FACULTY OF SCIENCE
UNIVERSITY OF COPENHAGEN

Master's Thesis

William Sprent – bsprent@gmail.com

Prize-Collecting Steiner Trees

From the Perspective of the Prize-Collecting Travelling Salesman Problem

Supervisors: Pawel Winter

Aug 2018

Abstract

bla bla bla

Contents

Chapter 1

Introduction

Chapter 2

Background

2.1 Combinatorial Optimisation

2.2 The Steiner Tree Problem

2.3 The Traveling Salesman Problem

bla bla

Chapter 3

Steiner Trees in Graphs

3.1 The Steiner Tree Problem

The Steiner Tree Problem (STP) is defined as follows. Given a graph $G = (V, N, E, c)$ with vertices V, terminal set $N \subseteq V$, edges E, and edge weights $c : E \to \mathbb{R}$, a Steiner Tree is a tree, $T \in E$, which spans N with minimal cost. We denote vertices which must be part of a feasible solution – that is $v \in N$ – as *terminals*, and vertice which are not required to be part of a solution – $v \in V \setminus N$ – as *Steiner points*.

When $N = V$, the STP is equivalent to the Minimum Spanning Tree Problen, and when $|N| = 2$ the STP is equivalent to finding the shortest path between two vertices. However, while these problems both have polynomial-time solutions, the STP is an NP-Hard problem, the decision variant being part of Edmund Karp's original 21 NP-Complete problems (Karp, 1972).

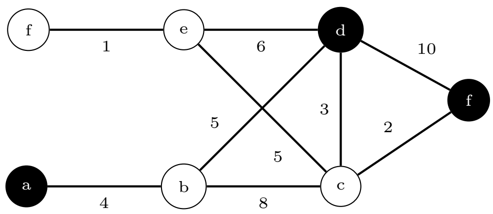


Figure 3.1: Instance of the Steiner Tree Problem. Terminals are coloured black and Steiner Points coloured white.

Figure 3.1 shows an instance of the STP with three terminals and four Steiner points. Since vertices A, C, and D are terminals, they must be spanned by any feasible solution. Figure 3.2a shows a feasible solution

$$T = \{(a, b), (b, d), (d, g)\}$$

with cost

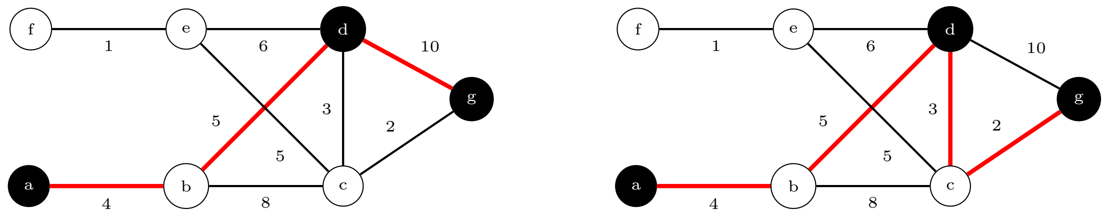
$$c(T) = 4 + 5 + 10 = 19.$$

However, T is not a Steiner tree as there exists at least one feasible solution with lower cost, i.e. the solution

$$T' = \{(a, b), (b, d), (d, c), (c, g)\}$$

in Figure 3.2b which has cost

$$c(T') = 4 + 5 + 3 + 2 = 14.$$



(a) Feasible but not optimal. (b) Minimal weight, optimal.

Figure 3.2: Solutions to the STP in 3.1. Red edges are part of the solution.

3.1.1 ILP Formulations

Cut Formulation Let x be a decision vector of length $|E|$ where $x_{ij} = 1$ implies that $(i,j) \in T$ and $x_{ij} = 0$ implies that $(i,j) \notin T$, and let c be a vector of node-weight s.t. $c_{ij} = c((i,j))$. Then define the function $x : E \to \mathbb{Z}$ as

$$x(E') = \sum_{i,j \in E'} x_{ij}$$

that is, $x(E)$ is equal to the number of selected edges in E. Finally let,

$$\delta(S) = \{(i,j) \mid i \in S \wedge j \in (E \setminus S)\}$$

be all edges which span the cut defined by S. Then we can formulate the STP as in ILP in terms of cuts (see Formulation 3.1).

$$\begin{aligned}
&\underset{x}{\text{minimize}} && c^T x && \text{(3.1a)}\\
&\text{subject to} && x(\delta(S)) \geq 1 \qquad \forall S \subset V && \text{(3.1b)}\\
& && \qquad\qquad\qquad\quad S \cap N \neq \emptyset &&\\
& && \qquad\qquad\qquad\quad S \cap (V \setminus N) \neq \emptyset &&\\
& && x \in \mathbb{B}. && \text{(3.1c)}
\end{aligned}$$

Formulation 3.1: The *Cut Formulation* of the STP.

3.2 Prize-Collecting Steiner Tree Problem

3.3 Steiner Aborescence Problem

3.4 Methods

3.4.1 Preprocessing

3.4.2 Primal Heuristics

3.4.3 Exact Frameworks

Bibliography

Richard M Karp. Reducibility among combinatorial problems. In *Complexity of computer computations*, pages 85–103. Springer, 1972.