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Lab 2: Mine Crafting Report

Section I: Introduction

In this report, I will discuss how I used JupyterLab to find different values relating to a test mass falling down a mineshaft. I first treated the mine shaft as four kilometers deep, and included drag, the Coriolis Force, and a variable acceleration due to gravity. Then I treated the shaft like it was infinitely deep and explored the physics of that model. I used Python to calculate the time at which the mass hit the bottom of the shaft, and when it hit the sides of the mineshaft due to the Coriolis Force. I also was able to calculate the time at which the mass crossed to the surface on the other side, considering an infinitely deep mineshaft, and the speed at which it crossed.

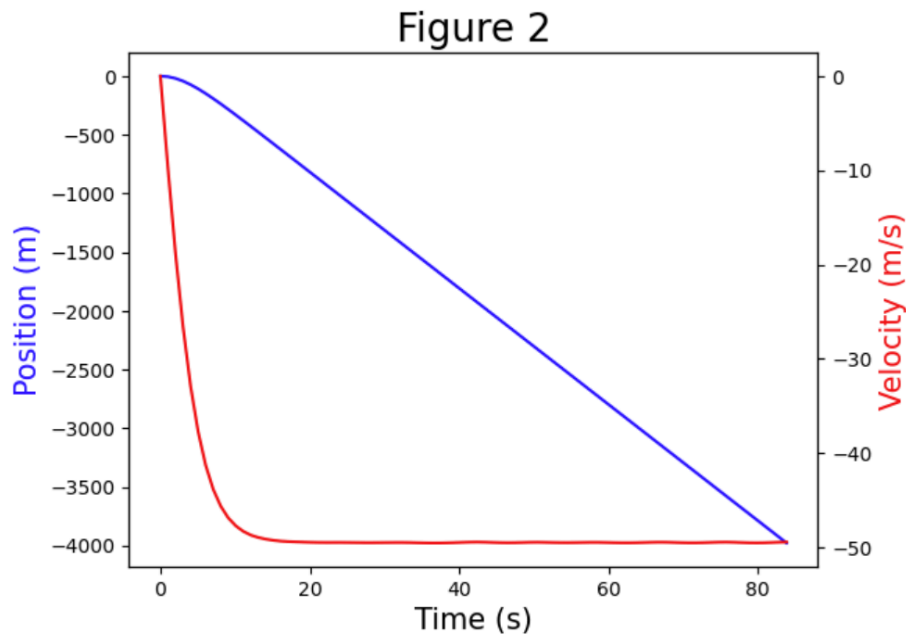
Section II: Calculation of Fall Time

First, the fall time without including drag, Coriolis Force, or variable acceleration, the mass will reach the bottom of the four kilometer mineshaft in about 28.6 seconds. I calculated this using a basic kinematic equation and setting time equal to the square root of 2 times 4,000 divided by 9.81. The 4,000 is the number of meters the mass will travel and the 9.81 is the acceleration due to gravity.

Next, including a variable acceleration that is dependent on the density of the Earth, the fall time is also 28.6 seconds, but there is a difference of about 0.002 seconds between this time and the constant acceleration time. The reason for the small difference in time is due to the fact that the radius of the Earth is 6,378,100 meters and the bottom of the shaft is only 4,000 meters. There is not a noticeable difference in density until about 3,000 kilometers, or 3,000,000 meters. The changes in density before that point are negligible. I was able to create an equation for the acceleration by setting 9.81 (normal acceleration ignoring density) times the depth of the mass divided by the radius of the Earth (6,378,100 meters).

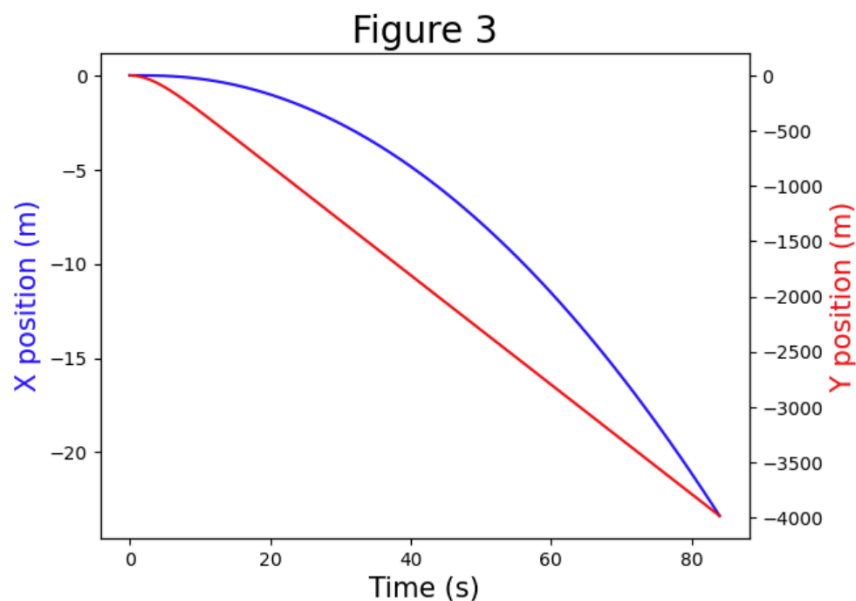
Finally, I included drag. I set the drag coefficient to 0.004 and the speed dependence of the drag to 2. With these parameters set, the terminal velocity of the test mass is 50 meters per second. After running code calibrated to these parameters, I calculated the fall time to be about 88.3 seconds. The equation I used to calculate this value was the acceleration due to gravity (considering changing density) minus the drag coefficient multiplied by the velocity to the power of the speed dependence.

In order to calculate these values, I used the command “solve_ivp” which solves an independent value problem from the library `scipy.integrate`. This command takes a derivative and computes the antiderivative value at certain time points. Below is a plot of the time (seconds) versus position (meters) and velocity (meters per second) of the mass, considering drag and variable acceleration, labeled “Figure 2.”



Section III: Feasibility of Depth Measurement

Of course, a valid measurement of the position must include the Coriolis Force, which is a result of the Earth's rotation. To simplify this, I made the assumption that the mine shaft is placed directly on the Earth's equator, which will have the highest Coriolis Force. The mass will hit the wall of the 5 meter wide shaft before it reaches the bottom. The mass hits the wall after about 29.7 seconds at a depth of about 2,703.5 meters. I was able to find this value using the same technique as earlier, using `scipy.integrate`'s command "solve_ivp" and setting the velocity in the x-direction equal to 2 times the rotation speed of the Earth (in radians) multiplied by the velocity in the y-direction. Below is a plot of the time (seconds) versus x-position and y-position (meters), labeled "Figure 3." This graph does not show the object bouncing off the wall, nor does it terminate when the object hits the wall. The mass should still reach the bottom, it will just take



longer due to the mass bouncing off the sides, because of this, I still recommend proceeding with this depth measurement technique.

Section IV: Calculation of Crossing Times

If we assume that the shaft is infinitely deep, that is, it crosses through to the other side of the Earth, I can calculate the time that the mass crosses the center of the Earth. First, we must consider the density concentrations of the Earth. The constant “n” determines the change in density, and I was able to find data for $n = 0, 1, 2$, and 9 . When this constant is higher, the Earth is denser towards the center, which reduces the fall time and increases the velocity when the mass crosses the center. When $n = 0$, the time it takes for the mass to reach the center is about 1,267.2 seconds and it crosses with a speed of about 7,906.0 meters per second. When $n = 9$, the time it takes for the mass to reach the center is about 943.9 seconds and it crosses with a speed of about 18,392 meters per second. When $n = 1$ or 2 , the time it takes is about 1,096.9 seconds and 1,035.1 seconds respectively, and it crosses with a speed of about 10,435.2 meters per second and 12,200.8 meters per second. If this were to be performed on, say, the moon, the speed would be decreased due to the decreased density of the lunar body.

Section V: Discussion and Future Work

I rounded all values in this report to the nearest tenth of a second, meter, or meter per second, but in reality, these values are much more precise. In order to make these calculations even more accurate, we must consider that the Earth is not a perfect sphere, but rather an oblate spheroid. This would change the data because an oblate spheroid bulges slightly at the equator, so if the shaft were at the equator, the radius would actually be higher. Also, the mass and of the Earth were significantly simplified to make calculations easier, but the real radius is not exactly 6,378,100 meters and the mass is not exactly 5.972×10^{24} kilograms.