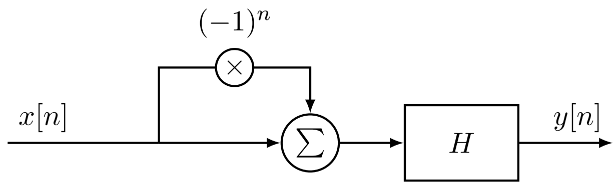
ECE 5210 quiz02

Name: SOLUTIONS

Consider the discrete-time system below where the subsystem H is an ideal low-pass filter with a passband gain of 1 and a cutoff frequency of $\omega_c = \frac{\pi}{4}$. Recall that $\frac{\sin(\omega_c n)}{\pi n} \Leftrightarrow \text{rect}\left(\frac{\omega}{2\omega_c}\right)$ and $\frac{\omega_c}{2\pi}\text{sinc}^2\left(\frac{\omega_c n}{2}\right) \Leftrightarrow \Delta\left(\frac{\omega}{2\omega_c}\right)$ where we define $\text{sinc}(x) = \sin(x)/x$.



a) If we have an input $x[n]$ that has a DTFT of

$$X(e^{j\omega}) = 2\Delta\left(\frac{\omega}{2\pi}\right)$$

from $-\pi < \omega \leq \pi$.

Solution: The input spectrum is flat with amplitude 2 over the entire frequency range (since $\Delta(\omega/2\pi)$ is 1 for $|\omega| < \pi$). The filter $H(e^{j\omega})$ is an ideal LPF with cutoff $\pi/4$.

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \begin{cases} 2 & |\omega| \leq \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\omega| \leq \pi \end{cases}$$

b) Find $y[n]$ in the time domain.

Solution: Using the inverse DTFT or recognizing the sinc function form:

$$y[n] = \frac{1}{\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi/4}^{\pi/4} = \frac{1}{\pi n j}(e^{j\pi n/4} - e^{-j\pi n/4}) = \frac{2}{\pi n}\sin(\frac{\pi n}{4})$$

For $n = 0$, using L'Hopital's rule or the integral directly:

$$y[0] = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} 2d\omega = \frac{1}{2\pi}(2)(\frac{\pi}{2}) = \frac{1}{2}$$

Using this, we can write the complete expression:

$$y[n] = \frac{2\sin(\frac{\pi n}{4})}{\pi n}$$

with $y[0] = \frac{1}{2}$. Or we could write it as as sinc function:

$$y[n] = \frac{1}{2}\text{sinc}\left(\frac{\pi n}{4}\right)$$