ECE 3210 Final Exam

Week of: December 9, 2019

Student's name and section: ______________________________

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You have 180 minutes for 6 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You may use three pages of notes and a calculator.

Problem	Score	Possible Points
1		25
2		20
3		20
4		20
5		20
6		20
Total score		125

1 Short answer

(a) If a system takes $x(t)$ as an input and

$$y(t) = x(t)\cos(10t)$$

is the system output. Is this system linear or non-linear? And is this system time-variant or time-invariant?

Circle one:	linear	nonlinear
Circle one:	time-variant	time-invariant

(b) Which of the following is an advantage of a Chebyshev filter compared to a Butterworth filter?

Circle your answer:

(i) Corner frequency amplitude is always -3dB.

(ii) Steeper rolloff.

(iii) Ripple in the passband

(c) The following systems are characterized by the following impulse functions or transfer functions. Write if they are stable, marginally stable, or unstable.

Write your answer next to the function/transfer function.

$h(t) = \cos(5\pi t)u(t)$

$h[k] = (-0.9)^k u[k]$

$H[z] = \frac{z(z-3)}{z^2+z-2}$

$H(s) = \frac{s(s-6)}{s^2-3+9}$

(d) True or false: is this Fourier transform pair valid?

$$2\text{sinc}(\pi t) \Longleftrightarrow \text{rect}\left(\frac{\omega}{2\pi}\right).$$

If it is, indicate in the answer box below. If it is false, indicate in the box in the box and write the correct transform. (*Hint:* using the Fourier transform tables will help.)

Circle one:	True	False

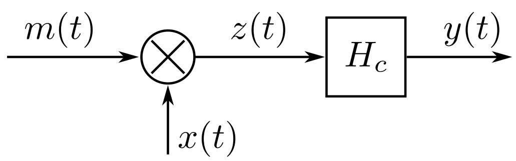
(e) Compute the following convolution by direct summation.

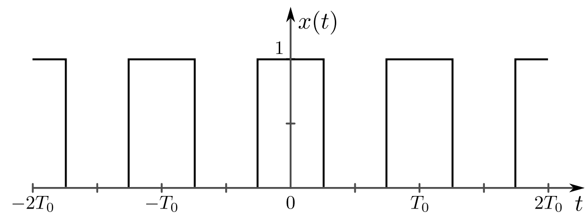
$$c[k] = (-0.9)^k u[k] * u[k]$$

$c[k] =$

2 Modulation

Thus far our discussion of modulation has largely focused on analog multiplication operation of our message signal $m(t)$ and some cosine waveform. However, this multiplication is largely difficult and expensive to implement in hardware. Fortunately, we can have a similar effect by switching $m(t)$ on and off with some duty cycle over some period T_0. This in turn can be represented by the diagram below, where $m(t)$ is the signal we want to transmit, and itsmultiplication with $x(t)$ represents the on/off operation.





Note that $T_0 = \frac{2\pi}{\omega_c}$ and $x(t)$ can be represented by an even Fourier series such that

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_c t).$$

(a) Derive an expression for the signal after it is has been modulated by the on/off modulation (point $z(t)$ on the figure). You will need to derive the Fourier series coefficients a_0 and a_n.

$z(t) =$

(b) If our perfect bandpass filter H_c has a frequency response of

$$H_c(\omega) = \begin{cases} 1 & -\omega_c - \omega_s \leq \omega \leq -\omega_c + \omega_s \\ 1 & \omega_c - \omega_s \leq \omega \leq \omega_c + \omega_s \\ 0 & \text{otherwise} \end{cases}$$

what is the signal $y(t)$? Assume that the original signal $m(t)$ is band-limited to $\pm\omega_s$ and there is no aliasing after modulation.

$y(t) =$

(c) Could the same scheme also be used for demodulation provided the bandpass filter H_c is replaced with a lowpass filter? Justify your answer.

Circle one:	yes	no

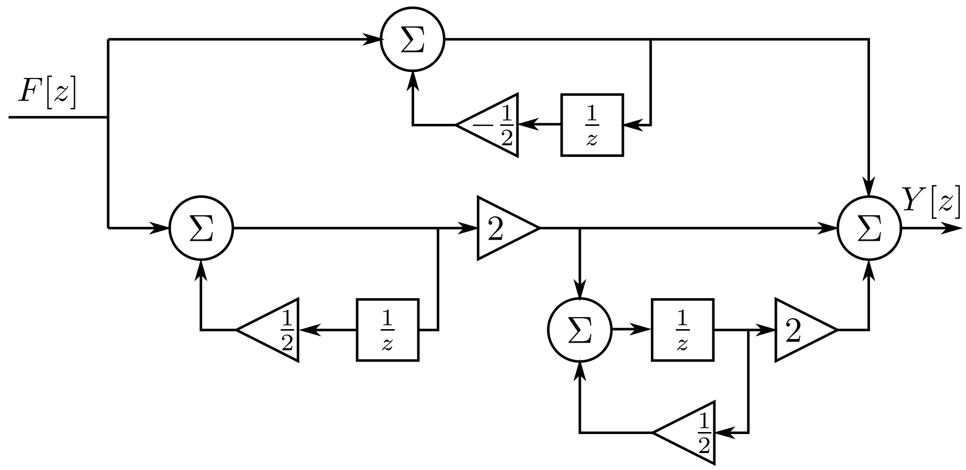
3 Block Diagram

(a) Given a system

$$H[z] = \frac{3z^2 - 2z}{z^2 - 5}$$

draw the block diagram system realization in canonical form below.

(b) Given the block diagram below, determine the system's impulse function.



$h[k] =$

4 DTFT

(a) Using the appropriate properties, find the DTFT of

$$f[k]=(k+1)a^k u[k].$$

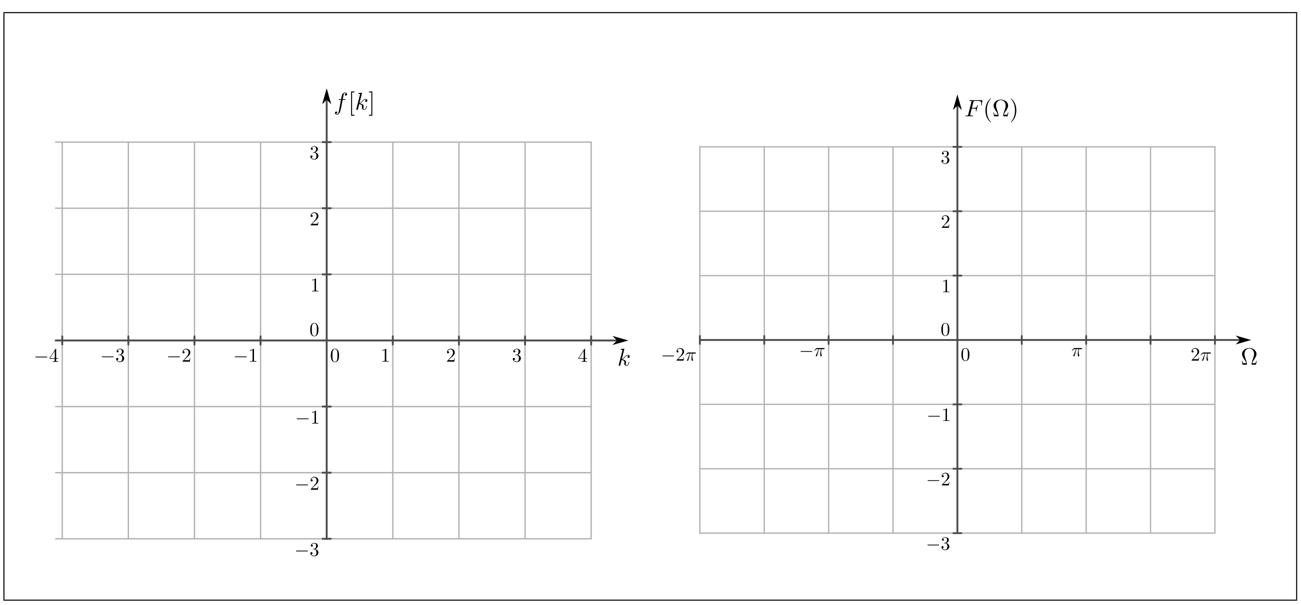
Assume ($|a|<1$).

$F(\Omega)=$

(b) Sketch the following signal and its discrete time Fourier transform

$$f[k] = \operatorname{sinc}\left(\frac{\pi k}{2}\right)\cos\left(\frac{\pi k}{2}\right).$$

It would be helpful to show some justification to support your answers.. (*Hint:* look at the zero-crossings. And plot the time-domain signal before you attempt the DTFT.)



5 Filter Design

Suppose we are interested in designing a low-pass filter. We want to build this filter to be within the following specifications:

(i) Passband gain to lie between 0dB and $\hat{G}_p = -2$dB for $0 \leq \omega \leq 100$.

(ii) Stopband gain not to exceed $\hat{G}_s = -20$dB for $\omega \geq 200$.

Please follow the prompts to design this filter.

(a) What is the order n of the filter? What is the 3dB bandwidth ω_c?

$n=$

(b) Determine the normalized transfer function $\tilde{H}(s)$. It might be beneficial later on to leave this as a cascade of 2^{nd} order filter transfer functions.

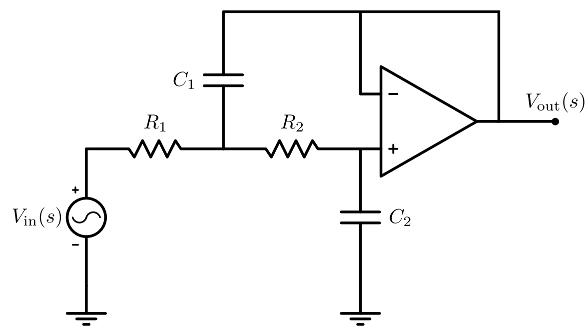
$\tilde{H}(s)=$

(c) Determine the final filter transfer function $H(s)$.

$H(s)=$

(d) Now suppose we want to build this circuit in hardware. We go to the ET 133-C lab and find that we have two op amps, a bunch of 10kΩ resistors, and any capacitor we want smaller than 10μF. Draw an appropriate filter that matches the transfer function you designed earlier.

Hint: It will be easisest if you use the Sallen-Key topology like we used in the lab. You might need to use multiple filters cascaded together. A single stage for this filter is



with a transfer function

$$H(s) = \frac{\omega_c^2}{s^2 + 2\zeta\omega_c s + \omega_c^2}$$

where $\omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$ and $2\zeta\omega_c = \frac{R_1 + R_2}{C_1 R_1 R_2}$.

Draw your filter here.

6 Difference Equation

A system is described by the difference equation

$$y[k+2]-\frac{5}{6}y[k+1]+\frac{1}{6}y[k]=5f[k+1]-f[k]$$

and its initial conditions $y[-1]=2$ and $y[-2]=0$ with an input $f[k]=2^{-k}u[k]$. .

(a) Solve for the zero-input response $y_{\text{zi}}(t)$.

$y_{\text{zi}}(t)=$

(b) Solve for the zero-state response $y_{\text{zs}}(t)$.

$y_{\text{zs}}(t) =$

(c) Solve for the total response $y(t)$.

$y(t) =$