ECE 2260 hw02

1. Op-amp circuit analysis

The op amp in the circuit of Fig. 1 is ideal.

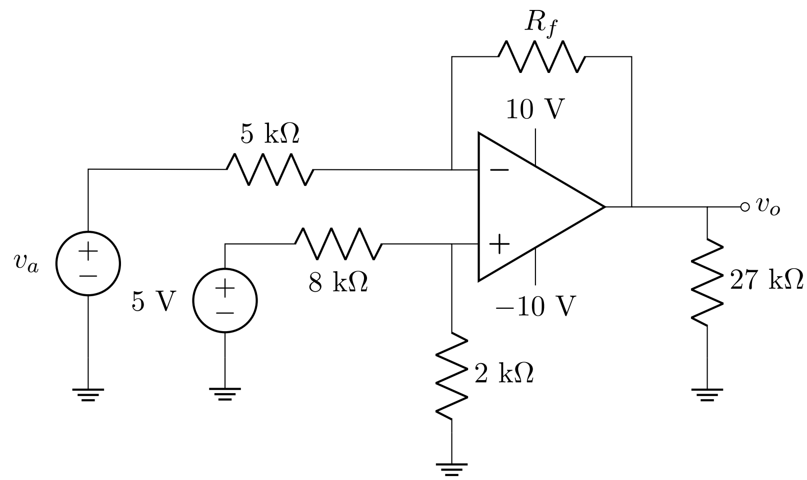

Figure 1: Op amp circuit

a) What op amp circuit configuration is this?

Solution: This is a difference amplifier circuit.

b) Find an expression for the output voltage v_o in terms of the input voltage v_a (assume that $R_f = 20$ kΩ).

Solution: Use the difference amplifier equation with $R_a = 5$ kΩ, $R_b = 20$ kΩ, $R_c = 8$ kΩ, $R_d = 2$ kΩ, and $v_b = 5$ V:

$$v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} v_b - \frac{R_b}{R_a} v_a$$

Substituting the values:

$$v_o = \frac{2(5+20)}{5(8+2)}(5) - \frac{20}{5} v_a = \frac{50}{50}(5) - 4v_a = 5 - 4v_a$$

c) Suppose $v_a = 2$ V. What value of R_f will cause the op amp to saturate?

Solution: With $v_a = 2$ V and R_f unknown (replacing R_b):

$$v_o = \frac{2000(5000 + R_f)}{5000(8000 + 2000)}(5) - \frac{R_f}{5000}(2)$$

$$v_o = \frac{5000 + R_f}{5000} - \frac{2R_f}{5000} = 1 - \frac{R_f}{5000}$$

The op amp saturates at ± 10 V.

Case 1: $1 - \frac{R_f}{5000} = 10 \implies -\frac{R_f}{5000} = 9 \implies R_f < 0$ (not possible).

Case 2: $1 - \frac{R_f}{5000} = -10 \implies \frac{R_f}{5000} = 11 \implies R_f = 5000(11) = 55\ \text{k}\Omega$.

2. Inductor current and voltage analysis

The current in a 150 µH inductor is known to be

$$i_L = 25te^{-500t}\,\mathrm{A} \quad \text{for } t \geq 0.$$

a) Find the voltage across the inductor for $t > 0$. (Assume the passive sign convention.)

Solution: $v = L\frac{di}{dt} = (150 \times 10^{-6})(25)[e^{-500t} - 500te^{-500t}] = 3.75e^{-500t}(1 - 500t)\,\mathrm{mV}$

b) Find the power (in µW) at the terminals of the inductor when $t = 5$ ms.

Solution: $i(5\text{ ms}) = 25(0.005)(e^{-2.5}) = 10.26$ mA
$v(5\text{ ms}) = 0.00375(e^{-2.5})(1 - 2.5) = -461.73$ µV
$p(5\text{ ms}) = vi = (10.26 \times 10^{-3})(-461.73 \times 10^{-6}) = -4.74$ µW

c) Is the inductor absorbing or delivering power at 5 ms?

Solution: delivering 4.74 µW

d) Find the energy (in µJ) stored in the inductor at 5 ms.

Solution: $i(5\text{ ms}) = 10.26$ mA (from part [b])
$w = \frac{1}{2}Li^2 = \frac{1}{2}(150 \times 10^{-6})(0.01026)^2 = 7.9$ nJ

e) Find the maximum energy (in µJ) stored in the inductor and the time (in ms) when it occurs.

Solution: The energy is a maximum where the current is a maximum:
$\frac{di_L}{dt} = 0$ when $1 - 500t = 0$ or $t = 2$ ms
$i_{\max} = 25(0.002)e^{-1} = 18.39$ mA
$w_{\max} = \frac{1}{2}(150 \times 10^{-6})(0.01839)^2 = 25.38$ nJ

3. Capacitor voltage and current analysis

The voltage at the terminals of the capacitor in Fig. 2 is known to be

$$v = \begin{cases} 60 \text{ V}, & t \leq 0; \\ 30 + 5e^{-500t}(6\cos 2000t + \sin 2000t) \text{ V}, & t \geq 0. \end{cases}$$

Assume $C = 120$ µF.

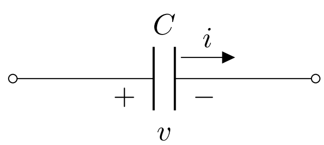

Figure 2: Capacitor circuit for problem.

a) Find the current in the capacitor for $t < 0$.

> **Solution:** $i = C\frac{dv}{dt} = 0, \quad t < 0$

b) Find the current in the capacitor for $t > 0$.

> **Solution:**
>
> $$\begin{aligned} i = C\frac{dv}{dt} &= 120 \times 10^{-6}\frac{d}{dt}[30 + 5e^{-500t}(6\cos 2000t + \sin 2000t)] \\ &= 120 \times 10^{-6}[5(-500)e^{-500t}(6\cos 2000t + \sin 2000t) + 5(2000)e^{-500t}(-6\sin 2000t + \cos 2000t)] \\ &= -0.6e^{-500t}[\cos 2000t + 12.5\sin 2000t] \text{ A}, \quad t \geq 0 \end{aligned}$$

c) Is there an instantaneous change in the voltage across the capacitor at $t = 0$?

> **Solution:** no,
> $v(0^-) = 60$ V
> $v(0^+) = 30 + 5(6) = 60$ V

d) Is there an instantaneous change in the current in the capacitor at $t = 0$?

> **Solution:** yes,
> $i(0^-) = 0$ A
> $i(0^+) = -0.6$ A

e) How much energy (in mJ) is stored in the capacitor at $t = \infty$?

Solution: $v(\infty) = 30$ V
$w = \frac{1}{2}Cv^2 = \frac{1}{2}(120 \times 10^{-6})(30)^2 = 54$ mJ

4. RL circuit with switch

The switch in the circuit in Fig. 3 has been open for a long time. At $t = 0$ the switch is closed.

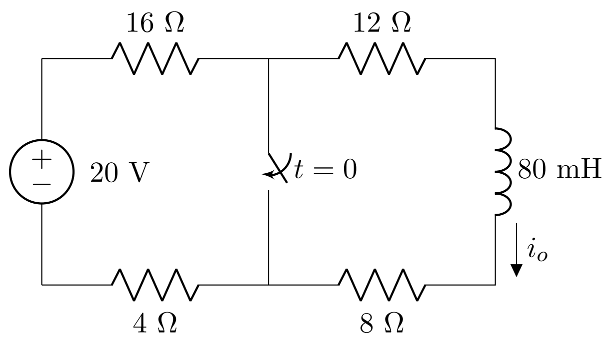

Figure 3: Circuit for Problem 7.1

a) Determine $i_o(0)$ and $i_o(\infty)$.

Solution:

$$i_o(0) = \frac{20}{16 + 12 + 4 + 8} = \frac{20}{40} = 0.5 \text{ A}$$
$$i_o(\infty) = 0 \text{ A}$$

b) Determine $i_o(t)$ for $t \geq 0$.

Solution:

$$i_o = 0.5e^{-t/\tau}; \quad \tau = \frac{L}{R} = \frac{80 \times 10^{-3}}{12 + 8} = 4 \text{ ms}$$
$$i_o = 0.5e^{-t/(4 \text{ ms})} \text{ A}, \quad t \geq 0$$

c) How many milliseconds after the switch has been closed will i_o equal 100 mA?

Solution:

$$0.5e^{-t/(4 \text{ ms})} = 0.1$$
$$e^{t/(4 \text{ ms})} = 5 \quad \Longrightarrow \quad t = 6.44 \text{ ms}$$

5. RL circuit with opening switch

The switch in the circuit in Fig. 4 has been closed for a long time. At $t = 0$ it is opened.

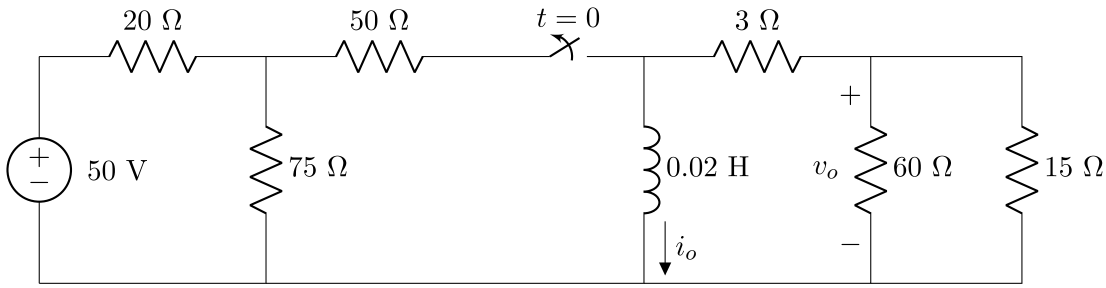

Figure 4: Circuit for Problem 7.2

a) Write the expression for $i_o(t)$ for $t \geq 0$.

Solution: First, we need to analyze the circuit with $t < 0$. The inductor will act as a short, and we can ignore the right-hand side of the circuit. Solving for the initial current gives $i_0(0^-) = 0.6$ A.

Looking at $t > 0$, we see that opening the switch will leave just the right-hand side of the circuit. We will need to find the Thevenin equivalent of the resistance which is $R_{\text{th}} = 63\ \Omega{+}60\ \Omega||15\ \Omega = 15\ \Omega$. This gives a time constant $\tau = 1.33$ ms.

Thus, the current is

$$i_o(t) = 0.6e^{-t/\tau} = 0.6e^{-t/(1.33\text{ms})}\ \text{A}$$

b) Write the expression for $v_o(t)$ for $t \geq 0^+$.

Solution: We know that $i_o(t)$ branches into a both resistors, so we can find the current flowing through the resistor using a current divider

$$i_{R_{60}}(t) = i_o(t) \cdot \frac{R_{15}}{R_{60} + R_{15}} = 0.6e^{-t/(1.33\text{ms})} \cdot \frac{15}{60 + 15} = 0.12e^{-t/(1.33\text{ms})}\ \text{A}$$

The voltage across the resistor $R = 60\ \Omega$ is then

$$v_o(t) = -i_{R_{60}}(t) \cdot R_{60} = -0.12e^{-t/(1.33\text{ms})} \cdot 60 = -7.2e^{-t/(1.33\text{ms})}\ \text{V}$$

Note the negative comes from the current $i_{R_{60}}(t)$ flowing in the opposite direction to the assumed voltage polarity across R_{60}.

6. Make-before-break switch

In the circuit shown in Fig. 5, the switch makes contact with position b just before breaking contact with position a. As already mentioned, this is known as a make-before-break switch and is designed so that the switch does not interrupt the current in an inductive circuit. The interval of time between "making" and "breaking" is assumed to be negligible. The switch has been in the a position for a long time. At $t = 0$ the switch is thrown from position a to position b.

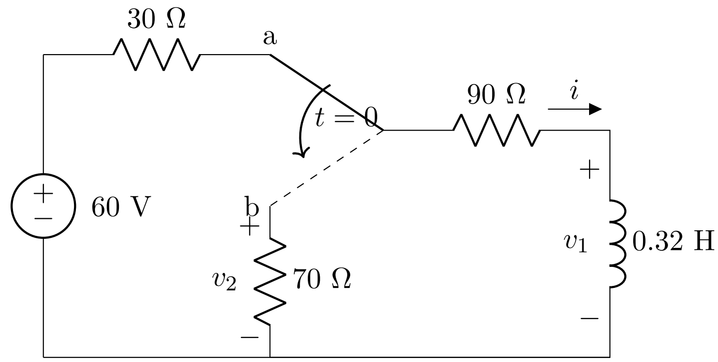

Figure 5: Circuit for Problem 7.3

a) Determine the initial current in the inductor.

> **Solution:** We see that with the swictch at position a, you have a simple loop such that $i(0^-) = \frac{V}{R} = \frac{60\ \text{V}}{120\ \Omega} = 500$ mA.

b) Determine the time constant of the circuit for $t > 0$.

> **Solution:** When the solution is at position b, the Thevenin resistance is simply $R_{\text{Th}} = 160\ \Omega$. Using this
>
> $$\tau = L/R_{\text{Th}} = \frac{0.32\ \text{H}}{160\ \Omega} = 2\ \text{ms}$$

c) Find i, v_1, and v_2 for $t \geq 0$.

> **Solution:** Finding $i(t)$ is trivial
>
> $$i(t) = 0.5e^{-t/(2\ \text{ms})}\ \text{A}.$$
>
> We can find the voltage over the inductor
>
> $$v_1(t) = L\frac{di(t)}{dt} = 0.32\ \text{H} \cdot \frac{d}{dt}\left(0.5e^{-t/(2\ \text{ms})}\right) = -80e^{-t/(2\ \text{ms})}\ \text{V}$$
>
> The voltage over the 70 Ω resistor is
>
> $$v_2(t) = i(t) \cdot R = 0.5e^{-t/(2\ \text{ms})} \cdot 70\ \Omega = -35e^{-t/(2\ \text{ms})}\ \text{V}$$

7. Switching circuit with inductor

The switch in the circuit in Fig. 6 has been in position 1 for a long time. At $t = 0$, the switch moves instantaneously to position 2. Find $v_o(t)$ for $t \geq 0^+$.

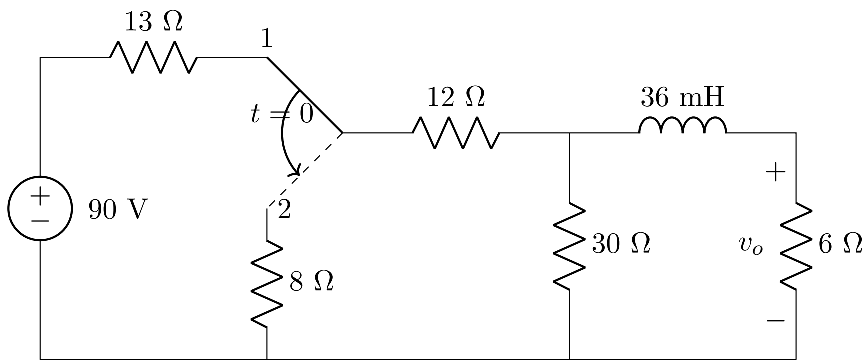

Figure 6: Switching circuit

Solution: First, we need to find the initial current through the inductor at $t = 0^-$. With the switch at position 1 for a long time, the inductor acts as a short circuit. The current through the inductor is then

$$i_L(0^-) = 2.5 \text{ A}$$

When the switch moves to position 2, we can find the Thevenin equivalent resistance seen by the inductor to find the time constant of the circuit. Turning off the voltage source and looking back into the circuit from the inductor terminals, we have

$$R_{\text{th}} = 18 \ \Omega$$

This gives a time constant of

$$\tau = \frac{L}{R_{\text{th}}} = 2 \text{ ms}$$

The current through the inductor for $t \geq 0$ is then

$$i_L(t) = 2.5e^{-t/\tau} = 2.5e^{-t/(2\text{ms})} \text{ A}.$$

We can use this current expression to find the voltage over the 6 Ω resistor

$$v_o(t) = i_L(t) \cdot R = 2.5e^{-t/(2\text{ms})} \text{ A} \cdot 6 \ \Omega = 15e^{-t/(2\text{ms})} \text{ V}.$$

8. Switched RC Circuit Analysis

The switch in the circuit in Fig. 7 has been in position a for a long time and $v_2 = 0$ V. At $t = 0$, the switch is thrown to position b.

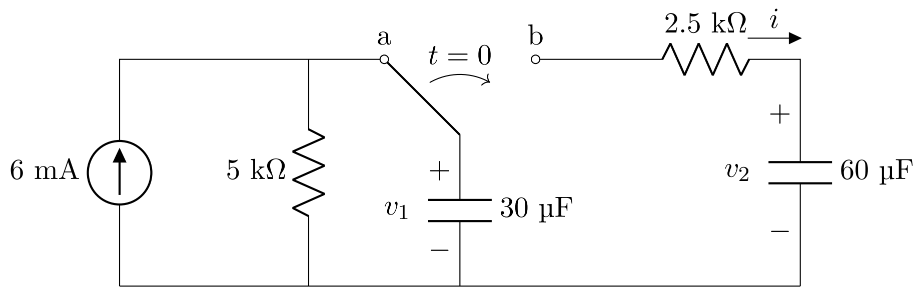

Figure 7: Circuit with mutliple capacitors

Calculate $i(t)$, $v_1(t)$, and $v_2(t)$ for $t \geq 0^+$.

Solution: First, we see that at $t = 0^-$, the switch has been in position a for a long time. Therefore, the capacitor at the switch pivot is fully charged and acts as an open circuit. The voltage over v_1 is then $v_1(0^-) = 30$ V, and the voltage over v_2 is $v_2(0^-) = 0$ V. After the switch moves, the equivalent circuit is a simple RC circuit with an equivalent capacitance of $C_{\text{eq}} = 20$ µF (the series combination of the 30 µF and 60 µF capacitors) and a resistance of $R = 2.5$ kΩ. The time constant of the circuit is then

$$\tau = RC_{\text{eq}} = 50 \text{ ms}.$$

The initial voltage across the equivalent capacitor at $t = 0^+$ is the same as the voltage across the 30 µF capacitor at $t = 0^-$, which is 30 V. Therefore, the voltage across the equivalent capacitor for $t \geq 0^+$ is

$$v_{\text{eq}}(t) = 30e^{-t/\tau} = 30e^{-t/(50\text{ms})} \text{ V}.$$

This is the same as the voltage across the resistor, we can find $i(t)$ by using Ohm's law

$$i(t) = \frac{v_{\text{eq}}(t)}{R} = \frac{30e^{-t/(50\text{ms})} \text{ V}}{2.5 \text{ k}\Omega} = 12e^{-t/(50\text{ms})} \text{ mA}.$$

Now we know the current through the capacitors, we can find $v_1(t)$ and $v_2(t)$. The voltage across the 30 µF capacitor is

$$v_1(t) = \frac{1}{C}\int -i(t)\,dt + 30 \text{ V} = -\frac{1}{C_1}\int_0^t 12e^{-x/\tau}\,dx + 30 \text{ V} = 20e^{-t/(50\text{ms})} + 10 \text{ V}.$$

From a voltage loop we see that $-v_1(t) + v_2(t) = v_{\text{eq}}(t)$. The voltage across the 60 µF capacitor is then

$$v_2(t) = v_{\text{eq}}(t) - v_1(t) = 30e^{-t/(50\text{ms})} - \left(20e^{-t/(50\text{ms})} + 10\right) = 10e^{-t/(50\text{ms})} - 10 \text{ V}.$$