ECE 3210 Midterm 1

Week of: October 2, 2023

Student's name: ____________________

Instructor:
Eric Gibbons
ericgibbons@weber.edu
801-626-6861

You have 2 hours for 5 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You may use one US letter-style size page of notes, front and back.

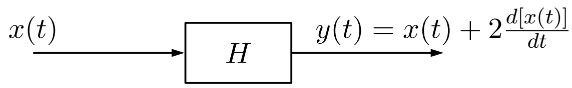
Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
Total score		100

1 Short answer

(a) What is the verb form of "convolution"?

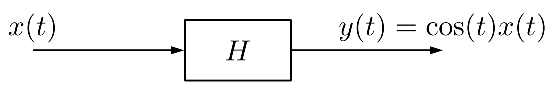
(b) Consider the following systems.

(i) Is the system below linear or non-linear?



Circle one: linear non-linear

(ii) Is the system below time-invariant?



Circle one: time-invariant time-variant

(c) The system described the differential equation $(D^2 + 6D + 18)y(t) = (D + 1)f(t)$ stable, marginally stable, or unstable?

Circle one: stable marginally stable unstable

(d) Consider the following implementation convolution implementation in Python code.

```
def py_convolve(f, t_f, x, t_x):

    dt = t_f[1] - t_f[0]

    t_y = dt*np.arange(len(f) + len(x) - 1)
    t_y += t_f[0] + t_x[0]

    y = np.zeros(len(f) + len(x) - 1)

    for n in range(len(x)):
        y[n:n+len(f)] += x[n]*f*dt

    return y, t_y
```

(i) Does this code accurately perform numerical convolution similar to what you did in Lab 3?

Circle one:	True	False

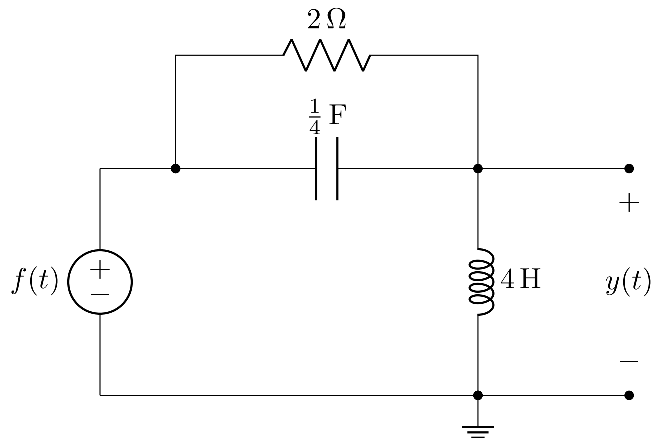
(ii) If arrays `f` and `t_f` had a length of 32 elements and arrays `x` and `t_x` had a length of 1096 elements, how could you speed up this code?

(e) Could the polynomial functions $x_0(t) = 1$, $x_1(t) = t$, and $x_2(t) = t^2$ be used as a set of basis functions on $t \in [-1, 1]$?

Circle one:	yes	no

2 Circuit analysis

Using the circuit below, please answer the following questions.



(a) Derive a differential equation relating the input $f(t)$ and the output $y(t)$. In your final answer, don't worry about including the units, just write the ODE in the form we have seen in class.

(b) Determine the zero-input response (we'll call it $y_2(t)$ for this system). Assume $y'(0) = 1$ and $y(0) = 0$.

$y_2(t) =$

(c) Determine the impulse response $h(t)$ for this system.

$h(t) =$

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3 Convolution

Using direct integration or graphical (i.e., "flip-and-drag") methods, solve for $y(t)$ by performing the following convolutions.

(a) $y(t) = e^{-2t}u(t-2) * e^{-t}(u(t-1) - u(t-4))$

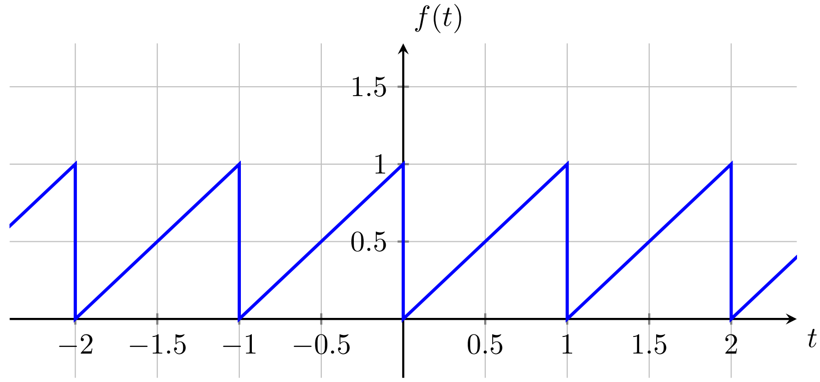
$y(t) =$

(b) $y(t) = u(t+1) * t(u(t) - u(t-2))$

$y(t) =$

4 Fourier series

(a) Determine the period and fundamental frequency ω_0 for the function $f(t)$ seen below.

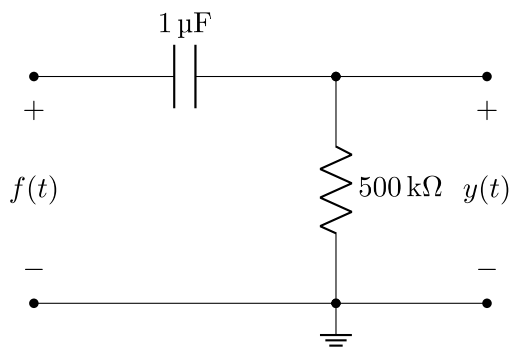


$T =$ $\qquad\qquad\qquad\qquad$ $\omega_0 =$

(b) Derive the *complex exponential* Fourier series for $f(t)$.

$f(t) =$

(c) Derive the transfer function $H(s)$ for the circuit seen below.



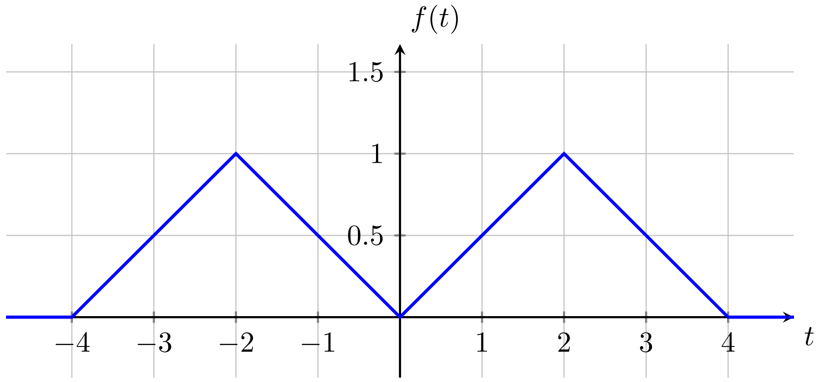
$H(s) =$

(d) Given the input $f(t)$ used in parts (a) and (b) for the circuit in part (c), what is the output $y(t)$?

$y(t) =$

5 Fourier transform

(a) Consider the time-domain signal $f(t)$ shown below, find the Fourier transform $F(\omega)$.



$F(\omega) =$

(b) Consider the frequency-domain

$$F(\omega) = 4\mathrm{sinc}\,(\omega)\cos\,(\omega)$$

Sketch the inverse Fourier tranform $f(t)$ in the plot below.