ECE 3210 Final Exam

Week of: December 14, 2023

Student's name: ______________________________

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You have 3 hours for 6 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You may use three US letter-style size page of notes, front and back.

Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
6		25
Total score		125

1 Short answer

(a) Is the discrete-time signal $x[n] = \cos(2n)$ periodic?

Circle one:	True	False

(b) Consider the signal of the form

$$x(t) = 2\cos^2(120\pi t).$$

This signal is sampled at a rate of 80 samples per second. No anti-aliasing filter was used. What is $x[n]$? (Make your answer as simplified as you are able.)

$x[n] =$

(c) Consider the signal

$$x[n] = u[n+2] - u[n-2].$$

Find the DTFT $X(\Omega)$.

$X(\Omega) =$

2 Convolution

Compute the following convolutions.

(a) $y(t) = e^{-3t}u(t-1) * (u(t+1) - u(t-1))$

$y(t) =$

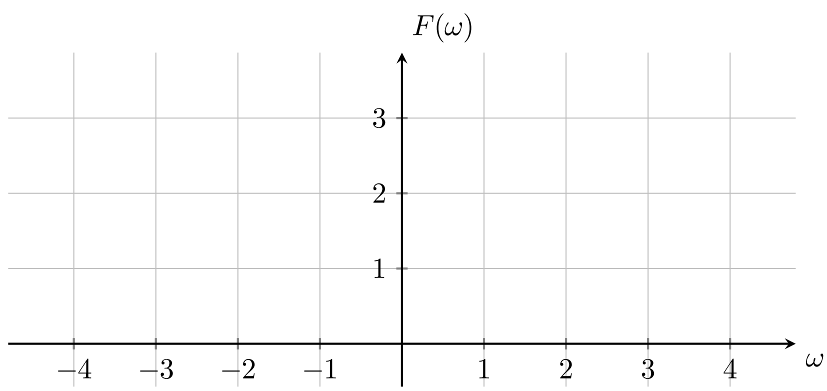
(b) $y[n] = (u[n] - u[n-5]) * \cos(0.4\pi n)$ (*Hint:* it might be helpful to look at this problem in the frequency domain.)

$y[n] =$

3 Sampling and modulation

Consider the signal $f(t) = \frac{1}{\pi}\mathrm{sinc}^2\left(\frac{t}{2}\right)$

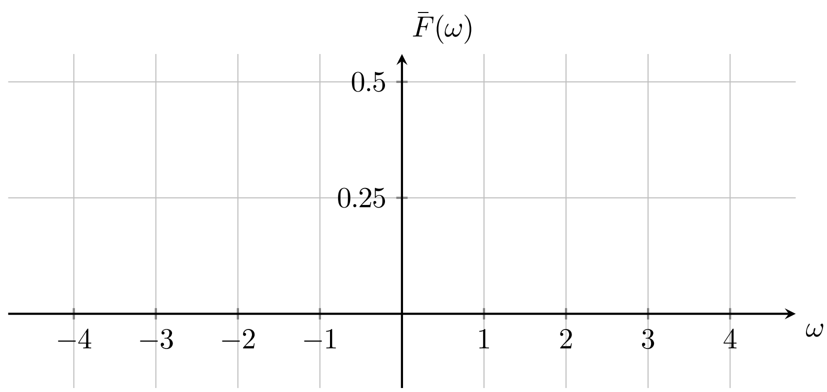
(a) Sketch the Fourier transform $F(\omega)$ below.



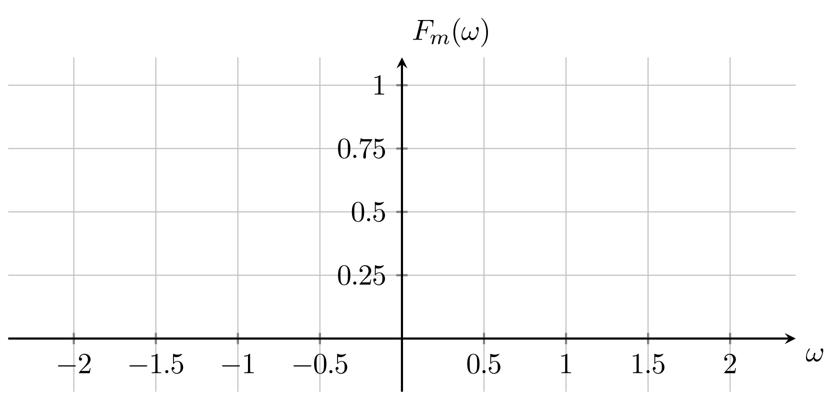
(b) What is the bandwidth and Nyquist sampling rate ($f_{s,\mathrm{Nyq}}$) for this signal in hertz?

BW= $f_{s,\mathrm{Nyq}} =$

(c) Suppose we were to sample at three quarters the Nyquist frequency (i.e., $0.75 f_{s,\mathrm{Nyq}}$), sketch the Fourier transform of the resulting signal $\bar{F}(\omega)$.

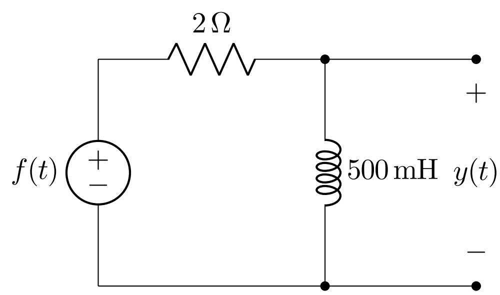


(d) Now suppose instead of sampling the signal you modulate $f(t)$ with a sinusoid such that $f_m(t) = f(t)\cos(0.5t)$. Sketch the resulting Fourier transform $F_m(\omega)$.



4 Frequency response

Consider the circuit below.



(a) Solve for the transfer function.

$H(s) =$

(b) If you were to use $f(t) = 3\sin(t - 60°)$ as input, what is the output $y(t)$?

$y(t) =$

(c) Suppose we were use the periodic signal $f(t)$ as an input where one period is described as

$$f(t)=\begin{cases}1 & 0<t\leq 1\\ 0 & 1<t\leq 2\end{cases}.$$

Find the complex exponential Fourier series representation of $f(t)$.

$f(t)=$

(d) If we use the $f(t)$ described in the previous part as an input, what is the output $y(t)$ of the system described above? Represent your answer as a summation of scaled complex exponentials.

$y(t)=$

5 Difference equation

Consider the difference equation

$$y[n+2]-y[n+1]+0.25y[n]=f[n+1]+f[n].$$

(a) Find the transfer function $H[z]$

$H[z]=$

(b) Sketch the system block diagram below.

(c) Given some input $f[n] = \cos(\pi n)u[n]$, find the system output $y[n]$. Assume initial conditions are zero.

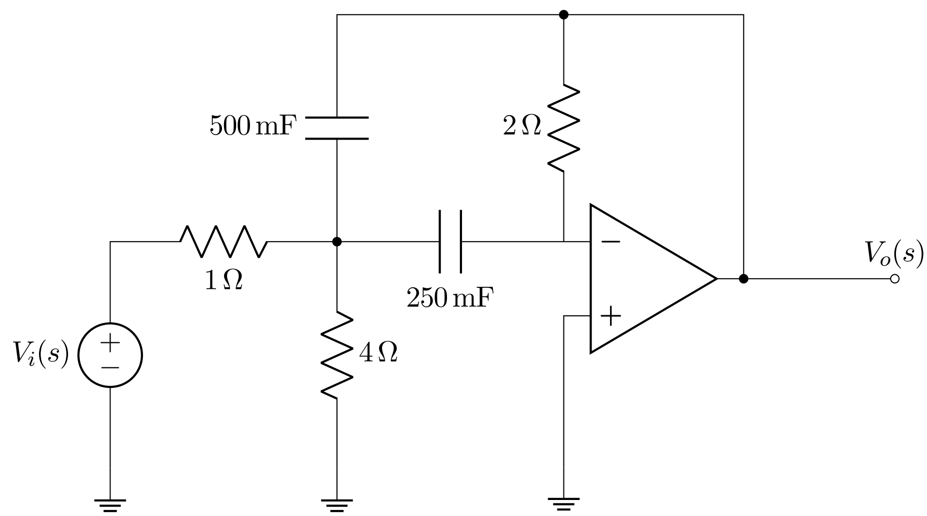
$y[n] =$

6 Filter design

(a) A multi-feedback active filter is another filter topology that is commonly used. It will typically have the following transfer function form

$$H(s) = \frac{2\zeta\omega_0 K s}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

One example of this circuit is given below. Find the values of K, ζ, and ω_0.



Your work continued...

$K =$	$\zeta =$	$\omega_0 =$

(b) Suppose you are tasked with designing a lowpass Butterworth filter that has a gain of -2dB at $f = 1$kHz (which defines the passband) and a gain of -20dB at $f = 4$kHz (which defines the stopband). (*Hint:* be careful with the units on your frequencies!)

(i) What is the order and ω_c (in rad/s) of this filter?

$n =$	$\omega_c =$

(ii) What is the transfer function $H(s)$?

$H(s) =$