ECE 5210 hw04

1. Sampling

a) If we have a continuous time domain signal

$$x_c(t) = \cos(40\pi t) + \sin(120\pi t)$$

and is sampled with a sampling period T to obtain a discrete-time signal

$$x[n] = \cos\left(\frac{2\pi n}{5}\right) - \sin\left(\frac{4\pi n}{5}\right).$$

What is T?

b) What is another value of T that will satisfy this C/D conversion?

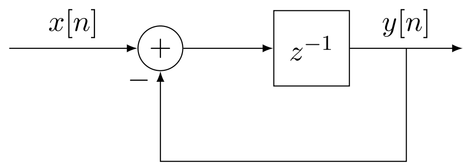
c) If we were to sample $x_c(t)$ with $T = 12.5$ ms it would result in a single sinusoid in the form

$$x[n] = A\cos(\omega n + \phi).$$

What is the amplitude A, the frequency ω, and the phase ϕ?

2. Delay System

Consider a system H, which is depicted in the figure below.



a) Determine the transfer function of the system $H(z)$.

b) Determine the impulse function of the system $h[n]$.

c) Given some input $x[n] = u[n]$, determine the z-transform of the output $Y(z)$.

d) Find the time-domain output $y[n]$.

3. Difference Equation

Consider a discrete LTI system which is described by the difference equation

$$y[n+2] - 5y[n+1] + 6y[n] = x[n+1]$$

a) Determine the transfer function $H(z)$.

b) Determine the z-transform of $x[n]$ if $x[n] = u[n]$.

c) Find $Y(z)$.

d) Find $y[n]$.

4. Inverse z-transform

Part A

Consider $X(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$.

a) Suppose $x[n]$ is causal. Find $x[n]$.

b) Suppose $x[n]$ is anti-causal. Find $x[n]$.

Part B

Consider $X(z) = \frac{5z^{-1}}{(1-2z^{-1})(3-z^{-1})}$.

a) Suppose $x[n]$ is causal. Find $x[n]$.

b) Suppose $x[n]$ is anti-causal. Find $x[n]$.

c) Suppose $x[n]$ is neither causal nor anti-causal. Find $x[n]$.

5. Notch Filtering

A significant problem in the recording of electrocardiograms (ECGs) is the appearance of unwanted 60 Hz interference in the output. The causes of this power line interference include magnetic induction, displacement currents in the leads on the patient's body, and equipment interconnections. Ultimately, we will want to get rid of this 60 Hz interference. Because this signal is analog, we could use an analog notch filter to remove it, but analog notch filters are a hassle to design. Instead, we are utilizing the C/D, DSP, and D/C signal processing approach discussed in class.

a) Assume that the bandwidth of the signal of interest is 240 Hz, that is (in radians per second),

$$X_a(j\Omega) = 0 \qquad |\Omega| > 2\pi \cdot 240\,\text{rad/s}$$

What should the minimum sampling period T_s be to satisfy Nyquist?

b) The analog signal is converted into a discrete-time signal with an ideal C/D converter operating at the sampling frequency Ω_s based on the sampling period T_s in the previous part. The resulting signal $x[n] = x_a(nT_s)$ is then processed with a discrete-time system H that is described by the difference equation

$$y[n] = H\{x[n]\} = x[n] + ax[n-1] + bx[n-2]$$

where a and b are some constants. Please find the frequency response $H(e^{j\omega})$ in terms of a and b.

c) Solve for $h[n]$. Please solve for a and b such that the 60 Hz frequency is notched. We can assume that this unwanted interference can take the form

$$w_a(t) = A\sin(120\pi t)$$

in continuous time. If $h[n]$ is designed correctly, $w_a(t)$ should not appear in the output of the D/C converter.