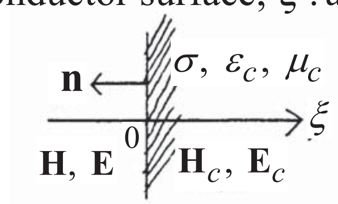
Chapter 8: Waveguides, Resonant Cavities, and Optical Fibers

8.1 Fields at the Surface of and Within a Good Conductor

The main results in Sec. 8.1 [(8.9), (8.10), (8.12), (8.14), and (8.15)] have been derived with a much simpler method in Ch. 7 [see Eqs. (37)-(51), Ch. 7 of lecture notes]. So, we will not cover this section (pp. 1-4) in class.

Notations: $\mathbf{H},\ \mathbf{E}$: fields outside the conductor; $\mathbf{H}_C,\ \mathbf{E}_C$: fields inside the conductor; $\mathbf{n}$: a unit vector $\perp$ to conductor surface; ξ : a normal coordinate into the conductor.



Assume: (i) fields $\sim e^{-i\omega t}$

(ii) good but not perfect conductor, i.e.

$\sigma \neq \infty$, but $\frac{\sigma}{\omega\varepsilon_b} \gg 1$ [See Ch. 7 of lecture notes, Eq. (37)].

(iii) $\mathbf{H}_{||}(\xi = 0)$ is known.

Find: $\mathbf{E}_c(\xi),\ \mathbf{H}_c(\xi),$ and power loss, etc. in terms of $\mathbf{H}_{||}(\xi = 0)$

Calculation of $\mathbf{E}_C(\xi),\ \mathbf{H}_C(\xi)$ **:** In the conductor, we have

$$\begin{cases} \nabla\times\mathbf{E}_c = -\frac{\partial}{\partial t}\mathbf{B}_c = i\omega\mu_c\mathbf{H}_c & (1) \\ \nabla\times\mathbf{H}_c = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D}_c = \sigma\mathbf{E}_c - i\omega\varepsilon_b\mathbf{E}_c \overset{\text{good conductor assumption}}{\simeq} \sigma\mathbf{E}_c & (2) \end{cases}$$

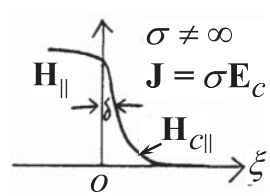
$$\nabla \simeq -\mathbf{n}\frac{\partial}{\partial\xi} \quad \left[\begin{array}{l}\text{In a good conductor, fields vary rapidly along the} \\ \text{normal to the surface, see Ch. 7 of lecture notes.}\end{array}\right] \qquad (3)$$

$$(1), (2), (3) \Rightarrow \begin{cases} \mathbf{E}_c \approx -\frac{1}{\sigma}\mathbf{n}\times\frac{\partial}{\partial\xi}\mathbf{H}_c & (4) \\ \mathbf{H}_c \approx \frac{i}{\mu_C\omega}\mathbf{n}\times\frac{\partial}{\partial\xi}\mathbf{E}_c & (5) \end{cases} \qquad \boxed{\delta \equiv \sqrt{\frac{2}{\mu_c\omega\sigma}} = \text{skin depth}}$$

Sub. (4) into (5): $\frac{\partial^2}{\partial\xi^2}(\mathbf{n}\times\mathbf{H}_c) + \frac{2i}{\delta^2}(\mathbf{n}\times\mathbf{H}_c) \approx 0$ (8.7)

$\Rightarrow \mathbf{n}\times\mathbf{H}_c(\xi) \approx \mathbf{n}\times\mathbf{H}_c(0)e^{-\frac{\xi}{\delta}}e^{\frac{i\xi}{\delta}}$ $\boxed{\text{b.c. at } \xi = 0\text{: } \mathbf{H}_{||}(0) = \mathbf{H}_{c||}(0)}$

$\Rightarrow \mathbf{H}_{c||}(\xi) \approx \mathbf{H}_{c||}(0)e^{-\frac{\xi}{\delta}}e^{\frac{i\xi}{\delta}} = \mathbf{H}_{||}(0)e^{-\frac{\xi}{\delta}}e^{\frac{i\xi}{\delta}}$ (6)



$\mathbf{n}\cdot(5) \Rightarrow\ \mathbf{n}\cdot\mathbf{H}_c(\xi) \approx 0 \Rightarrow \mathbf{H}_{c||}(\xi) \approx \mathbf{H}_c(\xi)$ (7)

Sub. (7) into (6) $\Rightarrow \mathbf{H}_c(\xi) \approx \mathbf{H}_{||}(0)e^{-\frac{\xi}{\delta}}e^{\frac{i\xi}{\delta}}$ (8.9)

Sub. $\mathbf{H}_c(\xi) \approx \mathbf{H}_{\|}(0) e^{-\frac{\xi}{\delta}} e^{\frac{i\xi}{\delta}}$ into $\mathbf{E}_c(\xi) \approx -\frac{1}{\sigma}\mathbf{n} \times \frac{\partial}{\partial \xi}\mathbf{H}_c(\xi)$

$$\Rightarrow \mathbf{E}_c(\xi) \approx \sqrt{\frac{\mu_C \omega}{2\sigma}}(1-i)\left[\mathbf{n} \times \mathbf{H}_{\|}(\xi=0)\right] e^{-\frac{\xi}{\delta}} e^{\frac{i\xi}{\delta}} \qquad (8.10)$$

$$\mathbf{E}_{\|}(\xi=0) = \mathbf{E}_{c\|}(\xi=0) \approx \mathbf{E}_c(\xi=0) \approx \sqrt{\frac{\mu_C \omega}{2\sigma}}(1-i)\left[\mathbf{n} \times \mathbf{H}_{\|}(\xi=0)\right] \qquad (8.11)$$

b.c. at $\xi = 0$ $\qquad$ $\mathbf{n}\cdot(4) \Rightarrow \mathbf{n}\cdot\mathbf{E}_C \simeq 0 \Rightarrow \mathbf{E}_{C\|} \simeq \mathbf{E}_C$

Power Loss Per Unit Area :

$$\frac{dP_{loss}}{da} = \text{time averaged power into conductor per unit area}$$

$$= -\frac{1}{2}\mathrm{Re}\left[\mathbf{n}\cdot\mathbf{E}(\xi=0)\times\mathbf{H}^*(\xi=0)\right]$$

$$= -\frac{1}{2}\mathrm{Re}\left[\mathbf{n}\cdot\mathbf{E}_{\|}(\xi=0)\times\mathbf{H}_{\|}^*(\xi=0)\right]$$

(8.11) →

$$= \frac{1}{4}\mu_C \omega\delta\left|\mathbf{H}_{\|}(\xi=0)\right|^2 = \frac{1}{2\sigma\delta}\left|\mathbf{H}_{\|}(\xi=0)\right|^2 \qquad (8.12)$$

$$\propto \mu_C^{\frac{1}{2}}\omega^{\frac{1}{2}}\sigma^{-\frac{1}{2}}\left|\mathbf{H}_{\|}(\xi=0)\right|^2$$

Alternative method to derive (8.12):

$$(8.10) \Rightarrow \mathbf{J}(\xi) = \sigma\mathbf{E}_c(\xi) \approx \frac{1}{\delta}(1-i)\left[\mathbf{n}\times\mathbf{H}_{\|}(\xi=0)\right] e^{-\frac{\xi(1-i)}{\delta}} \qquad (8.13)$$

$$\left[\begin{array}{l}\text{time averaged power}\\ \text{loss in conductor per}\\ \text{unit volume}\end{array}\right] = \frac{1}{2}\mathrm{Re}\left[\mathbf{J}(\xi)\cdot\mathbf{E}_c^*(\xi)\right] = \frac{1}{2\sigma}\left|\mathbf{J}(\xi)\right|^2$$

(8.13)

$$\frac{dP_{loss}}{da} = \frac{1}{2\sigma}\int_0^\infty d\xi\left|\mathbf{J}(\xi)\right|^2 = \frac{1}{\sigma\delta^2}\left|\mathbf{H}_{\|}(\xi=0)\right|^2\int_0^\infty e^{-\frac{2\xi}{\delta}}d\xi$$

$$= \frac{1}{2\sigma\delta}\left|\mathbf{H}_{\|}(\xi=0)\right|^2, \quad \text{same as (8.12)}$$

Effective surface current $\mathbf{K}_{eff}$:

$$\mathbf{K}_{eff} = \int_0^\infty \mathbf{J}(\xi)d\xi = \frac{1}{\delta}(1-i)\left[\mathbf{n}\times\mathbf{H}_{\|}(\xi=0)\right]\int_0^\infty e^{-\frac{\xi(1-i)}{\delta}}d\xi$$

$$= \mathbf{n}\times\mathbf{H}_{\|}(\xi=0) \qquad (8.14)$$

(8.13)

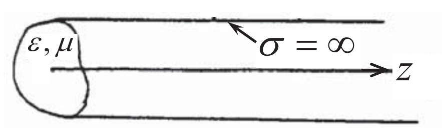
$$(8.12) \ \& \ (8.14) \Rightarrow \frac{dP_{loss}}{da} = -\frac{1}{2\sigma\delta}\left|\mathbf{K}_{eff}\right|^2 \qquad (8.15)$$

8.2-8.4 Modes in a Waveguide

Consider a hollow conductor of *infinite* length and *uniform* cross section of arbitrary shape (see figure). This is a structure which can be used to transport (high-power) EM waves, and hence is called a waveguide. Assume a *uniform*, *linear*, and *isotropic* filling medium and let $\begin{cases}\mathbf{E}(\mathbf{x},t)=\mathbf{E}(\mathbf{x})e^{-i\omega t}\\ \mathbf{B}(\mathbf{x},t)=\mathbf{B}(\mathbf{x})e^{-i\omega t}\end{cases}$ and $\begin{cases}\mathbf{B}(\mathbf{x})=\mu\mathbf{H}(\mathbf{x})\\ \mathbf{D}(\mathbf{x})=\varepsilon\mathbf{E}(\mathbf{x})\end{cases}$, where $\varepsilon,\ \mu,\ \mathbf{E}(\mathbf{x})$, $\mathbf{B}(\mathbf{x})$ are ω-space quantities (in general complex). t-space quantities, simply written as $\mathbf{E}(\mathbf{x},t)=\mathbf{E}(\mathbf{x})e^{-i\omega t}$, means $\mathbf{E}(\mathbf{x},t)=\mathrm{Re}[\mathbf{E}(\mathbf{x})e^{-i\omega t}]$

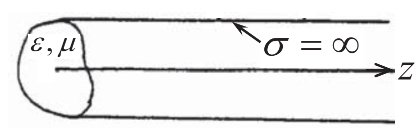
Under these assumptions, Maxwell eqs. are

$$\begin{cases}\nabla\times\mathbf{E}(\mathbf{x})=i\omega\mathbf{B}(\mathbf{x}) & (8)\\ \nabla\times\mathbf{B}(\mathbf{x})=-i\mu\varepsilon\omega\mathbf{E}(\mathbf{x}) & (9)\\ \nabla\cdot\mathbf{E}(\mathbf{x})=0 & (10)\\ \nabla\cdot\mathbf{B}(\mathbf{x})=0 & (11)\end{cases}$$



Assume further that the wall is a perfect conductor ($\sigma=\infty$, no field in conductor). $\Rightarrow$ Fields are only present in the hollow region.

$$\text{Rewrite}\quad\begin{cases}\nabla\times\mathbf{E}=i\omega\mathbf{B} & (8)\\ \nabla\times\mathbf{B}=-i\mu\varepsilon\omega\mathbf{E} & (9)\\ \nabla\cdot\mathbf{E}=0 & (10)\\ \nabla\cdot\mathbf{B}=0 & (11)\end{cases}$$



$$\nabla\times(8)\Rightarrow\nabla\times(\nabla\times\mathbf{E})=i\omega\nabla\times\mathbf{B}$$
$$\Rightarrow\nabla(\nabla\cdot\mathbf{E})-\nabla^2\mathbf{E}=\mu\varepsilon\omega^2\mathbf{E}$$
$$\Rightarrow\nabla^2\mathbf{E}+\mu\varepsilon\omega^2\mathbf{E}=0 \qquad (12a)$$

Similarly,

$$\nabla\times(9)\Rightarrow\nabla^2\mathbf{B}+\mu\varepsilon\omega^2\mathbf{B}=0 \qquad (12b)$$

(12a,b) have the same form as the wave equations in infinite space. However, (12a,b) are now subject to boundary conditions (b.c.'s) on the walls.

Most wave problems involving a boundary do not have exact solutions. The waveguide structure offers a rare case where exact solutions are possible (e.g. for rectangular and circular cross-sections).

Structure unchanged in $t \Rightarrow$ assume $e^{\pm i\omega t}$ dpendence. Similarly, structure unchanged in $z \Rightarrow$ assume $e^{\pm ik_z z}$ dpendence. Thus, we let

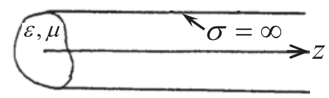
$$\begin{cases} \mathbf{E}(\mathbf{x}) = \mathbf{E}(\mathbf{x}_t)e^{\pm ik_z z} \\ \mathbf{B}(\mathbf{x}) = \mathbf{B}(\mathbf{x}_t)e^{\pm ik_z z} \end{cases},$$

$\mathbf{x}_t$: coordinates transverse to z, e.g. (x, y) or (r, θ)

k_z here $\leftrightarrow$ k in Jackson

(13)

then, $$\begin{cases} \mathbf{E}(\mathbf{x},t) = \mathbf{E}(\mathbf{x}_t)e^{\pm ik_z z - i\omega t} \\ \mathbf{B}(\mathbf{x},t) = \mathbf{B}(\mathbf{x}_t)e^{\pm ik_z z - i\omega t} \end{cases}$$



where, in general, ω and k_z are complex constants. Here, we assume (1) ω is real and positive, and (2) the real parts of k_z is positive. Then, $e^{ik_z z - i\omega t}$ and $e^{-ik_z z - i\omega t}$ have forward and backward *phase* velocities, respectively. Hence, we call $e^{ik_z z - i\omega t}$ a forward wave and $e^{-ik_z z - i\omega t}$ a backward wave, both of which are traveling waves.

With $e^{\pm ik_z z}$ dependence, we have

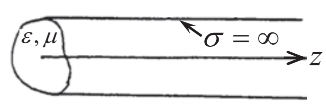
$$\begin{cases} \frac{\partial^2}{\partial z^2} \to -k_z^2 \\ \nabla^2 = \nabla_t^2 + \frac{\partial^2}{\partial z^2} = \nabla_t^2 - k_z^2 \end{cases}$$

$$\nabla_t^2 = \begin{cases} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, & \text{Cartesian} \\ \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}, & \text{cylindrical} \end{cases}$$

Thus, $$\begin{cases} \nabla^2\mathbf{E} + \mu\varepsilon\omega^2\mathbf{E} = 0 \ [(12a)] \\ \nabla^2\mathbf{B} + \mu\varepsilon\omega^2\mathbf{B} = 0 \ [(12b)] \end{cases}$$

$$\nabla^2 = \nabla_t^2 + \frac{\partial^2}{\partial z^2}$$

$$\Rightarrow \left(\nabla_t^2 + \mu\varepsilon\omega^2 - k_z^2\right)\begin{Bmatrix} \mathbf{E}(\mathbf{x}_t) \\ \mathbf{B}(\mathbf{x}_t) \end{Bmatrix} = 0 \qquad (8.19)$$

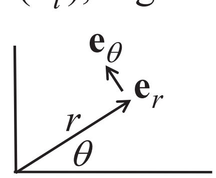


$$\Rightarrow \left(\nabla_t^2 + \mu\varepsilon\omega^2 - k_z^2\right)\begin{Bmatrix} E_z(\mathbf{x}_t) \\ B_z(\mathbf{x}_t) \end{Bmatrix} = 0 \quad \begin{bmatrix} \text{b.c.'s to be} \\ \text{given later} \end{bmatrix} \qquad (14)$$

So our strategy here is to solve (14) for $E_z(\mathbf{x}_t)$ and $B_z(\mathbf{x}_t)$, and then express the other field components $[\mathbf{E}_t(\mathbf{x}_t)$ and $\mathbf{B}_t(\mathbf{x}_t)]$ in terms of $E_z(\mathbf{x}_t)$ and $B_z(\mathbf{x}_t)$.

Note: It is in general not possible to obtain from (8.19) an isolated eq. like (14) for a transverse component of $\mathbf{E}(\mathbf{x}_t)$ and $\mathbf{B}(\mathbf{x}_t)$, e.g. in cylindrical coordinates, $\mathbf{E}(\mathbf{x}_t) = E_r\mathbf{e}_r + E_\theta\mathbf{e}_\theta + E_z\mathbf{e}_z$. (8.19) will lead to 2 coupled eqs. for E_r and E_θ due to

$$\frac{\partial}{\partial\theta}\mathbf{e}_r = \mathbf{e}_\theta, \quad \frac{\partial}{\partial\theta}\mathbf{e}_\theta = -\mathbf{e}_r$$



$$\text{Let}\begin{cases}\mathbf{E}(\mathbf{x}_t)=\mathbf{E}_t(\mathbf{x}_t)+E_z(\mathbf{x}_t)\mathbf{e}_z\\ \mathbf{B}(\mathbf{x}_t)=\mathbf{B}_t(\mathbf{x}_t)+B_z(\mathbf{x}_t)\mathbf{e}_z\\ \nabla=\nabla_t+\mathbf{e}_z\frac{\partial}{\partial z}=\nabla_t\pm ik_z\mathbf{e}_z\end{cases}\quad \boxed{\nabla_t=\begin{cases}\mathbf{e}_x\frac{\partial}{\partial x}+\mathbf{e}_y\frac{\partial}{\partial y}, & \text{Cartesian}\\ \mathbf{e}_r\frac{\partial}{\partial r}+\mathbf{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta}, & \text{cylindrical}\end{cases}}\tag{15}$$

$$\nabla\times\mathbf{E}=i\omega\mathbf{B}\ \Rightarrow\left(\nabla_t\pm ik_z\mathbf{e}_z\right)\times\left(\mathbf{E}_t+E_z\mathbf{e}_z\right)=i\omega\left(\mathbf{B}_t+B_z\mathbf{e}_z\right)\tag{16a}$$

$$\nabla\times\mathbf{B}=-i\omega\mu\varepsilon\mathbf{E}\Rightarrow\left(\nabla_t\pm ik_z\mathbf{e}_z\right)\times\left(\mathbf{B}_t+B_z\mathbf{e}_z\right)=-i\omega\mu\varepsilon\left(\mathbf{E}_t+E_z\mathbf{e}_z\right)\tag{16b}$$

Using the relations: $\begin{cases}(\nabla_t\times\mathbf{E}_t)\parallel\mathbf{e}_z\\ (\nabla_t\times E_z\mathbf{e}_z)\perp\mathbf{e}_z\end{cases}$, we obtain from the transverse components of (16a,b)

$$\begin{cases}\nabla_t\times E_z\mathbf{e}_z\pm ik_z\mathbf{e}_z\times\mathbf{E}_t=i\omega\mathbf{B}_t & (17)\\ \nabla_t\times B_z\mathbf{e}_z\pm ik_z\mathbf{e}_z\times\mathbf{B}_t=-i\mu\varepsilon\omega\mathbf{E}_t & (18)\end{cases}$$

Note: 1. In (15)-(18), the $\begin{Bmatrix}\text{upper}\\ \text{lower}\end{Bmatrix}$ sign refers to $\begin{Bmatrix}\text{forward}\\ \text{backward}\end{Bmatrix}$ wave.

2. (17), (18) are 2-D vector eqs. E_z, $\mathbf{E}_t$, $\mathbf{B}_t$ are functions of $\mathbf{x}_t$.

$$\text{Rewrite}\quad\begin{cases}\nabla_t\times E_z\mathbf{e}_z\pm ik_z\mathbf{e}_z\times\mathbf{E}_t=i\omega\mathbf{B}_t & [(17)]\\ \nabla_t\times B_z\mathbf{e}_z\pm ik_z\mathbf{e}_z\times\mathbf{B}_t=-i\mu\varepsilon\omega\mathbf{E}_t & [(18)]\end{cases}$$

In (17), (18), E_z, B_z can be regarded as known quantities, which have been solved from (14). Thus, (17), (18) give 4 *algebraic* (rather than differential) equations for 4 unknowns (e.g. $E_r, E_\theta, B_r, B_\theta$).

Hence, we just need to eliminate $\mathbf{B}_t$ from (17), (18) to express $\mathbf{E}_t$ in terms of E_z and B_z.

$$\mathbf{e}_z\times(17)\Rightarrow\ \mathbf{e}_z\times(\underbrace{\nabla_t\times E_z\mathbf{e}_z}_{\nabla_t E_z\times\mathbf{e}_z+E_z\underbrace{\nabla_t\times\mathbf{e}_z}_{0}})\pm ik_z\overbrace{\mathbf{e}_z\times\left(\mathbf{e}_z\times\mathbf{E}_t\right)}^{-\mathbf{E}_t}=i\omega\mathbf{e}_z\times\mathbf{B}_t$$

$$\boxed{\begin{array}{l}\nabla\times\psi\mathbf{a}=\nabla\psi\times\mathbf{a}+\psi\nabla\times\mathbf{a}\\ \text{If }\psi,\ \mathbf{a}\text{ are both independent of }z\text{, then}\\ \nabla_t\times\psi\mathbf{a}=\nabla_t\psi\times\mathbf{a}+\psi\nabla_t\times\mathbf{a}\end{array}}$$

$$\Rightarrow\ i\omega\mathbf{e}_z\times\mathbf{B}_t=\nabla_t E_z\mp ik_z\mathbf{E}_t\tag{19}$$

Rewrite $\begin{cases} \nabla_t \times B_z \mathbf{e}_z \pm i k_z \mathbf{e}_z \times \mathbf{B}_t = -i\mu\varepsilon\omega \mathbf{E}_t & [(18)] \\ i\omega \mathbf{e}_z \times \mathbf{B}_t = \nabla_t E_z \mp i k_z \mathbf{E}_t & [(19)] \end{cases}$

Sub. $\mathbf{e}_z \times \mathbf{B}_t$ from (19) into (18)

$$\Rightarrow \quad \underbrace{\nabla_t \times B_z \mathbf{e}_z}_{\nabla_t B_z \times \mathbf{e}_z} \pm i k_z \tfrac{1}{i\omega}\left(\nabla_t E_z \mp i k_z \mathbf{E}_t\right) = -i\mu\varepsilon\omega \mathbf{E}_t \tag{20}$$

Multiply (20) by $i\omega$: $i\omega \nabla_t B_z \times \mathbf{e}_z \pm i k_z \nabla_t E_z + k_z^2 \mathbf{E}_t = \mu\varepsilon\omega^2 \mathbf{E}_t$.

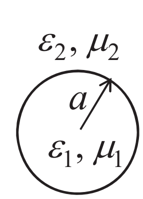
we obtain $\quad (\mu\varepsilon\omega^2 - k_z^2)\mathbf{E}_t = i\left(\omega \nabla_t B_z \times \mathbf{e}_z \pm k_z \nabla_t E_z\right)$,

which gives $\mathbf{E}_t(\mathbf{x}_t) = \dfrac{i\left[\pm k_z \nabla_t E_z(\mathbf{x}_t) - \omega \mathbf{e}_z \times \nabla_t B_z(\mathbf{x}_t)\right]}{\mu\varepsilon\omega^2 - k_z^2}$ (8.26a)

Similarly, $\mathbf{B}_t(\mathbf{x}_t) = \dfrac{i\left[\pm k_z \nabla_t B_z(\mathbf{x}_t) + \mu\varepsilon\omega \mathbf{e}_z \times \nabla_t E_z(\mathbf{x}_t)\right]}{\mu\varepsilon\omega^2 - k_z^2}$ (8.26b)

Once $E_z(\mathbf{x}_t)$ and $B_z(\mathbf{x}_t)$ are solved from (14), we may obtain $\mathbf{E}_t(\mathbf{x}_t)$ and $\mathbf{B}_t(\mathbf{x}_t)$ from (8.26).

Rewrite $\begin{cases} \left(\nabla_t^2 + \mu\varepsilon\omega^2 - k_z^2\right)\begin{Bmatrix} E_z(\mathbf{x}_t) \\ B_z(\mathbf{x}_t) \end{Bmatrix} = 0 & [(14)] \\ \mathbf{E}_t(\mathbf{x}_t) = \dfrac{i\left[\pm k_z \nabla_t E_z(\mathbf{x}_t) - \omega \mathbf{e}_z \times \nabla_t B_z(\mathbf{x}_t)\right]}{\mu\varepsilon\omega^2 - k_z^2} & [(8.26a)] \\ \mathbf{B}_t(\mathbf{x}_t) = \dfrac{i\left[\pm k_z \nabla_t B_z(\mathbf{x}_t) + \mu\varepsilon\omega \mathbf{e}_z \times \nabla_t E_z(\mathbf{x}_t)\right]}{\mu\varepsilon\omega^2 - k_z^2} & [(8.26b)] \end{cases}$



Discussion: With a generalized ε, (14) & (8.26) apply to any medium, e.g. a 2-medium waveguide (see figure).

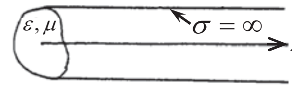
Case 1: A waveguide with a $\sigma = \infty$ wall (treated here).

Case 2; The outer medium ($r > a$) is a conductor with $\sigma \neq \infty$ [Huang, Chu, & Thumm, Phys. Plasmas 22, 013108 (2015)].

Case 3: The outer medium has a dielectric constant smaller than that of the inner medium (e.g. an optical fiber, see Sec. 8.11).

Case 4: The inner medium is a $\sigma \neq \infty$ conductor. The outer medium is free space [Chen, Chen, & Chu, AIP Advances 8, 115028 (2018)].

TE and TM Modes of a Hollow Waveguide :

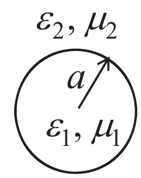


Rewrite
$$\begin{cases} \left(\nabla_t^2+\mu\varepsilon\omega^2-k_z^2\right)\begin{Bmatrix} E_z(\mathbf{x}_t) \\ B_z(\mathbf{x}_t)\end{Bmatrix}=0 & [(14)] \\ \mathbf{E}_t(\mathbf{x}_t)=\dfrac{i\left[\pm k_z\nabla_t E_z(\mathbf{x}_t)-\omega\mathbf{e}_z\times\nabla_t B_z(\mathbf{x}_t)\right]}{\mu\varepsilon\omega^2-k_z^2} & [(8.26a)] \\ \mathbf{B}_t(\mathbf{x}_t)=\dfrac{i\left[\pm k_z\nabla_t B_z(\mathbf{x}_t)+\mu\varepsilon\omega\mathbf{e}_z\times\nabla_t E_z(\mathbf{x}_t)\right]}{\mu\varepsilon\omega^2-k_z^2} & [(8.26b)] \end{cases}$$

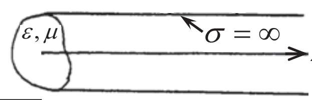
In (14), we set $B_z=0$. It can be shown that, if the b.c.'s on E_z is satisfied, so will the b.c.'s on $\mathbf{E}_t$ & $\mathbf{B}_t$. Hence, the set of solutions with $B_z=0$ are valid, which are called the TM (transverse magnetic) modes.

Similarly, setting $E_z=0$, we may obtain a set of valid solutions for B_z, $\mathbf{E}_t$, and $\mathbf{B}_t$, which are called the TE (transverse electric) modes.

Note : The separation of TE/TM modes is a special case. In general (e.g. for the 2-medium case in the right figure), we need both B_z and E_z in order to satisfy all b.c.'s at the interface between the 2 media.



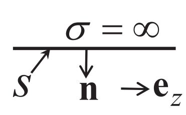
TM mode $(B_z=0)$: (see pp. 359-360)



$$\begin{cases} (\nabla_t^2+\gamma^2)E_z=0 \text{ with b.c. } E_z\big|_s=0 & (21) \\ \mathbf{E}_t=\pm\dfrac{ik_z}{\gamma^2}\nabla_t E_z & (21a) \\ \mathbf{H}_t=\pm\dfrac{\varepsilon\omega}{k_z}\mathbf{e}_z\times\mathbf{E}_t & (21b) \\ \gamma^2=\mu\varepsilon\omega^2-k_z^2 & (21c) \end{cases}$$

for $\sigma=\infty$ (perfectly conducting wall)

TE Mode $(E_z=0)$: (see pp. 359-360)



$$\begin{cases} (\nabla_t^2+\gamma^2)H_z=0 \text{ with b.c. } \frac{\partial}{\partial n}H_z\big|_s=0 & (22) \\ \mathbf{H}_t=\pm\dfrac{ik_z}{\gamma^2}\nabla_t H_z & (22a) \\ \mathbf{E}_t=\mp\dfrac{\mu\omega}{k_z}\mathbf{e}_z\times\mathbf{H}_t & (22b) \\ \gamma^2=\mu\varepsilon\omega^2-k_z^2 & (22c) \end{cases}$$

$B_\perp$ is continuous across wall surface S [(5.86)] and $B_\perp$ (inside conductor) $=0$.
$\Rightarrow \mathbf{H}_t\big|_s$ is tangential to $S \Rightarrow \mathbf{n}\cdot\mathbf{H}_t\big|_s=0$
$\downarrow$ (22a)
$\Rightarrow \mathbf{n}\cdot\nabla_t H_z\big|_s=0 \Rightarrow \frac{\partial}{\partial n}H_z\big|_s=0$

It can be said that fields of TM or TE modes are "generated" by E_z or B_z. The generating function (B_z or E_z) is denoted by ψ in Jackson.

Discussion:

$$\text{Rewrite}\begin{cases}(\nabla_t^2+\gamma^2)E_z=0 \text{ with b.c. } E_z\big|_s=0 & [(21)]\\(\nabla_t^2+\gamma^2)H_z=0 \text{ with b.c. } \frac{\partial}{\partial n}H_z\big|_s=0 & [(22)]\end{cases}$$

(i) Either (21) or (22) constitutes an eigenvalue problem (see lecture notes, Ch. 3, Appendix A). ∇_t^2 is Hermitian→The eigenvalue γ^2 will be an infinite set of discrete real values fixed by the b.c., representing an infinite set of orthogonal eigenmodes of the waveguide (See *Example* 1 below.)

(ii) (21b) and (22b) show that **E** is perpendicular to **B**.

(iii) (21b) and (22b) show that $\mathbf{E}_t$ and $\mathbf{B}_t$ are *in phase* if μ, ε, ω, k_z are all real.

(iv) $\gamma^2=\mu\varepsilon\omega^2-k_z^2$ [(21c) or (22c)] is the dispersion relation, which relates ω and k_z for a given mode.

Note: Due to b.c.'s, TE or TM mode has a component (B_z or E_z) in the direction of propagation [see similar case in Ch. 7, (24)].

Question: Have we missed any solution?

TEM Mode of Coaxial and Parallel-Wire Transmission Lines ($E_z=B_z=0$)**:** (see p. 358)

$$\text{Rewrite}\begin{cases}\mathbf{E}_t(\mathbf{x}_t)=\dfrac{i\left[\pm k_z\nabla_t E_z(\mathbf{x}_t)-\omega\mathbf{e}_z\times\nabla_t B_z(\mathbf{x}_t)\right]}{\mu\varepsilon\omega^2-k_z^2} & (8.26a)\\[2ex]\mathbf{B}_t(\mathbf{x}_t)=\dfrac{i\left[\pm k_z\nabla_t B_z(\mathbf{x}_t)+\mu\varepsilon\omega\mathbf{e}_z\times\nabla_t E_z(\mathbf{x}_t)\right]}{\mu\varepsilon\omega^2-k_z^2} & (8.26b)\end{cases}$$

These 2 equations fail for a different class of modes, for which $E_z=B_z=0$ (called TEM modes, TEM: transverse electromagnetic). However, they give the condition for the existence of this mode:

$$\boxed{\omega^2=\frac{k_z^2}{\mu\varepsilon}}\quad\left[\begin{array}{l}\text{Equations in rectangular boxes are}\\\text{basic equations for the TEM mode.}\end{array}\right]\qquad(8.27)$$

(8.27) is also the dispersion relation in infinite space. This makes the TEM mode very useful because it can propagate at any frequency.

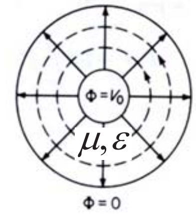
To calulate $\mathbf{E}_t$ and $\mathbf{B}_t$, we need to go back to Maxwell equations.

Let $\begin{Bmatrix} E_z = 0 \\ B_z = 0 \end{Bmatrix}$ and $\begin{Bmatrix} \mathbf{E}(\mathbf{x}) \\ \mathbf{B}(\mathbf{x}) \end{Bmatrix} = \begin{Bmatrix} \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) \\ \mathbf{B}_{\text{TEM}}(\mathbf{x}_t) \end{Bmatrix} e^{\pm i k_z z}$.

$$\boxed{\begin{matrix} \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) \perp \mathbf{e}_z \\ \mathbf{B}_{\text{TEM}}(\mathbf{x}_t) \perp \mathbf{e}_z \end{matrix}}$$

With $B_z = 0$, the z-component of $\nabla \times \mathbf{E}(\mathbf{x}) = i\omega \mathbf{B}(\mathbf{x})$ gives

$$\nabla_t \times \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) = 0 \Rightarrow \boxed{\mathbf{E}_{\text{TEM}}(\mathbf{x}_t) = -\nabla_t \Phi_{\text{TEM}}(\mathbf{x}_t)},$$



coaxial cable

[*Note*: $\nabla_t \times \mathbf{A}_t(\mathbf{x}_t) = 0 \Leftrightarrow \mathbf{A}_t(\mathbf{x}_t) = -\nabla_t \Phi(\mathbf{x}_t)$]

With $E_z = 0$, $\nabla \cdot \mathbf{E}(\mathbf{x}) = 0$ gives

$$\nabla_t \cdot \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) = 0 \Rightarrow \boxed{\nabla_t^2 \Phi_{\text{TEM}}(\mathbf{x}_t) = 0}.$$

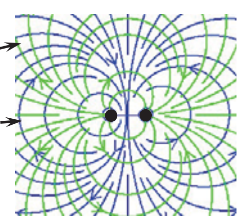
The transverse component of $\nabla \times \mathbf{E}(\mathbf{x}) = i\omega \mathbf{B}(\mathbf{x})$ gives

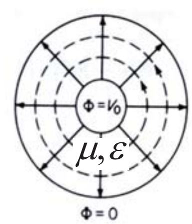
$$\boxed{\mathbf{B}_{\text{TEM}}(\mathbf{x}_t) = \pm \frac{k_z}{\omega} \mathbf{e}_z \times \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) = \pm \sqrt{\mu\varepsilon}\, \mathbf{e}_z \times \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) \qquad (8.28)}$$

Because $\mathbf{E}_{\tan} = 0$ ("tan": tangential) on the surface of a perfect conductor, we have Φ_{TEM}(on the boundary) $=$ *const*. This gives Φ_{TEM} $=$ *const*. or $\mathbf{E}_{\text{TEM}} = 0$ everywhere, if there is only one conductor. So, TEM modes exist only in 2-conductor configurations, such as coaxial and parallel-wire transmission lines.

In summary, the TEM modes are governed by

$$\begin{cases} \nabla_t^2 \Phi_{\text{TEM}}(\mathbf{x}_t) = 0 & (23) \\ \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) = -\nabla_t \Phi_{\text{TEM}}(\mathbf{x}_t) & (23a) \\ \mathbf{B}_{\text{TEM}}(\mathbf{x}_t) = \pm \dfrac{k_z}{\omega} \mathbf{e}_z \times \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) & (23b) \\ [\text{or } \mathbf{H}_{\text{TEM}} = \pm \dfrac{k_z}{\omega\mu} \mathbf{e}_z \times \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) = \pm \sqrt{\dfrac{\varepsilon}{\mu}} \mathbf{e}_z \times \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) = \pm Y \mathbf{e}_z \times \mathbf{E}_{\text{TEM}}] & \\ \omega^2 = \dfrac{k_z^2}{\mu\varepsilon} & (23c) \end{cases}$$





where Y $\left(= \sqrt{\frac{\varepsilon}{\mu}}\right)$ is the (intrinsic) admittance of the filling medium, which is defined in Ch. 7 of lecture notes following Eq. (26).

Since $\mathbf{E}_{\text{TEM}}$ and $\mathbf{B}_{\text{TEM}}$ are both $\perp \mathbf{e}_z$, the Poynting vector, $<\mathbf{S}>_t$ $= \frac{1}{2}\text{Re}[\mathbf{E}^*_{\text{TEM}} \times \mathbf{H}_{\text{TEM}}]$, is in the $\mathbf{e}_z$ direction.

Question: If an electron moves from $\Phi_{\text{TEM}} = 0$ to $\Phi_{\text{TEM}} = V_0$, does its energy change by eV_0?

Discussion:

(i) For the TEM modes, we solve a 2-D equation $\nabla_t^2 \Phi_{\text{TEM}}(\mathbf{x}_t) = 0$ for $\Phi_{\text{TEM}}(\mathbf{x}_t)$. But this is not a 2-D problem because Φ_{TEM} is not the full solution. The full solution is $\begin{Bmatrix} \mathbf{E}_t(\mathbf{x},t) \\ \mathbf{B}_t(\mathbf{x},t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) \\ \mathbf{B}_{\text{TEM}}(\mathbf{x}_t) \end{Bmatrix} e^{\pm ik_z z - i\omega t}$

with $\mathbf{E}_{\text{TEM}}(\mathbf{x}_t) = -\nabla_t \Phi_{\text{TEM}}(\mathbf{x}_t)$ and $\mathbf{B}_{\text{TEM}}(\mathbf{x}_t) = \pm \frac{k_z}{\omega} \mathbf{e}_z \times \mathbf{E}_{\text{TEM}}(\mathbf{x}_t)$.

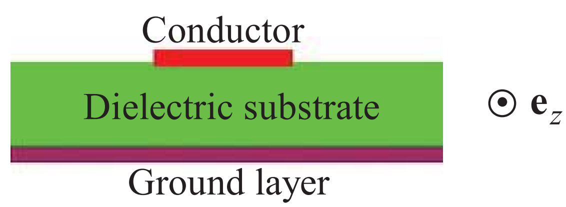
For an actual 2-D electrostatic problem $[\Phi(\mathbf{x}) = \Phi(\mathbf{x}_t)]$, we have $\nabla_t^2 \Phi(\mathbf{x}_t) = 0$, which gives the full solution $\mathbf{E}_t(\mathbf{x}_t) = -\nabla_t \Phi(\mathbf{x}_t)$.

(ii) Note the difference between the scalar potential discussed here and those defined in electrostatics and electrodynamics.

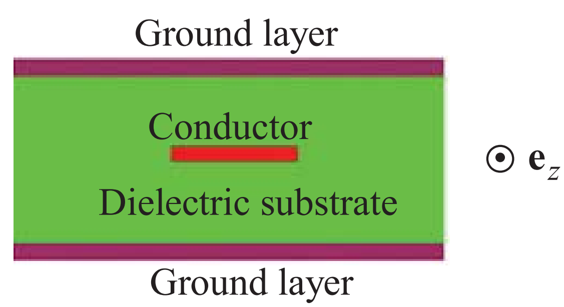
$$\begin{cases} \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) = -\nabla_t \Phi_{\text{TEM}}(\mathbf{x}_t), \text{ regard } \Phi_{\text{TEM}} \text{ as a mathematical tool.} \\ \mathbf{E}(\mathbf{x}) = -\nabla \Phi(\mathbf{x}) \text{ [electrostatics], regard } \Phi \text{ as a physical quantity.} \\ \mathbf{E}(\mathbf{x},t) = -\nabla \Phi(\mathbf{x},t) - \frac{\partial \mathbf{A}(\mathbf{x},t)}{\partial t}, \text{ regard } \Phi \text{ and } \mathbf{A} \text{ as mathematical tools.} \end{cases}$$

(iii) The microstrip and stripline (see figures) belong to the class of planar transmission lines, which are used in microwave integrated circuits (MIC) for low-power transmission in TEM modes (in the $\mathbf{e}_z$-direction as shown below).

Microstrip



Stripline



Example 1: TE mode of a rectangular waveguide

Rewrite the basic equations for the TE mode:

$$\begin{cases} (\nabla_t^2+\gamma^2)H_z(\mathbf{x}_t)=0 \text{ with boundary condition } \frac{\partial}{\partial n}H_z\big|_s=0 & [(22)] \\ \mathbf{H}_t(\mathbf{x}_t)=\pm\frac{ik_z}{\gamma^2}\nabla_t H_z(\mathbf{x}_t) & [(22a)] \\ \mathbf{E}_t(\mathbf{x}_t)=\mp\frac{\mu\omega}{k_z}\mathbf{e}_z\times\mathbf{H}_t(\mathbf{x}_t) & [(22b)] \\ \gamma^2=\mu\varepsilon\omega^2-k_z^2 & [(22c)] \end{cases}$$

Rectangular geometry $\Rightarrow$ Cartesian system $\Rightarrow \nabla_t^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

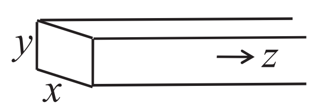
Hence, the wave equation in (22) becomes:

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\mu\varepsilon\omega^2-k_z^2\right]H_z(x,y)=0 \tag{24}$$

Rewrite $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\mu\varepsilon\omega^2-k_z^2\right]H_z(x,y)=0$ [(24)]

Assuming $e^{ik_x x+ik_y y}$ dependence for $H_z(x,y)$, we obtain

$$[\mu\varepsilon\omega^2-k_x^2-k_y^2-k_z^2]H_z(x,y)=0$$



In order for $H_z(x,y)\neq 0$, we must have

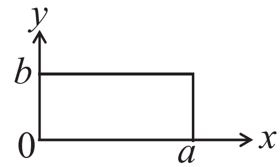
$$\mu\varepsilon\omega^2-k_x^2-k_y^2-k_z^2=0,$$

which is satisfied for both signs of k_x, k_y, & k_z. Since $(e^{ik_x x}, e^{-ik_x x})$, $(e^{ik_y y}, e^{-ik_y y})$, and $(e^{ik_z z}, e^{-ik_z z})$ are all linearly independent pairs, the full solution for H_z is

$$H_z(\mathbf{x},t)=e^{-i\omega t}\left(A_1e^{ik_x x}+A_2e^{-ik_x x}\right)\left(B_1e^{ik_y y}+B_2e^{-ik_y y}\right) \cdot\left(C_+e^{ik_z z}+C_-e^{-ik_z z}\right) \tag{25}$$

Rewrite $H_z(\mathbf{x},t)=e^{-i\omega t}\left(A_1e^{ik_xx}+A_2e^{-ik_xx}\right)\left(B_1e^{ik_yy}+B_2e^{-ik_yy}\right)$
$\cdot\left(C_+e^{ik_zz}+C_-e^{-ik_zz}\right)$ [(26)]

Apply b.c.'s : $\frac{\partial}{\partial n}H_z\Big|_s=0$ [(22)]



$$\begin{cases}\frac{\partial}{\partial x}B_z(x=0)=0\Rightarrow ik_xA_1-ik_xA_2=0\Rightarrow A_1=A_2\\ \frac{\partial}{\partial y}B_z(y=0)=0\Rightarrow ik_yB_1-ik_yB_2=0\Rightarrow B_1=B_2\end{cases}$$

$$\Rightarrow H_z(\mathbf{x},t)=\cos k_xx\cos k_yy(C_+e^{-i\omega t+ik_zz}+C_-e^{-i\omega t-ik_zz})$$

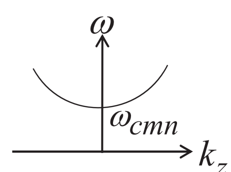
$$\begin{cases}\frac{\partial}{\partial x}B_z(x=a)=0\Rightarrow \sin k_xa=0\Rightarrow k_x=m\pi/a,\ m=0,1,2,\ldots\\ \frac{\partial}{\partial y}B_z(y=b)=0\Rightarrow \sin k_yb=0\Rightarrow k_y=n\pi/b,\ n=0,1,2,\ldots\end{cases}$$

$$\Rightarrow H_z(\mathbf{x},t)=\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}(\underbrace{C_+e^{ik_zz-i\omega t}}_{\text{forward wave}}+\underbrace{C_-e^{-ik_zz-i\omega t}}_{\text{backward wave}}) \tag{26}$$

Sub. $k_x=\frac{m\pi}{a}$, $k_y=\frac{n\pi}{b}$ into $\mu\varepsilon\omega^2-k_x^2-k_y^2-k_z^2=0$, we obtain

$$\mu\varepsilon\omega^2-k_z^2-\pi^2\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)=0,\ m,n=0,1,2,\ldots \tag{27}$$

Rewrite $\mu\varepsilon\omega^2-k_z^2-\pi^2\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)=0$ [(27)] as



$$\mu\varepsilon\omega^2-k_z^2-\mu\varepsilon\omega_{cmn}^2=0\ \left[\text{for complex } \mu \text{ and } \varepsilon\right], \tag{28a}$$

where $\omega_{cmn}=\frac{\pi}{\sqrt{\mu\varepsilon}}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^{1/2}$, $m,n=0,1,2,\ldots$ (28b)

(28) is the exact expression of the TE_{mn} mode dispersion relation for a rectangular waveguide with infinite wall conductivity and a uniform dielectric filling medium of (in general complex) ε and μ.

Each pair of $(m,\ n)$ gives a normal mode (TE_{mn} mode) of the waveguide. The mode indices m and n cannot *both* be 0, because that will make the denominator in (22a) vanish.

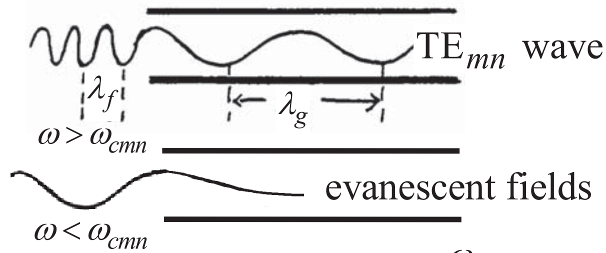
Common case: Unfilled waveguide ($\varepsilon=\varepsilon_0$, $\mu=\mu_0$) good approx. to air medium

We have $\mu\varepsilon=\mu_0\varepsilon_0=\frac{1}{c^2}$, and (28a,b) can be written

$$\omega^2-k_z^2c^2-\omega_{cmn}^2=0\quad\left[\text{for unfilled waveguide}\right], \tag{29a}$$

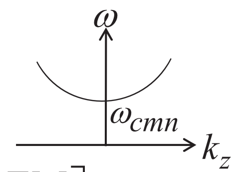
where $\omega_{cmn}=\pi c\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^{1/2}$, $m,n=0,1,2,\ldots$ (29b)

Rewrite $\begin{cases} \omega^2 - k_z^2 c^2 - \omega_{cmn}^2 = 0 \\ \omega_{cmn} = \pi c\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^{\frac{1}{2}} \end{cases}$



Definitions and terminology :

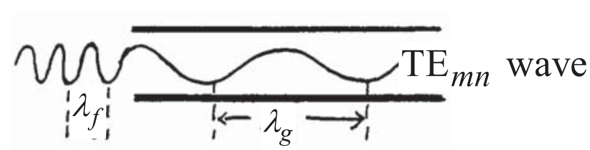
$\begin{cases} \omega > \omega_{cmn} \Rightarrow k_z = \text{real} \Rightarrow \underline{\text{propagating waves}} \\ \omega < \omega_{cmn} \Rightarrow k_z = \text{imaginary} \Rightarrow \underline{\text{evanescent fields}}. \end{cases}$

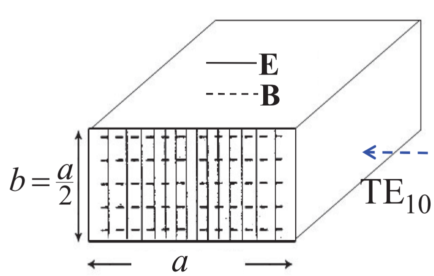


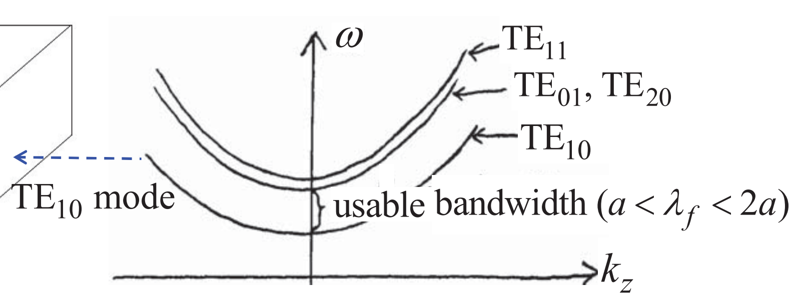
$\begin{cases} \lambda_f = \underline{\text{free space wavelength}} \equiv \frac{2\pi c}{\omega} \quad \left[\begin{array}{l}\text{wavelength of an EM} \\ \text{wave in free space}\end{array}\right] \\ \lambda_g = \underline{\text{guide wavelength}} \equiv \frac{2\pi}{k_z} \quad \left[\begin{array}{l}\text{wavelength of the TE}_{mn} \\ \text{wave in the waveguide}\end{array}\right] \end{cases}$

$\omega_{cmn} = \underline{\text{cutoff frequency}}$ $\left[\begin{array}{l}\text{lowest } \omega \text{ allowed to enter the waveguide} \\ \text{as a TE}_{mn} \text{ wave } (k_z = 0 \text{ when } \omega = \omega_{cmn}) \\ \text{The corresponding free-space wavelength} \\ (\lambda_c = \frac{2\pi c}{\omega_{cmn}}) \text{ is called the } \underline{\text{cutoff wavelength}}. \end{array}\right]$

Rewrite $\begin{cases} \omega^2 - k_z^2 c^2 - \omega_{cmn}^2 = 0 \\ \omega_{cmn} = \pi c\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^{\frac{1}{2}} \\ \lambda_f = \frac{2\pi c}{\omega} \quad [\text{wavelength of an EM wave in free space}] \\ \lambda_g = \frac{2\pi}{k_z} \quad [\text{wavelength of the TE}_{mn} \text{ wave in the waveguide}] \end{cases}$







Lowest TE & TM mode: $\omega_c(\text{TE}_{10}) = \frac{\pi c}{a}$; $\omega_c(\text{TM}_{11}) = \pi c\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^{\frac{1}{2}}$.

Question: Can we use a waveguide to transport a 60-Hz wave?

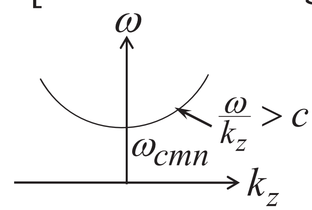
Other quantities of interest:

(1) Rewrite (29a): $\omega^2 - k_z^2 c^2 - \omega_{cmn}^2 = 0$ [for unfilled waveguide]

$$\Rightarrow v_{ph} = \frac{\omega}{k_z} > c \quad \text{[phase velocity]}$$

$$\frac{d}{dk_z}\text{(29a)} \Rightarrow 2\omega\frac{d\omega}{dk_z} - 2k_z c^2 = 0$$

$$\Rightarrow v_g = \frac{d\omega}{dk_z} = \frac{k_z c^2}{\omega} \quad \text{[group velocity]} \tag{30}$$



$$\Rightarrow \text{(i) } v_g < c;\ \text{(ii) } v_{ph} v_g = c^2;\ \text{(iii) } \begin{cases} v_{ph} \to \infty \\ v_g \to 0 \end{cases} \text{as } \omega \to \omega_{\text{cmn}}$$

Note: (1) Since there is no loss, k_z = real and positive. (2) For waves with $e^{\pm ik_z z - i\omega t}$ dependence, v_{ph} and v_g are along $\pm \mathbf{e}_z$ (see figure).

(2) The remaining field components (E_x, E_y, H_x, and H_y) can be obtained from $H_z(x, y)$ through

$$\mathbf{H}_t(x, y) = \pm\frac{ik_z}{\gamma^2}\nabla_t H_z(x, y) \quad \text{[(22a)]}$$

$$\mathbf{E}_t(x, y) = \mp\frac{\mu\omega}{k_z}\mathbf{e}_z \times \mathbf{H}_t(x, y) \quad \text{[(22b)]}$$

$$\left[\begin{array}{l} \gamma^2 = \mu_0\varepsilon_0\omega^2 - k_z^2 = \dfrac{\omega_{cmn}^2}{c^2} \\ \text{see (22c) and (29a).} \end{array}\right]$$

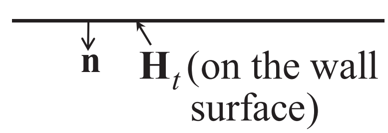
(3) Wall current

The waves in a waveguide are guided by currents on the walls. When we apply boundary conditions to calculate the fields, effects of the wall currents are automatically accounted for. It can be easily calculated using $\mathbf{H}_t$ on the wall surface.

For infinite conductivity, the skin depth is zero. Hence, the wall current flows entirely on the surface. It is given by

$$\mathbf{J}_s = \mathbf{n} \times \mathbf{H}_t \text{(on the wall surface)}$$

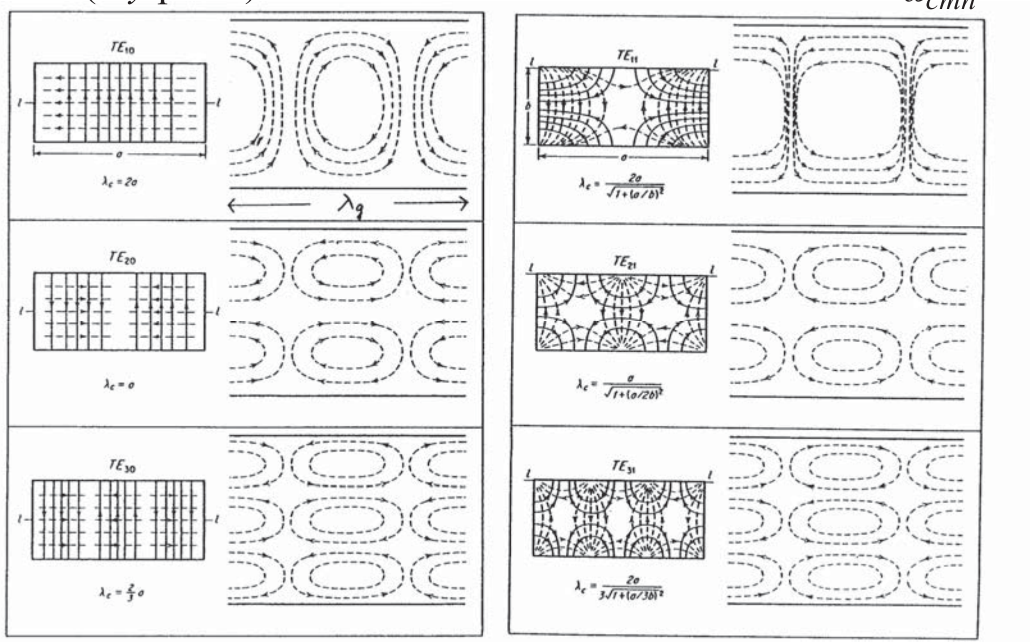
[See lecture notes, Ch. 7, Eq. (45) or Jackson Eq. (8.14)]



For $\sigma \neq \infty$, the wall current flows within approximately a skin depth. Then, $\mathbf{J}_s$ above is an integrated value [denoted by $\mathbf{K}_{eff}$ in (45) of lecture notes in Ch. 7 or (8.14)].

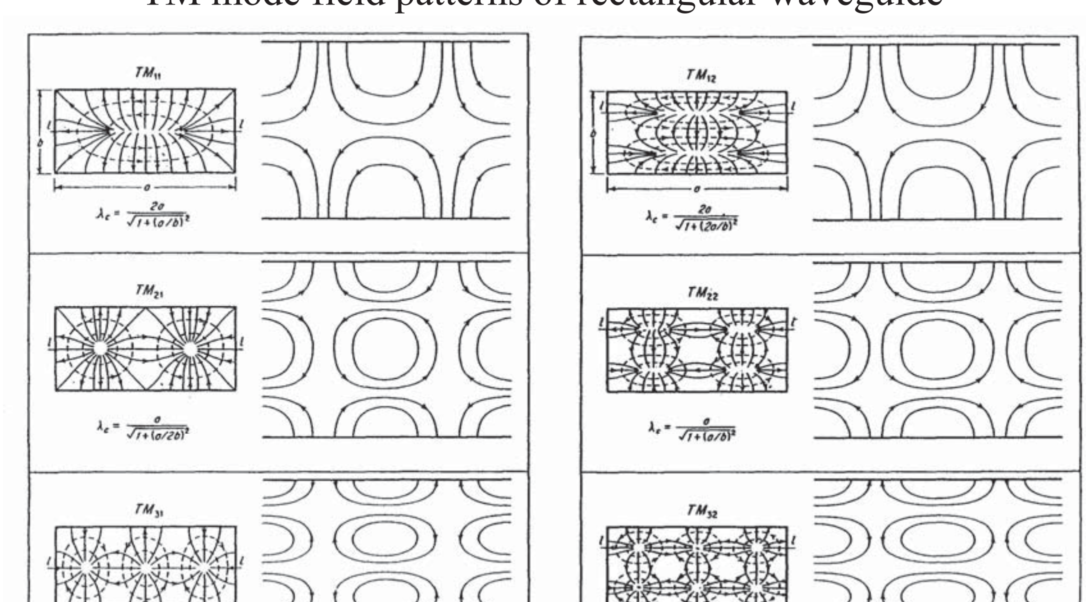
TE mode field patterns of rectangular waveguide

cross-section view (x-y plane) side view λ_c (cutoff frequency) $= \dfrac{2\pi c}{\omega_{cmn}}$



E. L. Ginzton, "Microwave measurements", p. 486.
solid curves: **E**-field lines; dashed curves: **B**-field lines

TM mode field patterns of rectangular waveguide

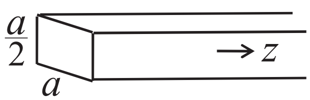


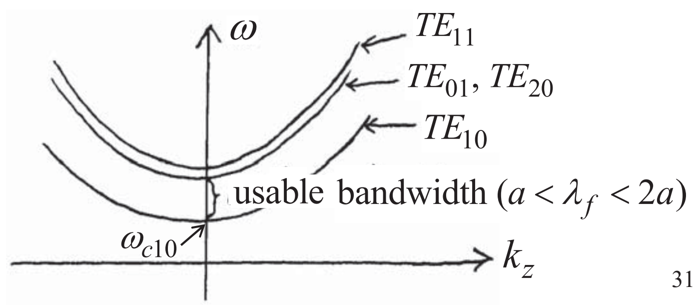
E. L. Ginzton, "Microwave measurements", p. 488.
solid curves: **E**-field lines; dashed curves: **B**-field lines

Waveguide and microwaves

If $b \le \frac{1}{2}a$, there is a maximum usable bandwidth ($\Delta\omega$) for the TE_{10} mode. It is "usuable" in the sense that only the TE_{10} mode can propagate (no possibility of mode conversion). $\Delta\omega$ is "maximum" in the sense $\Delta\omega$ ($= 2\omega_{c10} - \omega_{c10}$) has the maximum value reletive to ω_{c10}. A typical waveguide has $b = \frac{1}{2}a$ for maximum power capability and usable $\Delta\omega$. The waveguide is capable of handling high power and hence is used in high power systems (such as long-range radars).

Microwaves are normally transported by the TE_{10} mode in the usable $\Delta\omega$. Waveguides come in different practical sizes (0.1 cm $< a <$ 100 cm) to cover the entire microwave band (300 MHz to 300 GHz).

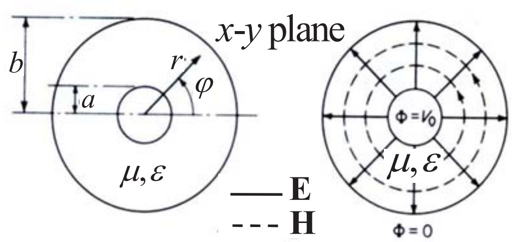




Example 2: TEM modes of a coaxial transmission line

TEM modes are governed by the following set of equtions:

$$\begin{cases} \nabla_t^2 \Phi_{\text{TEM}}(\mathbf{x}_t) = 0 & (23) \\ \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) = -\nabla_t \Phi_{\text{TEM}}(\mathbf{x}_t) & (23a) \\ \mathbf{H}_{\text{TEM}}(\mathbf{x}_t) = \pm Y \mathbf{e}_z \times \mathbf{E}_{\text{TEM}}(\mathbf{x}_t) & (23b) \\ \omega^2 = \dfrac{k_z^2}{\mu\varepsilon} & (23c) \end{cases}$$



(23) gives $\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Phi_{\text{TEM}}}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 \Phi_{\text{TEM}}}{\partial \varphi^2} = 0.$

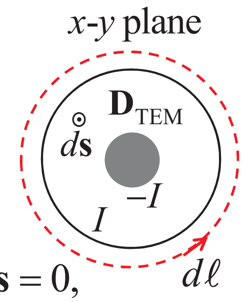
Neglect the $\frac{\partial \Phi}{\partial \varphi} \ne 0$ modes $\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Phi_{\text{TEM}}}{\partial r}) = 0 \Rightarrow \Phi_{\text{TEM}} = C_1 \ln(r) + C_2$

Apply b.c. $\begin{cases} \Phi_{\text{TEM}}(r=a) = V_0 \\ \Phi_{\text{TEM}}(r=b) = 0 \end{cases} \Rightarrow \begin{cases} C_1 = V_0 / \ln(a/b) \\ C_2 = -C_1 \ln(b) \end{cases} \Rightarrow \Phi_{\text{TEM}} = V_0 \frac{\ln(r/b)}{\ln(a/b)}$

(23a, b) then give $\begin{cases} \mathbf{E}_{\text{TEM}}(\mathbf{x},t) = \frac{V_0}{\ln(b/a)}\frac{1}{r} e^{\pm i k_z z - i\omega t}\mathbf{e}_r \\ \mathbf{H}_{\text{TEM}}(\mathbf{x},t) = \pm\frac{Y V_0}{\ln(b/a)}\frac{1}{r} e^{\pm i k_z z - i\omega t}\mathbf{e}_\varphi \; [Y = \sqrt{\frac{\varepsilon}{\mu}}] \end{cases}$ (31)

Exercise: Prove $I(\text{inner conductor}) = -I(\text{outer conductor})$

Consider a loop (dashed line) on the x-y plane in the field-free region outside the outer conductor. We have $\mathbf{H}_{\text{TEM}}(\mathbf{x}_t) = 0$ on the loop.



Ampere's law (for $e^{-i\omega t}$ dependence):

$$\oint_{loop} \underbrace{\mathbf{H}_{\text{TEM}}(\mathbf{x}_t)}_{\text{outside } (=0)} \cdot d\ell = \int_A [\mathbf{J}_{\text{TEM}}(\mathbf{x}_t) - i\omega \underbrace{\mathbf{D}_{\text{TEM}}(\mathbf{x}_t)}_{\varepsilon \mathbf{E}_{\text{TEM}}(\mathbf{x}_t)}] \cdot d\mathbf{s} = 0,$$

where $\begin{cases} \mathbf{J}_{\text{TEM}} \text{ is the current density on the two conductors.} \\ A \text{ is the area on the } x\text{-}y \text{ plane enclosed by the loop.} \\ d\mathbf{s} \text{ is a differential surface area on } A \ (d\mathbf{s} \parallel \mathbf{e}_z). \end{cases}$

TEM wave $\Rightarrow$ $\mathbf{D}_{\text{TEM}}(\mathbf{x}_t) \perp \mathbf{e}_z \Rightarrow \int_A \mathbf{D}_{\text{TEM}}(\mathbf{x}_t) \cdot d\mathbf{s} = 0$

$\Rightarrow \int_A \mathbf{J}_{\text{TEM}}(\mathbf{x}_t) \cdot d\mathbf{s} = 0,$ impling that the *total* (surface) current on the inner conductor is equal in magnitude and opposite in sign to that on the outer conductors, i.e. $I(\text{outer conductor}) = -I(\text{inner conductor})$.

Coaxial-to-waveguide transition: It is often necessay to couple waves from a coaxial line to a waveguide (or vice versa) with a transition. Two coupling methods are used. The coupling efficiency can be close to 100%.

1. Probe coupling: The tip of the coaxial line's inner conductor acts as a probe to produces an E-field matching with the TE_{10} mode, hence sending TE_{10} waves in $\pm\mathbf{e}_z$ directions. The $-\mathbf{e}_z$ wave is reflected at the shorted end with 180^{o} phase change in E_y. As it reaches the probe after a $\lambda_{\text{g}}/2$ round trip, there is another 180^{o} phase change. So, it superposes most constructively with the newly entered $+\mathbf{e}_z$ wave at the probe.

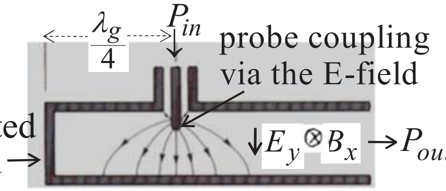


Fig. 5.1 of R. S. Elliott's book

2. Loop coupling: A loop at the end of the coaxial line generates a TE_{10} wave by producing a B-field matching that of the TE_{10} mode.

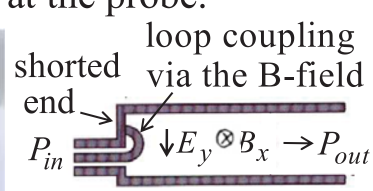


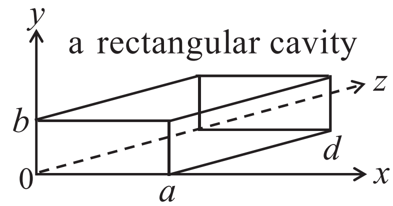
Fig. 5.2 of R. S. Elliott's book

8.7 Modes in Cavities

We consider the example of a rectangular cavity (i.e. a rectangular waveguide with two ends closed by conductors), for which we have two additional boundary conditions at the ends: $z = 0$ and d.

Rewrite $H_z(\mathbf{x},t) = \cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}(C_+ e^{ik_z z - i\omega t} + C_- e^{-ik_z z - i\omega t})$ [(26)]

b.c. (i): $H_z(z=0) = 0 \;\Rightarrow\; C_+ = -C_-$



$\Rightarrow H_z = H_{z0}e^{-i\omega t}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\sin k_z z$

b.c. (ii): $H_z(z=d) = 0$

$$\Rightarrow \sin k_z d = 0 \;\Rightarrow\; k_z = \frac{l\pi}{d},\; l = 1, 2, \ldots \tag{32}$$

$$\Rightarrow H_z(\mathbf{x},t) = H_{z0}e^{-i\omega t}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\sin\frac{l\pi z}{d}\quad \begin{bmatrix} m, n = 0, 1, 2, \ldots \\ l = 1, 2, \ldots \end{bmatrix} \tag{33}$$

Sub. (32) into $\omega^2 - k_z^2 c^2 - \omega_{cmn}^2 = 0$ [(29a)], where $\omega_{cmn} = \pi c(\frac{m^2}{a^2} + \frac{n^2}{b^2})^{\frac{1}{2}}$

$$\Rightarrow \omega = \omega_{mnl} = \pi c(\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{l^2}{d^2})^{1/2}\quad \begin{bmatrix} \omega_{mnl}\text{ : resonant frequency of} \\ \text{TE}_{mnl}\text{ mode of } \textit{unfilled}\text{ cavity} \end{bmatrix} \tag{34}$$

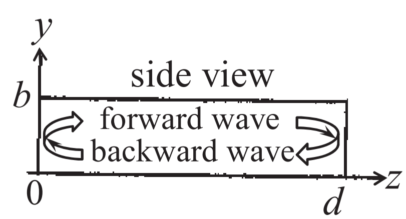
In (26), let $C_+ = -C_- = C$

$$\Rightarrow \quad H_z(\mathbf{x},t) = \cos\frac{m\pi x}{a}\cos\frac{m\pi y}{b}(\overbrace{Ce^{ik_z z - i\omega t}}^{\text{forward wave}} - \overbrace{Ce^{-ik_z z - i\omega t}}^{\text{backward wave}})$$

$\Rightarrow$ A cavity mode consists of a forward wave and a backward wave of equal amplitude. The forward wave is reflected at the right end to become a backward wave, while the backward wave is simultaneously reflected at the left end to become a forward wave.

The 2 waves superpose into a standing wave given by (33):

$H_z(\mathbf{x},t) = H_{z0}e^{-i\omega t}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\sin\frac{l\pi z}{d}$

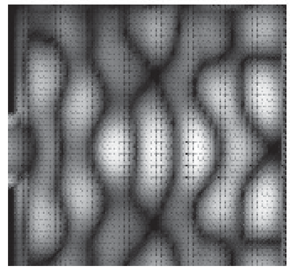


Comparison with vibrational modes of a string:

	dependent variable(s)	independent variables	mode index
string	x (oscillation amp.)	$z,\ t$	l
cavity	$E_x,\ E_y,\ B_x,\ B_y,$ E_z (or B_z)	$x,\ y,\ z,\ t$	$m,\ n,\ l$

Applications of cavities:

1. Microwave oven cavity (size $\gg$ wavelength)

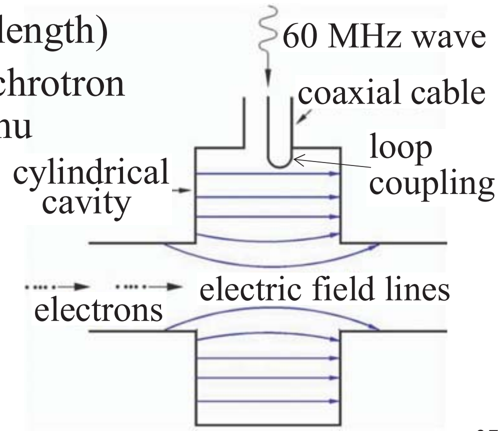


Standing wave pattern in a 2.45 GHz microwave oven cavity

2. Accelerating cavity (size $\approx$ wavelength)

The cavity used in the National Synchrotron Radiation Research Center in Hsinchu

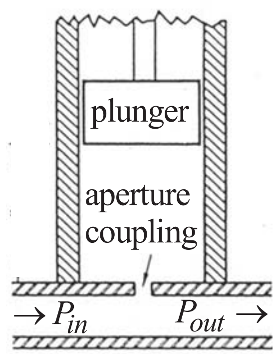
The lowest order TM mode is used. By (21b), $\mathbf{E}_t \perp \mathbf{B}_t$. Hence, B-field is in the θ-direction ($\Rightarrow$ loop coupling).



3. Frequency Meter

The figure to the right shows a cavity-type frequency meter (also called a wavemeter). It is inserted into a waveguide to measure the frequency of the passing wave.

The frequency meter consists of a cavity with its resonant frequency tunable by the up-and-down motion of a "plunger".



The cavity is (weakly) coupled to the waveguide through a hole (called "aperture coupling"). When the cavity resonant frequency is tuned to that of the passing wave, the cavity will be excited to reach the maximum field, which draws power from the passing wave to compensate for wall losses. Hence, the power in the waveguide will show a dip ($P_{out} < P_{in}$). The plunger position can be calibrated for direct reading of the wave frequency when a dip is detected.

8.5 Energy Flow and Attenuation in Waveguides

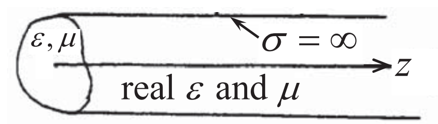
(Next 5 pages give an elegant general formalism, which we will skip.)

Power in a Lossless Waveguide: Conside a TM mode ($\mathbf{E} = \mathbf{E}_t + E_z\mathbf{e}_z$, $\mathbf{H} = \mathbf{H}_t$) in a medium with real ε, μ (hence real ω, k_z).

$$\mathbf{S}_{\text{TM}} = \tfrac{1}{2}\mathbf{E}\times\mathbf{H}^* = \tfrac{1}{2}[\mathbf{E}_t\times\mathbf{H}_t^* + E_z\mathbf{e}_z\times\mathbf{H}_t^*] \quad \text{[complex Poynting vector]}$$

$$\overset{\boxed{(21b)}}{=} \tfrac{1}{2}\frac{\varepsilon\omega}{k_z}[\underbrace{\mathbf{E}_t\times(\mathbf{e}_z\times\mathbf{E}_t^*)}_{\mathbf{e}_z|\mathbf{E}_t|^2} + E_z\,\underbrace{\mathbf{e}_z\times(\mathbf{e}_z\times\mathbf{E}_t^*)}_{-\mathbf{E}_t^*}] \quad \text{[for TM modes]}$$

$$\overset{\boxed{(21a)}}{=} \frac{\varepsilon\omega}{2k_z}[\mathbf{e}_z\frac{k_z^2}{\gamma^4}|\nabla_t E_z|^2 + \frac{ik_z}{\gamma^2}E_z\nabla_t E_z^*]$$



$$= \frac{\omega k_z\varepsilon}{2\gamma^4}[\mathbf{e}_z|\nabla_t E_z|^2 + \frac{i\gamma^2}{k_z}E_z\nabla_t E_z^*]$$

P_{TM} = time averaged power in the z-direction

$$= \int_A \mathbf{e}_z\cdot[\text{Re}\,\mathbf{S}_{\text{TM}}]da \quad [A\text{: cross-sectional area}]$$

$$= \frac{\omega k_z\varepsilon}{2\gamma^4}\int_A(\nabla_t E_z^*\cdot\nabla_t E_z)da \tag{35}$$

Green's first identity: $\int_V(\phi\nabla^2\psi + \nabla\phi\cdot\nabla\psi)d^3x = \oint_S\phi\frac{\partial\psi}{\partial n}da$ (1.34)

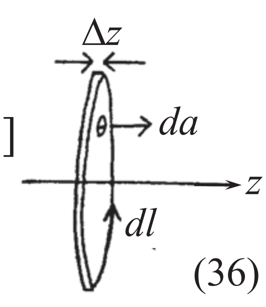
Let ϕ and ψ be independent of z and apply (1.34) to a slab of end surface area A (on the x-y plane) and infinistesimal thickness Δz in z,

$$\Delta z\int_A(\phi\nabla_t^2\psi + \nabla_t\phi\cdot\nabla_t\psi)da = \Delta z\oint_C\phi\frac{\partial\psi}{\partial n}dl + \left[\begin{array}{l}\text{surface integrals on}\\ \text{two ends of the}\\ \text{slab, which vanish.}\end{array}\right]$$

$$\Rightarrow \int_A(\phi\nabla_t^2\psi + \nabla_t\phi\cdot\nabla_t\psi)da = \oint_C\phi\frac{\partial\psi}{\partial n}dl$$

Let $\phi = E_z^*$ and $\psi = E_z$, then

$$\int_A(\nabla_t E_z^*\cdot\nabla_t E_z)da = [\oint_C \underbrace{E_z^*\tfrac{\partial}{\partial n}E_z}_{\substack{=0\\ \text{by b.c.}}}dl - \int_A E_z^*\underbrace{\nabla_t^2 E_z}_{\substack{-\gamma^2 E_z\\ \text{by (14)}}}da]$$



$$= \gamma^2\int_A|E_z|^2 da. \tag{36}$$

Sub. (36) into (35): $P_{\text{TM}} = \frac{\omega k_z\varepsilon}{2\gamma^4}\int_A(\nabla_t E_z^*\cdot\nabla_t E_z)da$, we obtain

$$P_{\text{TM}} = \frac{\omega k_z\varepsilon}{2\gamma^2}\int_A|E_z|^2 da, \quad [\text{where } \gamma^2 = \mu\varepsilon\omega^2 - k_z^2] \tag{37}$$

$$\gamma^2 = \mu\varepsilon\omega^2 - k_z^2 \;\Rightarrow \omega_c = \frac{\gamma}{\sqrt{\mu\varepsilon}} \quad \begin{bmatrix} \omega_c \text{(i.e. } \omega \text{ at } k_z = 0\text{) is the} \\ \text{cutoff freq. of the mode.} \end{bmatrix} \tag{38}$$

$$\Rightarrow \; k_z = (\mu\varepsilon\omega^2 - \gamma^2)^{\frac{1}{2}} = \sqrt{\mu\varepsilon}\,\omega\big(1 - \tfrac{\omega_c^2}{\omega^2}\big)^{\frac{1}{2}} \tag{39}$$

Sub. (38) and (39) into (37)

$$P_{\text{TM}} = \tfrac{1}{2}\sqrt{\tfrac{\varepsilon}{\mu}}\big(\tfrac{\omega}{\omega_c}\big)^2\big(1 - \tfrac{\omega_c^2}{\omega^2}\big)^{\frac{1}{2}} \int_A |E_z|^2\, da \quad \text{[cf. (8.51)]} \tag{40}$$

Similarly, for the TE mode and real μ, ε, ω, and k_z, we obtain from (22), (22a), and (22b),

$$\mathbf{S}_{\text{TE}} = \tfrac{\omega k_z \mu}{2\gamma^4}\big[\mathbf{e}_z |\nabla_t H_z|^2 - \tfrac{i\gamma^2}{k_z} H_z^* \nabla_t H_z\big] \tag{41}$$

$$P_{\text{TE}} = \int_A \mathbf{e}_z \cdot [\text{Re}\,\mathbf{S}_{\text{TE}}]da = \tfrac{\omega k_z \mu}{2\gamma^2}\int_A |H_z|^2\, da$$

$$= \tfrac{1}{2}\sqrt{\tfrac{\mu}{\varepsilon}}\big(\tfrac{\omega}{\omega_c}\big)^2\big(1 - \tfrac{\omega_c^2}{\omega^2}\big)^{\frac{1}{2}} \int_A |H_z|^2\, da \quad \text{[cf. (8.51)]} \tag{42}$$

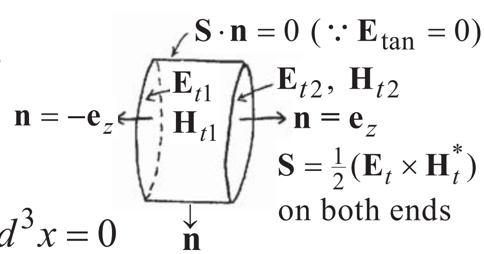
Note: P_{TM} and P_{TE} are expressed in terms of the generating function.

Energy in a Lossless Waveguide:

$$\oint_S \mathbf{S}\cdot\mathbf{n}da + \tfrac{1}{2}\int_V \mathbf{J}^* \cdot \mathbf{E} d^3x + 2i\omega \int_V (w_e - w_m) d^3x = 0 \tag{6.134}$$

$$\begin{cases} w_e = \tfrac{1}{4}\mathbf{E}\cdot\mathbf{D}^* = \tfrac{1}{4}\varepsilon|E|^2 \\ w_m = \tfrac{1}{4}\mathbf{B}\cdot\mathbf{H}^* = \tfrac{1}{4\mu}|B|^2 \end{cases} \begin{bmatrix} \text{if } \varepsilon,\ \mu \text{ are real, } w_e \text{ and } w_m \text{ are} \\ \text{also real and represent time} \\ \text{averaged field energy densities.} \end{bmatrix} \tag{6.133}$$

Apply (6.134) to a section of a lossless waveguide [i.e. μ, ε are real and the wall conductivity $\sigma = \infty$].



$\sigma = 0$ (inside volume) $\Rightarrow \mathbf{J} = 0 \Rightarrow \int_V \mathbf{J}^* \cdot \mathbf{E} d^3x = 0$

$\mathbf{E}_{\tan} = 0$ ("tan": tangential) on the side wall $\Rightarrow \mathbf{S}\cdot\mathbf{n} = 0$ on the side wall.

μ, ε (hence ω, k_z) are real $\Rightarrow \mathbf{E}_t$ and $\mathbf{H}_t$ are in phase [by (21b)&(22b)]

$\Rightarrow \mathbf{E}_t \times \mathbf{H}_t^*$ is real $\Rightarrow \mathbf{S}$ is real on both ends $\Rightarrow \oint_S \mathbf{S}\cdot\mathbf{n}da$ is real

$$\begin{cases} \text{Re}[(6.134)] \Rightarrow \oint_S \mathbf{S}\cdot\mathbf{n}da = 0 \;\; \text{(no net power into or out of volume)} \\ \text{Im}[(6.134)] \Rightarrow \int_V w_e d^3x = \int_V w_m d^3x \;\; \text{(B-field energy = E-field energy)} \end{cases}$$

For the TM mode ($H_z = 0$):

U_{TM} = field energy per unit length

$$= \int_A (w_e + w_m) da = 2\int_A w_m da = \frac{\mu}{2}\int_A |\mathbf{H}_t|^2 da \overset{(21b)}{=} \frac{\mu}{2}\frac{\varepsilon^2\omega^2}{k_z^2}\int_A |\mathbf{E}_t|^2 da$$

$$\underset{(21a)}{=} \frac{\mu}{2}\frac{\varepsilon^2\omega^2}{k_z^2}\frac{k_z^2}{\gamma^4}\underbrace{\int_A |\nabla_t E_z|^2 da}_{\substack{\gamma^2\int_A |E_z|^2 da \\ \text{by (36)}}} = \frac{\mu\varepsilon^2\omega^2}{2\gamma^2}\int_A |E_z|^2 da \underset{\boxed{\gamma^2 = \mu\varepsilon\omega_c^2}}{=} \frac{\varepsilon}{2}\left(\frac{\omega}{\omega_c}\right)^2 \int_A |E_z|^2 da \quad (43)$$

Similarly, for the TE mode ($E_z = 0$):

$$U_{\text{TE}} = 2\int_A W_e da = \frac{\varepsilon}{2}\int_A |\mathbf{E}_t|^2 da = \frac{\mu}{2}\left(\frac{\omega}{\omega_c}\right)^2 \int_A |H_z|^2 da \quad (44)$$

Use (22a,b) and Green's 1st identity

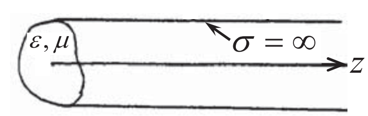
From (40), (42), (43), and (44)

$$\frac{P_{\text{TM}}}{U_{\text{TM}}} = \frac{P_{\text{TE}}}{U_{\text{TE}}} = \frac{1}{\sqrt{\mu\varepsilon}}\left(1 - \frac{\omega_c^2}{\omega^2}\right)^{\frac{1}{2}} \underset{(39)}{=} \frac{k_z}{\mu\varepsilon\omega} \underset{\frac{d}{dk_z}(21c)}{=} v_g \quad (8.53)$$

$$v_p = \omega / k_z$$

$$\Rightarrow v_p v_g = 1/\mu\varepsilon \quad (8.54)$$

Attenuation in Waveguides:



Rewrite $\mu\varepsilon\omega^2 - k_z^2 - \mu\varepsilon\omega_{cmn}^2 = 0$ [(28a)]
which is valid for infinitely conducting walls ($\sigma = \infty$) and arbitrary (complex) values of μ and ε. For simplicity, we assume μ is real.

In general, ε of the medium is a complex number ($\varepsilon = \varepsilon' + i\varepsilon''$). Also, the walls have finite resistivity. Thus, there are 2 types of losses: dielectric medium loss and wall loss. We assume that both are small pertubations to a lossless solution (i.e. $\varepsilon'' = 0$ and $\sigma = \infty$). Hence, the two loss terms are uncoupled and can be treated separately.

Dielectric medium loss: (28a) gives $k_z = \sqrt{\mu\varepsilon}\sqrt{\omega^2 - \omega_{cmn}^2}$.

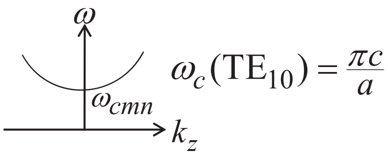
Write $\varepsilon = \varepsilon' + i\varepsilon''$, assume $\varepsilon'' \ll \varepsilon'$, and expand $\sqrt{\varepsilon}$ to obtain

$$\sqrt{\varepsilon} = \sqrt{\varepsilon'}\left(1 + i\frac{\varepsilon''}{\varepsilon'}\right)^{1/2} \approx \sqrt{\varepsilon'}\left(1 + i\frac{1}{2}\frac{\varepsilon''}{\varepsilon'}\right)$$

Thus, $k_z = \sqrt{\mu\varepsilon'}\sqrt{\omega^2 - \omega_{cmn}^2}\left(1 + i\frac{1}{2}\frac{\varepsilon''}{\varepsilon'}\right)$ and the field attenuation constant is

$$k_{zi} = \frac{1}{2}\sqrt{\mu\varepsilon'}\sqrt{\omega^2 - \omega_{cmn}^2}\,\frac{\varepsilon''}{\varepsilon'}$$

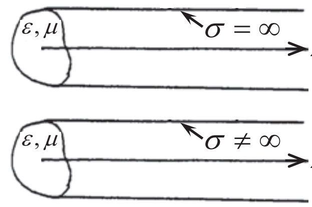
Wall loss: For $\sigma = \infty$ waveguide, we have



$$\mu\varepsilon\omega^2 - k_z^2 - \mu\varepsilon\omega_{cmn}^2 = 0 \quad [(28a)]$$

At a fixed ω, express k_z for a lossless ($\sigma = \infty$) and lossy ($\sigma \neq \infty$) wall as

$$k_z = \begin{cases} k_z^{(0)} \ [\text{solution of (28a)}], \ \sigma = \infty \\ k_z^{(0)} + \alpha + i\beta, \ \sigma \neq \infty \end{cases} \tag{8.55}$$



For the $\sigma \neq \infty$ expression in (8.55), we assumes that the wall loss is so small that it modifies $k_z^{(0)}$ by a *small* real part α and a *small* imaginary part β, where α and β can be treated as *perturbations*.

Physical reason for α: The wave sees a waveguide size larger by $\sim \delta$ (skin depth), hence a lower ω_{cmn} $[= \pi c(\frac{m^2}{a^2} + \frac{n^2}{b^2})^{1/2}$ (29b)].

$\omega^2 - k_z^2 c^2 - \omega_{cmn}^2 = 0$ [(29a)] $\Rightarrow$ larger k_z (for the same ω) $\Rightarrow \alpha > 0$.

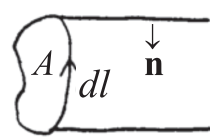
Physical reason for β: Ohmic dissipation on the wall.

In $k_z = k_z^{(0)} + \alpha + i\beta$, α modifies the guide wavelength slightly. It makes only a quantitative difference to $k_z^{(0)}$. However, β results in wave attentuation, which is qualitatively different from the $\sigma = \infty$ case. We outline below how β can be evaluated.

$P(z)$ = power flow integrated over the cross-sectional area A

$$= \frac{1}{2}\int_A \text{Re}[\mathbf{E}_t e^{ik_z z} \times \mathbf{H}_t^* e^{-ik_z^* z}]da = P_0 e^{-2k_{zi}z} = P_0 e^{-2\beta z} \tag{8.56}$$

where $\quad P_0 = P(z=0) = \frac{1}{2}\int_A \text{Re}[\mathbf{E}_t \times \mathbf{H}_t^*]da \quad (46)$



$$\Rightarrow \beta = -\frac{1}{2P}\frac{dP}{dz} = \text{field } \underline{\text{attenuation constant}} \tag{8.57}$$

[dP/dz : total power dissipated on the wall per unit length]

$$\begin{cases} (8.15) \Rightarrow \frac{dP}{dz} = -\frac{1}{2\sigma\delta}\oint_C |\mathbf{K}_{eff}|^2 dl \\ (8.14) \Rightarrow \mathbf{K}_{eff} = \mathbf{n} \times \mathbf{H} \end{cases} \Rightarrow \frac{dP}{dz} = -\frac{1}{2\sigma\delta}\oint_C |\mathbf{n} \times \mathbf{H}|^2 dl \tag{8.58}$$

Note: 1. (8.14), (8.15) are (45), (50) in lecture notes, Ch. 7.

2. $\mathbf{E}(\mathbf{x}_t) = \mathbf{E}_t + E_z\mathbf{e}_z$; $\mathbf{H}(\mathbf{x}_t) = \mathbf{H}_t + H_z\mathbf{e}_z$ [see (15)] with E_z or $H_z = 0$

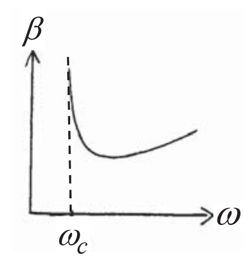
3. $\mathbf{n} \perp$ wall $\Rightarrow \mathbf{n} \times \mathbf{H} = \mathbf{n} \times \mathbf{H}_{\|}$ ($\mathbf{H}_{\|}$ is the component of $\mathbf{H}$ $\|$ to the wall)

Rewrite the field attenuation constant: $\beta = -\frac{1}{2P}\frac{dP}{dz}$ [(8.57)]

Since the wall loss can be treated as a small perturbation, we may use the zero-order $\mathbf{H}$ derived for $\sigma = \infty$ in Secs. 8.2-4 to calculate $\frac{dP}{dz}$ by (8.58) and calculate P by (8.56) and (46). (8.57) then gives β.

Formulae for β for rectangular and cylindrical waveguides are tabulated in many microwave textbooks, e.g. R. E. Collin, "Foundation of Microwave Engineering" (2nd Ed.), p. 189 & p.197 (where the attenuation constant is denoted by α instead of β).

Note: (i) β has been calculated by a perturbation method, which assumes $k_z = k_z^{(0)} + \alpha + i\beta$ and requires $\alpha,\ \beta \ll k_z^{(0)}$. The method is invalid near the cutoff frequency because $k_z^{(0)} \to 0$ as $\omega \to \omega_c$. In fact, the perturbation method is so invalid that it gives an infinite β as $\omega \to \omega_c$ (see figure). Sec 8.6 gives a method which calculates both α and β near the cutoff frequency.



(ii) There are different definitions and notations for the attenuation constant:

In Ch. 8 of Jackson, the attentuation constant for the waveguide is denoted by β and it is defined as

$$\beta = -\frac{1}{2P}\frac{dP}{dz}, \tag{8.57}$$

This is the *field* attentuation constant, i.e.

$$\mathbf{E}, \mathbf{B} \propto e^{-\beta z}.$$

In Ch. 7 of Jackson, the attentuation constant for EM waves in an infinite medium is denoted by α [see (7.53)] and it is defined as

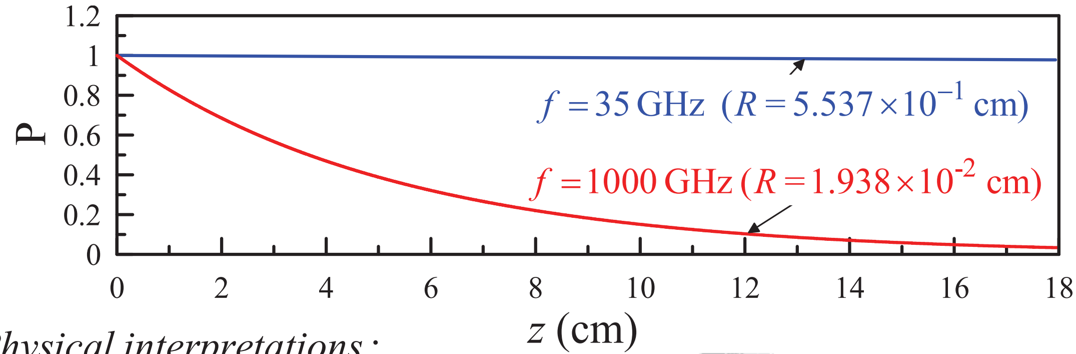
$$\alpha = -\frac{1}{P}\frac{dP}{dz}$$

This is the *power* attentuation constant, i.e.

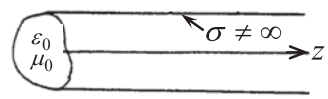
$$P \propto e^{-\alpha z}$$

Obviously, the power attentuation constant is twice the value of the field attentuation constant.

Example : TE_{01} wave attentuation in circular cross section copper waveguides (radius $= R$, $f = 1.06 \times$ cutoff frequency)



Physical interpretations :



1. $\dfrac{dP_{loss}}{da} = \frac{1}{2}\sqrt{\dfrac{\mu_0\omega}{2\sigma}}\left|H_0\right|^2$ [Ch.7, (48)]

   Higher $\omega \Rightarrow$ higher $\dfrac{dP_{loss}}{da}$ [$\because$ same K_{eff} flows in smaller $\delta (= \sqrt{\dfrac{1}{\mu_0\sigma\omega}})$]

2. $\begin{cases} \text{Circumference} \propto R \\ \text{Cross-sectional area} \propto R^2 \end{cases} \Rightarrow$ $\begin{cases} \text{For smaller } R\text{, there is more surface} \\ \text{area (which dissipates power) for} \\ \text{a unit cross-sectional area (which} \\ \text{transports power), hence higher loss.} \end{cases}$

8.8 Cavity Power Loss and *Q*

Definition of Q : Waves *propagates* in a waveguide. Hence, its attenuation is represented by a complex k_z. Since fields are *stored* in a cavity, any loss results in damping in time. The damping is then represented by a complex ω. We consider only the wall loss for now.

Assume fields at any point in the cavity have the time dependence:

$$E(t) = \begin{cases} E_0 e^{-i\omega_0 t}, & \sigma = \infty \\ E_0 e^{-i(\omega_0 + \Delta\omega - i\frac{\omega_0}{2Q})t} = E_0 e^{-i(\omega_0+\Delta\omega)t - \frac{\omega_0}{2Q}t}, & \sigma \neq \infty \end{cases} \quad (8.88)$$

where ω_0 is the resonant frequency [e.g. (34)] without the wall loss.

(8.88) assumes that the wall loss modifies ω_0 by a small real part $\Delta\omega$ and a small imaginary part $\frac{\omega_0}{2Q}$, where $\Delta\omega$ and Q are to be dertermined, and ω_0 is a refernece value (for $\sigma = \infty$).

Physical reason for $\Delta\omega$: Effective cavity size increases by $\sim \delta$. A larger cavity has a lower resonant ω_{mnl} [see (34)]. Hence, $\Delta\omega < 0$.

Physical reason for Q : Ohmic dissipation on the wall

$U =$ stored energy in the cavity $[\propto |E|^2 \propto e^{-i\omega t} \cdot e^{i\omega^* t} = e^{2\omega_i t} = e^{-\frac{\omega_0 t}{Q}}]$

$$= U_0 e^{-\frac{\omega_0 t}{Q}}$$

$$\boxed{\begin{aligned} &E(t) = E_0 e^{-i(\omega_0+\Delta\omega)t - \frac{\omega_0}{2Q}t} \quad [(8.88)] \\ &\Rightarrow \omega_i = -\frac{\omega_0}{2Q} \end{aligned}}$$

$$\Rightarrow \frac{dU}{dt}\ (\text{power loss}) = -\frac{\omega_0}{Q} U \tag{8.87}$$

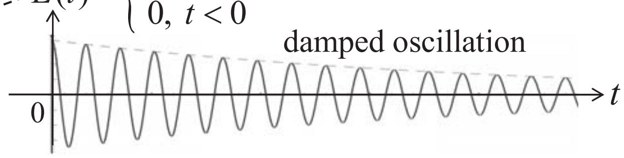
$$\Rightarrow Q = \omega_0 \frac{U\ (\text{stored energy})}{P_{loss}\ (\text{power loss})} \quad [\underline{\text{time-space definition of } Q}] \tag{8.86}$$

To examine the damped oscillation in ω-space, we write

$$E(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(\omega) e^{-i\omega t} d\omega,$$

where

$$E(t) = \begin{cases} E_0 e^{-i(\omega_0+\Delta\omega)t - \frac{\omega_0}{2Q}t}, & t \geq 0 \quad [(8.88)] \\ 0, & t < 0 \end{cases}$$



$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt$$

$$= \frac{1}{\sqrt{2\pi}} E_0 \int_0^{\infty} e^{-\frac{\omega_0}{2Q}t + i(\omega - \omega_0 - \Delta\omega)t} dt = \frac{1}{\sqrt{2\pi}} \frac{E_0}{-i(\omega - \omega_0 - \Delta\omega) + \frac{\omega_0}{2Q}}$$

$\boxed{\text{Let } E(t<0) = 0 \text{ (a realistic assumption)}}$

Rewrite $E(\omega) = \frac{1}{\sqrt{2\pi}} \frac{E_0}{-i(\omega - \omega_0 - \Delta\omega) + \frac{\omega_0}{2Q}}$, which gives

$$|E(\omega)|^2 \propto \frac{1}{(\omega - \omega_0 - \Delta\omega)^2 + (\frac{\omega_0}{2Q})^2} = \begin{cases} \max, & \omega = \omega_0 + \Delta\omega \\ \frac{1}{2}\max, & \omega = \omega_0 + \Delta\omega \pm \frac{\omega_0}{2Q} \end{cases} \tag{8.90}$$

$$\Rightarrow \delta\omega = \begin{bmatrix} \text{full width at} \\ \text{half-maximum points} \end{bmatrix} = \frac{\omega_0}{Q}$$

$$\Rightarrow Q = \frac{\omega_0}{\delta\omega} \quad (\underline{\text{frequency-space definition of } Q}) \tag{8.91}$$

Note: ω_0 (a reference value) is the resonant frequency with no wall loss $(\sigma = \infty)$, while $\omega_0 + \Delta\omega$ is the resonant frequency with wall loss $(\sigma \neq \infty)$. In almost all cases, $|\Delta\omega| \ll \omega_0$. So we let

$$\omega_0 + \Delta\omega \approx \omega_0$$

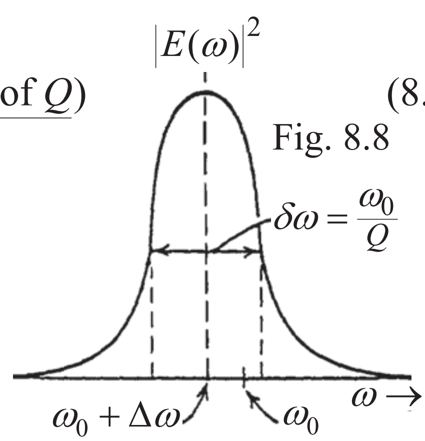


Fig. 8.8

Physical Interpretation of Q :

(i) Use the time-space definition: $Q = \omega_0 \dfrac{\text{stored energy}}{\text{power loss}}$ [(8.86)]

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{\tau_0} \leftarrow \boxed{\text{wave period}} \tag{47a}$$

$$\frac{\text{stored energy}}{\text{power loss}} \approx \tau_d \leftarrow \boxed{\text{decay time of stored energy}} \tag{47b}$$

Sub. (47a,b) into (8.86) $\Rightarrow$ $\quad Q \approx 2\pi \dfrac{\tau_d}{\tau_0}$ (48)

(48) shows that Q, which results from the power loss, is approximately 2π times the number of oscillations during the decay time. A larger Q value implies that the field energy can be stored in the cavity for a longer time. Hence, Q is commonly referred to as the quality factor.

However, in some cases, e.g. high-power microwave generation, a low Q value may often be desired. This is arranged by a structure which couples wave out of the cavity (not by high Ohmic loss).

(ii) Use the frequency-space definition: $Q = \dfrac{\omega_0}{\delta\omega}$ (see Fig. 8.8)

For a lossy cavity, a resonant mode can be excited not just at one ω (as a lossless cavity) but over the range of $\delta\omega$. The resonant frequency ($\omega_0 + \Delta\omega$, right fig.) is the frequency at which the cavity field has the largest amplitude when excited by a source of the same power.

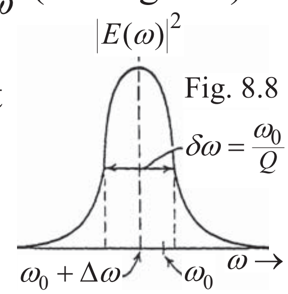
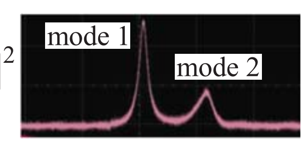


Fig. 8.8

Measurement of Q :

Source of variable ω at a constant power $\rightarrow$ Cavity $\nearrow$ $|E(\omega)|^2$



$\rightarrow \omega$

A Comparison : In Ch.7, the eq. of motion of a bound electron is

$$m\frac{d^2}{dt^2}\mathbf{x} = -e\mathbf{E}(\mathbf{x},t) - \gamma m \frac{d}{dt}\mathbf{x} - m\omega_0^2\mathbf{x} \quad [(7.49)] \quad (\omega_0\text{: binding freq.})$$

$$\begin{cases} \mathbf{x}(t) = \mathbf{x}_0 e^{-i\omega t} \\ \mathbf{E}(\mathbf{x},t) = \mathbf{E}_0 e^{-i\omega t} \end{cases} \Rightarrow \mathbf{x}_0 = -\frac{e}{m}\frac{\mathbf{E}_0}{\omega_0^2 - \omega^2 - i\omega\gamma} \Rightarrow |\mathbf{x}_0|^2 \text{ also exhibits}$$

similar effect of "resonant frequency broadening" due to damping (γ).

Evaluation of Q $(= \omega_0 \dfrac{U}{P_{loss}})$:

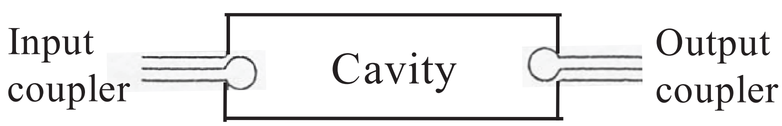
We can evaluate Q (but not $\Delta\omega$) due to the ohmic loss as follows. First calculate the zero-order $\mathbf{E}$ and $\mathbf{H}$ of a specific cavity assuming $\sigma = \infty$, then use the zero-order $\mathbf{E}$ and $\mathbf{H}$ to calculate U and P_{loss},

$$U \text{ (stored energy)} = \int_v (w_e + w_m) d^3x = \underbrace{\begin{cases} 2\int_v w_e d^3x = \frac{\varepsilon}{2}\int_v |\mathbf{E}|^2 d^3x \\ 2\int_v w_m d^3x = \frac{\mu}{2}\int_v |\mathbf{H}|^2 d^3x \end{cases}}_{\text{Assume } \varepsilon,\ \mu \text{ are real}}$$

$$\begin{aligned} P_{loss} \text{ (power loss)} &\overset{\boxed{(8.15)}}{=} \frac{1}{2\sigma\delta}\oint_s |\mathbf{K}_{eff}|^2 da \\ &\overset{\boxed{(8.14)}}{=} \frac{1}{2\sigma\delta}\oint_s |\mathbf{n}\times\mathbf{H}|^2 da \end{aligned} \tag{49}$$

Formulae for Q (due to ohmic loss) for rectangular and cylindrical cavities can be found in, for example, R. E. Collin, "Foundation of Microwave Engineering", p. 503 and p. 506.

Q due to other types of losses :



If there are several types of power losses in a cavity (e.g. due to a lossy filling medium or leakage through a coupling structure), Q can be expressed as

$$Q = \omega_0 \frac{\text{stored energy}}{\sum_n (\text{power loss})_n} \tag{50}$$

n-th type of power loss

$$\Rightarrow \frac{1}{Q} = \sum_n \frac{1}{Q_n} \tag{51}$$

where Q_n (Q due to the n-th type of power loss) is given by

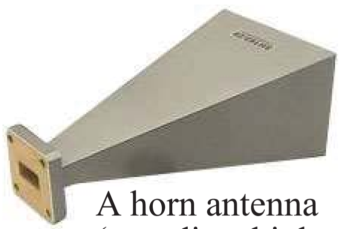
$$Q_n = \omega_0 \frac{\text{stored energy}}{(\text{power loss})_n}$$

Example: Q due to ohmic loss on the wall [see (49)] is commonly written as Q_{ohm}.

Waveguides and Cavities – A Comparison

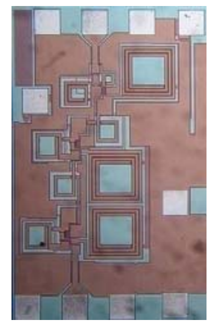
	Waveguide	Cavity
Function	Transport EM energy	Store EM energy
Characterization	Dispersion relation and attenuation constant	Resonant frequency and Q
Examples of applications	Feed high power to an antenna	(1) Particle acceleration (2) Frequency measurement

High- and Low-Power Microwave Devices – A Comparison



A horn antenna (to radiate high power)

A low-noise amplifier used in low-power devices, such as cell phones. All components form a single monolithic* microwave integrated circuit (MMIC)

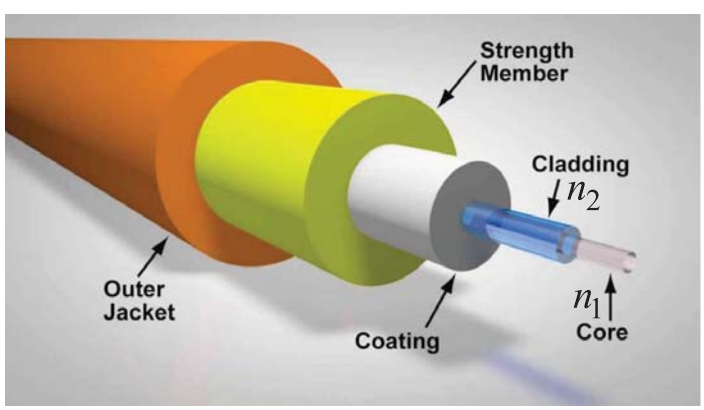


*Having all components manufactured into or on top of a single chip.

8.10 Multimode Propagation in Optical Fibers

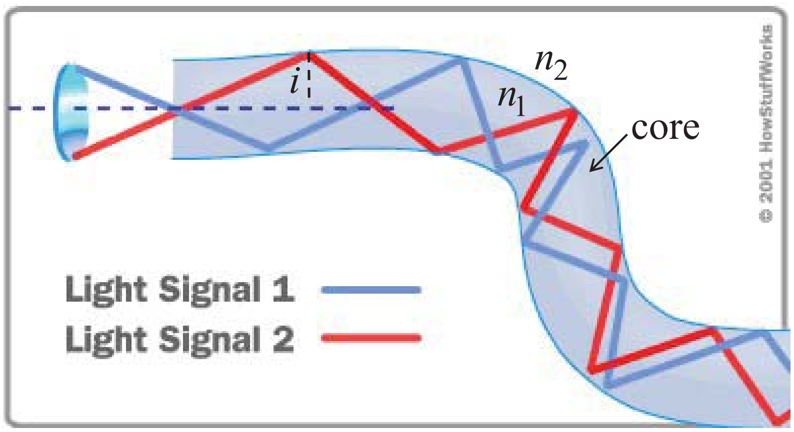
Optical fibers are widely used in high-speed, high-capacity, and long-distance telecomunications. An optical fiber consists mainly of a "core" (the region of most energy flow) surrounded by a "cladding", both of low-loss materials. The core is about as thin as a human hair.

If the index of refraction of the core (n_1) is greater than that of the cladding (n_2), total internal reflection can confine the light wave within the core, allowing extremely low-loss propagation along the fiber (e.g. < 1 dB/km).



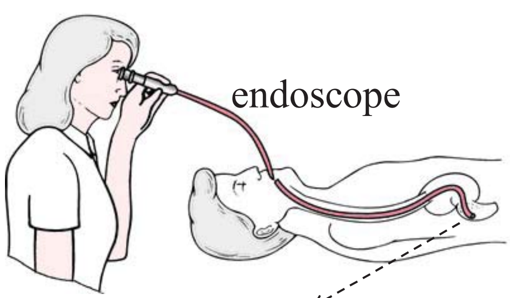
For multiple (high-order) modes propagation, geometrical optics applies. Let the incidence angle of a light ray (from core to cladding) be i (see figure below). If $i > i_0 (\equiv \sin^{-1}\frac{n_2}{n_1}$, the critical angle), the ray will be totally reflected at the cladding.

An optical fiber can carry many independent channels, each at a different wavelength of light. 1000's of fibers can be bundled into an optical fiber cable, which can be used for image transmission, etc.

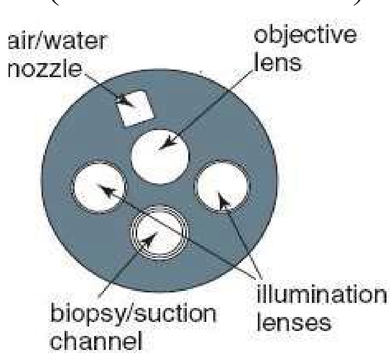


optical fiber cable

Endoscope : medical application of the optical fiber cable



srtructure of the cable tip
(~1 cm in diameter)



How it works

Some of the optical fibers take light down to the tip of the endoscope which shines inside the body. Other optical fibers collect images of organs with a lens. The images are sent back along the fibers for viewing.

The image-transmitting fibers must be lined up in the same order from end to end in order to preserve the image. Up to 10,000 fibers are needed to produce a clear image.

The endoscope can also take organ tissues for analysis.

Charles K. Kao
was awarded the 2009 Nobel Prize in Physics

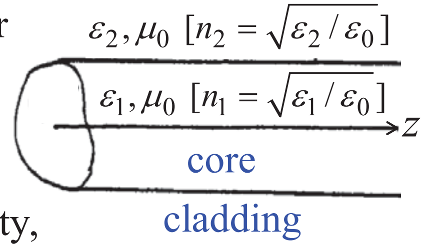
"for groundbreaking achievements concerning the transmission of light in fibers for optical communication"

高錕 (1933-2018)

The rapid transmission of signals over long distances is fundamental to the flow of information in our time. Since the 1930s, thin filaments, or fibers, of glass have been used to see inside the body, but these had long remained unusable for long-distance information transfer because too much light was lost along the way. In the 1960s, Charles Kao presented a solution: fibers of very pure glass transported sufficient light. Together with laser technology, his solution has made telecommunication using optical fibers possible.

8.11 Modes in Dielectric Waveguides (pp. 388-389)

The core and cladding of an optical fiber form a dielectric waveguide. We model the core by a non-magnetic ($\mu = \mu_0$) dielectric cylinder of *infinite* length with a *uniform* cross section (arbitrary shape). For simplicity, let its permittivity ε_1 be a constant (Jackson assumes ε_1 to be a function of the transverse coordinates). Outside the core is an infinite, non-magnetic cladding with constant permittivity ε_2.



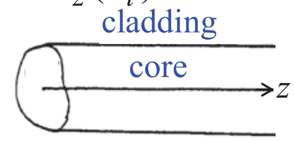
Let $\begin{cases} \mathbf{E}(\mathbf{x},t) = \mathbf{E}(\mathbf{x}_t)e^{ik_z z - i\omega t} \\ \mathbf{B}(\mathbf{x},t) = \mathbf{B}(\mathbf{x}_t)e^{ik_z z - i\omega t} \end{cases}$, we obtain [as in obtaining (14)]

$$(\nabla_t^2 + \mu_0\varepsilon_{1,2}\omega^2 - k_z^2)\begin{Bmatrix} E_z(\mathbf{x}_t) \\ B_z(\mathbf{x}_t) \end{Bmatrix} = 0 \quad \begin{bmatrix} \varepsilon_1\text{: inside the core} \\ \varepsilon_2\text{: outside the core} \end{bmatrix} \tag{51}$$

$$n_{1,2} \equiv \sqrt{\frac{\mu_{1,2}\varepsilon_{1,2}}{\mu_0\varepsilon_0}} = \sqrt{\frac{\varepsilon_{1,2}}{\varepsilon_0}} \;\; [\mu_{1,2} = \mu_0 \text{ by assumption}] \Rightarrow \varepsilon_{1,2} = n_{1,2}^2\varepsilon_0 \tag{52}$$

$$\Rightarrow (\nabla_t^2 + \frac{n_{1,2}^2\omega^2}{c^2} - k_z^2)\begin{Bmatrix} E_z(\mathbf{x}_t) \\ B_z(\mathbf{x}_t) \end{Bmatrix} = 0 \quad \boxed{\begin{array}{l} n_1 \text{ \& } n_2\text{: index of refraction of} \\ \text{core \& cladding, respectively} \end{array}} \tag{53}$$

Following the same treatment for a metallic waveguide as in (8.26), $\mathbf{E}_t(\mathbf{x}_t)$ & $\mathbf{B}_t(\mathbf{x}_t)$ can be expressed in terms of $E_z(\mathbf{x}_t)$ & $B_z(\mathbf{x}_t)$:



$$\begin{cases} \mathbf{E}_t(\mathbf{x}_t) = \dfrac{i\left[k_z \nabla_t E_z(\mathbf{x}_t) - \omega \mathbf{e}_z \times \nabla_t B_z(\mathbf{x}_t)\right]}{\dfrac{n_{1,2}^2\omega^2}{c^2} - k_z^2} \\ \mathbf{B}_t(\mathbf{x}_t) = \dfrac{i\left[k_z \nabla_t B_z(\mathbf{x}_t) + \omega \varepsilon_0 n_{1,2}^2 \mathbf{e}_z \times \nabla_t E_z(\mathbf{x}_t)\right]}{\dfrac{n_{1,2}^2\omega^2}{c^2} - k_z^2} \end{cases} \quad (8.126)$$

subscript "1": core region
subscript "2": cladding region

With $n_{1,2}^2 = \varepsilon_{1,2}/\varepsilon_0$ [by (52)], (8.126) can be cast into the same form as (8.26) (for hollow waveguides), but with 2 differences:

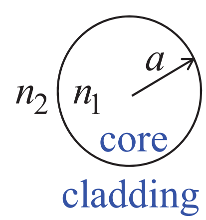
1. There are 2 regions of fields: inside the core [(8.126) with subscript "1"] and outside the core [(8.126) with subscript "2"].

2. Fields in general do not separate into TE and TM modes. They are "HE modes" with both E_z & B_z components (p.388). However, a circular fiber with fields independent of θ has TE and TM modes (but no TEM modes). An example is provided below.

Example: Circular cross section with no θ-dependence for fields
Assume the core radius is a. Consider the TE_{0n} mode ($E_z = 0$).

$$(53) \Rightarrow \left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} + \frac{n_{1,2}^2\omega^2}{c^2} - k_z^2\right) B_z(\mathbf{x}_t) = 0$$

$\boxed{\frac{\partial}{\partial\theta} = 0}$



$$\Rightarrow \begin{cases} \left(\dfrac{d^2}{d\rho^2} + \dfrac{1}{\rho}\dfrac{d}{d\rho} + \gamma^2\right) B_z(\rho) = 0, \quad \rho < a & (54a) \\ \left(\dfrac{d^2}{d\rho^2} + \dfrac{1}{\rho}\dfrac{d}{d\rho} - \beta^2\right) B_z(\rho) = 0, \quad \rho > a & (54b) \end{cases}$$

$$\text{where} \begin{cases} \gamma^2 \equiv \dfrac{n_1^2\omega^2}{c^2} - k_z^2 & (55a) \\ \beta^2 \equiv k_z^2 - \dfrac{n_2^2\omega^2}{c^2} & (55b) \end{cases}$$

We have defined γ^2, β^2 this way so that they are both positive numbers, as will be shown in (63) and (64).

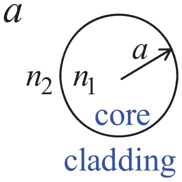
$$\text{Solu. for (54a,b):} \begin{cases} B_z(\rho) = J_0(\gamma\rho), \quad \rho < a & (56a) \\ B_z(\rho) = AK_0(\beta\rho), \quad \rho > a & (56b) \end{cases}$$

Question: Have we used any b.c.?

where J_0 is the Bessel function of the 1st kind and K_0 is the modified Bessel function of the 2nd kind (Sec. 3.7). A is an arbitrary constant.

In (8.126), set $E_z = 0$. Use the notations $\gamma^2 = \frac{n_1^2\omega^2}{c^2} - k_z^2$ [(55a)] &

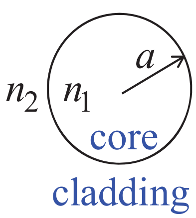
$$\beta^2 = k_z^2 - \frac{n_2^2\omega^2}{c^2} \text{[(55b)]} \Rightarrow \begin{cases} \mathbf{E}_t(\mathbf{x}_t) = \frac{-i}{\gamma^2}\omega\mathbf{e}_z \times \nabla_t B_z(\mathbf{x}_t), & \rho < a \\ \mathbf{B}_t(\mathbf{x}_t) = \frac{i}{\gamma^2} k_z \nabla_t B_z(\mathbf{x}_t), & \rho < a \\ \mathbf{E}_t(\mathbf{x}_t) = \frac{i}{\beta^2}\omega\mathbf{e}_z \times \nabla_t B_z(\mathbf{x}_t), & \rho > a \\ \mathbf{B}_t(\mathbf{x}_t) = \frac{-i}{\beta^2} k_z \nabla_t B_z(\mathbf{x}_t), & \rho > a \end{cases} \quad (57)$$



Sub. $\begin{cases} B_z(\rho) = J_0(\gamma\rho), \ \rho < a & \text{[(56a)]} \\ B_z(\rho) = AK_0(\beta\rho), \ \rho > a & \text{[(56b)]} \end{cases}$ into (57) and

$$\text{use} \begin{cases} \frac{d}{dx} J_0(x) = -J_1(x) \\ \frac{d}{dx} K_0(x) = -K_1(x) \end{cases} \Rightarrow \begin{cases} B_\rho(\rho) = -\frac{ik_z}{\gamma} J_1(\gamma\rho), & \rho < a & (58a) \\ B_\rho(\rho) = \frac{ik_z}{\beta} AK_1(\beta\rho), & \rho > a & (58b) \\ E_\theta(\rho) = \frac{i\omega}{\gamma} J_1(\gamma\rho), & \rho < a & (59a) \\ E_\theta(\rho) = -\frac{i\omega}{\beta} AK_1(\beta\rho), & \rho > a & (59b) \end{cases}$$

Dispersion relation: Apply b.c.'s at $r = a$ to all fields:



$$\begin{cases} B_z(\rho) = J_0(\gamma\rho), & \rho < a & \text{[(56a)]} \\ B_z(\rho) = AK_0(\beta\rho), & \rho > a & \text{[(56b)]} \\ B_\rho(\rho) = -\frac{ik_z}{\gamma} J_1(\gamma\rho), & \rho < a & \text{[(58a)]} \\ B_\rho(\rho) = \frac{ik_z}{\beta} AK_1(\beta\rho), & \rho > a & \text{[(58b)]} \\ E_\theta(\rho) = \frac{i\omega}{\gamma} J_1(\gamma\rho), & \rho < a & \text{[(59a)]} \\ E_\theta(\rho) = -\frac{i\omega}{\beta} AK_1(\beta\rho), & \rho > a & \text{[(59b)]} \end{cases}$$

Continuity of $H_z \Rightarrow J_0(\gamma a) = AK_0(\beta a) \Rightarrow A = \frac{J_0(\gamma a)}{K_0(\beta a)}$ (60)

Continuity of B_ρ and $E_\theta \Rightarrow -\frac{1}{\gamma} J_1(\gamma a) = \frac{1}{\beta} AK_1(\beta a)$ (61)

(60), (61) give the dispersion relation (relation between k_z & ω):

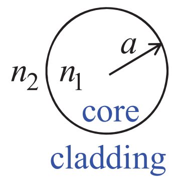
$$\begin{cases} \frac{\gamma J_0(\gamma a)}{J_1(\gamma a)} + \frac{\beta K_0(\beta a)}{K_1(\beta a)} = 0 \quad \text{[Jackson Prob. 8.17, } m = 0\text{]} \quad (62) \\ \text{with } \gamma^2 \equiv \frac{n_1^2\omega^2}{c^2} - k_z^2 \ \text{[(55a)] \& } \beta^2 \equiv k_z^2 - \frac{n_2^2\omega^2}{c^2} \text{[(55b)]} \end{cases}$$

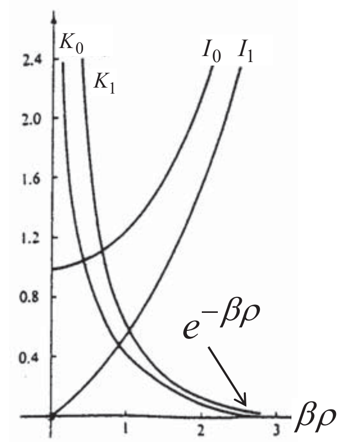
Condition for total internal reflection at $r = a$:

As in a metallic waveguide, there are an infinite no of modes in a fiber. Their existene requires total internal reflection of the core fields (or the energy will radiate away, no mode).

In (56)-(59), $K_{0,1}(\beta\rho) \sim e^{-\beta\rho}$ as $\rho \to \infty$. [see (3.104)]. Thus, for total internal reflection at $r = a$, we must have

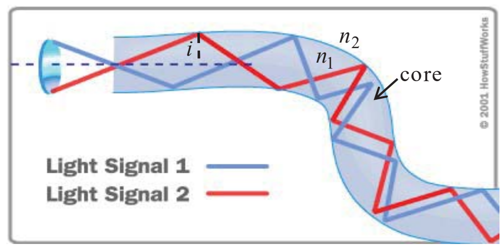
$$\beta = \sqrt{k_z^2 - n_2^2\omega^2/c^2} > 0 \quad (63)$$



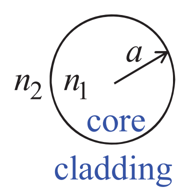


so that the fields in the cladding are evenescent (exponentially small).

To satisfy (63), k_z must be sufficiently large. This is similar to the case of geometrical optics (right figure); a large k_z produces a large incident angle (i) at the cladding for total internal reflection.



Cutoff frequencies (ω_c) of TE_{0n} modes:



$$\text{Rewrite}\begin{cases} \dfrac{\gamma J_0(\gamma a)}{J_1(\gamma a)} + \dfrac{\beta K_0(\beta a)}{K_1(\beta a)} = 0 \quad \begin{bmatrix}\text{dispersion}\\ \text{relation}\end{bmatrix} \; [(62)] \\ \text{with } \gamma^2 \equiv \dfrac{n_1^2\omega^2}{c^2} - k_z^2 \; [(55a)] \; \& \; \beta^2 \equiv k_z^2 - \dfrac{n_2^2\omega^2}{c^2} \, [(55b)] \end{cases}$$

The cutoff frequency (ω_c) occurs when $\beta = 0$. The corresponding γ (denote by γ_c) is obtained by setting $\beta = 0$ in $\dfrac{\gamma J_0(\gamma a)}{J_1(\gamma a)} + \dfrac{\beta K_0(\beta a)}{K_1(\beta a)} = 0$.

$$\Rightarrow J_0(\gamma_c a) = 0 \;\Rightarrow\; \gamma_c a = 2.405,\; 5.520,\; 8.654\cdots \text{ (see p. 114)} \qquad (64)$$

$\Rightarrow$ An infinite no of roots, each forming a TE_{0n} mode.

To obtain ω_c, let $\gamma = \gamma_c$ in (55a) and $\beta = 0$ in (55b). Then, (55a,b) give

$$\omega_c = \frac{\gamma_c c}{\sqrt{n_1^2 - n_2^2}} \quad \left[\text{e.g. for TE}_{01} \text{ mode}: \omega_{c01} = \frac{2.405c}{\sqrt{n_1^2 - n_2^2}\,a}, \text{ p.389}\right] \quad (65)$$

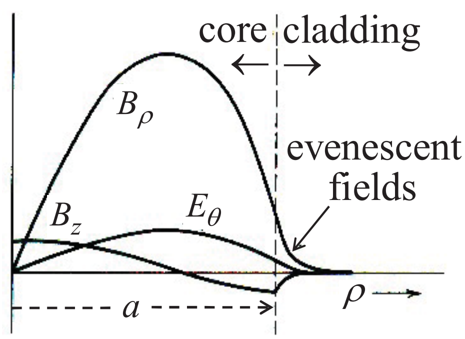
$$\textit{Example}: \begin{cases} n_1 = 1.5 \\ n_2 = 1.485 \\ a = 3\ \mu\text{m} \end{cases} \Rightarrow \begin{cases} \omega_{c01} \approx 1.1\times10^{15} \text{ rad/s} \\ \approx 1.8\times10^{14} \text{ Hz} \end{cases} \quad \begin{bmatrix} \textit{Question}: \text{Why let} \\ n_1 \;\&\; n_2 \text{ be so close?} \end{bmatrix}$$

$$
Field\ profiles: \begin{cases}
B_z(\rho) = J_0(\gamma\rho), \quad \rho < a & [(56a)] \\
B_z(\rho) = AK_0(\beta\rho), \quad \rho > a & [(56b)] \\
B_\rho(\rho) = -\dfrac{ik_z}{\gamma} J_1(\gamma\rho), \quad \rho < a & [(58a)] \\
B_\rho(\rho) = \dfrac{ik_z}{\beta} AK_1(\beta\rho), \quad \rho > a & [(58b)] \\
E_\theta(\rho) = \dfrac{i\omega}{\gamma} J_1(\gamma\rho), \quad \rho < a & [(59a)] \\
E_\theta(\rho) = -\dfrac{i\omega}{\beta} AK_1(\beta\rho), \quad \rho > a & [(59b)]
\end{cases}
$$

The dispersion relation gives ω as a function of k_z. Each set of (ω, k_z) values gives a set of (γ, β) values. Sub. the set of (γ, β) values into the above field eqs., we obtain the corresponding field profiles.

Field profiles of the TE_{01} mode (at $\omega > \omega_{c01}$) are illustrated to the right.

Fields of the TE_{01} mode



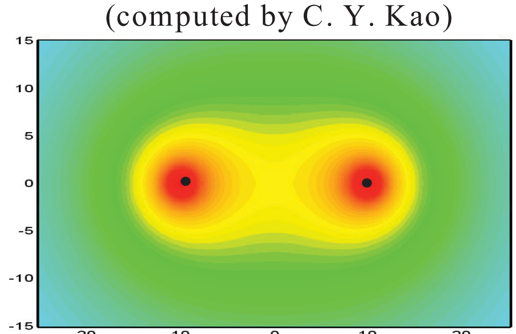
以下節錄自「高壓線、微波爐及電磁武器Q&A 」(作者: 朱國瑞)
刊載於「物理雙月刊」第 廿八卷第二期 (2006年4月)

高壓線

Q1：高壓線如何傳輸能量?

A：高壓線是兩條平行導線構成，它的規格通常用電流及電壓表示，這兩個量可以用來計　算傳輸的功率，但不足以顯現能量的傳輸機制。從物理角度來看，高壓線是藉著周圍的電場和磁場來傳輸能量。高壓線附近的電磁場稱為近場(near field)，其中的磁場主要由導線中的電流產生(電流方向一正一反)，電場主要由二線之間的電壓差產生，二者正好相互垂直，以光速沿線流動(稱為Poynting vector)，因此可以傳輸能量。電壓和電流以60 Hz的頻率變化，但因為電場和磁場的方向一起跟著變，能量維持在同一方向傳輸。

(computed by C. Y. Kao)



除了高壓線外，只要是傳輸能量的一對導線，不論是否平行排列，都是利用線外的電磁場作為傳輸工具，從插座接到家用電器的兩條導線，是常見的例子。即使是直流電，也有電壓差及電流，二線周圍的靜電場和靜磁場照樣可以藉 Poynting vector 傳輸能量，否則汽、機車電瓶裡的能量就無法送到前端的照明燈了。

這樣看來，高壓線的四周像是一條電磁場的大洪流，高場強區的橫截面半徑大約等於二線之間的距離。反而是導線內的電磁場係垂直於表面向內傳播，並且迅速變為熱能，所以導線的裡面，不僅不能傳輸能量，還會造成歐姆損耗，甚至連線內損耗掉的能量也是從線外的電磁場傳輸進去的。

Q2：高壓線如何使人觸電？

A：相對於地面，高壓線上的電壓高達幾萬甚至幾十萬伏。電場值是電壓差除以距離，假如一個人腳著地，把手伸向一條高壓線，等於把地面移向高壓線，手和線越接近，其間的電場越強，接近到某個程度，電場會將空氣中少數的游離電子加速到足夠能量，把中性分子中的電子撞出來，再將這些電子一起加速，撞出更多電子，產生連鎖效應，頓時在手和線之間造成一條導電通道，而人體和地面對60 Hz的低頻電源也是導體，電流因此經由通道和人體傳入地面，人就像被雷擊一樣。兩條高壓線之間要有相當距離，就是為了避免彼此之間的電場太強，而高壓線的危險性不僅在於碰到線會致命，而是一進到電磁場的洪流區就有觸電的可能。

Q3：住在高壓線附近的人，雖然不會觸電，但長期曝露在高壓線產生的電磁場中，是不是比較容易罹患癌症？

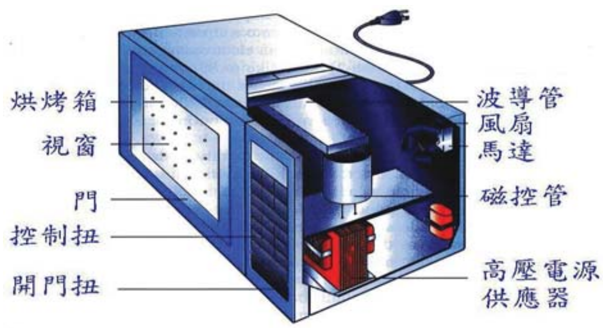
A：有些個案研究認為高壓線有導致某種癌症的危險，也有報導指出，某一區住在高壓線附近的居民，癌症患者人數高於平均，因而造成許多的訴訟案件，而電力公司也不知為此作了多少疏通的工作，訴訟和疏通的費用都非常龐大。

高壓線是否會導致癌症，是一項極為艱巨和費時的研究工作，雖然目前仍是一個全球性的研究課題，但究竟已進行了數十年之久，相關文獻已具參考價值。美國物理學會在檢視現有的疾病分佈和生物研究資料，以及參閱其他小組對高壓線致癌的評估報告後，認為這些資料和報告並未顯示高壓線和癌症之間的科學關連性，因此於1995年及2005年兩度發表聲明，要點是：「曝露在任何環境因素之下，都不可能絕對證明其無害於健康，但是要作出某一環境因素有害健康的結論，仍必須舉證二者之間一致的、有意義的、和互為因果的關係。基於此一觀點，高壓線導致癌症的臆測尚無科學依據」(“While it is impossible to prove that no deleterious health effects occur from exposure to any environmental factor, it is necessary to demonstrate a consistent, significant, and causal relationship before one can conclude that such effects do occur. From this standpoint, the conjectures relating cancer to power line fields have not been scientifically substantiated.”)。這份措辭嚴謹的聲明也反映了其他一些學術和公益團體對此問題的看法。

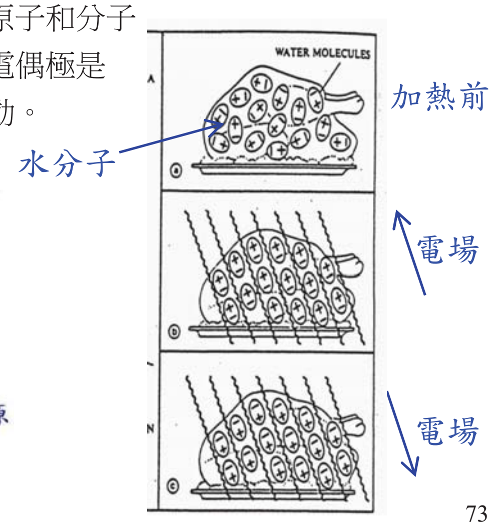
微波爐

Q4：微波爐如何加熱食物?

A：微波進入任何物質，都會帶動裡面的電子，撞擊鄰近分子，或多或少都有加熱的效果。但介電質中的電子被原子核束縛住，位移極小，電場對它作的功也很少。微波爐的快速加熱，靠的是水分子的大動作。水分子和一般原子、分子不同，它天生就有電偶極。沒有外加電場的時候，各個水分子的電偶極排列很紊亂。把微波爐打開，裡面就有電磁場，電偶極在電場中受到力矩作用，就會轉動，電場的方向不斷改變，水分子的方向也跟著改變，不斷的反覆轉動。相較之下，一般的原子和分子只有在電場中才會感應出電偶極，而感應產生的電偶極是朝著電場方向的，受到的力矩為零，因此不會轉動。



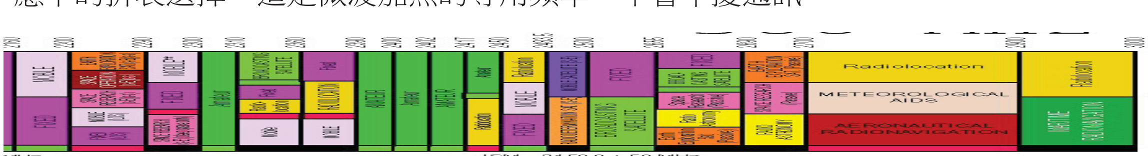
(取自L. A. Bloomfield, “How Things Work”)



多轉動一次，並不表示就會增多一點能量。水分子在交流電場中反覆轉動，是有序的「受力振盪(forced oscillation)」，能吸收也能回傳能量，很快就會達到平衡狀態。並且，這種有序的動能也不叫作熱能。熱能是亂無秩序的動能(例如空氣中分子的運動)，有序的動能要靠碰撞，才能轉換為熱能。所以水分子在轉動的時候，必須和其他分子(包括其他水分子)擠在一起，相互擦撞，才會產生熱能。一旦成為熱能，就無法再回傳給電場，溫度因此持續上升。比如一塊肉，電場讓裡面的水分子轉動，去擠動旁邊的肉分子，水和肉都熱了起來。一杯水也可以加熱，換成水蒸氣就無法加熱了。冰塊是固體，裡面的水分子轉動不易，加熱很慢，但表面多少會融化出一點液體，所以冷凍食物在微波爐裡加熱，常常外熱內冷。

Q5：微波爐為什麼選擇2.45 GHz的電磁波? 是否會干擾通訊?

A：微波的頻率範圍很廣，介於0.3 GHz 和300 GHz之間。用微波加熱，不但要微波能進到食物裡面，還要能被吸收。頻率太低，吸收太慢；頻率太高，吸收太快，以致在表面上就被吸收，進不到裡面。微波爐的頻率(大都是2.45 GHz)，就是在這兩個考慮下的折衷選擇，這是微波加熱的專用頻率，不會干擾通訊。



Q6: 為什麼加熱速度快? 為什麼省電?

A: 傳統烤箱是先加熱食物表面，再靠傳導加熱內部。微波則能直接進到食物裡面，各處同時加熱，所以加熱速度快得多。加熱腔的四周是金屬腔壁，2.45 GHz的微波可以滲入腔壁約0.002毫米深度(skin depth)，電場在這個薄層內驅動電流，其強度恰好能把微波反射回去，由於腔壁是良導體，電流的歐姆加熱效應很小，腔壁熱不起來，因此微波大部份都在加熱食物。這種加熱方法效率高達50%左右，比傳統烤箱的加熱效率(約10%)大得多，所以也省電。

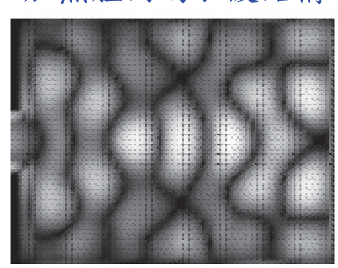
某些原子或分子對某些特定頻率反應特別激烈，產生共振吸收現象。2.45 GHz並非水分子的共振頻率，共振吸收也非(但常被誤解為)微波爐的加熱原理。

Q7：用微波爐加熱食物為什麼有時冷熱不均？

A：前面提到，2.45 GHz的微波，不但能進到食物裡面，還能被快速吸收。儘管如此，還是不能面面顧到。例如微波爐內部的加熱空間是一個共振腔(cavity)，微波在裡面形成駐波結構，有些地方強，有些地方弱，造成加熱不均。食物放置在一個轉盤上，就是為了避免它一直停留在駐波最強或最弱的地方。共振腔壁不用弧形的，是避免弧面的聚焦作用。

另外，食物所含的水份有多有少，吸波係數有很大的差別，若吸收特別快，裡面還沒加到熱，微波就已經被表層吸收掉了。食物內部的水份也可能分佈不均，加熱後，裡面反而特別燙。加熱不均不只是影響口味，還可能傷到身體，例如不小心吞下外溫內燙的食物，灼傷腸胃，或者有的地方還有病菌未被殺死，就吃進肚裡。

加熱腔內的駐波結構



Q8：微波爐漏出的輻射對人體會不會造成傷害?

A：就頻率而言，微波屬於非游離性輻射，亦即其光子能量太小，沒辦法破壞身體的細胞組織，但輻射太強仍會造成灼傷。相較之下，游離輻射(如X光)的光子能量大，會破壞細胞組織的化學鍵，導致癌症等，不過也需累積到一定的劑量才會如此。微波是以瞬間強度(單位面積通過的功率)為安全標準，游離輻射則以累積劑量為安全標準，例如單位體積的空氣中，在照射時間內，被游離分子的累積總數。

高強度微波造成灼傷，屬於顯而易見的熱效應(thermal effect)。輕度微波不會灼傷人，但長期曝露其下，是否會產生諸如頭痛、焦慮、疲勞、失眠等非熱效應(non-thermal effect)，則是一個爭議極大，目前尚無定論的問題。微波的非熱效應，是設定微波爐安全標準的因素之一，由於各國對此效應的認知不同，所規定的安全標準也不相同。美國的標準是離開微波爐表面5公分處，輻射強度不得高於每平方公分5毫瓦，俄國的標準則嚴得多。在政府單位的管制下，正常微波爐外洩的輻射強度是低於規定值的。

微波爐有時受到損壞，門關不緊，自動斷電機制也失靈，不小心拿來用，大量微波就會露了出來。最可能發生的情況是人感覺到燙，馬上閃開，不致造成重大灼傷。體內的血液流動還有冷卻效果，可以保護身體。但是微波外洩絕非小事，嚴重的灼傷，無法痊癒，眼球的水晶體中沒有血管，比較容易受到灼傷(例如造成白內障)，尤其要避免微波的照射。

微波爐的發明，其實還得歸因於微波的外洩。1946年，美國Raytheon公司的雷達工程師Dr. Percy LaBaron Spencer在測試微波發射器時，受到輕度的微波照射，口袋裡的糖融化了，使他想到微波加熱的用途。他的公司於1947年製造出第一個微波爐，幾乎有一個人高，重達三百多公斤，價格5000美元，還需要用水冷卻。二十年後，微波爐才成為實用的家電產品。1999年，Dr. Spencer被選入發明家名人堂，成為歷史人物。

Q9：微波加熱會不會破壞食物的分子，對人體造成間接傷害？

A：用微波爐烹煮食物，特別熱的地方，食物的分子結構可能會被破壞。其實煎和烤破壞的程度更厲害，不過都不如吃進肚子後，遭到腸胃分解的破壞來得大，所以破壞不一定就是有害，目前並無確切証據顯示微波加熱會產生有害食物。烹飪技術之一就是如何把食物的分子結構破壞得宜，讓它口感更好。就美食觀點而言，微波加熱達到的溫度較低，對分子的破壞力較小，即使是名廚，也難以發揮手藝。

Q10：電視上曾看到，用微波爐加熱的水會突然「爆炸」，這是什麼原因？

A：某些情況下，微波爐可以把一杯水加熱到沸點以上，卻完全沒有沸騰的跡象，拿出來輕輕一碰就突然大量噴出，像爆炸一樣。因為意想不到，常常燙傷人。

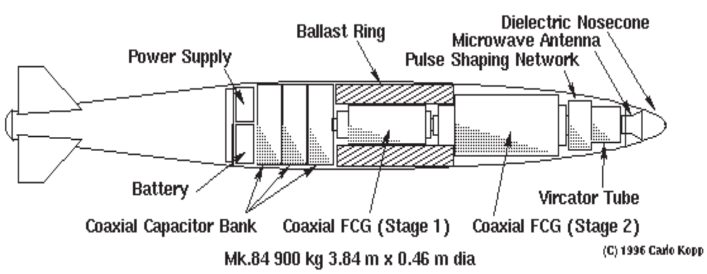
水隨時都在蒸發，但那只是表面上和大氣接觸的水。內部的水沒有汽化的空間，若要化成蒸氣，需要「種子」來促成，這些種子，就是汽泡。極燙的容器表面、粗糙的容器表面、或雜質附近，有助於形成汽泡。汽泡產生後，它四週界面上的水就有了汽化的空間，於是蒸發到汽泡內；溫度越高，蒸發越多。

在一般火爐上燒開水，因為鍋底溫度特高，容易產生汽泡。汽泡在浮出水面時，沿途的水不斷蒸發到汽泡內，最後一起逸出水面，水溫到了沸點後，汽泡源源不絕的逸出，熱量全部用在蒸發上，溫度不再上升。相較之下，水在微波爐中加熱均勻，沒有特別熱的接觸點，也沒有對流，若杯面平滑，水中雜質也不多，氣泡就無法形成，內部的水因此無法靠蒸發散熱，溫度可以一直上升到沸點以上[稱為「超熱(superheated)」狀態]。超過沸點的水，外表看來，沒有一點動靜，一旦受到觸動，內部會產生一些汽泡，大量的水驟然蒸發到汽泡裡面，使汽泡瞬間漲大，幾乎將整杯水擠了出來，造成電視上看到的爆炸現象。水和咖啡等飲料，放在微波爐裡加熱，千萬要小心。若是熱過了沸點，擺一塊糖進去或用湯匙攪動，就會觸發這種現象。

電磁武器

Q11：是否有一種武器，可以產生很強的脈衝電磁波，破壞電子器材？

A：這種武器俗稱電磁炸彈(E–bomb)，射出的是電磁波，可依頻率分為兩類：朝特定方向發射的微波炸彈(頻率約幾個GHz，波長10公分上下)和有如閃電四射的低頻波炸彈(頻率低於1 MHz，波長300公尺以上)，功率都高達10^{10}瓦以上，比一個城市的供電量還高，所以只能維持一瞬間(微秒至毫秒)，在高空，得用炸藥提供瞬間能源。



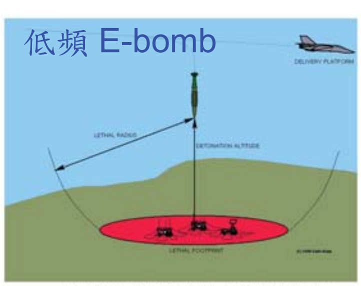


FIG.7 LETHAL FOOTPRINT OF LOW FREQUENCY E– BOMB IN RELATION TO ALTITUDE

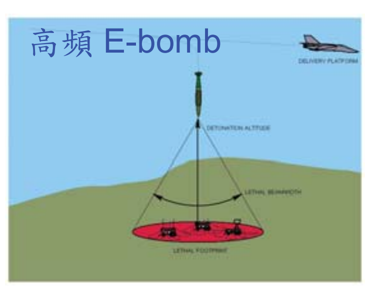


FIG.8 LETHAL FOOTPRINT OF A HPM E -BOMB IN RELATION TO ALTITUDE

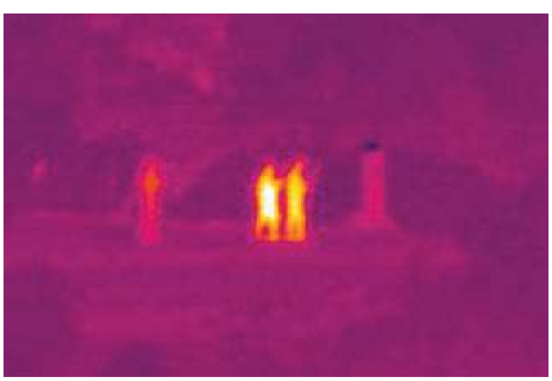
電磁炸彈射出的若是微波，可以直接進入電子器材(甚至還會在裡面短暫累積，形成駐波，有如微波爐的共振腔)，或者在銜接於器材的電路上激發駐波，再耦合到器材裡面，藉放電或歐姆加熱等機制，燒壞裡面的敏感零件(例如電晶體)。射出的若是低頻脈衝波，由於波長太長，無法直接進入器材，但可以在輸電及通訊網路上，藉磁通量的快速變化，感應出仟伏以上的尖峰電壓，從電源或信號線，滲入與網路連線的電子器材，使器材不勝負荷。據一般評估，微波炸彈的威力較強。

在敏感儀器的外圍覆蓋電磁波屏蔽，可以減少微波進入。但通訊及雷達器材總得留一個微波出入口(天線)，屏蔽上也有冷卻洞口，所以還是有一些微波進得去。器材深藏在地下，微波也可以沿著通風洞進去。至於經由電源或訊號線進來的脈衝電壓，可以用隔離電路保護，即使如此，也可能連同器材一起被破壞。將電磁炸彈用飛機或巡曳飛彈投射到目標區上空引爆，有潛力破壞數百公尺範圍內的雷達、通訊及電腦等器材，在大舉侵犯前就癱瘓敵人，並且不造成人員的傷亡。

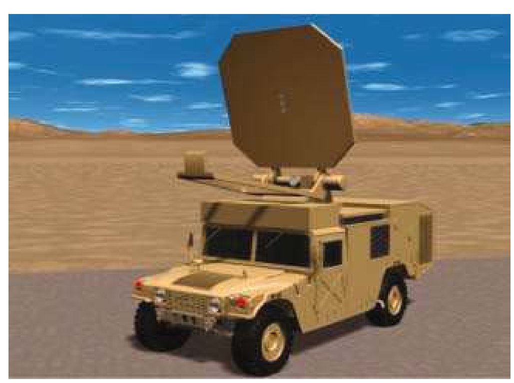
這種武器確實很吸引人，許多國家都投入了高額經費作這方面研究，有的國家業已進行了幾十年，報紙上也常常把它描述為超級武器。電磁炸彈是先進國家的秘密武器，相信已有雛型存在，可是要達到大家想像中的威力，據悉還需要不少物理和技術上的的突破，例如功率和脈衝時間的提昇、功率的有效輸出和避免彈體內部放電等等，都是困難重重的問題。

Q12：是否有一種用電磁波照射人體的武器?

A：美國已經發展出這樣的武器，準備用來驅退群眾，並且試驗過，但是還沒有實地使用過。它用一種稱為迴旋管(gyrotron)的新型波源，射出95 GHz的毫米波。頻率選擇95 GHz，一方面是因為大氣在這個頻率附近的吸收率較低，另方面是波長只有3毫米左右，可以用輕便的碟形天線控制方向，俾可將整個系統裝在卡車上。目前還在設法將尖峰功率提昇至百萬瓦級，輕便度也有待改進。



Infrared image of silhouette targets



Vehicle-mounted ADT (Active Denial Technology) concept

美國軍方曾在慎密控制的情況下，用這種武器對志願者作過實驗。人的身體被照射了約兩秒鐘，就好像被熱鍋燙到一樣的痛苦，可是電磁波只集中在表皮約0.3毫米的深度，剛好觸及痛覺神經最敏感的地方，進入體內的能量並不多，關掉後疼痛即止，沒有什麼傷害，眼睛會直覺式地閉下，也沒有受到傷害。在真實情況下使用，傷害恐怕還是難免。這種武器和前述的微波炸彈通稱為「定向能量武器(directed energy weapon)」，前者對人，後者對器材，類似的武器還有超高功率雷射等，均被歸類為非致命性武器。

中華民國101年3月13日／星期二 ▌新聞投訴專線：02-23064553、02-23087111 轉5550 國際新聞 A12 中國時報

非致命電磁波武器 讓人一心想逃

尹德瀚／綜合報導

美國軍方日前對媒體展示了一種最新的非致命武器「主動拒止系統」（Active Denial System，ADS），它可放出強力電磁波束，讓接觸到的人在瞬間產生難以忍受的灼熱感，只想儘快逃離。這種武器可用於驅散暴動群眾、周邊維安、區域拒止（area denial）、檢查哨維安及保護基礎設施等多種用途。

美軍聯合非致命武器管理局局長塔福拉上校，在陸戰隊關地哥基地主持這項展示，她解釋說，ADS看不到、聽不到、聞不到，卻可以感覺到。在場記者親身體驗時，立即產生快逃的本能反應。

塔福拉表示，ADS威力雖然強大，且射程達一千公尺，卻是軍方最「安全」的非致命武器，開發歷時十五年，尚未用於實戰，二〇一〇年短暫部署於阿富汗，但不曾實際使用。

這項科技引起安全顧慮，可能是其波束容易與加熱食物的微波混淆。負責測量生物效應的軍方研究人員米勒說，ADS的頻率為九十五吉赫（GHz），只能穿透〇．四公釐，安全上沒有問題。相較之下，微波頻率約一吉赫，但波速更快，穿透更深。

米勒說，軍方對一萬一千多人做過實驗，只有兩人受傷，需要接受治療，後來完全復原，沒有併發症。另一位研究員說，ADS放出的電磁波束，即使強度為微波的一百倍，也無法讓玉米粒變成爆米花。

塔福拉說，主動拒止系統可安裝於卡車上執行任務，其發射鈕有自動閉鎖裝置，超過三秒就會停止發射。國防部尚未訂購ADS，但塔福拉表示，只要五角大廈開口，廠商立刻可以交貨。

▲美軍展示最新型非致命武器「主動拒止系統」，以強力電磁波束讓接觸到的人只想逃之夭夭。一位記者親身體會它的威力。（法新社）