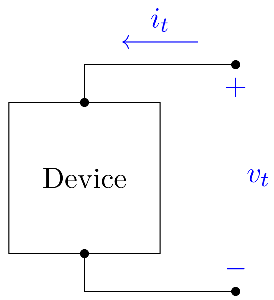
ECE 2260 hw01

1. Basic model

The voltage and current were measured at the terminals of the device shown in Fig. 1 (a). The results are tabulated in Fig. 1 (b).



(a)

v_t (V)	i_t (A)
50	0
66	2
82	4
98	6
114	8
130	10

(b)

Figure 1: Device under test

a) Construct a circuit model for this device using an ideal voltage source in series with a resistor.

Solution: Resistor is $R = 8\ \Omega$ and voltage source is $V = 50$ V.

b) Use the model to predict the value of i_t when v_t is zero.

Solution: The model is then

$$v_t = V - i_t R.$$

Setting $v_t = 0$ gives

$$0 = 50 - i_t(8) \implies i_t = -6.25 \text{ A}.$$

2. Power dissipated in resistor

Find the power dissipated in the 5 Ω resistor in the current divider circuit in Fig. 2.

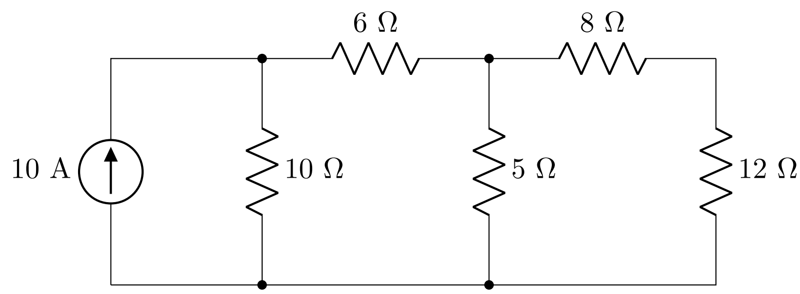


Figure 2: circuit

Solution: Using mesh analysis, we have the following mesh equations:

$$\begin{aligned}
&\text{Mesh 1:}(i_1 - 10\text{ A}) \cdot 10\ \Omega + (i_1 - i_2) \cdot 6\ \Omega + (i_1 - i_2) \cdot 5\ \Omega = 0\text{ v} \\
&\text{Mesh 2:}(i_2 - i_1) \cdot 5\ \Omega + i_2 \cdot 20\ \Omega = 0\text{ V.}
\end{aligned}$$

Rearranging these equations gives:

$$\begin{aligned}
\text{Mesh 1:}\quad & i_1(21\ \Omega) + i_2(-5\ \Omega) = 100\text{ V} \\
\text{Mesh 2:}\quad & i_1(-5\ \Omega) + i_2(25\ \Omega) = 0\text{ V.}
\end{aligned}$$

Solving these equations simultaneously, we get $i_1 = 5$ A and $i_2 = 1$ A. The current through the resistor is then 4 A.The power dissipated in the 5 Ω resistor is then:

$$P = I^2R = (4\text{ A})^2 \cdot 5\ \Omega = 20\text{ W.}$$

3. Specifying resistors in current divider

Specify the resistors in the current divider circuit in Fig. 3 to meet the following design criteria:

$$i_g = 50 \text{ mA}; \quad v_g = 25 \text{ V}; \quad i_1 = 0.6 i_2;$$

$$i_3 = 2 i_2; \quad \text{and} \quad i_4 = 4 i_1.$$

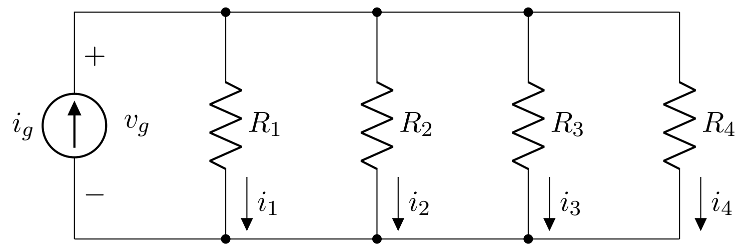


Figure 3: current divider circuit

Solution: First, we note that the total current supplied by the current source is the sum of the branch currents:

$$i_g = i_1 + i_2 + i_3 + i_4.$$

Substituting the given relationships between the branch currents, we have:

$$i_g = 0.6 i_2 + i_2 + 2 i_2 + 4(0.6 i_2) = 6 \cdot i_2.$$

Solving for i_2

$$i_2 = \frac{i_g}{6} = \frac{50 \text{ mA}}{6} \approx 8.33 \text{ mA}.$$

Now we can find the other branch currents

$$i_1 = 0.6 i_2 = 0.6 \cdot 8.33 \text{ mA} = 5 \text{ mA},$$

$$i_3 = 2 i_2 = 2 \cdot 8.33 \text{ mA} = 16.67 \text{ mA},$$

$$i_4 = 4 i_1 = 4 \cdot 5 \text{ mA} = 20 \text{ mA}.$$

Next, we can find the voltage across each resistor using Ohm's law. Since all resistors are in parallel, they all have the same voltage drop $v_g = 25$ V. Now we can calculate each resistor value:

$$R_1 = \frac{v_g}{i_1} = \frac{25 \text{ V}}{5 \text{ mA}} = 5 \text{ k}\Omega,$$

$$R_2 = \frac{v_g}{i_2} = \frac{25 \text{ V}}{8.33 \text{ mA}} = 3 \text{ k}\Omega,$$

$$R_3 = \frac{v_g}{i_3} = \frac{25 \text{ V}}{16.67 \text{ mA}} = 1.5 \text{ k}\Omega,$$

$$R_4 = \frac{v_g}{i_4} = \frac{25 \text{ V}}{20 \text{ mA}} = 1.25 \text{ k}\Omega.$$

4. Node-voltage method power dissipation

Use the node-voltage method to find the total power dissipated in the circuit in Fig. 4.

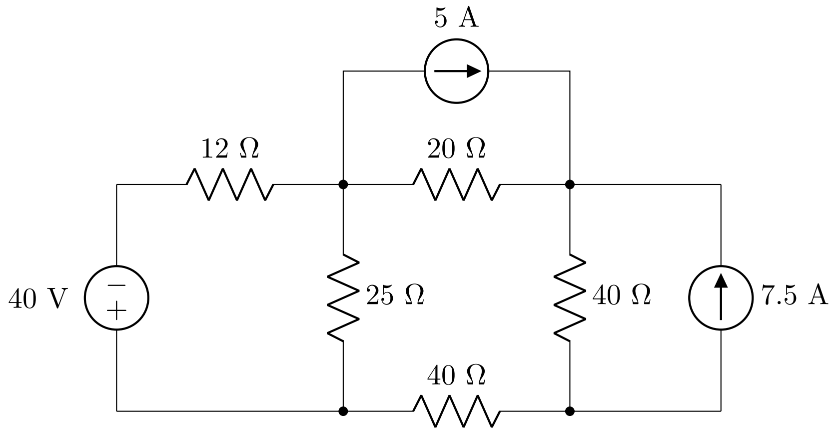


Figure 4: Node voltage circuit

Solution: Solving for three nodes (top left: 1, top right: 2, bottom right: 3) using node-voltage method, we have $v_1 = -10$ V, $v_2 = 132$ V, and $v_3 = -84$ V.
We can find the power for each element:

$$\begin{aligned}
p_{5\text{A}} &= -710 \text{ W} \\
p_{7.5\text{A}} &= -1620 \text{ W} \\
p_{40\text{V}} &= -100 \text{ W} \\
p_{12\Omega} &= 75 \text{ W} \\
p_{25\Omega} &= 4 \text{ W} \\
p_{20\Omega} &= 1008.2 \text{ W} \\
p_{40\Omega,l} &= 176.4 \text{ W} \\
p_{40\Omega,r} &= 1166.4 \text{ W}
\end{aligned}$$

The power dissipated in the resistors is then

$$p_{\text{dissipated}} = 75 \text{ W} + 4 \text{ W} + 1008.2 \text{ W} + 176.4 \text{ W} + 1166.4 \text{ W} = 2430 \text{ W}.$$

You can verify that the total power supplied equals the total power dissipated:

$$p_{\text{total}} = -2430 \text{ W} + 2430 \text{ W} = 0 \text{ W}.$$

5. Node-voltage method branch currents

Use the node-voltage method to find the branch currents i_1, i_2, and i_3 in the circuit in Fig. 5.

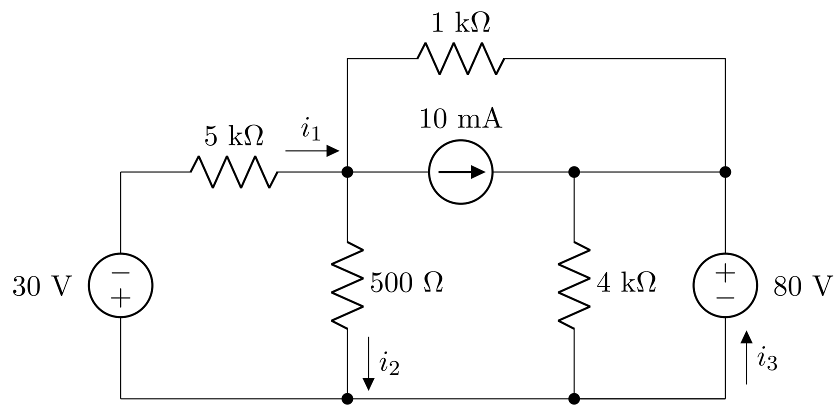


Figure 5: Circuit

Solution: We notice there is only one node on the center left. We can write a node-voltage equation for that node (call it v_1)

$$\frac{v_1 + 30\text{ V}}{5\text{ k}\Omega} + \frac{v_1 - 80\text{ V}}{1\text{ k}\Omega} + 10\text{ mA} + \frac{v_1}{500\ \Omega} = 0\text{ A}$$

We can isolate v_1 such that

$$v_1\left(\frac{1}{5\text{ k}\Omega} + \frac{1}{1\text{ k}\Omega} + \frac{1}{500\ \Omega}\right) = -10\text{ mA} + 80\text{ mA} - 6\text{ mA}.$$

Solving gives $v_1 = 20$ V. Using this, we can find each branch current indiviually. Solving for i_1 is just

$$i_1 = \frac{-30\text{ V} - 20\text{ V}}{5\text{ k}\Omega} = -10\text{ mA}.$$

Solving for i_2 is

$$i_2 = \frac{20\text{ V}}{500\ \Omega} = 40\text{ mA}.$$

Lastly, we look at the center-right node and notice that the voltage referenced to a bottom ground is 80 V. Thus, we can solve for i_3 as

$$i_3 = -10\text{ mA} + \frac{80\text{ V} - 20\text{ V}}{1\text{ k}\Omega} + \frac{80\text{ V}}{4\text{ k}\Omega} = 70\text{ mA}.$$

6. DC Thevenin equivalent

Find the Thevenin equivalent with respect to the terminals a,b for the circuit seen in Fig. 6.

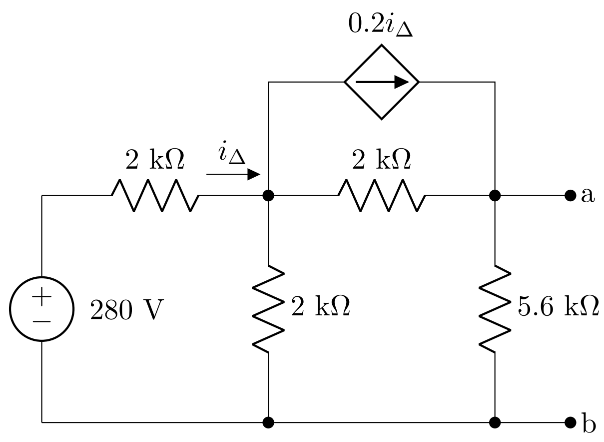


Figure 6: Circuit for Thevenin equivalent problem

Solution: We being by finding the Thevenin voltage v_{th} across terminals a and b. We will need to find the voltage over a and b with no load connected. We see there are two nodes and a dependent current source, so we will have a three equation system to solve. Solving this gives us $v_1 = 120$ V, $v_{\text{Th}} = 112$ V, and $i_\Delta = 80$ mA.

Next, we need to find the Thevenin resistance R_{th}. To do this, we short a and b and solve for the current over the short i_{sc}. Here we will use mesh current with two mesh currents i_Δ and i_{sc}. Solving this system gives us $i_\Delta = 100$ mA and $i_{\text{sc}} = 60$ mA.

Finally, we can find the Thevenin resistance as

$$R_{\text{th}} = \frac{v_{\text{th}}}{i_{\text{sc}}} = \frac{112 \text{ V}}{60 \text{ mA}} = 1.867 \text{ k}\Omega.$$

7. AC Thevenin equivalent

Use source transformations to find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.45.

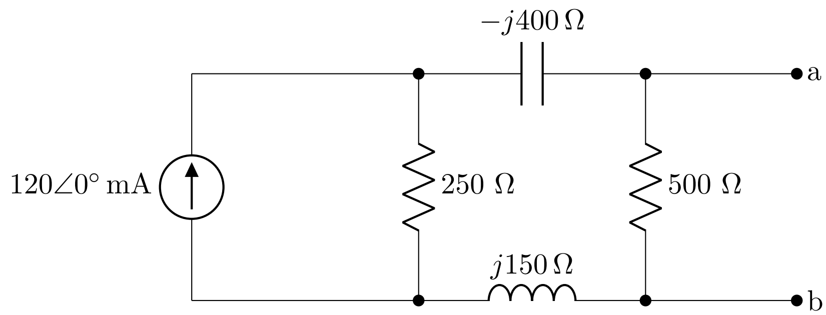


Figure 7: Circuit for AC Thevenin equivalent problem

Solution: After a series of source transformations and simplifications, we find the Thévenin equivalent circuit with respect to terminals a and b to be a voltage source of $V_{th} = (18 + \text{j}6)$ V with an impedance of $Z_{th} = (200 - \text{j}100)\ \Omega$.

8. Phasor voltage

Find the phasor voltage $\mathbf{V}_g$ and phasor current $\mathbf{I}_g$ in the circuit shown in Fig. 8.

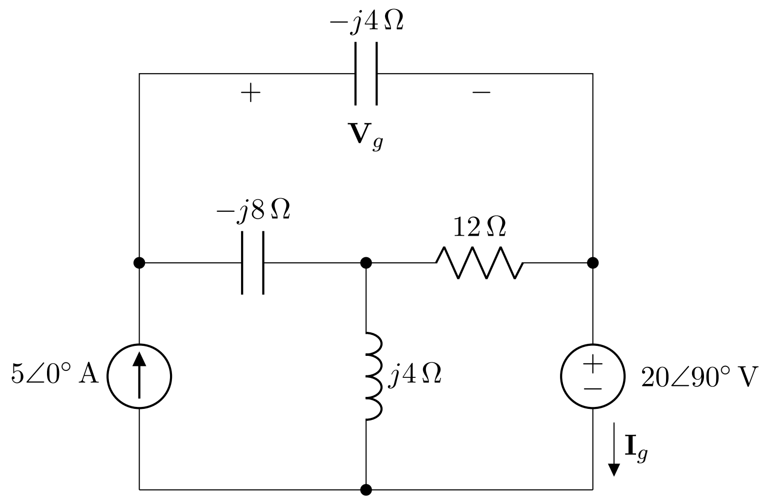


Figure 8: Circuit for phasor voltage problem

Solution: We can solve this using either mesh or nodal analysis. Here, we will use mesh analysis. We define mesh currents as shown in Fig. 8.

$$\text{Mesh 1: } I_1(-\text{j}4\ \Omega) + (I_1 - I_g)(12\ \Omega) + (I_1 - 5\ \text{A})(-\text{j}8\ \Omega) = 0\ \text{V}$$
$$\text{Mesh 2: } (I_g - 5\ \text{A})(\text{j}4\ \Omega) + (I_g - I_1)(12\ \Omega) + \text{j}20\ \text{V} = 0\ \text{V}$$

Rearranging, we have

$$((12 - \text{j}12)\ \Omega)\, I_1 + (-12)\, I_g = -\text{j}40\ \text{V}$$
$$(-12)\, I_1 + ((12 + \text{j}4)\ \Omega)\, I_g = 0\ \text{V}$$

Solving this system of equations, we get $\mathbf{I}_1 = (4.667 - \text{j}0.667)$ A and $\mathbf{I}_g = (4 - \text{j}2)$ A. To find $\mathbf{V}_g$, we can use

$$\mathbf{V}_g = \mathbf{I}_1 - \text{j}4\ \Omega = (-2.667 - \text{j}18.667)\ \text{V}.$$

9. AC power transfer

The load impedance Z_L for the circuit shown in Fig. 9 is adjusted until maximum average power is delivered to Z_L. What is the impedance Z_L in ohms?

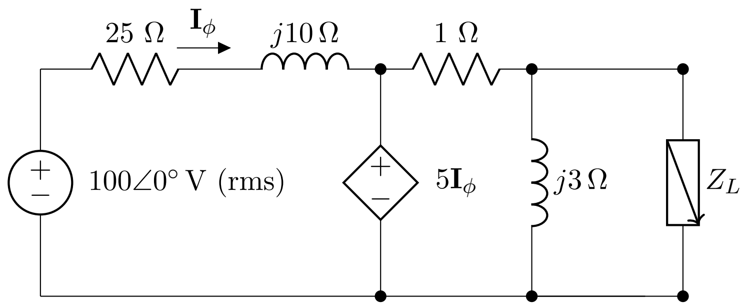


Figure 9: Circuit for AC power transfer problem

Solution: The average power delivered will be maximized when Z_L is the complex conjugate of the Thevenin impedance seen from the load terminals. We can start by finding the open circuit voltage at the terminals where Z_L is connected. To do this, we first need to find the current $\mathbf{I}_\phi$ supplied by the voltage source. We can first do a voltage loop on the left-hand side of the circuit

$$-100\text{ V} - I_\phi(25 + \mathrm{j}10)\ \Omega + 5I_\phi = 0.$$

We can solve for $I_\phi = (-4 + \mathrm{j}2)$ A. Now we can find the voltage over the dependent source and use a voltage divider to find the open circuit voltage V_{Th}

$$V_{\mathrm{Th}} = 5I_\phi \cdot \frac{\mathrm{j}3\ \Omega}{(1 + \mathrm{j}3)\ \Omega} = (-21 + \mathrm{j}3)\ \text{V}.$$

Next we can short the load terminals and find the short circuit current I_{SC}. We can use mesh analysis to find the current through the short. Notice this doesn't impact I_ϕ since it is on the left side of the circuit. We notice that the voltage supplied by the dependent source is also unaffected, so we can simply find I_{SC} by

$$I_{\mathrm{SC}} = \frac{5I_\phi}{1\ \Omega} = (-20 + \mathrm{j}10)\ \text{A}.$$

Now we can find the Thevenin impedance

$$Z_{\mathrm{Th}} = \frac{V_{\mathrm{Th}}}{I_{\mathrm{SC}}} = (0.9 + \mathrm{j}0.3)\ \Omega.$$

Finally, we can find the load impedance for maximum power transfer by taking the complex conjugate

$$Z_L = Z_{\mathrm{Th}}^* = (0.9 - \mathrm{j}0.3)\ \Omega.$$

10. AC Thevenin equivalent

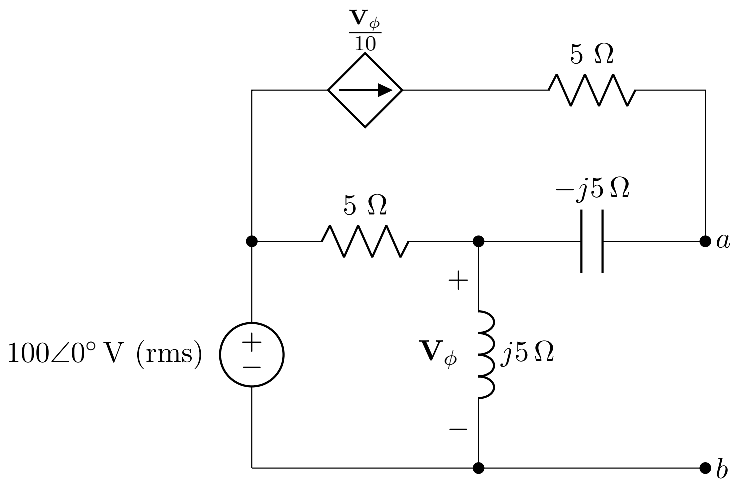


Figure 10: Circuit for AC for Thevenin equivalent problem

Solution: To find the Thevenin voltage, we can use the node at the center of the circuit. The nodal analysis equation at this node is

$$\frac{V_\phi}{\text{j5 }\Omega} + \frac{V_\phi - 100}{5\ \Omega} - \frac{V_\phi}{10} = 0\ \text{A}.$$

We can solve $\mathbf{V}_\phi = (40 + \text{j}80)\ \Omega$. To get the Thevenin voltage, we can use

$$V_{\text{Th}} = V_\phi + (0.1V_\phi)(-\text{j}5\ \Omega) = (80 + \text{j}60)\ \text{V}.$$

To solve for the Thevenin impedance, we can short between a and b to solve for $\mathbf{I}_{\text{sc}}$. Using nodal analysis again, we have

$$\frac{V_phi - 100\ \text{V}}{5\ \Omega} + \frac{V_\phi}{\text{j5 }\Omega} + \frac{V_\phi}{-\text{j5 }\Omega} = 0\ \text{A}.$$

Solving, we find $\mathbf{V}_\phi = 100$ V. Solving for the short-circuit current, we have

$$I_{\text{sc}} = \frac{V_\phi}{-\text{j5 }\Omega} + \frac{V_\phi}{10} = (10 + \text{j}20)\ \text{A}.$$

Finally, we can find the Thevenin impedance as

$$Z_{\text{Th}} = \frac{V_{\text{Th}}}{I_{\text{sc}}} = \frac{(80 + \text{j}60)\ \text{V}}{(10 + \text{j}20)\ \text{A}} = (4 - \text{j}2)\ \Omega.$$