ECE 3210 Midterm 2

Week of: November 8, 2024

Student's name: ______________________________

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You have 2.5 hours for 5 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You may use two US letter-style size page of notes, front and back.

Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
Total score		100

1 Differential equations

Consider the following differential equation

$$\frac{d^2y}{dt} + 6\frac{dy}{dt} + 34y(t) = 10\frac{df}{dt}$$

with initial conditions $y(0^-) = 0$ and $y'(0^-) = 5$ as well as an input $x(t) = \delta(t)$.

(a) Find the zero-input response using the Laplace transform.

$y_{zi}(t) =$

(b) Find the zero-state response using the Laplace transform.

$y_{zs}(t) =$

(c) Find the total response.

$y(t) =$

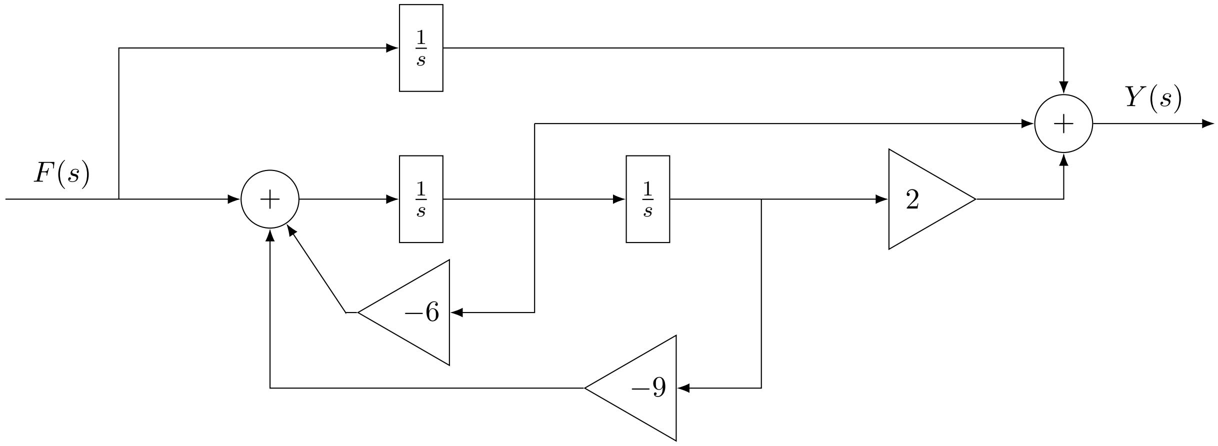
2 Block diagrams

(a) Consider the transfer function

$$H(s) = \frac{2s^2 - 1}{s^3 + 2s^2 + s}.$$

Draw the canonical block diagram for this system.

(b) Consider the block diagram below.



(i) What is the transfer function $H(s)$? *Hint:* see if you can identify the second order system in the block diagram rather than trying to describe it entirely in terms of first order systems.

$H(s) =$

(ii) What is its impulse response $h(t)$?

$h(t) =$

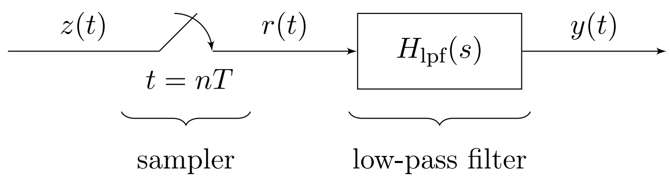
3 Demodulation with an ideal sampler

Multiplying a signal $f(t)$ with $\cos(\omega_c t)$ produces a modulated signal

$$z(t) = f(t)\cos(\omega_c t)$$

where ω_c is the carrier frequency. In class we learned that one way to demodulate this signal and recover $f(t)$ is to multiply $z(t)$ by $\cos(\omega_c t)$, and lowpass filter the result.

An alternative approach uses an ideal sampler. The block diagram of the receiver is where the ideal



sampler is drawn as a switch that closes instantaneously every T seconds to acquire a new sample.

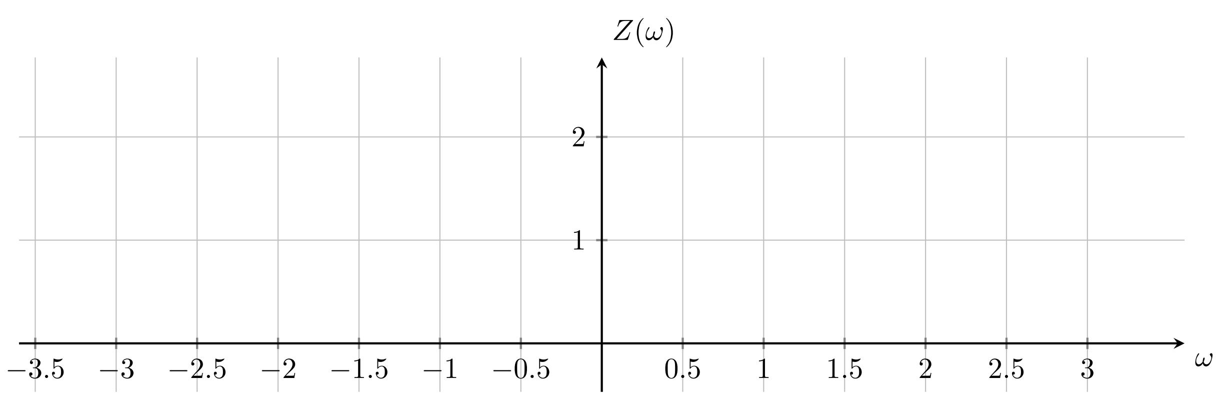
(a) Assume that

$$f(t) = \frac{1}{2\pi}\operatorname{sinc}^2\left(\frac{t}{4}\right).$$

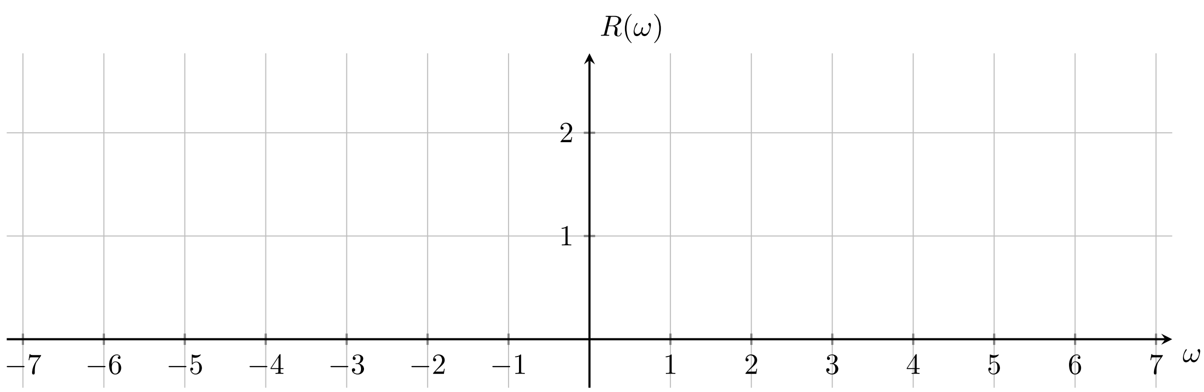
Find an expression for the Fourier transform $F(\omega)$.

$F(\omega) =$

(b) Assume that the carrier frequency of the modulating cosine is $\omega_c = 2$. Sketch the spectrum of the modulated signal $Z(\omega)$.



(c) Draw the spectrum of the signal right before the lowpass filter, $R(\omega)$. Assume we are sampling at a rate of $\frac{1}{\pi}$ Hz.

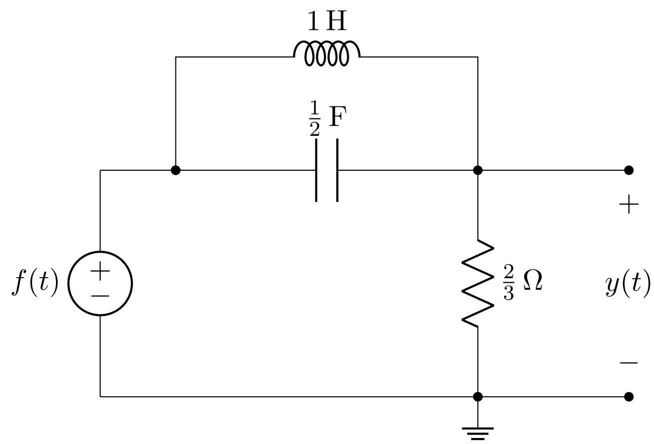


(d) What is the largest sampling period T at which the sampler can operate, and still recover $f(t)$?

$T =$

4 Transfer functions

Consider the following circuit. The input is $f(t)$ and the output is $y(t)$.



(a) Derive the transfer function $H(s)$ for this circuit.

$H(s) =$

(b) Find the impulse response $h(t)$ for this system.

$h(t) =$

(c) Given an input $x(t) = u(t)$, find the zero-state response for $y(t)$.

$y_{zsr}(t) =$

5 DFT and circular convolution

(a) Consider the following discrete-time signal

$$x[k] = \begin{cases} 0.5 + \cos(\pi k) + \sin\left(\frac{7\pi k}{4}\right) & \text{for } 0 \leq k \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

Find the 8-point DFT $X[r]$ for $0 \leq k \leq 8$.

$X_0 =$	$X_1 =$	$X_2 =$	$X_3 =$
$X_4 =$	$X_5 =$	$X_6 =$	$X_7 =$

(b) Consider the following discrete signals

$$f_k = \{2, -1, 3, 1, 1\}$$
$$g_k = \{1, -2, 1\}.$$

(i) Find the linear convolution of f_k and g_k.

$y_{k,\text{lin}} =$

(ii) Find the *5-point circular convolution* of f_k and g_k.

$y_{k,\text{cir}} =$