ECE 5210 Final

Week of: April 22, 2025

Instructor:
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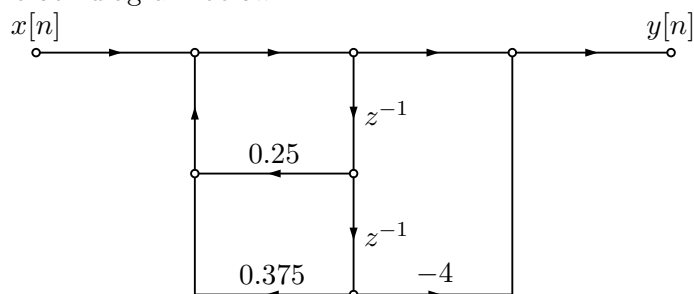801-626-6861

You have 3 hours for 6 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.

- Put your final answers in the boxes at the bottom of the page.

You are allowed THREE pages of notes (front and back) for this exam. You may use a graphing calculator of your choice. Consulting with any third party is considered cheating.

Problem	Score	Possible Points
1		20
2		25
3		20
4		20
5		20
6		20
Total score		125

1   System decomposition
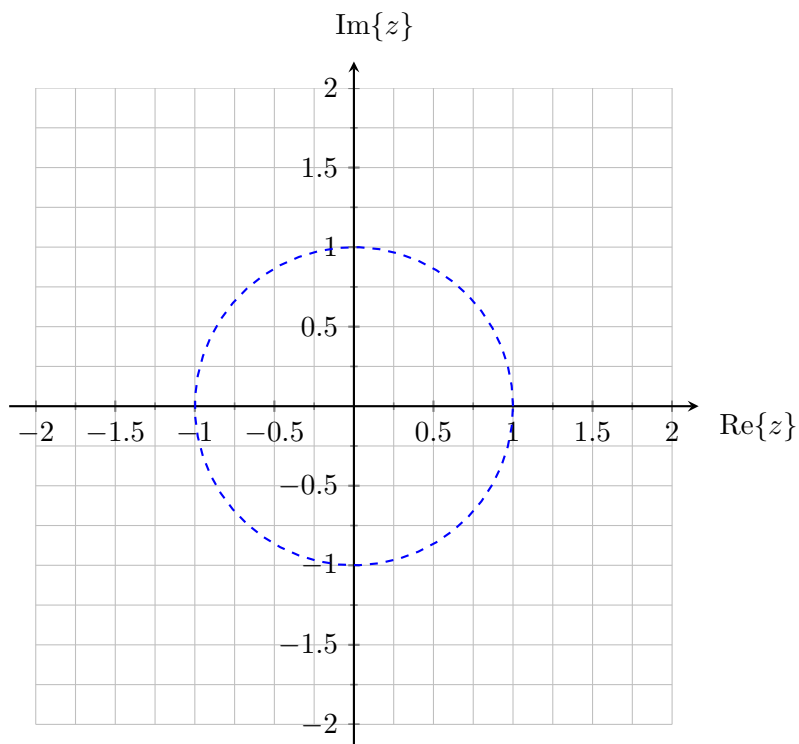
(a) Consider the block diagram below.

Find the transfer function $H(z)$ of the system. Assuming the system is causal, please denote the region of convergence (ROC) of the system.

$$H(z) = \qquad\qquad\qquad\qquad \mathrm{ROC} =$$

(b) Plot the poles and zeros of the system on the z-plane. Indicate the ROC of the system.
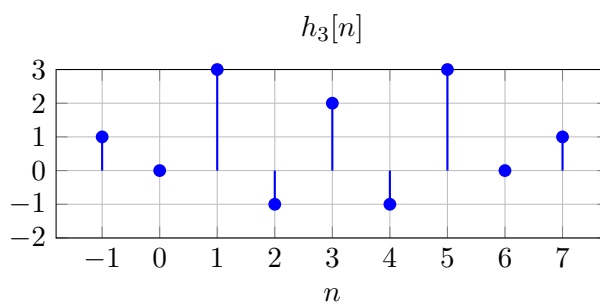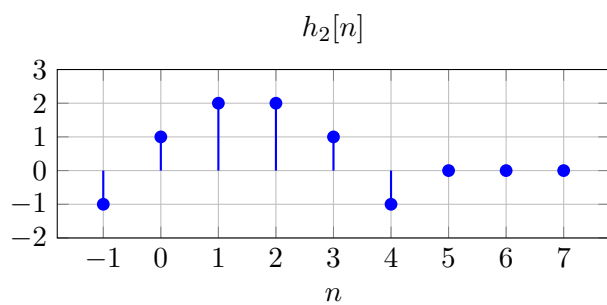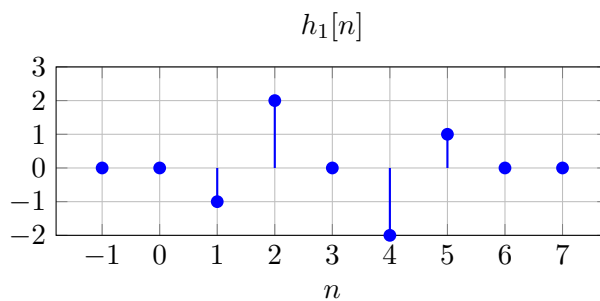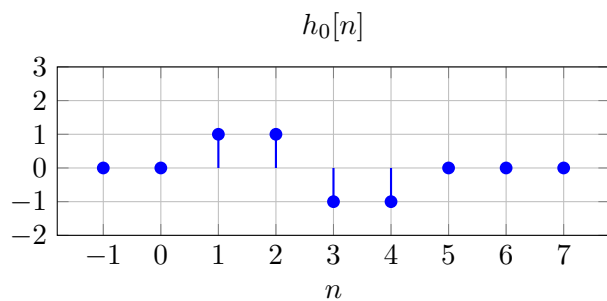
(c) Determine the minimum phase $H_{\min}(z)$ and the all-pass $H_{\mathrm{ap}}(z)$ components of the system.

$H_{\min}(z) =$ $H_{\mathrm{ap}}(z) =$

2 FIR filters

Below are the impulse response of four FIR filters. Complete the table below by filling in the missing information.

$h_0[n]$

$h_1[n]$

$h_2[n]$

$h_3[n]$

Please fill out the following table. ("GLP Type" refers to the type of generalized linear phase filter: "Type I", "Type II", "Type III", or "Type IV". If the filter is not a generalized linear phase filter, please write "N/A" in the table. The filter is causal if $h[n]$ is non-zero for $n \geq 0$ and anti-causal if $h[n]$ is non-zero for $n < 0$. The values of $H(e^{j\omega})$ at $\omega = 0$ and $\omega = \pi$ are the values of the frequency response at those frequencies.)

Filter	GLP Type	M	Causal?	$H(e^{j\omega})\vert_{\omega=0}$	$H(e^{j\omega})\vert_{\omega=\pi}$
$h_0[n]$					
$h_1[n]$					
$h_2[n]$					
$h_3[n]$					

3 Window-based FIR filter design

(a) Consider the following ideal lowpass filter with cutoff frequency $\omega_c = \frac{\pi}{4}$.

$$H_{id}(e^{j\omega}) = \begin{cases} 1 & |\omega| < \frac{\pi}{4} \\ 0 & \text{otherwise.} \end{cases}$$

$h[n] =$

(b) When using windows to design low-pass filters, there is a tradeoff between transition width and peak stop-band attenuation. However for increasing filter length, N, the transition band improves while the peak stop-band attenuation does not change much and can be fit as a constant value. The table below shows this best fit characteristic for various window types. The tranisition width is defined as $\Delta\omega$ where N is the total filter length.

Window Type	$\Delta\omega$	$\max(\delta_s)$
Rectangular	$\pi/(0.4069N)$	-21 dB
Hann	$\pi/(0.1470N)$	-44 dB
Hamming	$\pi/(0.1398N)$	-54 dB
Gaussian	$\pi/(0.0891N)$	-60 dB
Blackman	$\pi/(0.0704N)$	-75 dB

Which windows can be used to design a window-based LPF with a transition band of 0.15π and peak stopband attenuation less than -50 dB?
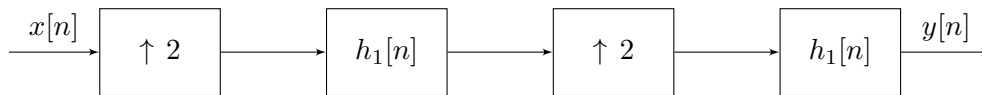
(c) Which window would you pick to optimize your filter design? Explain your reasoning.

(d) Write an expression for the filter $h[n]$ using the window you selected in the previous question. You will need to determine the filter length N and the window function $w[n]$ to satisfy the design criteria of transition band of 0.15π and peak stopband attenuation less than -50 dB.

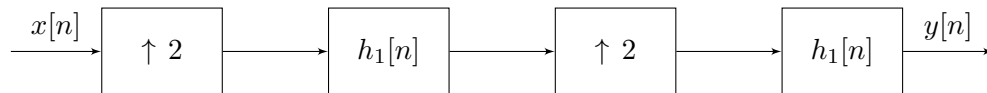$h[n] =$

4   Upsampling

(a) Consider the system shown below.

$$x[n] \rightarrow \boxed{\uparrow 2} \rightarrow \boxed{h_1[n]} \rightarrow \boxed{\uparrow 2} \rightarrow \boxed{h_1[n]} \xrightarrow{y[n]}$$

where $h_1[n] = (-1)^n \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$. Determine the impulse response $h[n]$ in the figure below so that the I/O relationship of the system is exactly the same as the I/O relationship of the system in the figure above.

$$x[n] \rightarrow \boxed{\uparrow 4} \rightarrow \boxed{h[n]} \xrightarrow{y[n]}$$

Hint: Analyze the system in the frequency domain with the interchange identities.

$$h[n] =$$

(b) Now consider a similar system

$$x[n] \longrightarrow \boxed{\uparrow 2} \longrightarrow \boxed{h_1[n]} \longrightarrow \boxed{\uparrow 2} \longrightarrow \boxed{h_1[n]} \longrightarrow y[n]$$
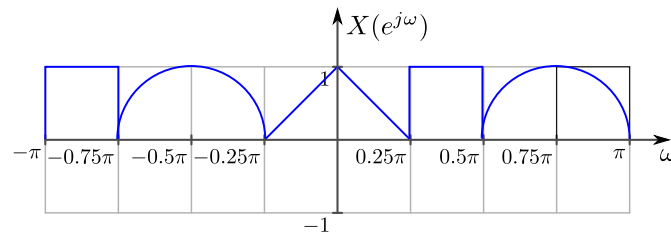
where $h_1[n] = \delta[n] - \delta[n-1]$. Determine the impulse response $h[n]$ in the figure below so that the I/O relationship of the system is exactly the same as the I/O relationship of the system in the figure above.

$$x[n] \longrightarrow \boxed{\uparrow 4} \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$
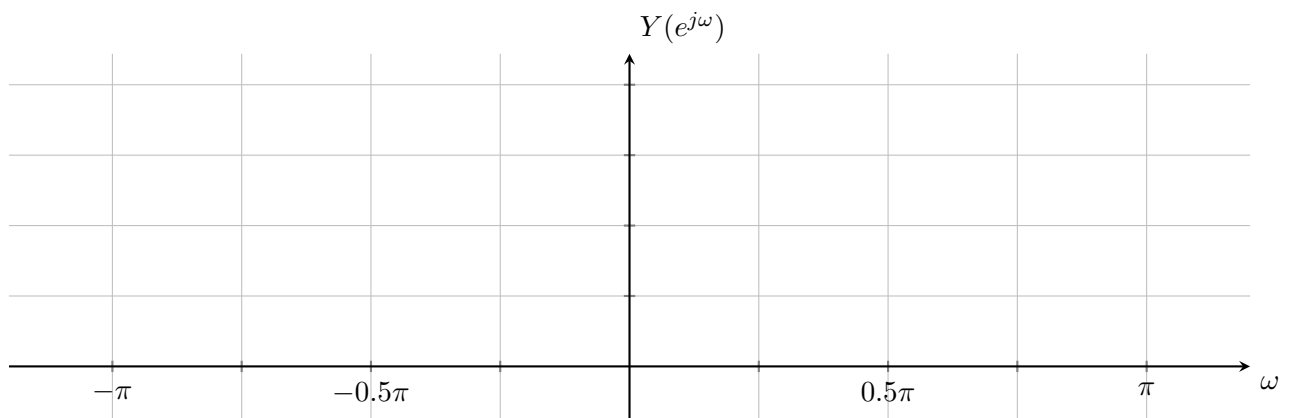
$h[n] =$

5   Sampling

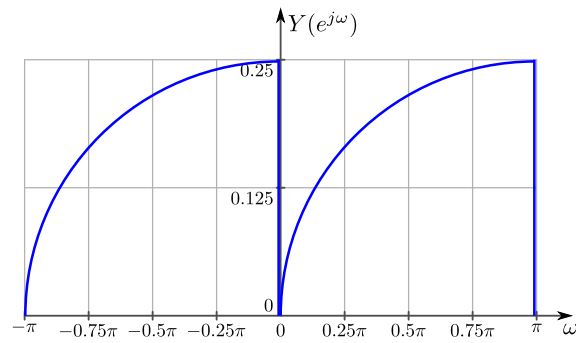Consider the signal $x[n]$ with the DTFT $X(e^{j\omega})$, which is shown below.

The following system has two downsampling stages as well as two filters. The filters $h_{\text{lpf}}[n]$ and $h_{\text{hpf}}[n]$ are ideal low-pass and high-pass filters, respectively, with cutoff frequencies $\omega_c = 0.5\pi$.
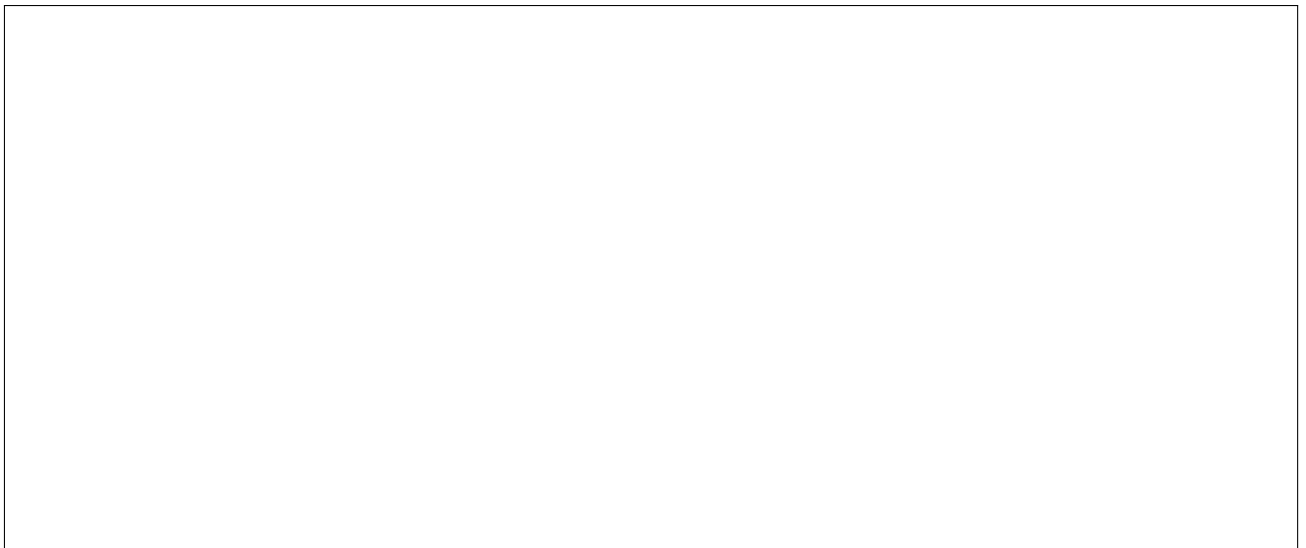

(a) Determine and sketch $Y(e^{j\omega})$, the frequency response of the system output $y[n]$ given some input $x[n]$. Make sure to label values on the y-axis.

(b) Now consider the frequency response $Y(e^{j\omega})$ which is the output of a *different* cascaded downsampling system given the same $x[n]$—and associated $X(e^{j\omega})$—as described above.

If you have the same components as described above—downsample by 2, the same $h_{\text{lpf}}[n]$, and the same $h_{\text{hpf}}[n]$—how would you design this cascaded system? Determine and sketch this system below.

6   DFT

(a) Consider the 32-point sequence $x[n] = \cos\left(\frac{6\pi}{32}n\right)$ for $0 \leq n < 32$ and $x[n] = 0$ otherwise. Compute the 32-point DFT of $x[n]$. Represent your answer a sum of shifted impulse functions in k.

$X[k] =$

(b) Now consider a new signal that is an everlasting sinusoid $x[n] = \cos\left(\frac{14\pi}{32}n\right)$ for $-\infty < n < \infty$. Compute the DTFT of $x[n]$.

$X(e^{j\omega}) =$

(c) Let's window the everlasting sinusoid with a rectangular window $w[n] = u[n] - u[n-32]$. Compute the DTFT of $\tilde{x}[n] = x[n]w[n]$.

$$\tilde{X}(e^{j\omega}) =$$

(d) Now compute the 32-point DFT of $x[n]$ using the rectangular window $w[n]$. This needs to be left as a function of k.

$$\tilde{X}[k] =$$