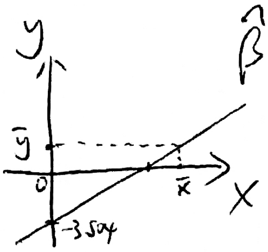
Econ 571 Midterm 2019S

I. 6' a. $\hat{\beta}_2 = \dfrac{\Sigma(y_i-\bar{y})(X_i-\bar{X})}{\Sigma(X_i-\bar{X})^2} = \dfrac{106.4}{215.4} = 0.494$

$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \cdot \bar{X} = \dfrac{\Sigma y_i}{n} - \hat{\beta}_2 \cdot \bar{X} = \dfrac{21.9}{20} - 0.494 \cdot \dfrac{186.2}{20} = \boxed{-3.504}$



5' b. $\hat{\sigma}^2 = \dfrac{SSE}{n-2}$

$= \dfrac{SST-SSR}{n-2}$

$= \dfrac{\Sigma(y_i-\bar{y})^2 - \hat{\beta}_2^2 \cdot \Sigma(X_i-\bar{X})^2}{n-2}$

$= \dfrac{86.9 - 0.494^2 \cdot 215.4}{18}$

$= 1.908$

6' c. $\text{Est Var}(\hat{\beta}_2) = \dfrac{\hat{\sigma}^2}{\Sigma(X_i-\bar{X})^2} = \dfrac{1.908}{215.4} = 8.858 \times 10^{-3}$

$\text{Est Var}(\hat{\beta}_1) = \left[\dfrac{1}{n} + \dfrac{\bar{X}_2^2}{\Sigma(X_i-\bar{X})^2}\right]\hat{\sigma}^2 = \left[\dfrac{1}{20} + \dfrac{(186.2/20)^2}{215.4}\right] \cdot 1.908$

$= 0.863$

6' d. $\mu_i \overset{iid}{\sim} N(0, \sigma^2)$

$y_i \overset{iid}{\sim} N(\beta_1 + \beta_2 X_i, \sigma^2)$

$\hat{\beta}_1 \sim N\left\{\beta_1, \sigma^2\left[\frac{1}{n} + \frac{\bar{X}^2}{\Sigma(X_i-\bar{X})^2}\right]\right\}$, $\hat{\beta}_2 \sim N\left\{\beta_2, \frac{\sigma^2}{\Sigma(X_i-\bar{X})^2}\right\}$

e. $H_0: \beta_2 = 0$, $H_A: \beta_2 \neq 0$; $\alpha = 0.05$

2' (i) $t^\circ = \frac{\hat{\beta}_2}{se(\hat{\beta}_2)} \sim t_{n-2}$

3' (ii) Significance level α is Pr(Type I error)

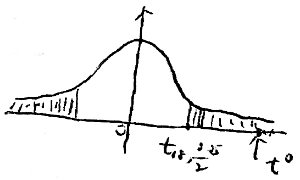
$= Pr(\text{Reject } H_0 \mid H_0 \text{ is true})$.

4' (iii) $t^\circ = \frac{0.494}{\sqrt{8.858\times10^{3}}} = 5.249$

3' (iv) Choose $\alpha = 0.05$, two tail test

$t_{18, \frac{0.05}{2}} = 2.101$

5' (v) $t^\circ > t_{18, \frac{0.05}{2}}$ Reject H_0



6' f. $R^2 = \frac{SSR}{SST} = 0.605$

60.5% variability of y is explained by x.

15' g. $\hat{y}^f = \hat{\beta}_1 + \hat{\beta}_2 \cdot X_{n+1} = 6.376$

Est Var $(\hat{y}^f) = \hat{\sigma}^2 \cdot \left[\frac{1}{n} + \frac{(\bar{X} - X_{n+1})^2}{\Sigma(X_i - \bar{X})^2}\right] = 1.908\left[\frac{1}{20} + \frac{(9.31-20)^2}{215.4}\right] = 1.108$

CI: $6.376 \pm t_{18,0.025} \cdot se(\hat{y}^f)$

$[4.165, 8.587]$

CI is narrower at the sample mean.

II. 1) $\frac{\partial \ln(price)}{\partial \ln(lotsize)} = \beta_2 + 2\beta_{11}$

10'

$\frac{\partial \ln(price)}{\partial\, bedrooms} = \beta_3 + \beta_{12} \cdot stories$

$\frac{\partial \ln(price)}{\partial\, stories} = \beta_{10} + \beta_{12} \cdot bedrooms$

. . . . . .

Expect all positive

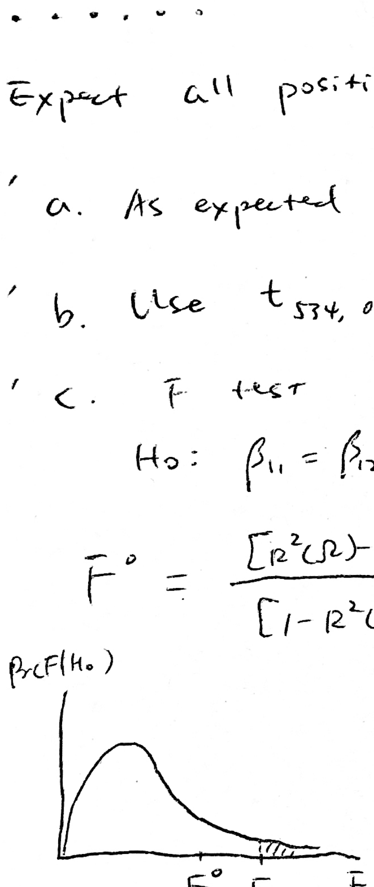
2) 1' a. As expected except β_{12} & β_{11}

11' b. Use $t_{534,\ 0.05/2} = 1.96$ as the critical value.

7' c. F test

$H_0: \beta_{11} = \beta_{12} = 0$ H_A: negation

$$F^{\circ} = \frac{[R^2(\Omega) - R^2(\omega)]/q}{[1 - R^2(\Omega)]/(n-k)} = \frac{(0.6708 - 0.6679)/2}{(1 - 0.6708)/534} = 2.352$$



$F_c = F_{534,\ 2,\ \alpha = 0.05} = 3.00$

Accept H_0.

2' d. (3) is the "best" model. $R^2 = 0.6679$

9' e. 1% ↑ lotsize => 0.329% ↑ price

~~1 more bedroom/bathroom => 2.9% / ~~

An additional bedroom / bathroom / garage / story => 2.9% / 16.7% / 5.7% / 10.1% ↑ price

Has recroom/full base/AC/prefered area => 5.9%/10.9%/15.2%/13.8% ↑ price