

# ECE 3210 Midterm 1

*Week of: September 28, 2021*

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You have 2 hours for 5 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You may use one US letter-style size page of notes, front and back.

Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
<b>Total score</b>		100

**1 Short answer**

- (a) What is the verb form of “convolution”?

- (b) Given the following system differential equations or impulse response functions, determine if each system is stable, marginally stable, or unstable. Show work if needed.

(i)  $h(t) = tu(t)$

(ii)  $(D^2 + 9)y(t) = (D + 3)f(t)$

(iii)  $(D^3 + 2D^2 + D)y(t) = f(t)$

(i)

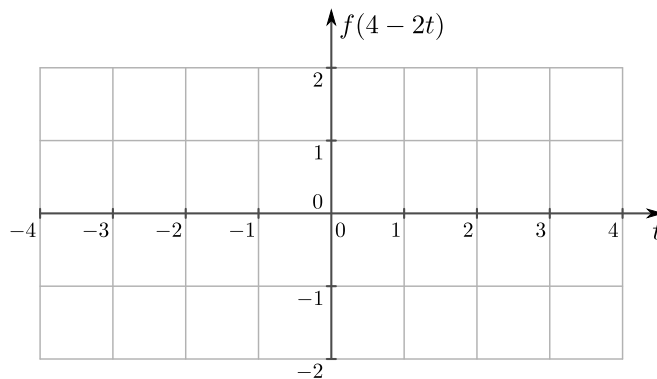
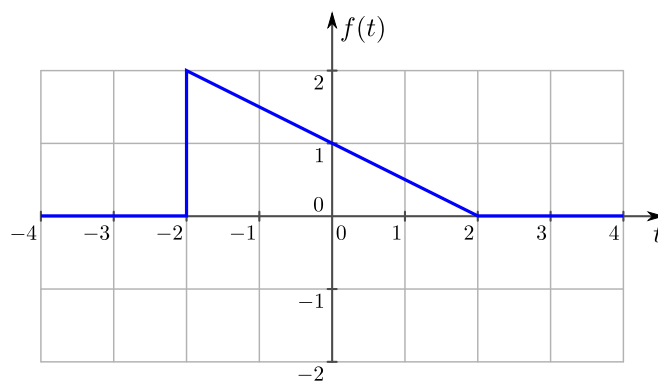
(ii)

(iii)

- (c) True or False: are the functions  $f_1(t) = t \cos(t)$  and  $f_2(t) = t \sin(t)$  orthogonal on the interval  $t \in [0, 2\pi]$ ?

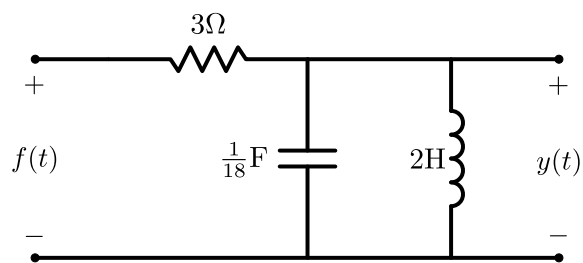
True or False:

- (d) Given  $f(t)$  below, plot  $f(2t + 2)$ .



## 2 Circuit analysis

Using the circuit below, please answer the following questions.



- (a) Derive a differential equation relating the input  $f(t)$  and the output  $y(t)$ . In your final answer, don't worry about including the units, just write the ODE in the form we have seen in class.

- (b) Determine the zero-input response (we'll call it  $y_2(t)$ ) for this system. Assume  $y'(0) = 1$  and  $y(0) = 0$ .

$y_2(t) =$

- (c) Determine the impulse response  $h(t)$  for this system.

$h(t) =$

### 3 Convolution

Using direct integration or graphical (i.e., “flip-and-drag”) methods, solve for  $y(t)$  by performing the following convolutions.

(a)  $y(t) = \sin(t)(u(t) - u(t - 2\pi)) * u(t)$

$y(t) =$

(b)  $y(t) = u(t) * t(u(t+1) - u(t-2))$

$y(t) =$



#### 4 Fourier series

(a) Consider the function

$$f(t) = 0.5 + 5 \sin(0.75t) + 2 \cos(t) + 7 \cos(-0.5t) + \sin(-0.25t).$$

Determine if this function is periodic. If it is, find the first five terms of the trigonometric Fourier series. (*Hint:* if you are using integrals here, you are doing it wrong...)

$$a_0 =$$

$$a_1 =$$

$$a_2 =$$

$$a_3 =$$

$$a_4 =$$

$$a_5 =$$

$$b_1 =$$

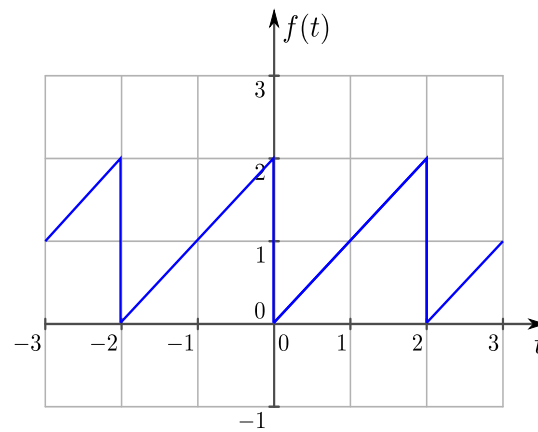
$$b_2 =$$

$$b_3 =$$

$$b_4 =$$

$$b_5 =$$

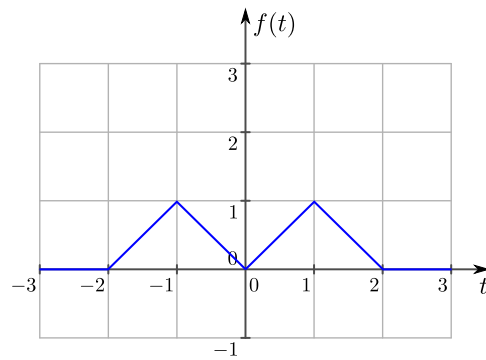
- (b) Derive the *complex exponential* Fourier series for  $f(t)$  which is periodic and is shown below.



$f(t) =$

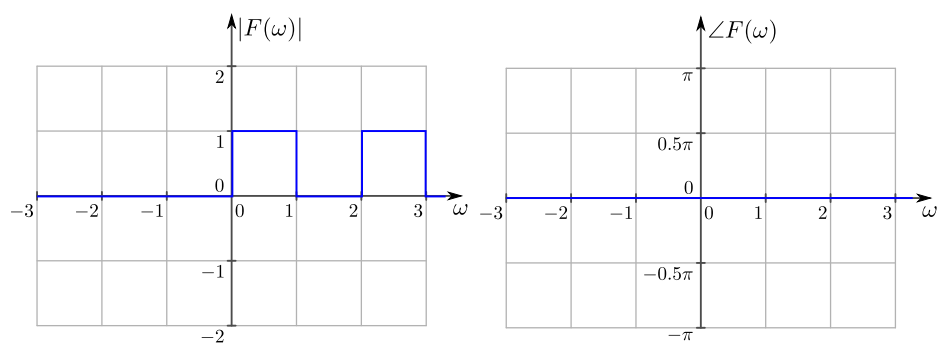
**5 Fourier transforms**

- (a) Find the Fourier transform for  $f(t)$  which is defined in the plot below.



$$F(\omega) =$$

- (b) Find the inverse transform  $f(t)$  if  $|F(\omega)|$  and  $\angle F(\omega)$  are given below. Note: the phase  $\angle F(\omega)$



extends linearly for all  $\omega$ .

$f(t) =$