ECE 5210 hw02

1. Convolution

Compute the following convolutions.

a) Compute the convolution $y[n] = n(u[n-1] - u[n-3]) * u[n-1]$ for $n = -2, \ldots, 5$.

b) Compute the convolution $y[n] = (u[n-1] - u[n-4]) * u[n+1]$ for $n = -2, \ldots, 5$.

c) Compute the convolution $y[n] = \sin\left(\frac{\pi n}{2}\right) u[n] * u[n+2]$ for $n = -2, \ldots, 5$.

d) Compute the convolution $y[n] = (u[n-1] - u[n-5]) * 0.5^n(u[n] - u[n-8])$. Express your answer as a piecewise function $y[n]$.

2. Difference Equation

An engineer is asked to evaluate a simple signal processing system with a single digital filter. The input $x[n]$ is obtained a continuous-time signal at a sampling rate of $1/T$. The goal for $H(e^{j\omega})$ is to be a linear-phase FIR filter, and ideally it should have the following amplitude response such that it acts as a bandlimited differentiator

$$|H_{id}(e^{j\omega})| = \begin{cases} -\omega/T & \omega < 0 \\ \omega/T & \omega > 0. \end{cases}$$

For one implementation of $H(e^{j\omega})$, referred to as $H_1(e^{j\omega})$, the designer, motivated by the definition

$$\frac{d[x(t)]}{dt} = \lim_{\Delta t \to 0} \frac{x(t) - x(t - \Delta t)}{\Delta t},$$

chooses the system impulse response $h_1[n]$ so the input-output relationship is

$$y[n] = \frac{x[n] - x[n-1]}{T}.$$

a) Find $H_1(e^{j\omega})$.

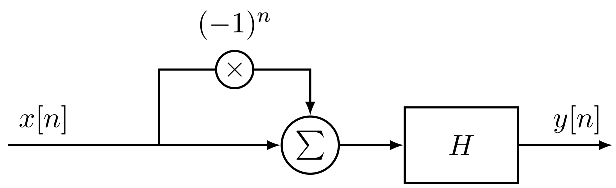
b) We are interested in how well this approximation matches against the ideal response $H_{id}(e^{j\omega})$. Find the difference in the squared magnitudes between the two responses, i.e.,

$$\text{error } = |H_1(e^{j\omega})|^2 - |H_{id}(e^{j\omega})|^2.$$

Represent your answer as 6th order polynomial function of ω.

3. DTFT Systems

Consider the discrete-time system below where the subsystem H is an ideal low-pass filter with a passband gain of 1 and a cutoff frequency of $\omega_c = \frac{\pi}{4}$.



If we have an input $x[n]$ that has a DTFT of

$$X(e^{j\omega}) = 2\Delta\left(\frac{\omega}{2\pi}\right)$$

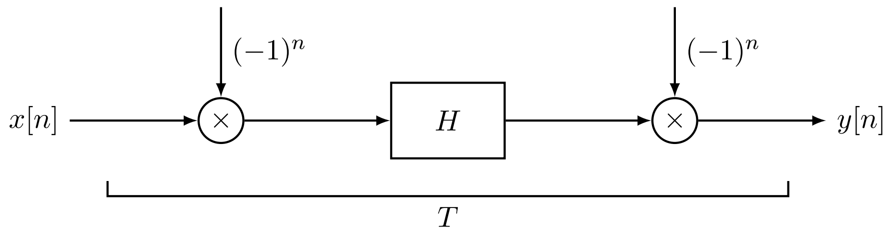
from $-\pi < \omega \leq \pi$.

a) Find an expression for the output $Y(e^{j\omega})$ for $-\pi < \omega \leq \pi$.

b) Find $y[n]$ in the time domain.

4. More DTFT Systems

Consider the system below. The entire system $y[n] = T\{x[n]\}$ will have the effect of being an ideal high-pass filter with a cutoff frequency of $\omega_c = \frac{\pi}{2}$. H is a subsystem within the larger system T.



a) Derive an expression for $H(e^{j\omega})$ such that the entire system will behave as an ideal high-pass filter as described above.

b) Find $h[n]$ that would satisfy such a system.

5. DTFT Pairs

The DTFT pair

$$a^n u[n] \Longleftrightarrow \frac{1}{1-ae^{-j\omega}} \qquad |a|<1$$

is given.

a) Determine the DTFT, $X(e^{j\omega})$, of the sequence

$$x[n] = -b^n u[-n-1] = \begin{cases} -b^n & n \le -1 \\ 0 & n \ge 0 \end{cases}.$$

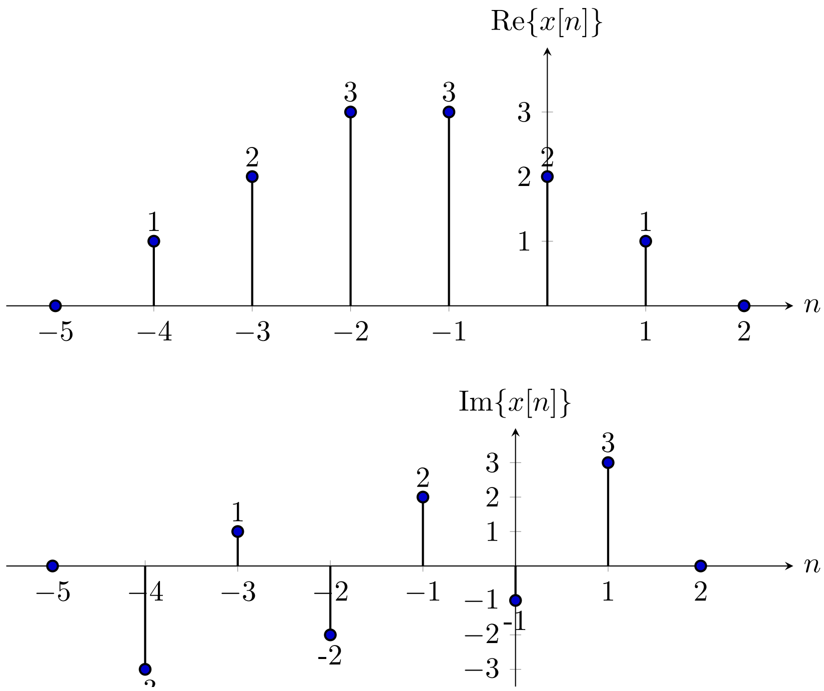
b) What restrictions must you put on b to make this a valid DTFT?

c) Determine the sequence $y[n]$ whose DTFT is

$$Y(e^{j\omega}) = \frac{2e^{-j\omega}}{1+2e^{-j\omega}}.$$

6. DTFT Values

$X(e^{j\omega})$ denotes the Fourier transform of the complex-valued signal $x[n]$, where the real and imaginary parts of $x[n]$ are given below. (The sequence is zero outside the interval shown.)



a) Evaluate $X(e^{j\omega})|_{\omega=0}$.

b) Evaluate $X(e^{j\omega})|_{\omega=\pi}$.

c) Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$.

7. Determining $h[n]$

Consider some input signal $x[n] = -\delta[n] + \delta[n-1]$ which passes through some causal system H which gives some output signal

$$y[n] = \delta[n] + \delta[n-1] - \delta[n-2] - \delta[n-3].$$

a) Find the impulse response $h[n]$ for this LTI system.

b) Find $y_2[n]$, which is the system response to the signal $x_2[n] = \delta[n] - \delta[n-5]$.