Econ571

Some Conclusions and Implications: The Seemingly Unrelated Regression (SUR) Model

1) $\underline{y}_\ell = X_\ell \underline{\beta}_\ell + \underline{\mu}_\ell, \ell = 1,2,\ E\underline{\mu}_\ell = 0, E\underline{\mu}_\ell \underline{\mu}_\ell' = \sigma_{\ell\ell}, E\underline{\mu}_\ell \underline{\mu}_m' = \sigma_{\ell m}, \ell, m = 1,2,$ i=1,......., n.

A.OLS Equation-by-Equation is feasible when there are no cross equation restrictions and can be a useful starting point event with cross-equation restrictions.

2) $\hat{\underline{\beta}}_\ell = (X_\ell' X_\ell)^{-1} X_\ell' y_\ell, Var(\hat{\underline{\beta}}_\ell) = \sigma_{\ell\ell}(X_\ell' X_\ell)^{-1}, \hat{\sigma}_{\ell m} = (\underline{y}_\ell - X_\ell \hat{\underline{\beta}}_\ell)'(\underline{y}_m - X_m \hat{\underline{\beta}}_m)/n$ (or $n-k$)

B.If no cross-equation restrictions and no tests of coefficients across equations, the potential exists to fit SUR model as OLS equation by equation or by the stacked 2-equation model.

i) If $X_1 = X_2'$, there is no gain over OLS EQBE in going to the stacked model

ii) If $X_1 \neq X_2$ and μ_1 and μ_2 are heteroscedastic $\sigma_{11} \neq \sigma_{22}$ and/or correlated $\sigma_{12} \neq 0$, then there is potential for efficiency gain by fitting the two equation model as one stacked model by GLS or FGLS.

iii) If $\underline{\mu}_1$ and $\underline{\mu}_2$ have the same variance and are uncorrelated, then perform OLS EQBE.
NOTE: SSE for the stacked model unrestricted: $SSE(\Omega_Y) = SSE(\Omega_1) + SSE(\Omega_2)$.

C.If have cross-equation restrictions, then the SUR model must be fitted by the stacked model. Also, this must be used if one is testing or imposing cross-equation restrictions. $\underline{Y} = \underline{X}\,\underline{\mathbb{B}} + \underline{U}$ st. $C\underline{\mathbb{B}} = \underline{c}$

i)If $\underline{\mu}_1$ and $\underline{\mu}_2$ are homoscedastic and uncorrelated correlated across equations, estimate the stacked model by OLS subject to a set of q linear restrictions.

Remark: This could happen if you have two different data sets on the same econometric model. By fitting the stacked model, you can perform a test that $\underline{\beta}_1 = \underline{\beta}_2$, i.e., the regressions coefficients are the same in the two data sets.

ii) If $\underline{\mu}_1$ and $\underline{\mu}_2$ are heteroscedastic and/or correlated across equations, then estimate the stacked model by GLS/FGLS.

Remark: With cross-equation restrictions, the same transformation must be performed on both equations in the stack; FGLS subject to q linear restrictions.

Remark: Testing here uses the t and F statistics for the stacked model.

$$\underset{2n\times 1}{\underline{Y} = \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix}}, \quad \underset{2n\times 2k}{\underline{X} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}}; \quad \underset{2n\times 1}{\underline{U} = \begin{bmatrix} \underline{\mu}_1 \\ \underline{\mu}_2 \end{bmatrix}}, \quad \underset{2k\times 1}{\underline{\mathbb{B}} = \begin{bmatrix} \underline{\beta}_1 \\ \underline{\beta}_2 \end{bmatrix}}, \quad n_1 = n_2 = n$$

$$F^{c} = \frac{[SSE(\hat{\omega}) - SSE(\hat{\Omega})]/q}{SSE(\hat{\Omega})/(2n-2k)} \sim F_{q,2n-2k}$$