ECE 5210 Midterm 2

Week of: March 28, 2024

Student's name: ______________________________

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You have 2 hours for 5 problems.

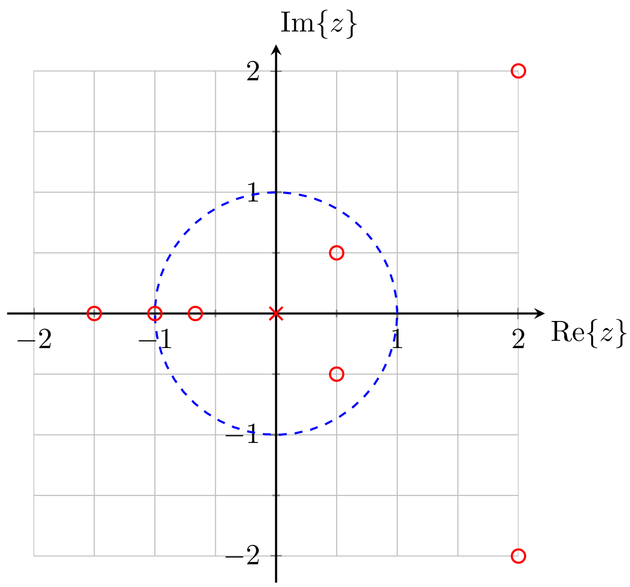
- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You are allowed TWO pages of notes (front and back) for this exam. You may use a graphing calculator of your choice. Consulting with any third party is considered cheating.

Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
Total score		100

1 Short answer

(a) Consider the pole-zero plot for some system $H(z)$. Note that the pole at the origin is repeated seven times.



(i) Is the system stable?

yes	no

(ii) Does the system have minimum phase?

yes	no

(iii) Does the system have linear phase? If so, what type of linear phase filter is this?

Type I	Type II	Type III	Type IV	Not linear phase

(iv) Is the system causal?

yes	no

(b) The signal

$$x_c(t) = \cos(2\pi(600)t)$$

was sampled with a sampling period $T = 1/300\,\mathrm{s}$ to obtain a discrete-time signal $x[n]$. What is the resulting sequence $x[n]$?

$x[n] =$

(c) Suppose we had a continuous signal $x_c(t)$ and wanted to filter it with some impulse response

$$h(t) = 2e^{-t/3}\cos(12\pi t)u(t).$$

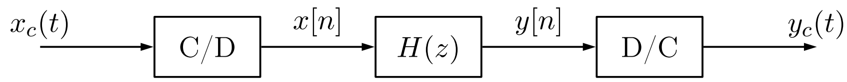
(i) If you implemented this on a discrete-time system using a sampling rate $T = 0.1$, what would the discrete-time impulse response $h[n]$ be?

$h[n] =$

(i) What is the issue with implementing this system using this sampling rate?

2 Sampling and reconstruction

Consider the signal processing system below, where the C/D and D/C converters are ideal. Let the input $x_c(t) = 2\cos(30\pi t)$ and the sampling rate be $f_s = 40\,\text{Hz}$.

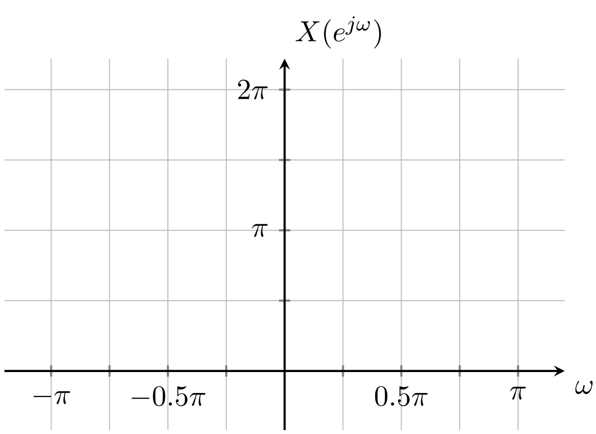


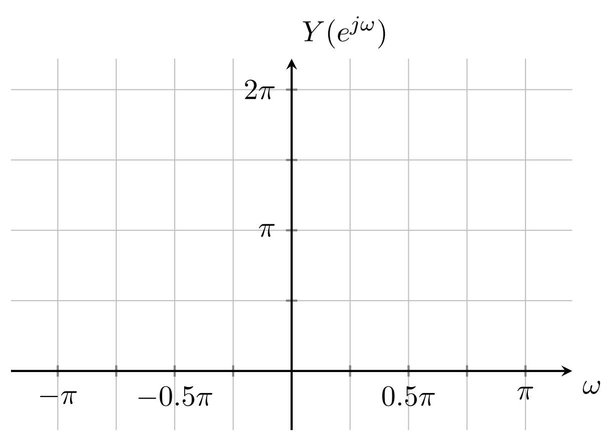
The system $H(z)$ is described by the input/output relationship $y[n] = x^2[n]$.

(a) Is the subsystem (and only the subsystem) $H(z)$ linear and/or time-invariant?

Circle one:	linear	non-linear
Circle one:	time-invariant	time-variant

(b) Sketch $X(e^{j\omega})$ and $Y(e^{j\omega})$ from $\omega = -\pi$ to π.





(c) What is the output $y_c(t)$?

$y_c(t) =$

(d) For which frequencies of Ω of $x_c(t) = 2\cos(\Omega t)$ will $y_c(t) = x_c^2(t)$?

$\Omega :$

3 Minimum phase and all-pass systems

Consider a FIR system with one zero at $z = 0.5$, one zero at $z = 0.8e^{j\frac{\pi}{5}}$, and all associated zeros needed for a Type I linear phase filter.

(a) What is the transfer function $H(z)$ of the overall system? You do not need to simplify the transfer function beyond second-order stages.

$H(z) =$

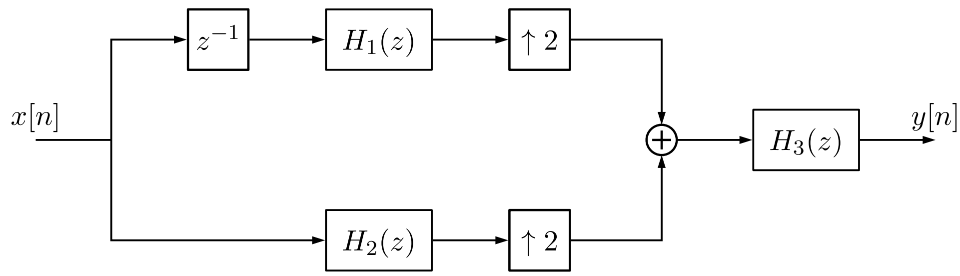
(b) Decompose this system to a minimum phase system $H_{\text{min}}(z)$ and an all-pass system $H_{\text{ap}}(z)$.

$H_{\text{min}}(z) =$

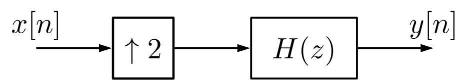
$H_{\text{ap}}(z) =$

4 Interpolation

Consider the system below.



This system can also be implemented as a cascade as an expander followed by a simple FIR filter $H(z)$. This simplified system is shown below.



The filter $H(z)$ is given as

$$H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}.$$

(a) Please find $H(z)$ in terms of H_1, H_2, H_3.

$H(z) =$

(b) The filters $H_1(z)$, $H_2(z)$, and $H_3(z)$ each have two taps. Thus, we can represent

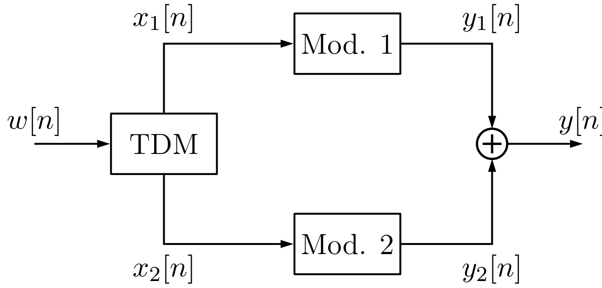
$$\begin{aligned} H_1(z) &= h_1[0] + h_1[1]z^{-1} \\ H_2(z) &= h_2[0] + h_2[1]z^{-1} \\ H_3(z) &= h_3[0] + h_3[1]z^{-1}. \end{aligned}$$

Find the values $h_1[0]$, $h_1[1]$, $h_2[0]$, $h_2[1]$, $h_3[0]$, and $h_3[1]$. There are multiple correct solutions. I will check your answer in a Python script to verify it works.

$h_1[0] =$	$h_1[1] =$
$h_2[0] =$	$h_2[1] =$
$h_3[0] =$	$h_3[1] =$

5 Time-division multiplexing

Communication systems often require conversion from time-division multiplexing (TDM) to frequency-division multiplexing (FDM). In this problem, we examine a simple example of such a system. The full block diagram of the system is given below.



(a) The TDM input, $w[n]$, is the sequence of interleaved samples of two signals, $x_1[n]$ and $x_2[n]$, such that

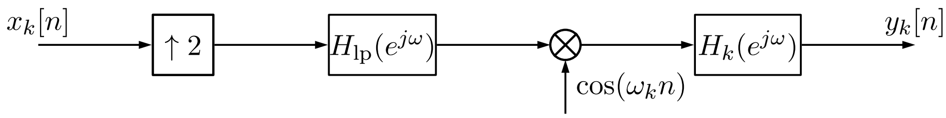
$$w[n] = \begin{cases} x_1[n/2] & \text{if } n \text{ is even} \\ x_2[(n-1)/2] & \text{if } n \text{ is odd.} \end{cases}$$

Design and draw a block diagram of a TDM demultiplexer that separates $w[n]$ into $x_1[n]$ and $x_2[n]$. You may use only the following blocks:

- delay
- decimator by a factor of M ($\downarrow M$, if used specify M)
- expander by a factor of L ($\uparrow L$, if used specify L)
- ideal low-pass filter with cutoff frequency ω_c, if used specify ω_c

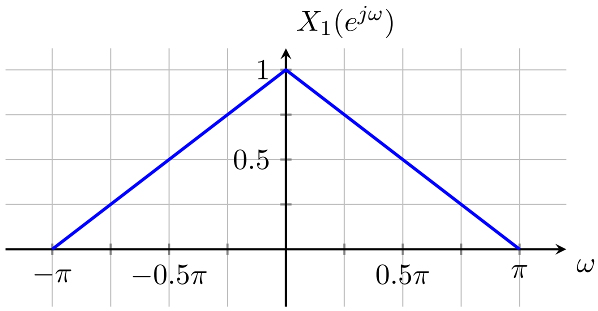
Your work continued. . .

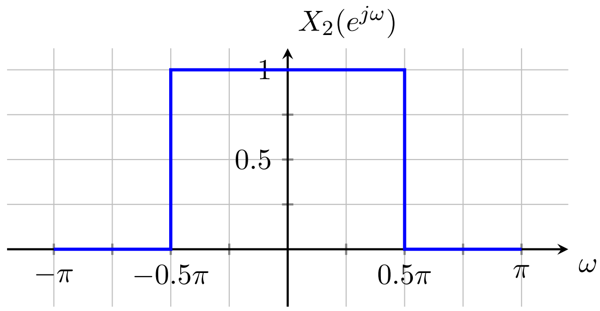
(b) Each modulator system ($k = 1$ or 2) is given by the block diagram below.



All filters are ideal. The low-pass filter $H_{\text{lpf}}(e^{j\omega})$ is the same for both channels and has a gain of 2 and a cutoff freuqency $\omega_c = \pi/2$. The high pass filters $H_k(e^{j\omega})$ have unity gain and cutoff frequencies $\omega_1 = \pi/4$ and $\omega_2 = 3\pi/4$ for $k = 1$ and $k = 2$, respectively.

The DTFT of $x_1[n]$ and $x_2[n]$ are plotted from $\omega \in [-\pi, \pi]$. Sketch the DTFT of the full system $y[n]$. Sketch the DTFT at each point in the modulator system for partial credit.





Your work continued...