ECE 3210 Final Exam

Week of: December 7, 2020

Student's name: ______________________________

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You have 3 hours for 6 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You may use three US letter-style size page of notes, front and back.

Problem	Score	Possible Points
1		25
2		20
3		20
4		20
5		20
6		20
Total score		125

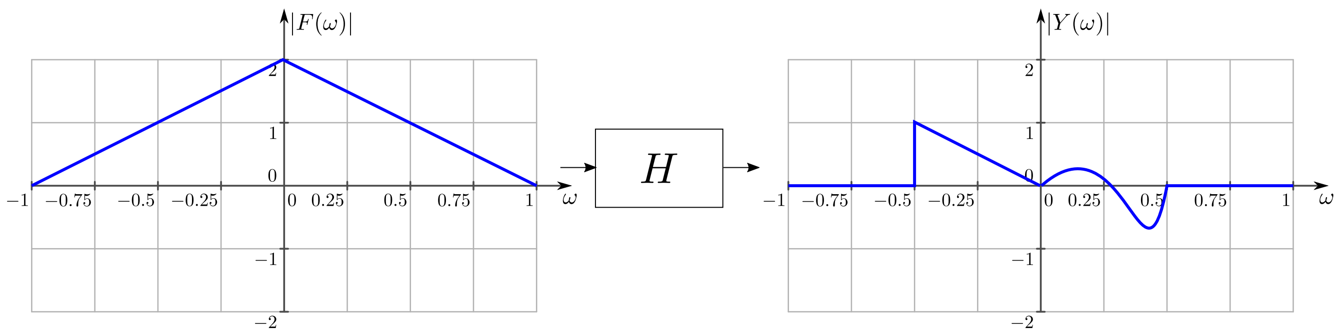
1 Short answer

(a) Evaluate the following convolution using any method you know

$$e^{-3t}u(t)*(u(t)-u(t-9))$$

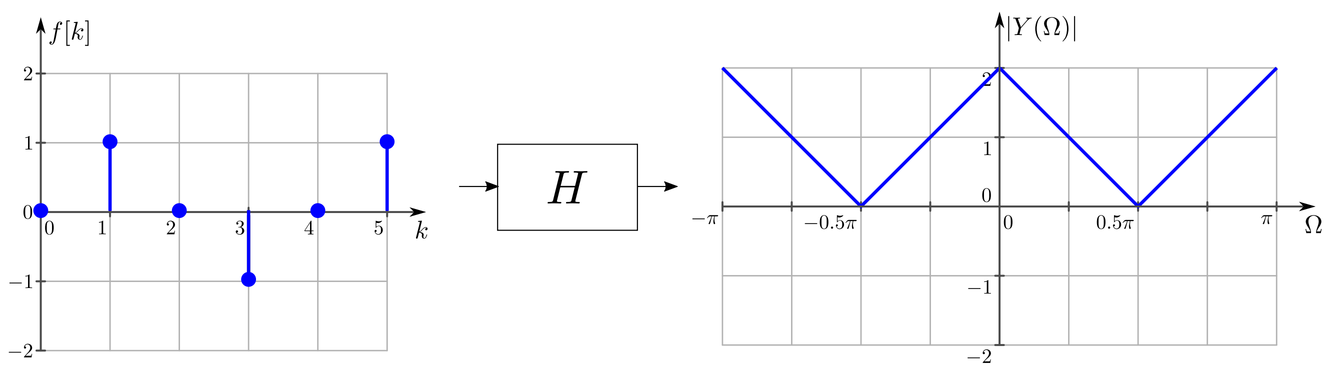
$e^{-3t}u(t)*(u(t)-u(t-9))=.$

(b) Determine if the following system is LTI.



LTI	not LTI

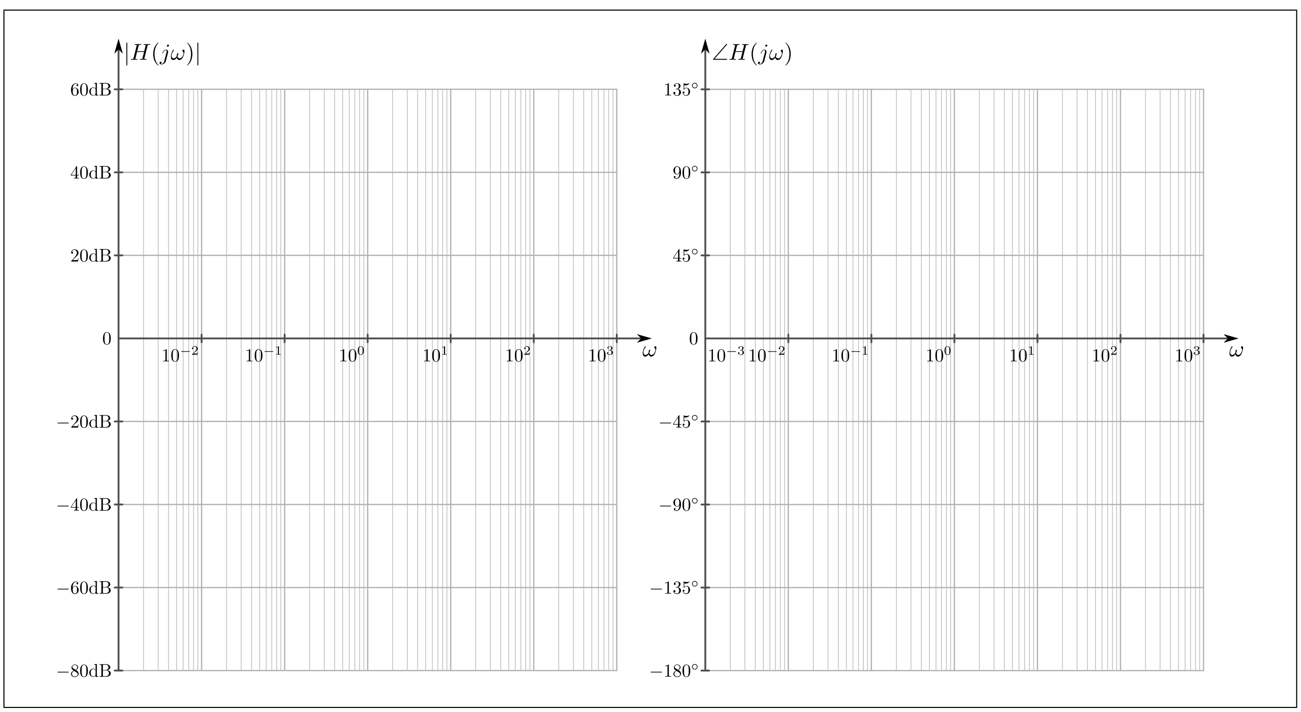
(c) Determine if the following system is LTI. Note: the signal represented as $f[k]$ is periodic with a period of 4.



LTI	not LTI

(d) Sketch the Bode plot given the following transfer function

$$H(s) = 1000\frac{s}{s^2 + 110s + 1000}.$$



2 Difference equations

Consider the following difference equation

$$4y[k+2]+4y[k+1]+y[k]=f[k+1]$$

with initial conditions $y[-1]=0$, $y[-2]=1$ and input $f[k]=u[k]$.

(a) Solve for the zero-input solution.

$y_{zi}[k]=$

(b) Solve for the zero-state solution.

$y_{zs}[k] =$

(c) Solve for the total solution.

$y[k] =$

3 Fourier transforms

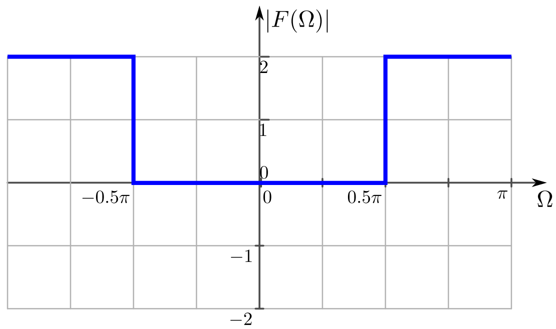
(a) Find the DTFT of

$$f[k] = \left(-\frac{1}{3}\right)^k (u[k] - u[k-10]).$$

You are welcome to use tables or a brute-force summation.

$F(\Omega) =$

(b) Given $F(\Omega)$ below, derive an expression for $f[k]$, which is the inverse DTFT of $F(\Omega)$.



$f[k] =$

4 Sinusoidal response

(a) For an LTIC system described by the transfer function

$$H(s) = \frac{s+2}{s^2+5s+4}$$

find the response ($y(t)$) to the following everlasting sinusoidal input

$$f(t) = 10\sin(2t+45^\circ).$$

$y(t) =$

(b) Suppose a signal $f(t) = \sin(1750\pi t - 90°) + \cos(1250\pi t)$ is sampled at 500Hz. Write the sampled signal $f[k]$ below. Please simplify your expression as much as possible.

$f[k] =$

5 Filter design

Suppose we are interested in designing a low-pass filter. We want to build this filter to be within the following specifications:

(i) Passband gain to lie between 0dB and $\hat{G}_p = -2$dB for $0 \leq \omega \leq 50$.

(ii) Stopband gain not to exceed $\hat{G}_s = -20$dB for $\omega \geq 200$.

Please follow the prompts to design this filter.

(a) What is the order n of the filter? What is the 3dB bandwidth ω_c?

$n=$	$\omega_c=$

(b) Determine the normalized transfer function $\tilde{H}(s)$. It might be beneficial later on to leave this as a cascade of 2$^{\text{nd}}$ order filter transfer functions.

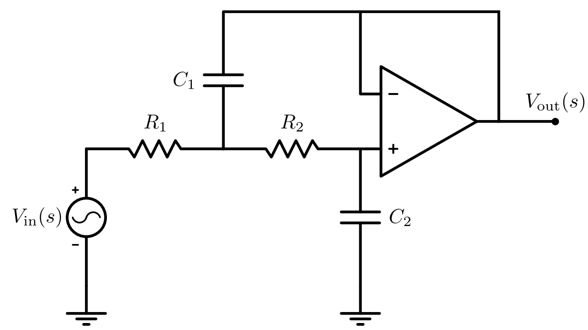
$\tilde{H}(s)=$

(c) Determine the final filter transfer function $H(s)$.

$H(s)=$

(d) Now suppose we want to build this circuit in hardware. We go to the ET 133-C lab and find that we have two op amps, a bunch of 10kΩ resistors, and any capacitor we want smaller than 100μF. Draw an appropriate filter that matches the transfer function you designed earlier.

Hint: It will be easisest if you use the Sallen-Key topology like we used in the lab. You might need to use multiple filters cascaded together. A single stage for this filter is



with a transfer function

$$H(s) = \frac{\omega_c^2}{s^2 + 2\zeta\omega_c s + \omega_c^2}$$

where $\omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$ and $2\zeta\omega_c = \frac{R_1+R_2}{C_1 R_1 R_2}$.

6 Discrete-time system implementation

A major problem in recording of electrocardiograms (ECGs) is the appearance of unwanted 60Hz interference in the output. We want a clean signal hope to knock out this 60Hz interference. Because this signal is analog, we could design a notch filter to remove it, but analog notch filters are a hassle to design. Instead, we are going to sample the signal and use a digital filter to clean it up.

(a) Assume that the bandwidth of the signal of interest is 180Hz, what should the minimum sampling period T_s be to satisfy Nyquist (you might want to recall the relationship between sampling period and frequency is $f_s = {}^1/_{T_s}$.

$T_s =$

(b) The analog signal is sampled at the sampling period T_s that you just solved for in the previous part. The resulting signal $f[k]$ is then processed with a discrete-time system H that is described by the impulse response

$$h[n] = \delta[n] - \delta[n-1] + \delta[n-2].$$

Is the unwanted interference removed? Justify your answer. (*Hint:* look at $H(\Omega_i)$ where Ω_i is the digital frequency of the interference.)

yes	no

(c) Find the z-transform of $h[n]$.

$H[z] =$

(d) Is this an IIR or FIR system?

IIR FIR

(e) Sketch out the realization (aka block diagram) of this system

(f) Assume all system intermediate points are set to zero. Given some input $f[k] = [5, 0, -1, 7, -3]^T$, what is the system output $y[k]$?

$y[k] =$