_{j \in V} y_{kj} = 1 \qquad \forall k \in V && \text{(1d)}\\
& y_{ik} \leq y_{kk} \qquad \forall i, k \in V && \text{(1e)}\\
& y_{kk} \leq \sum_{i \in \delta(k)} x_{ik} \qquad \forall k \in V && \text{(1f)}\\
& \mathbf{x} \in \mathbb{B}^{|E|} && \text{(1g)}\\
& \mathbf{y} \in \mathbb{B}^{|V \times V|} && \text{(1h)}
\end{aligned}$$

Median Tree Problem

Valid Inequalities

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Forced self-assignment:

$$y_{ii} \geq x_{ji} \qquad \forall i \in V, \forall j \in \delta(i).$$

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Degree of Nonterminals:

$$\sum_{j \in \delta(i)} x_{ij} \geq 2x_{ik} \qquad \forall i \in N,\ \forall k \in \delta(i)$$

Median Tree Problem

The Solver

- Based on the Gurobi MIP solver.

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- Callbacks written in Python 3.
- Applies methods from the PCSTP survey:
  - Primal heuristic from the DHEA solver.
  - User cuts based on GSEC separation from an article by Lucena and Resende for the PCSTP.

Results

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- Primal Heuristics: Ambivalent performance

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- Valid Inequalities: Ambivalent performance

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What did I manage to do?

-

Reflections

Improvements

Reflections

Improvements

Reflections

Futher Work

haha

Reflections

Lessions Learned

Reflections

Lessions Learned

Postface

Questions?