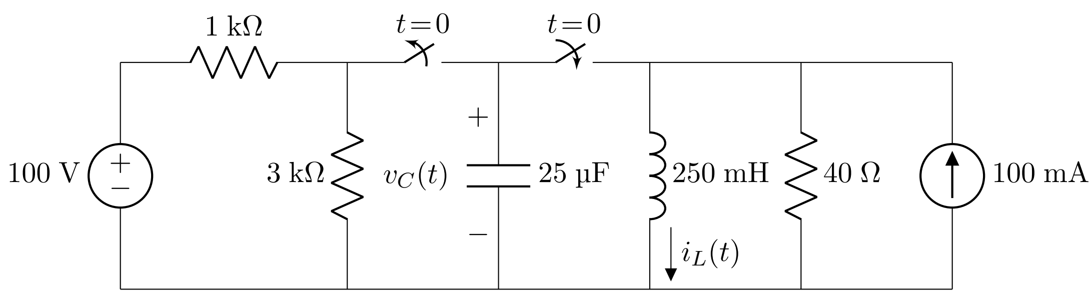
ECE 2260 quiz04

Name: SOLUTIONS

The left switch in the circuit below has been closed for a long time. The right switch has been open for a long time. At time $t=0$, the left switch opens, and the right switch closes simultaneously. Find an expression for $i_L(t)$.



Solution: First, we need to find the initial voltage over the capacitor. This is a voltage divider

$$v_C(0^-)=v_C(0^+)=\frac{3\text{ k}\Omega}{1\text{ k}\Omega+3\text{ k}\Omega}\cdot 100\text{ V}=75\text{ V}.$$

We can also find the initial current through the inductor. The current source forces 100 mA through the right side of the circuit, so

$$i_L(0^-)=i_L(0^+)=100\text{ mA}.$$

We can also find the characteristic roots of the circuit. We see that $\alpha=\frac{1}{2RC}=\frac{1}{2\cdot 40\cdot 25\times 10^{-6}}=500\text{ s}^{-1}$, and $\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{250\times 10^{-3}\cdot 25\times 10^{-6}}}=400\text{ s}^{-1}$. Since $\alpha>\omega_0$, we have an over-damped circuit. The roots are

$$s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}=-500\pm\sqrt{500^2-400^2}=-500\pm 300,$$

so $s_1=-200$ and $s_2=-800$. Thus, the response is

$$i_{L,n}(t)=I_f+A_1e^{-200t}+A_2e^{-800t}.$$

The final current through the inductor is simply the current source value, so $I_f=100$ mA. To find A_1 and A_2, we use the initial conditions. At $t=0$,

$$i_L(0)=I_f+A_1+A_2=100\text{ mA}+A_1+A_2=100\text{ mA},$$

so $A_1+A_2=0$. We also know that $v_L(t)=v_C(t)$ and that $v_L(t)=L\frac{di_L(t)}{dt}$. We can say that $\frac{di_L(t)}{dt}=\frac{v_L(t)}{L}$. At $t=0$ this would be $i_L'(0^+)=\frac{v_C(0^+)}{L}=300\text{ As}^{-1}$. Thus, we see that $i_L'(t)=-200A_1e^{-200t}-800A_2e^{-800t}$, and at $t=0$,

$$i_L'(0)=-200A_1-800A_2=300\text{ As}^{-1}.$$

We now have two equations and two unknowns:

$$A_1+A_2=0,$$

$$-200A_1-800A_2=300\text{ As}^{-1}.$$

Solving, we find that $A_1=500$ mA and $A_2=-500$ mA. Thus, the final expression for $i_L(t)$ is

$$i_L(t)=\begin{cases}100\text{ mA}, & t<0\\ 100+500e^{-200t}-500e^{-800t}\text{mA}, & t\geq 0\end{cases}.$$

We know that the current through the inductor cannot change instantaneously, so $i_L(0^-)=i_L(0^+)=100$ mA, which is confirmed by our expression.