HW2

(1) $$\hat{\beta}_2 = \frac{\Sigma(y_i-\bar{y})(X_{i2}-\bar{X}_2)}{\Sigma(X_{i2}-\bar{X}_2)^2} = 5.6501$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2\bar{X}_2 = 7.7242$$

$$\hat{\sigma}^2 = \frac{\Sigma \hat{\mu}_i^2}{8-2} = 4.0057$$

$$se(\hat{\beta}_2) = \sqrt{\left(\frac{1}{8} + \frac{\bar{X}_2^2}{\Sigma(X_{i2}-\bar{X}_2)^2}\right)\hat{\sigma}^2} = 1.9733$$

$$se(\hat{\beta}_1) = \sqrt{\frac{1}{\Sigma(X_{i2}-\bar{X}_2)^2}\hat{\sigma}^2} = 6.3784$$

(2) Positive relationship.

Test $H_0: \beta_2 = 0$ against $H_A: \beta_2 \neq 0$

$t = 2.863 > t_{6,0.025} = 2.447$

Reject H_0.

(3) $R^2 = 1 - SSE/SST$

$$= 1 - \frac{\Sigma \hat{\mu}_i^2}{\Sigma(y_i-\bar{y})^2}$$

$$= 0.5774$$

(4) "Forecast of an individual's outcome" for $X_{02} = 3.0$ and $X_{02} = \bar{X}_2 = 3.3125$

- For $X_{02} = 3.0$

$\hat{y}^f = 24.6744$

$se(\hat{y}^f) = \sqrt{\left[1 + \frac{1}{8} + \frac{(\bar{X}_2 - X_{02})^2}{\Sigma(X_{i2} - \bar{X}_2)^2}\right]\hat{\sigma}^2} = 2.1638$

PI is $[19.3796, 29.9691]$

- For $X_{02} = 3.3125$

$\hat{y}^f = 25.875$

$se(\hat{y}^f) = \sqrt{\left[1 + \frac{1}{8} + \frac{(\bar{X}_2 - X_{02})^2}{\Sigma(X_{i2} - \bar{X}_2)^2}\right]\hat{\sigma}^2} = \sqrt{\frac{9}{8}\hat{\sigma}^2} = 2.1228$

PI is $[20.6806, 31.0694]$

* Can also calculate forecast at the mean

$se(\hat{y}^f) = \sqrt{\hat{\sigma}^2/8} = 0.7076$

CI is $[24.1435, 27.6065]$

$\hat{y}^f \pm t_c \cdot se(\hat{y}^f)$