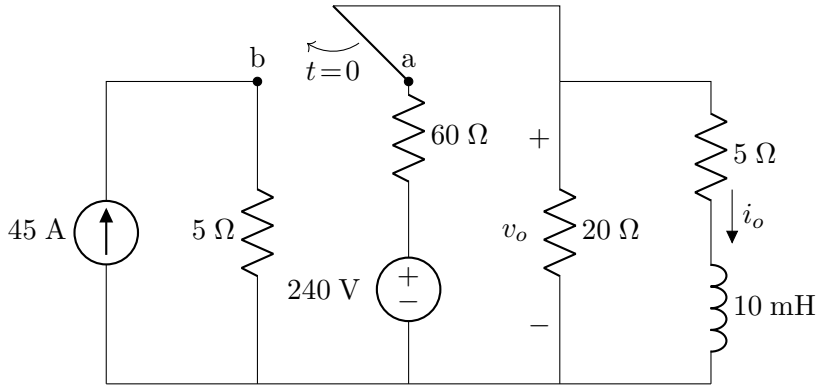
ECE 2260 quiz03

Name: SOLUTIONS

The switch in the circuit below has been in position a for a long time. At $t=0$, the switch moves instantaneously to position b.



Find the numerical expressions for $i_o(t)$ and $v_o(t)$ for all values of t.

Solution: First, we need to find $i_o(0^-)$, which is the current through the inductor just before the switch moves. This is $i_o(0^-)=3$ A.

Next, we need to find the Thevenin equivalent circuit seen by the inductor branch after the switch moves to position b at $t=0$. This will give us $V_{th}=180$ V and $R_{th}=9\ \Omega$. This will give us a time constant $\tau=\frac{L}{R_{th}}=\frac{10\text{ mH}}{9\ \Omega}=\frac{1}{900}$ s.

We need the final value of the current $i_o(\infty)=\frac{V_{th}}{R_{th}}=\frac{180}{9}=20$ A.

Putting this all together, we can write the expression for $i_o(t)$

$$i_o(t)=i_o(\infty)+[i_o(0^-)-i_o(\infty)]e^{-t/\tau}=20+(3-20)e^{-900t}=20-17e^{-900t}\quad\text{A},\quad t\geq 0.$$

For all values of t,

$$i_o(t)=\begin{cases}3\text{ A}, & t<0,\\ 20-17e^{-900t}\text{ A}, & t\geq 0.\end{cases}$$

The voltage $v_o(t)$ is the voltage across the $20\ \Omega$ resistor. However, looking at this, we see this is also the voltage over the $5\ \Omega$ resistor, which makes them in parallel with an equivalent resistance of $4\ \Omega$. We know current in this equivalent branch is

$$i_{eq}(t)=45\text{ A}-i_o(t)=45-(20-17e^{-900t})=25+17e^{-900t}\text{ A}.$$

Therefore, the voltage across this equivalent resistance is

$$v_o(t)=4\ \Omega i_{eq}(t)=4(25+17e^{-900t})=100+68e^{-900t}\text{ V},t\geq 0.$$

Then for all values of t,

$$v_o(t)=\begin{cases}15\text{ V}, & t<0,\\ 100+68e^{-900t}\text{ V}, & t\geq 0.\end{cases}$$