ECE 5210 hw03

1. DTFT

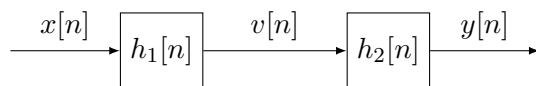a) Suppose you were to implement a three sample moving average filter. The impulse response is

$$h[n] = \frac{1}{3}(\delta[n] + \delta[n-1] + \delta[n-2])$$

What is the frequency response of this filter?

b) Suppose we were to send a signal $x[n]$ into this filter that is some cosine $x[n] = \cos\left(\frac{2\pi}{3}n\right)$. What is the output $y[n]$?

2.   System response of cascade of LTI systems

Consider the system below with input $x[n]$ and output $y[n]$.

$$x[n] \rightarrow \boxed{h_1[n]} \xrightarrow{v[n]} \boxed{h_2[n]} \xrightarrow{y[n]}$$

Subsystems $h_1[n]$ and $h_2[n]$ are known to be LTI. The output of $h_1[n]$ is $v[n]$ and can be described by the difference equation

$$v[n] = x[n] - x[n-1].$$

Additionally, subsystem $h_2[n]$ is described by

$$h_2[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n}.$$
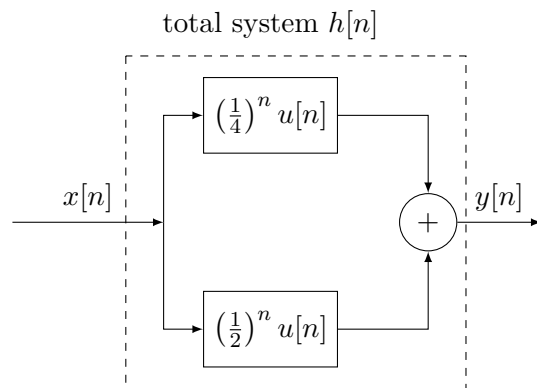
a) Find the impulse response $h[n]$.

b) Consider some input

$$x[n] = \cos(0.4\pi n) + \sin(0.6\pi n) + u[n].$$

What is the output $y[n]$? Note: The cosine and sine terms are everlasting in that they are not multiplied by $u[n]$.

3. Parallel system

Consider the system below.

total system $h[n]$

a) Find the impulse response for the entire system $h[n]$.

b) Find the DTFT of the entire system $H(e^{j\omega})$.

c) Find the difference equation for this system.

d) Suppose we were to send an input signal $x[n] = \cos(n)$ into this system. What is the output $y[n]$?

4. Deconvolution

Oftentimes we know some signal $y[n]$, which is the output of some LTI system H. We might even be lucky enough to know the system's impulse function $h[n]$. However, we are interested in the input signal $x[n]$, which is unknown. This process is called *deconvolution* and is very common in image processing (e.g., removing camera shake blurring from photos).

Suppose we have a system with an impulse function

$$h[n] = 3\delta[n] + 2\delta[n-1] + \delta[n-2]$$

and we have some output

$$y[n] = 3\delta[n-1] - \delta[n-2] + 5\delta[n-3] + 3\delta[n-4] + 2\delta[n-5].$$

Please determine the input $x[n]$.

5. Inverse DTFT

An LTI discrete-time system has a frequency response given by

$$H(e^{j\omega}) = \frac{(1 - je^{-j\omega})(1 + je^{-j\omega})}{1 - 0.8e^{-j\omega}}$$

a) Find the impulse response $h[n]$ of the system.

b) From the frequency response, determine the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$.

c) If the input to this system is

$$x[n] = 4 + 2\cos(\omega_0 n)$$

for $-\infty < n < \infty$, for what value of ω_0 will the output be of the form $y[n] = A$ where A is a constant for all $-\infty < n < \infty$? What is the constant A? Do not use ω_0 as multiples of 2π.

6. z-Transform

a) Determine the unilateral z-transform for: $\delta[n-1]$. Find the corresponding ROC.

b) Determine the unilateral z-transform for: $\left(\frac{1}{2}\right)^{n-1} u[n-1]$. Find the corresponding ROC.