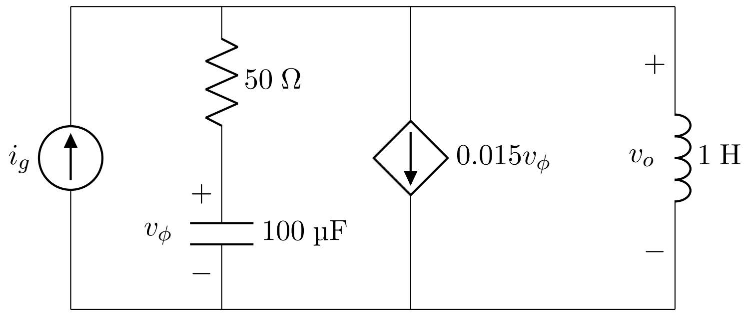
ECE 2260 quiz07

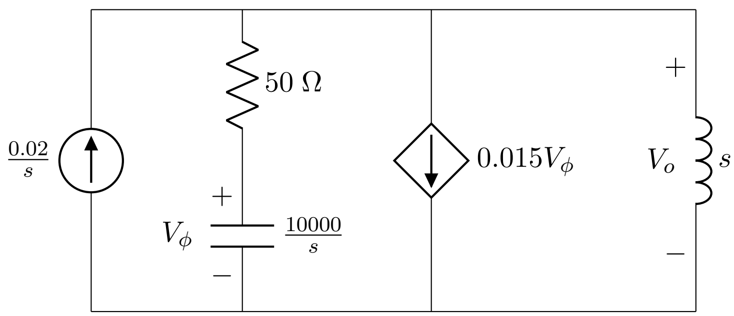
Name: SOLUTIONS

Consider the circuit below if $i_g = 20u(t)$ mA. There is no energy stored in the circuit at $t = 0$.



a) Draw the circuit in the s-domain.

Solution:



b) Find $V_o(s)$.

Solution: Using nodal analysis at the top, we have

$$-\frac{0.02}{s} + \frac{V_o}{50 + 10000/s} + 0.015V_\phi + \frac{V_o}{s} = 0$$

The voltage V_ϕ is the voltage across the capacitor in the first branch. Using the voltage divider rule with V_o:

$$V_\phi = \frac{V_o}{50 + 10000/s} \cdot \frac{10000/s}{1} = \frac{10000V_o}{50s + 10000}$$

Substitute this expression for V_ϕ back into the KCL equation:

$$-\frac{0.02}{s} + \frac{V_o s}{50s + 10000} + 0.015\left(\frac{10000V_o}{50s + 10000}\right) + \frac{V_o}{s} = 0$$

$$\frac{V_o s}{50s + 10000} + \frac{150V_o}{50s + 10000} + \frac{V_o}{s} = \frac{0.02s}{s}$$

Cleaning it up

$$V_o s^2 + V_o \cdot 150s + V_o(50s + 10000) = 0.02(50s + 10000)$$

Factor out V_o and solve

$$V_o\left(\frac{s + 150}{50s + 10^4} + \frac{1}{s}\right) = \frac{20 \times 10^{-3}}{s}.$$

Combine the terms inside the parentheses:

$$\frac{s(s + 150) + (50s + 10^4)}{s(50s + 10^4)} = \frac{s^2 + 150s + 50s + 10^4}{s(50s + 10^4)}$$

$$= \frac{s^2 + 200s + 10^4}{s(50s + 10^4)}$$

So the equation becomes:

$$V_o \frac{s^2+200s+10^4}{s(50s+10^4)} = \frac{20\times10^{-3}}{s}$$

Solving for V_o:

$$\begin{aligned} V_o &= \frac{20\times10^{-3}}{s}\cdot\frac{s(50s+10^4)}{s^2+200s+10^4} \\ &= \frac{20\times10^{-3}(50s+10^4)}{s^2+200s+10^4} \\ &= \frac{1000\times10^{-3}s+200\times10^{-3}\times10^4}{s^2+200s+10^4} \\ &= \frac{s+200}{s^2+200s+10^4} \end{aligned}$$

Notice that the denominator is a perfect square: $s^2+200s+10^4=(s+100)^2$.

$$V_o = \frac{s+200}{s^2+200s+10000} = \frac{s+200}{(s+100)^2}$$

c) Find $v_o(t)$. Recall $e^{-at}u(t) \Longleftrightarrow \frac{1}{s+a}$ and $te^{-at}u(t) \Longleftrightarrow \frac{1}{(s+a)^2}$.

Solution: Perform partial fraction expansion:

$$V_o = \frac{K_1}{(s+100)^2} + \frac{K_2}{s+100}$$

Find coefficients:

$$s+200 = K_1 + K_2(s+100)$$

Set $s=-100$:

$$-100+200 = K_1 \Longrightarrow K_1 = 100$$

Equate coefficients of s:

$$1 = K_2 \Longrightarrow K_2 = 1$$

Thus:

$$V_o(s) == \frac{100}{(s+100)^2} + \frac{1}{s+100}$$

Taking the inverse Laplace transform

$$v_o(t) = (100te^{-100t}+e^{-100t})u(t) = e^{-100t}(100t+1)u(t) \text{ V}$$