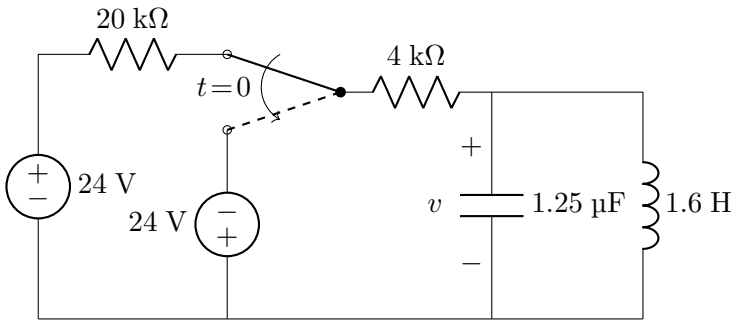
ECE 2260 quiz05

Name: SOLUTIONS

For the following circuit, find $v(t)$.



Solution: First, we should notice that we need to get the circuit in the parallel form with a source transformation after $t \geq 0$. This would give us a current source with a parallel resistor, capacitor, and inductor. The current source value is given by $I_s = V/R = 24\text{ V}/20\text{ k}\Omega = 1.2\text{ mA}$. The parallel resistance is simply $R = 20\text{ k}\Omega$.
We should figure out some initial conditions. The initial current through the inductor is $i_L(0^-) = \frac{24\text{ V}}{24\text{ k}\Omega} = 1\text{ mA}$. The initial voltage across the capacitor is $v_C(0^-) = 0\text{ V}$. We can find $i'(0^+) = \frac{v(0^+)}{L} = \frac{0\text{ V}}{1.6\text{ H}} = 0\text{ A/s}$. Lastly, we can find $I_f = \frac{-24\text{ V}}{4\text{ k}\Omega} = -6\text{ mA}$.
We next need to determine the natural response characteristic roots

$$S_{1,2} = -100 \pm j700\text{rad/s}.$$

We can then find the natural response

$$i_L(t) = I_f + e^{-100t}(B_1'\cos(700t) + B_2'\sin(700t)).$$

We can find B_1' using the initial condition

$$i_L(0^+) = I_f + B_1' = 1\text{ mA} \Longrightarrow B_1' = 7\text{ mA}.$$

We can find B_2' using the derivative initial condition

$$i_L'(0^+) = -100B_1' + 700B_2' = 0\text{ A/s} \Longrightarrow B_2' = 1\text{ mA}.$$

Putting it together

$$i_L(t) = -6 + e^{-100t}(7\cos(700t) + \sin(700t))\text{mA}.$$

However, we are looking for $v(t)$! We can find $v(t)$ using $v(t) = L\frac{di_L(t)}{dt}$.

$$\begin{aligned} v(t) &= 1.6\text{ H}\big(-100e^{-100t}(0.007\text{ A}\cos(700t) + 0.001\text{ A}\sin(700t)) + e^{-100t}(-0.007\text{ A}\cdot 700\sin(700t) + 0.001\text{ A}\cdot 700\cos(700t))\big)\text{V} \\ &= -8\text{ V}e^{-100t}\sin(700t) \end{aligned}$$

We can also solve this from the voltage side. The characteristic equation is the same, so we can write the voltage as

$$v(t) = e^{-100t}(B_1\cos(700t) + B_2\sin(700t))$$

We can find B_1 using the initial condition:

$$v(0^+) = B_1 = 0\text{ V}$$

We can find B_2 using the derivative initial condition

$$v'(0^+) = \frac{i_c(0^+)}{C} = \frac{-7\text{ mA}}{1.25\text{ µF}} = -5600\text{ V/s} \Longrightarrow 700B_2 = -5600\text{ V/s} \Longrightarrow B_2 = -8\text{ V}$$

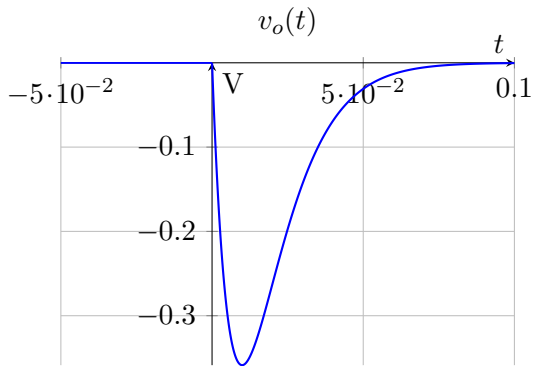
Putting it together

$$v(t) = -8\text{ V}e^{-100t}\sin(700t)t > 0.$$

Let's give a more complete final answer

$$v(t) = \begin{cases} 0\text{ V}, & t < 0 \\ -8\text{ V}e^{-100t}\sin(700t), & t \geq 0 \end{cases}$$

It might be helpful to visualize $v(t)$ with a plot.


$v_o(t)$
t
$-5\cdot10^{-2}$
V
$5\cdot10^{-2}$
0.1
-0.1
-0.2
-0.3