ECE 3210 Midterm 2

Week of: November 2, 2021

Student's name: ______________________________

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You have 2 hours for 5 problems.

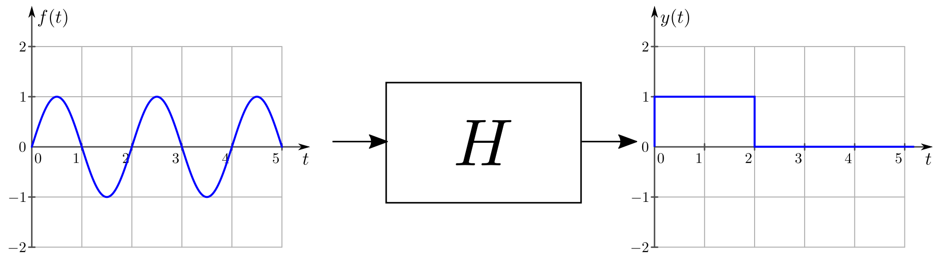
- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You may use two US letter-style size page of notes, front and back.

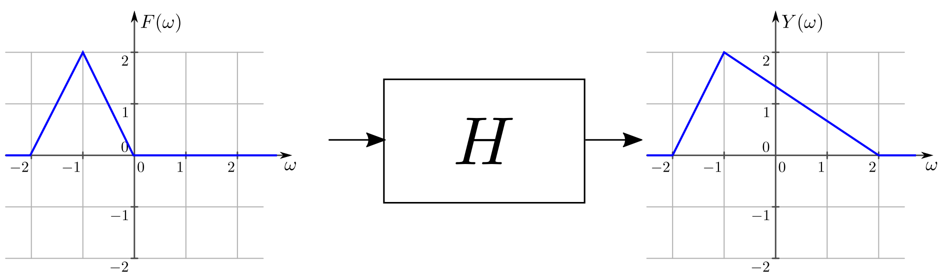
Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
Total score		100

1 Grab bag of short answer stuff

(a) Given the shown inputs and outputs, is the following system LTI?



(b) Given the shown inputs and outputs, is the following system LTI?



(c) Consider this implementation of the FFT algorithm. What simple change could we make to make it run faster (assuming we don't have access to Numba or something exotic like that...and that the `reverse_bits( )` is optimized already)? Just mark in the code what you would do and write a quick sentence what you are trying to accomplish. Don't get hung up on the Python/NumPy syntax, look at what the algorithm is doing.

```
def fft(x):
    reverse_bits(x)
    N = len(x)
    dft_pt = 2

    while N >= dft_pt:
        x = x.reshape((dft_pt,-1),order="F")

        Gr = x[:dft_pt//2,:]

        theta = 2*np.pi/dft_pt

        x_out = np.zeros_like(x).astype(complex)
        x_out[:dft_pt//2,:] = Gr + W[:,None]*x[dft_pt//2:,:]
        x_out[dft_pt//2:,:] = Gr - W[:,None]*x[dft_pt//2:,:]

        x = x_out.flatten(order="F")
        dft_pt <<= 1

    return x
```

(d) Consider bandlimited signals f_1 with a bandwidth of 75 Hz and f_2 with a bandwidth of 150 Hz.

(i) What is the Nyquist sampling rate for $f_1 + f_2$?

$f_s =$

(ii) What is the Nyquist sampling rate for $f_1 * f_2$?

$f_s =$

2 Differential equations

Consider the following differential equation

$$(D^2 + 4D + 4)y(t) = (3D^2 + 13D + 12)f(t)$$

with initial conditions $y(0^-) = 2$ and $y'(0^-) = -7$ as well as an input $f(t) = u(t)$.

(a) Find the zero-input response using the Laplace transform.

$y_{zi}(t) =$

(b) Find the zero-state response using the Laplace transform.

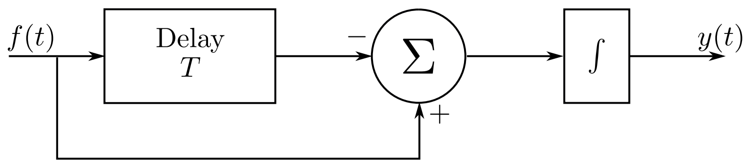
$y_{zs}(t) =$

(c) Find the total response.

$y(t) =$

3 Sampling

Consider a zero-order hold circuit, which is a practical way to reconstruct sampled signals.



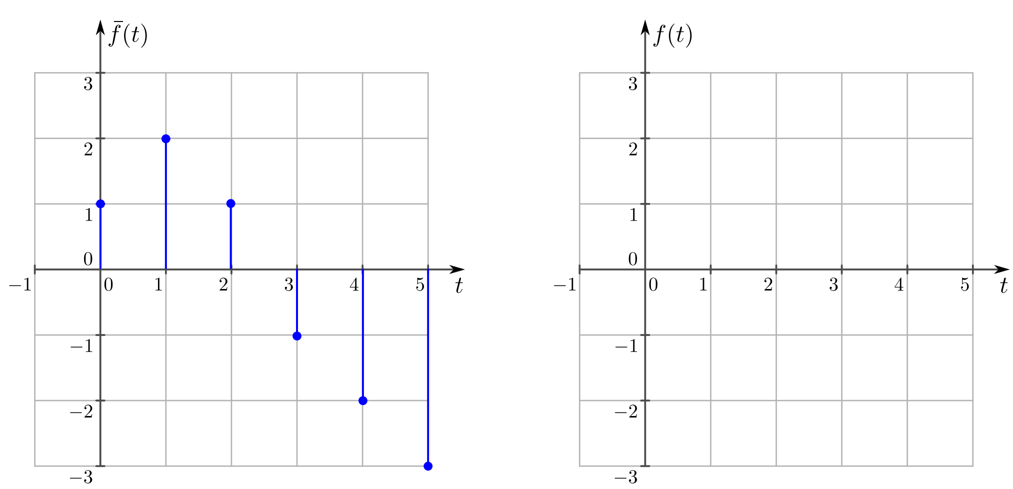
(a) Find the unit impulse response of this system. (*Hint:* the impulse response $h(t)$ is the output of the system when $f(t) = \delta(t)$, or maybe look at it in Laplace domain...lots of options here...)

$h(t) =$

(b) Find the frequency response $H(\omega)$.

$H(\omega) =$

(c) Given some sampled signal $\bar{f}(t)$, sketch the system output $y(t)$. Please sketch $f(t)$ on the plot marked $f(t)$.

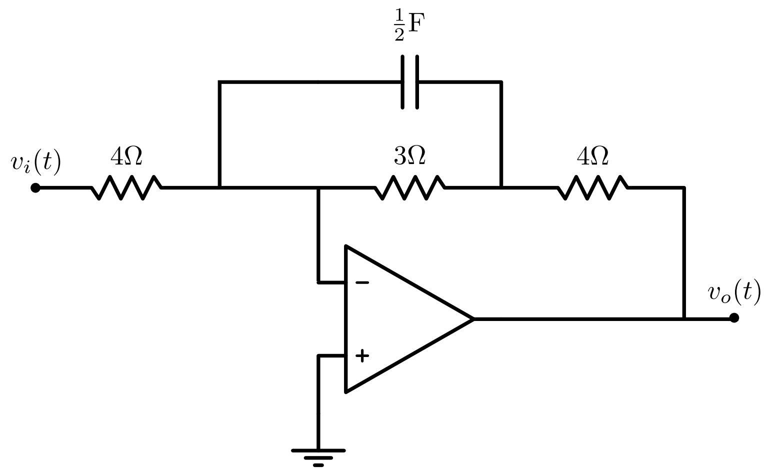


4 Transfer functions

(a) Draw the canonical block diagram representation of the transfer function

$$H(s) = \frac{s^2 - s}{s(s+1)^2}$$

(b) The following circuit is a simplified bass control circuit (i.e., what you might find in a subwoofer). Find the impulse response using Laplace techniques.



$h(t) =$

5 Discrete convolution

Consider the following discrete-time signals

$$f_k = \{-1, 0, 1, -1, 0, 1\}$$
$$g_k = \{1, 1, 1, 1, 1, 1\}.$$

(a) Given two discrete-time signals f_k and g_k above, please find the *linear* convolution $y_{k,\text{lin}} = f_k * g_k$.

$y_{k,\text{lin}} =$

(b) Find the circular convolution of f_k and g_k.

$y_{k,\mathrm{cir}} =$