ECE 3210 Final Exam

Week of: December 15, 2022

Student's name: ______________________________

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You have 3 hours for 6 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

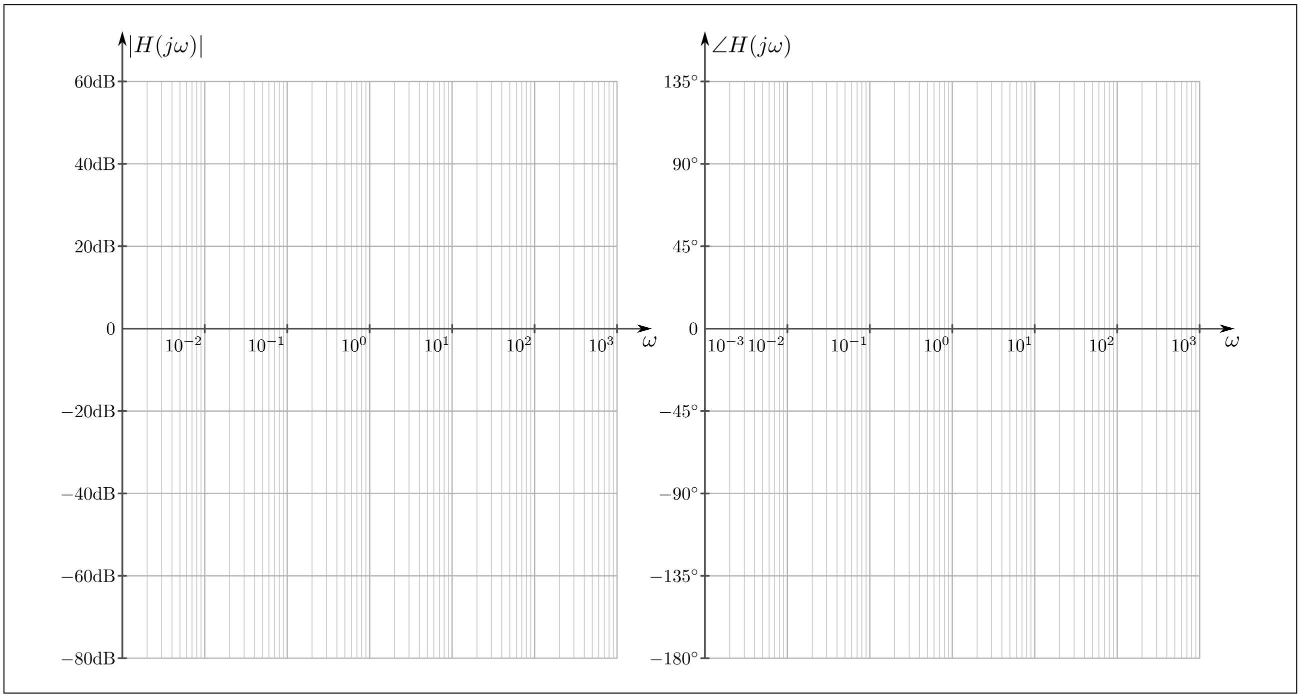
You may use three US letter-style size page of notes, front and back.

Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
6		25
Total score		125

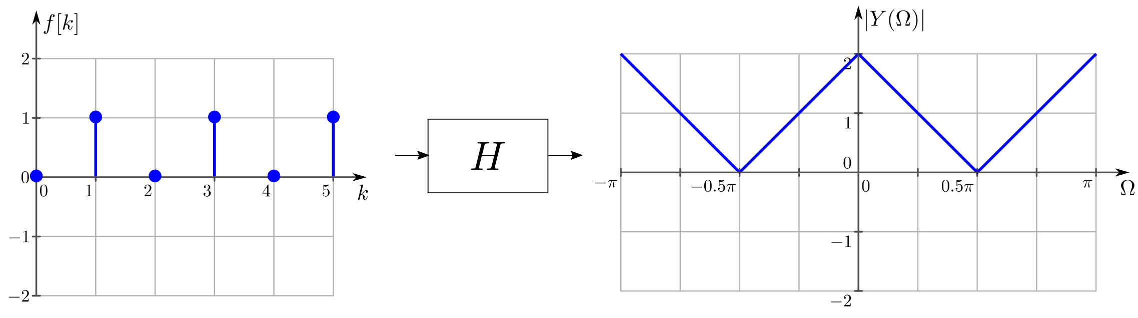
1 Short answer

(a) Sketch the Bode plot for the following transfer function

$$H(s) = \frac{10s+1000}{s^2+11s+10}.$$



(b) Determine if the following system could be LTI. (Assume that $f[k]$ is at steady state and will continue this way for $k \in (-\infty, \infty)$.)



Circle one:	LTI	not LTI

(c) For an LTIC system described by the transfer function

$$H(s) = \frac{s+3}{(s+2)^2}$$

find the response to $2\cos(2t+60°)$.

$y(t) =$

(d) Evaluate the integral $\int_{-\infty}^{\infty} \text{sinc}^2(t)dt$.

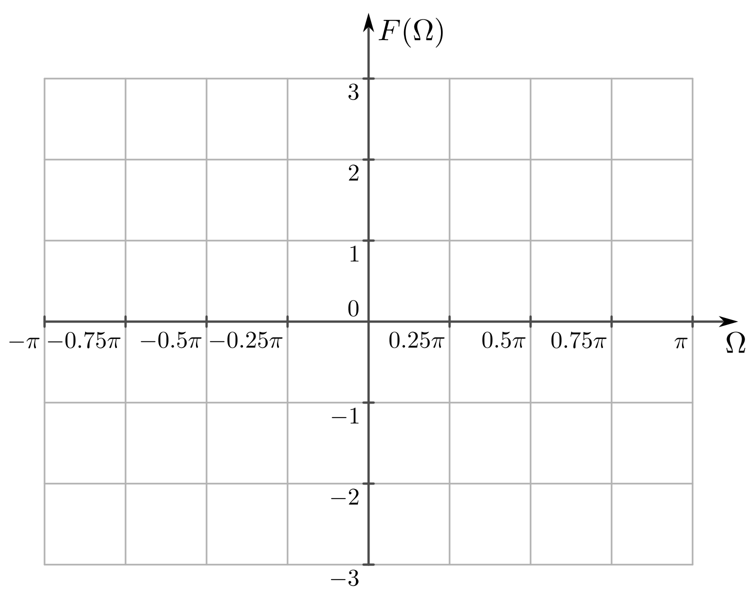
$\int_{-\infty}^{\infty} \text{sinc}^2(t)dt =$

2 DTFT

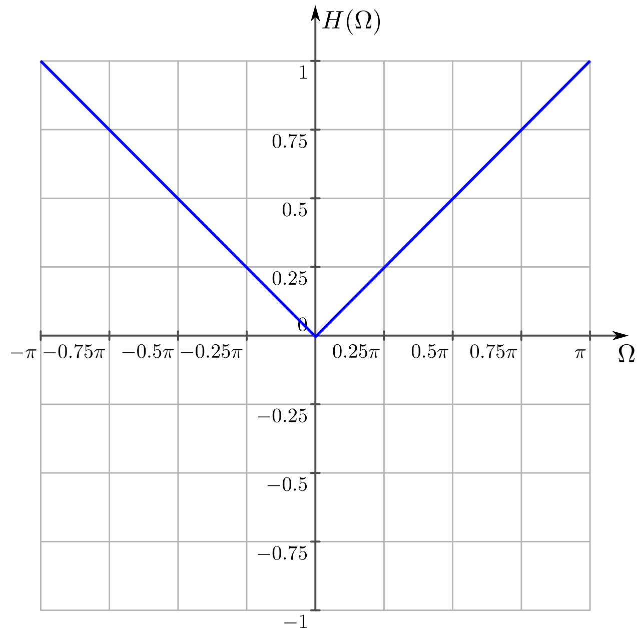
(a) On the plot below, sketch the DTFT $F(\Omega)$ of

$$f[n] = \text{sinc}(\pi n)$$

which is a discrete-time signal. It might help to think about what the values of $f[n]$ really are.



(b) A common digital filter used in medical imaging is the ramp filter, which is used to reconstruct a computed tomography (CT) image from x-ray data. An example ramp filter is given below. Oftentimes the filtering is computed using convolution with $h[n]$, which is the inverse DTFT of $H(\Omega)$. Please determine $h[n]$.



$h[n] =$

3 Convolution

(a) Please compute $u(t-2)*t(u(t)-u(t-3))$ in the time domain (do not use the Laplace transform, though you can if you want to check your work).

$(u(t-2))*t(u(t)-u(t-3))=$

(b) Please compute $\left(-\frac{1}{2}\right)^n u[n-1] * u[n+1]$ in the time domain (do not use the z-transform, though you can if you want to check your work).

$\left(-\frac{1}{2}\right)^n u[n-1] * u[n+1] =$

4 Analog filter design

Suppose we are interested in designing a low-pass filter. We want to build this filter to be within the following specifications:

(i) Passband gain to lie between 0dB and $\hat{G}_p = -2$dB for $0 \leq \omega \leq 100$.

(ii) Stopband gain not to exceed $\hat{G}_s = -20$dB for $\omega \geq 400$.

Please follow the prompts to design this filter.

(a) What is the order n of the filter? What is the 3dB bandwidth ω_c?

$n =$	$\omega_c =$

(b) Determine the normalized transfer function $\tilde{H}(s)$. It might be beneficial later on to leave this as a cascade of 1st and 2nd order filter transfer functions.

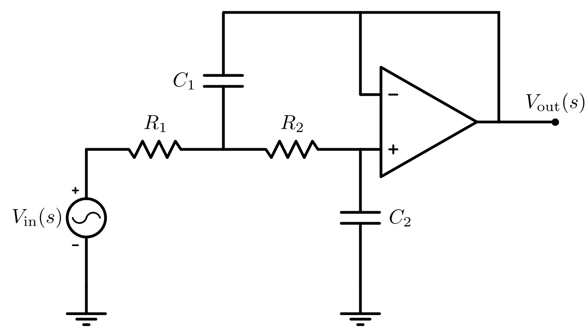
$\tilde{H}(s) =$

(c) Determine the final filter transfer function $H(s)$.

$H(s) =$

(d) Now suppose we want to build this circuit in hardware. We go to the ET 133-C lab and find that we have two op amps, a bunch of 10kΩ resistors, and any capacitor we want smaller than 100μF. Draw an appropriate filter that matches the transfer function you designed earlier.

Hint: It will be easisest if you use the Sallen-Key topology like we used in the lab for a 2$^{\text{nd}}$ order stage. You might need to use multiple filters cascaded together. A single stage for this filter is



with a transfer function

$$H(s) = \frac{\omega_c^2}{s^2 + 2\zeta\omega_c s + \omega_c^2}$$

where $\omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$ and $2\zeta\omega_c = \frac{R_1 + R_2}{C_1 R_1 R_2}$.

5 Difference equations

Consider the following difference equation

$$y[n+2]+3y[n+1]+2y[n]=f[n]$$

with initial conditions $y[-1]=0$, $y[-2]=1$ and input $f[n]=(-1)^n u[n]$.

(a) Solve for the zero-input solution for $n\geq 0$.

$y_{zi}[n]=$

(b) Solve for the zero-state solution for $n \geq 0$.

$y_{zs}[n] =$

(c) Solve for the total solution for $n \geq 0$.

$y[n] =$

6 Discrete-time system realization

Doppler weather radar is susceptible to interference due to 5 GHz Wi-Fi sources. We are interested in a discrete-time stopband filter that will remove the 5GHz interference and retain the rest of the signal.

(a) Suppose the Wi-Fi interference can be represented as a sinusoid $\cos(\omega_i t)$ in continuous-time and after sampling $\cos(\Omega_i n)$ in discrete-time. If $\omega_i = 2\pi \cdot 5$GHz, what is the digital frequency of the interference Ω_i of the noise signal after sampling with a period of $T = 40$ps? Remember, one picosecond is 10^{-12}s.

$\Omega_i =$

(b) Will we satisfy the Nyquist criteria to properly reconstruct the noise signal?

Circle one:	yes	no

(c) If we were to design a digital filter with the transfer function

$$H[z] = \frac{0.3z^2 - 0.2z + 0.3}{z^2 - 0.2z - 0.3}$$

how much would this filter attenuate the noise? Please give your answer in decibels. In other words, compute the system magnitude response at the interference frequency $|H(\Omega_i)|$ in decibels.

$\|H(\Omega_i)\| =$

(d) Draw a block diagram of the canonical block diagram realization of this system.

(e) What type of filter is this?

Circle one:	FIR	IIR

(f) If we have some input signal $x[n] = \{1, 4, -2\}$ for samples $n = 0, \ldots, 2$, what is the output $y[n]$ for $n = 0, \ldots, 2$? Assume zero-state for your solution.

$y[n] =$