Master's Thesis Defense

The Prize-Collecting Steiner Tree Problem and Related Problems

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The fun we are having today:

Outline

Overview

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- I read a stack of research papers about the PCSTP.
- I read a smaller stack of research papers about related problems.
- *I was indecisive.*
- I worked on a solver for the Median Tree Problem.

Motivation

Why the PCSTP?

- It's a hard problem. Hard problems are worth solving.
- It relates to the Steiner Tree Problem, and shares some of its characteristica.

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- It's a hard problem. Hard problems are worth solving.
- It relates to the Steiner Tree Problem, and shares some of its characteristica.
- Finally: *It is an interesting problem.*

Motivation

The Survey

What makes a survey necessary?

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What makes a survey necessary?

- A lot is written about the PCSTP, but it is unstructured and disjoint.
- Some of these papers touch on very complex subjects and are sometimes short and unintuitive.
- The PCSTP is a good “case study” for an ILP problem.

What makes a survey necessary?

- A lot is written about the PCSTP, but it is unstructured and disjoint.
- Some of these papers touch on very complex subjects and are sometimes short and unintuitive.
- The PCSTP is a good "case study" for an ILP problem.

— **There is a lot to learn.**

Motivation

Why the Median Tree Problem?

Motivation

Why the Median Tree Problem?

We'll get back to that later.

Outline

The Prize-Collecting Steiner Tree Problem

Short History

- Defined by Egon Balas in 1988 as a side effect.
- Subject to steady focus in the late 90’s and early 00’s.
- One of the subjects of the 11th DIMACs Implementation Challenge in 2014.

Given an undirected graph

$$G = (V, E, c, p)$$

where $c : E \rightarrow \mathbb{R}^+$ defines edge weights, and $p : V \rightarrow \mathbb{R}^+$ defines vertex *prizes*, then the solution to the *PCSTP* is a tree

$$T = (V_T, E_T, c, p) \subseteq G$$

which minimizes

$$c(T) = \min_T GW(T) = \sum_{(i,j) \in E_T} c_{ij} + \sum_{v \in (V \setminus V_T)} p_v.$$

I looked at:

1. The history of solving the PCSTP.
2. Two heuristic algorithms. LP-based and search based.
3. An approximation algorithm: the GW Algorithm.
4. How to separate GSECs.
5. The DHEA and SCIP-Jack solvers.

The Survey

Main Points - What have we learned?

The PCSTP is well covered.

Points

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Points

1. Preprocessing is *very* good for the PCSTP.
2. Directed formulations of the problem are preferable for branch and bound.
3. Heuristics are aplenty.

The Survey

Main Points - Curiosities

- Not a lot of focus on applications.
- Linear progress?
- Somewhat general methods besides preprocessing.

Outline

Prize-Collecting Tours

Three main variants:

1. The Prize-Collecting Travelling Salesman Problem
2. The Orienteering Problem
3. The Profitable Tour Problem

Prize-Collecting Tours

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Some notes:

- Main point of the Balas paper in 88.
- Shares approximation algorithms.
- Apart from the OP, not well covered.

Median Subgraphs

- Assignment Problem.
- Different shapes of facility.
- Median Trees.

Summary

- Two axis' of similarity: structure and prize function.
- PCSTP is the most well researched problem in the family.

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What now?

Outline

Motivation

Options

- Stay with the PCSTP.
- Look at the Profitable Tour Problem.
- Look at the Median Tree Problem.

Motivation

Median Trees over Prize-Collecting Tours

- A feasible solution to the PCSTP is a feasible solution to the MTP.
- Collect Prize vs. Assignment: similar – although not the exact same – trade offs.

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- Collect Prize vs. Assignment: similar — although not the exact same — trade offs.

And a splash of subjectivity.

Median Tree Problem

Problem Definition

Let $G = (V, E, c, d)$ be an undirected graph. Denote $c : E \to \mathbb{R}^+$ as an *edge cost* function and $d : V \times V \to \mathbb{R}^+$ be an *assignment cost* function where we have

$$d_{ii} = 0.$$

Then the *Median Tree Problem* is defined as finding a *connected subgraph* $T = (V_T, E_T)$ of G where $V_T \subseteq V$ and $E_T \subseteq E$ which minimises the cost function,

$$c(T) = \sum_{ij \in E_T} c_{ij} + \sum_{i \in V} \min_{j \in V_T} d_{ij}.$$

Median Tree Problem

ILP Formulation

$$\begin{aligned}
\underset{\mathbf{x},\mathbf{y}}{\text{minimize}} \quad & \sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in V} d_{ij}y_{ij} && \text{(1a)}\\
\text{subject to} \quad & \sum_{ij\in E} x_{ij} = \sum_{i\in V} y_{ii} - 1 && \text{(1b)}\\
& x(E(S)) \leq \sum_{i\in S\setminus\{s\}} y_{ii} \; \forall S \subseteq V, s \in S && \text{(1c)}\\
& \sum_{j\in V} y_{kj} = 1 \qquad \forall k \in V && \text{(1d)}\\
& y_{ik} \leq y_{kk} \qquad \forall i,k \in V && \text{(1e)}\\
& y_{kk} \leq \sum_{i\in\delta(k)} x_{ik} \qquad \forall k \in V && \text{(1f)}\\
& \mathbf{x} \in \mathbb{B}^{|E|} && \text{(1g)}\\
& \mathbf{y} \in \mathbb{B}^{|V\times V|} && \text{(1h)}
\end{aligned}$$

Median Tree Problem

Valid Inequalities

Forced self-assignment:

$$y_{ii} \geq x_{ji} \qquad \forall i \in V, \forall j \in \delta(i).$$

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Degree of Nonterminals:

$$\sum_{j \in \delta(i)} x_{ij} \geq 2x_{ik} \qquad \forall i \in N,\ \forall k \in \delta(i)$$

The Solver

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- Based on the Gurobi MIP solver.
- Callbacks written in Python 3.
- Applies methods from the PCSTP survey:
  - Primal heuristic from the DHEA solver.
  - User cuts based on GSEC separation from an article by Lucena and Resende for the PCSTP.

Results

Outline

What did I manage to do?

haha

Improvements

The Survey

Improvements

The Solver

Further Work

Solve a real problem

haha

Lessons Learned

Postface

Questions?