

Master's Thesis Defense

The Prize-Collecting Steiner Tree Problem and Related Problems

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The fun we are having today:

- 1 Introduction
- 2 The Prize-Collecting Steiner Tree Problem
- 3 The Survey
- 4 The Median Tree Problem
- 5 Reflections

Outline

- 1 Introduction
- 2 The Prize-Collecting Steiner Tree Problem
- 3 The Survey
- 4 The Median Tree Problem
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Introduction

Overview

Initial Problem Statement

Apply results from research on the more covered Prize-Collecting Traveling Salesman Problem to the lesser covered Prize-Collecting Steiner Tree problem.

Introduction

Overview

What I spent six months doing?

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- I read a stack of research papers about the PCSTP.
- I read a smaller stack of research papers about problems related to the PCSTP.

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Overview

What I spent six months doing?

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- I read a smaller stack of research papers about problems related to the PCSTP.
- *I was indecisive.*
- I worked on a solver for the Median Tree Problem.

Introduction

Overview

Revised Problem Statement

Apply results from research on the more covered Prize-Collecting Steiner Tree Problem to the lesser covered Median Tree Problem.

Goals

- 1 Survey research on the PCSTP
- 2 Identify methods worth *porting*
- 3 Implement these methods in a solver for the MTP

Introduction

Motivation

What makes a survey necessary?

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- Some of these papers touch on very complex subjects and are sometimes short and unintuitive.
- The PCSTP is a good “case study” for an Graph Optimisation and ILP problem.

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 - Some of these papers touch on very complex subjects and are sometimes short and unintuitive.
 - The PCSTP is a good “case study” for an Graph Optimisation and ILP problem.
- **There is a lot to learn.**

Introduction

Motivation

Why the Median Tree Problem?

Introduction

Motivation

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We'll get back to that later.

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The Prize-Collecting Steiner Tree Problem

Preface

Short (Meta) History

- Defined by Daniel Bienstock in 1993 as a side note

The Prize-Collecting Steiner Tree Problem

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The Prize-Collecting Steiner Tree Problem

Preface

Short (Meta) History

- Defined by Daniel Bienstock in 1993 as a side note
- Subject to steady focus in the late 90's and early 00's
- One of the subjects of the 11th DIMACs Implementation Challenge in 2014

The Prize-Collecting Steiner Tree Problem

Problem Definition

Given an undirected graph

$$G = (V, E, c, p)$$

where $c : E \rightarrow \mathbb{R}^+$ defines edge weights, and $p : V \rightarrow \mathbb{R}^+$ defines vertex *prizes*, then the solution to the *PCSTP* is a tree

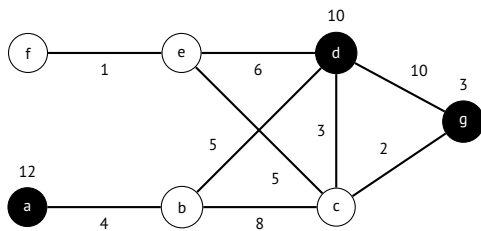
$$T = (V_T, E_T, c, p) \subseteq G$$

which minimizes

$$c(T) = \min_T \sum_{(i,j) \in E_T} c_{ij} + \sum_{v \in (V \setminus V_T)} p_v.$$

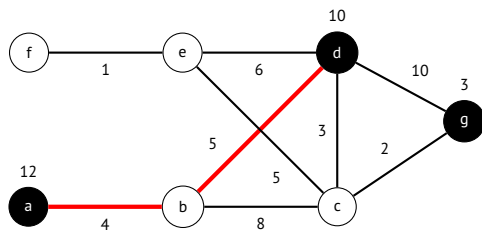
The Prize-Collecting Steiner Tree Problem

Example



The Prize-Collecting Steiner Tree Problem

Example



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The Survey

Quick Summary

Contents of the PCSTP Survey:

- 1 The history of solving the PCSTP.
- 2 Preprocessing routines.
- 3 Two heuristic algorithms. LP-based and search based.
- 4 An approximation algorithm: the GW Algorithm.
- 5 How to separate GSECs.
- 6 The DHEA and SCIP-Jack solvers.

The Survey

Main Points

What have we learned?

The Survey

Main Points

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- 1 The PCSTP is well covered all things considered.

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- 3 Directed formulations of the problem are preferable for branch and bound.

The Survey

Main Points

What have we learned?

- 1 The PCSTP is well covered all things considered.
- 2 Preprocessing is *very* good for the PCSTP.
- 3 Directed formulations of the problem are preferable for branch and bound.
- 4 Heuristics are aplenty.

The Survey

Main Points

Curiosities

The Survey

Main Points

Curiosities

- Not a lot of focus on applications
- Linear progress turns to lateral progress
- Somewhat general methods besides preprocessing

The Survey

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On to other problems.

The Survey

Prize-Collecting Tours

Three main variants:

- 1 The Prize-Collecting Travelling Salesman Problem
- 2 The Orienteering Problem
- 3 The Profitable Tour Problem

The Survey

Prize-Collecting Tours

Three main variants:

- 1 The Prize-Collecting Travelling Salesman Problem
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Some Notes:

- Shares approximation algorithms
- Apart from the OP, not well covered

The Survey

Median Subgraphs

Notes

- Assignment Problem
- Different shapes of facility
- Median Trees are only research on shaped graphs

The Survey

Related Problems

Summary

- MTP and PTP are the “adjacent-most” problems to the PCSTP
- Neither the MTP or PTP are well researched
- In fact, PCSTP is the most well researched problem in the whole family

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Median Tree Problem

Motivation

Question: How do I best make use of the survey?

Median Tree Problem

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Options

- Stay with the PCSTP

Median Tree Problem

Motivation

Question: How do I best make use of the survey?

Options

- Stay with the PCSTP
- Look at the Profitable Tour Problem

Median Tree Problem

Motivation

Question: How do I best make use of the survey?

Options

- Stay with the PCSTP
- Look at the Profitable Tour Problem
- Look at the Median Tree Problem

Median Tree Problem

Motivation

Median Trees instead of Prize-Collecting Tours

- A feasible solution to the PCSTP is a feasible solution to the MTP
- Collect Prize vs. Assignment: similar – although not the exact same – trade offs

Median Tree Problem

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Median Trees instead of Prize-Collecting Tours

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- Collect Prize vs. Assignment: similar – although not the exact same – trade offs

And, of course, a splash of subjectivity.

Median Tree Problem

Problem Definition

Let $G = (V, E, c, d)$ be an undirected graph. Denote $c : E \rightarrow \mathbb{R}^+$ as an *edge cost* function and $d : V \times V \rightarrow \mathbb{R}^+$ be an *assignment cost* function where we have

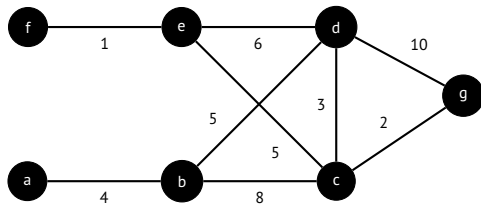
$$d_{ii} = 0.$$

Then the *Median Tree Problem* is defined as finding a *connected subgraph* $T = (V_T, E_T)$ of G where $V_T \subseteq V$ and $E_T \subseteq E$ which minimises the cost function,

$$c(T) = \sum_{ij \in E_T} c_{ij} + \sum_{i \in V} \min_{j \in V_T} d_{ij}.$$

Median Tree Problem

Example



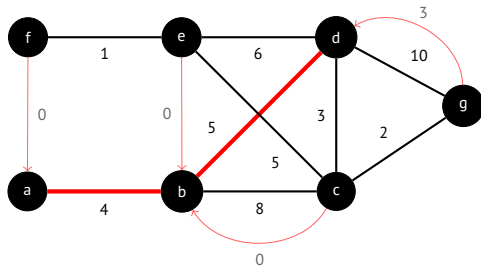
Median Tree Problem

Example

d_{ij}	a	b	c	d	e	f	g
a	0	12	12	12	12	12	12
b	0	0	0	0	0	0	0
c	0	0	0	0	0	0	0
d	10	10	10	0	10	10	10
e	0	0	0	0	0	0	0
f	0	0	0	0	0	0	0
g	3	3	3	3	3	3	0

Median Tree Problem

Example



Median Tree Problem

ILP Formulation

$$\begin{array}{ll} \underset{\mathbf{x}, \mathbf{y}}{\text{minimize}} & \sum_{ij \in E} c_{ij} x_{ij} + \sum_{i, j \in V} d_{ij} y_{ij} \end{array} \quad (1a)$$

$$\begin{array}{ll} \text{subject to} & \sum_{ij \in E} x_{ij} = \sum_{i \in V} y_{ii} - 1 \end{array} \quad (1b)$$

$$x(E(S)) \leq \sum_{i \in S \setminus \{s\}} y_{ii} \quad \forall S \subseteq V, s \in S \quad (1c)$$

$$\sum_{j \in V} y_{kj} = 1 \quad \forall k \in V \quad (1d)$$

$$y_{ik} \leq y_{kk} \quad \forall i, k \in V \quad (1e)$$

$$y_{kk} \leq \sum_{i \in \delta(k)} x_{ik} \quad \forall k \in V \quad (1f)$$

$$\mathbf{x} \in \mathbb{B}^{|E|} \quad (1g)$$

$$\mathbf{y} \in \mathbb{B}^{|V \times V|} \quad (1h)$$

Median Tree Problem

Valid Inequalities

Median Tree Problem

Valid Inequalities

Forced self-assignment:

$$y_{ii} \geq x_{ji} \quad \forall i \in V, \forall j \in \delta(i).$$

Median Tree Problem

Valid Inequalities

Forced self-assignment:

$$y_{ii} \geq x_{ji} \quad \forall i \in V, \forall j \in \delta(i).$$

Degree of Nonterminals:

$$\sum_{j \in \delta(i)} x_{ij} \geq 2x_{ik} \quad \forall i \in N, \forall k \in \delta(i)$$

Median Tree Problem

The Solver

Specifications

- Based on the Gurobi MIP solver.

Median Tree Problem

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Median Tree Problem

The Solver

Specifications

- Based on the Gurobi MIP solver.
- Callbacks written in Python 3.
- Applies methods from the PCSTP survey:
 - Primal heuristic from the DHEA solver
 - User cuts based on GSEC separation from an article by Lucena and Resende for the PCSTP

Median Tree Problem

Computational Experience

Results

Median Tree Problem

Computational Experience

Results

- Primal Heuristics: Ambivalent performance

Median Tree Problem

Computational Experience

Results

- Primal Heuristics: Ambivalent performance
- Valid Inequalities: Ambivalent performance

Median Tree Problem

Computational Experience

Results

- Primal Heuristics: Ambivalent performance
- Valid Inequalities: Ambivalent performance
- User Cuts: *Terrible* performance

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Why?

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Reflections

Summary

What did I manage to do?

- Survey of the PCSTP
- ILP formulation for the MTP
- Dataset for the MTP
- Solver for the MTP

Reflections

Improvements

Survey criticism

- I approach approximation algorithms, exact algorithms, and heuristics the same way

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- More figures outside the preprocessing section

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- Sometimes there's a lapse in "intuition" (see the equation on p. 42)

Reflections

Improvements

Survey criticism

- I approach approximation algorithms, exact algorithms, and heuristics the same way
- More figures outside the preprocessing section
- Sometimes there's a lapse in "intuition" (see the equation on p. 42)
- Too little focus on application – may be a feature of the body of research

Reflections

Improvements

Solver section criticism

- Python is fun and fast for me, but slow for the PC

Reflections

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- Python is fun and fast for me, but slow for the PC
- Something is deeply wrong with the GSEC seperation routine

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- I should have measured performance in other ways than end to end execution time

Reflections

Improvements

Solver section criticism

- Python is fun and fast for me, but slow for the PC
- Something is deeply wrong with the GSEC separation routine
- I should have measured performance in other ways than end to end execution time
- The dataset is a bit uninteresting

Reflections

Lessons Learned

Thoughts about the subject field

- There is a limit on how much research on an arbitrary problem is interesting without real world motivation

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- Same with laterally defining new problems

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Lessons Learned

Thoughts about the subject field

- There is a limit on how much research on an arbitrary problem is interesting without real world motivation
- Same with laterally defining new problems
- There could be a greater focus on generalising results as widely as possible

Reflections

Further Work

Make the solver as good as it can be

Perform a more thorough investigation of the shortcomings in the MTP solver, and use these results to reimplement the solver.

Apply

Operationalise the research done on the PCSTP (or MTP) for a real-world problem field, and apply it to a specific scenario.

Postface

Postface

Questions?