ECE 3210 Midterm 1

Week of: October 7, 2023

Student's name: ____________________

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You have 2.5 hours for 5 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You may use one US letter-style size page of notes, front and back.

Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
Total score		100

1 Short answer

(a) What is the verb form of "convolution"?

(b) Consider the following systems. Determine if they are stable, marginally stable, or unstable.

(i) $(D^2 + 4)y(t) = (D + 1)f(t)$

Circle one: stable marginally stable unstable

(ii) $h(t) = e^{-t}\sin(t)u(t)$.

Circle one: stable marginally stable unstable

(iii) $(D^2 - 6D + 18)y(t) = (D + 1)f(t)$

Circle one: stable marginally stable unstable

(c) Consider the following systems. Determine if they are linear and/or time-invariant. No need to write out a formal proof but feel free to use the space to work through the problem if needed.

(i) $y(t) = \cos(t)x(t) + a$

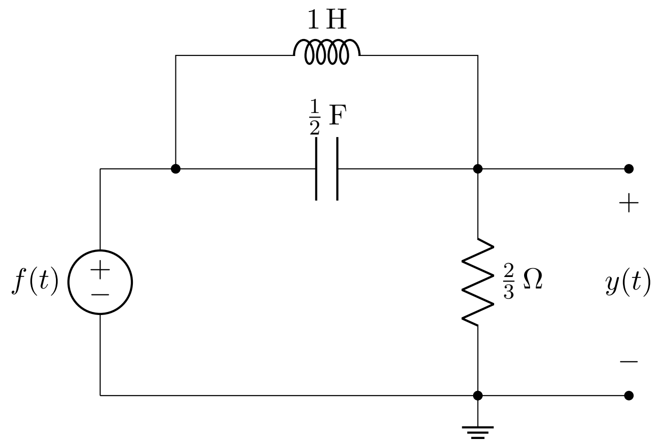
Circle one:	linear	non-linear
Circle one:	time-invariant	time-variant

(ii) $y(t) = |x(t)|$

Circle one:	linear	non-linear
Circle one:	time-invariant	time-variant

2 Impulse response

Consider the following circuit.



(a) Derive a differential equation relating the input $f(t)$ and the output $y(t)$. In your final answer, don't worry about including the units, just write the ODE in the form we have seen in class.

(b) Determine the zero-input response (we'll call it $y_2(t)$ for this system). Assume $y'(0) = 1$ and $y(0) = 0$.

$y_2(t) =$

(c) Determine the impulse response $h(t)$ for this system.

$h(t) =$

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3 Convolution

Using direct integration or graphical (i.e., "flip-and-drag") methods, solve for $y(t)$ by performing the following convolutions.

(a) $y(t) = 2u(t) * t(u(t) - u(t-1))$

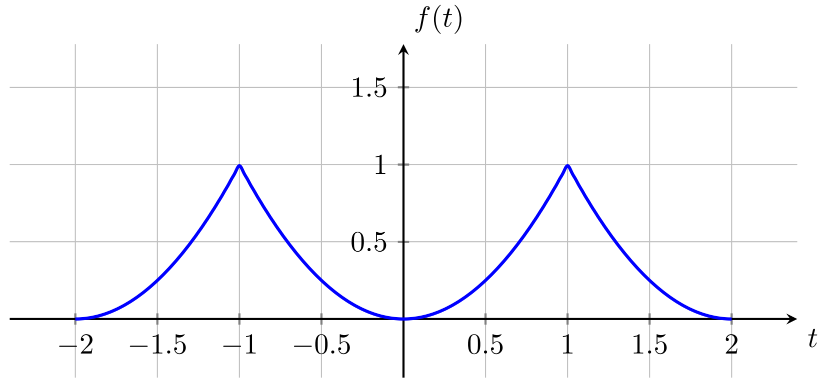
$y(t) =$

(b) $y(t) = \text{rect}\left(\frac{t}{4}\right) * \sin(\pi t)\text{rect}\left(\frac{t}{2}\right)$

$y(t) =$

4 Fourier series

Consider the periodic function $f(t)$ seen below.



Note that between $-1 \leq t \leq 1$, $f(t)$ is defined as $f(t) = t^2$.

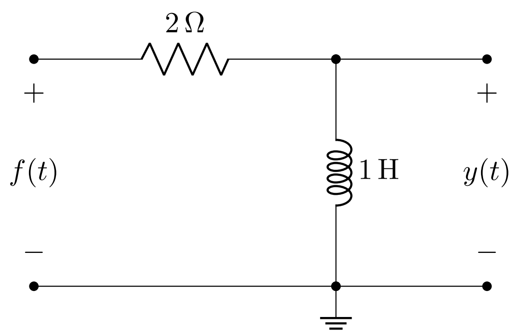
(a) Determine the period and fundamental frequency ω_0 for the function $f(t)$.

$T =$	$\omega_0 =$

(b) Derive the *complex exponential* Fourier series for $f(t)$.

$f(t) =$

(c) Derive the transfer function $H(s)$ for the circuit seen below.



$H(s) =$

(d) Given the input $f(t)$ used in parts (a) and (b) for the circuit in part (c), what is the output $y(t)$? If you couldn't solve part (b), you can just use "D_n" in your series expansion.

$y(t) =$

5 Fourier transforms

(a) Consider the system described by an input/output pair.

$$x(t) = \cos(t) \longrightarrow \boxed{H} \longrightarrow y(t) = t(u(t+1) - u(t-1))$$

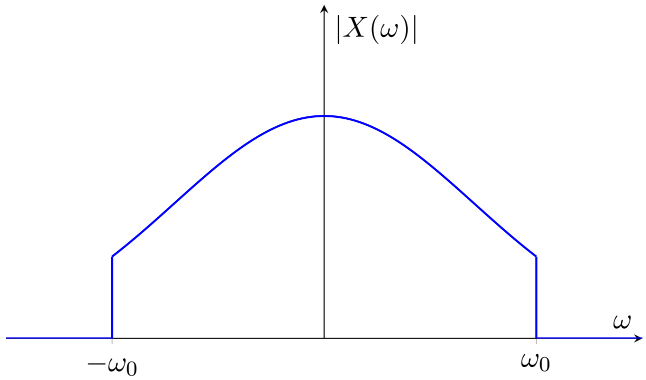
(a) Find $Y(\omega)$, the Fourier transform of the output $y(t)$. Feel free to use Fourier properties and known transforms or brute-force integration. Simplify your answer as much as possible.

$Y(\omega) =$

(b) Given this input/output relationship, is this system linear and time-invariant?

Circle one: LTI not LTI

(b) Let $x(t)$ be a signal whose spectrum is identically zero outside the range $-\omega_0 \leq \omega \leq \omega_0$. An example of such a spectrum is shown below. The actual values of $X(\omega)$ are not important for this problem, but the range of ω for which $X(\omega)$ is non-zero is.



For the following signals, determine the range of ω (in terms of ω_0 over which their spectrum is non-zero.

(i) $y(t) = x(t) + x(t-1)$

(ii) $y(t) = x(t)\cos(\omega_0 t)$.