ECE 3210 Final Exam

Week of: December 12, 2024

Student's name: ______________________________

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You have 3 hours for 6 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You may use three US letter-style size page of notes, front and back.

Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
6		25
Total score		125

1 Short-ish answer

(a) The following system has a transfer function

$$H[z] = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}.$$

(i) Is this system stable?

Circle one:	stable	unstable

(ii) Is this system FIR or IIR?

Circle one:	FIR	IIR

(b) The following system has a transfer function

$$H[z] = \frac{z^3 + z}{z^2 - 2z + 1.25}.$$

(i) Is this system stable?

Circle one:	stable	unstable

(ii) Is this an FIR or IIR system?

Circle one:	FIR	IIR

(iii) Is this system causal?

Circle one:	causal	non-causal

(c) Consider the following signal

$$f[k] = \cos(5k).$$

Is this signal periodic? If so, what is the period of this signal?

Circle one:	Periodic	Aperiodic
Period =		

(d) Evaluate the following integral

$$\int_{-\infty}^{\infty} \operatorname{sinc}^2(t) dt.$$

$\int_{-\infty}^{\infty} \operatorname{sinc}^2(t) dt =$

2 Convolution

(a) Perform the following convolution

$$y(t) = (u(t-1) - u(t-3)) * t(u(t) - u(t-3))$$

$y(t) =$

(b) Perform the following convolution

$$y[k] = u[k] * k(u[k] - u[k-10]).$$

It might be helpful to remember that $\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$.

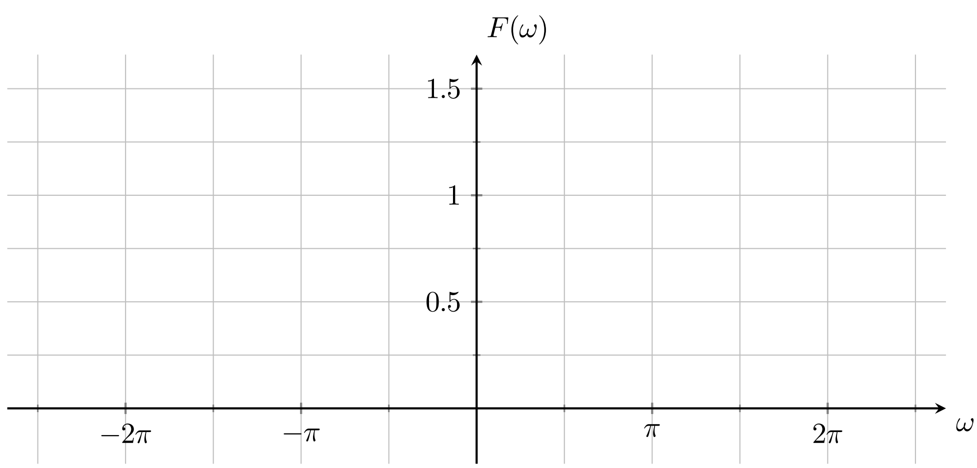
$y[k] =$

3 Sampling

(a) Suppose we have some time-domain signal

$$f(t) = \frac{1}{2}\operatorname{sinc}^2\left(\frac{\pi t}{2}\right)e^{j\pi t}$$

which is complex-valued. Please sketch the Fourier transform of this signal.



(b) If we were to define the bandwidth of the signal to be the highest frequency represented in this signal, what is the bandwidth B of this signal in hertz?

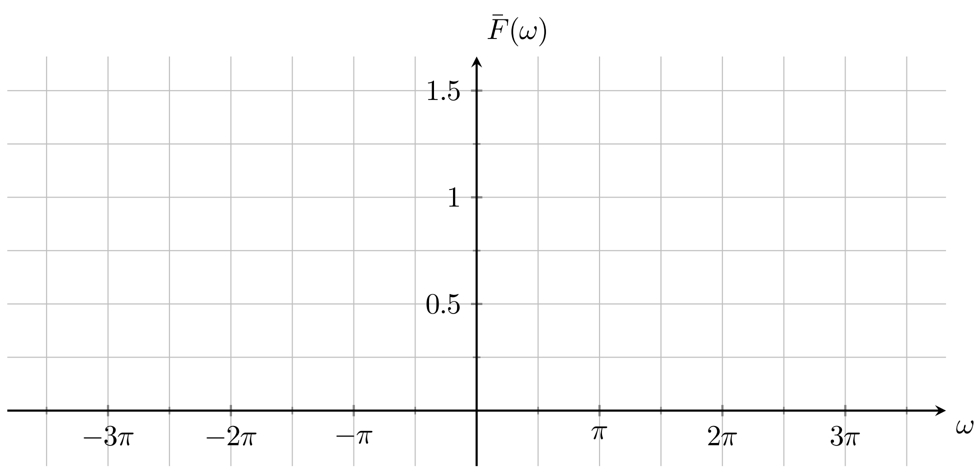
$B =$

(c) If we were to sample this signal $f(t)$, what is the longest period T we could use to sample this signal without aliasing? What is the corresponding sampling rate f_s in hertz?

Note: sampling a complex-valued is called "quadrature sampling" and is done by sampling the real and imaginary parts of the signal separately. The frequency domain behavior is the same as with real-valued signals, however. This is common in communications systems and MRI imaging.

$T =$ $\qquad\qquad$ $f_s =$

(d) Please sketch the Fourier transform of the sampled signal $\bar{f}(t)$.



(e) If we sample at this sampling rate, did we violate the Nyquist criterion? Why or why not?

4 DTFT

(a) Some signal $f[k]$ is non-zero only from $k=0,\ldots,3$. The DTFT of this signal ($F(\Omega)$) is given by its real and imaginary parts

$$\mathrm{Re}\{F(\Omega)\}=1+\cos(\Omega)+\cos(2\Omega)+\cos(3\Omega)$$
$$\mathrm{Im}\{F(\Omega)\}=-(\sin(\Omega)+\sin(2\Omega)+\sin(3\Omega)).$$

Find the 4-point DFT of $f[k]$.

$F_0=$	$F_1=$
$F_2=$	$F_3=$

(b) Consider the DTFT of a (not necessarily LTI) system that has an input/output relationship $y[k] = \mathcal{H}\{x[k]\}$. The DTFT of these signals are related by the equation

$$Y(\Omega) = 2X(\Omega) + e^{-j\Omega}X(\Omega) - \frac{dX(\Omega)}{d\Omega}.$$

(i) Is this system linear?

Circle one:	linear	non-linear

(ii) Is this system time-invariant?

Circle one:	time-invariant	non-time-invariant

(iii) What is $y[k]$ if $x[k] = \delta[k]$?

$y[k] =$

5 Difference equations

Consider the difference equation below

$$y[k+2]-1.6y[k+1]+0.64y[k]=f[k+1]+f[k]$$

with initial conditions $y[-2]=2$ and $y[-1]=1$ and input $f[k]=\cos(\pi k)u[k]$.

(a) Find the zero-input solution $y_{\text{zir}}[k]$.

$y_{\text{zir}}[k]=$

(b) Find the zero-state solution $y_{\text{zsr}}[k]$.

$y_{\text{zsr}}[k] =$

(c) Find the total solution $y[k]$.

$y[k] =$

6 Filter design

(a) We are interested in designed a Butterworth low-pass filter. We want the passband to go up to 7.5kHz with a maximum attenuation of −2dB and a stopband starting at 90kHz with a minimum attenuation of −40dB. Find the filter order and the cutoff frequency of the filter.

$n=$	$\omega_c=$

(b) The Butterworth polynomial is given by

$$B_n(s) = s^n + a_{n-1}s^{n-1} + ... + a_1 s + 1$$

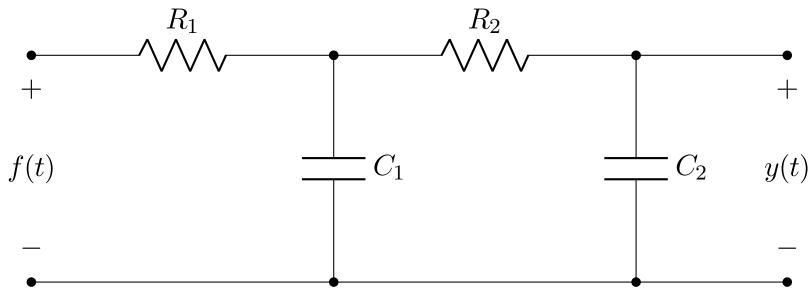
where the coefficients a_i are determined by the filter order n. A very abbreviated table of these coefficients is given below.

n	$B_n(s)$
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	s^3+2s^2+2s+1

Find the transfer function for a Butterworth filter that will satisfy the requirements given above.

$H(s)=$

(c) In lab we designed our low-pass filter using active circuits via the Sallen-Key topology. However, sometimes we are interested in implementing these filters using a passive circuit (i.e., no op-amps). One common topology is an RC-RC ladder circuit, which gives a transfer function that is similar to the Sallen-Key topology. Consider the RC-RC ladder circuit below, find the transfer function $H(s)$ for this circuit in terms of R_1, R_2, C_1, and C_2.



$H(s)=$

(d) Suppose we are limited in our resistor values to 10kΩ and 250kΩ and we fix the capacitor values with 0.1nF and 1.4nF. Select resistor and capacitor values that will give us the closest approximation to the Butterworth filter we designed above.

$R_1 =$	$R_2 =$
$C_1 =$	$C_2 =$

(e) Let's see how well this matches the given spec. Find the passband and stopband attenuation of the filter you designed (in dB) in previous part (using the RC-RC ladder circuit your selected component values).

$\lvert H(\omega_p)\rvert =$	$\lvert H(\omega_s)\rvert =$