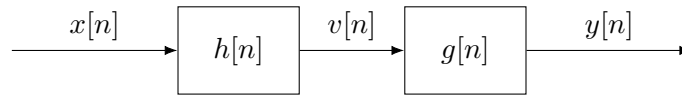


ECE 5210 quiz04

Name: _____ SOLUTIONS

A stable and causal system is shown below, consisting of a cascade of two LTI discrete-time filters such that the overall system response will yield $y[n] = x[n]$. The first filter has an unknown impulse response $h[n]$. The second filter is defined



by the difference equation

$$y[n] = v[n] - \frac{3}{4}v[n-1] + \frac{1}{8}v[n-2].$$

a) Find the transfer function $G(z)$.

Solution: Taking the Z-transform of the difference equation, we have

$$Y(z) = V(z) - \frac{3}{4}z^{-1}V(z) + \frac{1}{8}z^{-2}V(z).$$

We see that

$$G(z) = \frac{Y(z)}{V(z)} = 1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}$$

b) Find the impulse response $g[n]$.

Solution: If we know that $G(z) = 1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}$, we can simply take the inverse Z-transform to find $g[n]$

$$g[n] = \delta[n] - \frac{3}{4}\delta[n-1] + \frac{1}{8}\delta[n-2].$$

c) Find the transfer function $H(z)$.

Solution: Since the overall system is such that $y[n] = x[n]$, we have that the overall transfer function is

$$H(z)G(z) = 1 \implies H(z) = \frac{1}{G(z)} = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

d) Find the impulse response $h[n]$.

Solution: We can factor the denominator of $H(z)$ as

$$H(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})}.$$

Using partial fraction expansion, we have

$$H(z) = \frac{2}{1 - \frac{1}{2}z^{-1}} - \frac{1}{1 - \frac{1}{4}z^{-1}}.$$

Taking the inverse Z-transform, we find

$$h[n] = 2\left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n].$$