ECE 5210 Midterm 1

Week of: February 12, 2024

Student's name: ______________________________

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You have 2 hours for 5 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You are allowed ONE page of notes (front and back) for this exam. You may use a graphing calculator of your choice. Consulting with any third party is considered cheating.

Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
Total score		100

1 Short answer

For each part below, circle True if the statement is *always* true. Otherwise, circle False.

(a) You can uniquely determine $h[n]$ if you know the location of all the poles and zeros of $H(z)$ (and their multiplicity), and the value of $H(z_0)$ at a single, non-singular value z_0.

True	False

(b) If a system is FIR, then $H(z)$ has no poles, except perhaps at $z = 0$.

True	False

(c) The discrete-time signal $x[n] = \cos(3n)$ is periodic.

True	False

(d) If you know $H(z)$, and whether a single point of the z-plane is inside or outside the ROC, then you can completely specify the ROC.

True	False

(e) If $H(z)$ has more poles than zeros, the system is causal.

True	False

(f) If $H(z)$ has all its poles and zeros inside the unit circle, it is stable.

True	False

(g) The system $y[n] = 2x[n] + 1$ is linear.

True	False

(h) The system $y[n] = x[2n]$ is time-invariant.

True	False

(i) The system $y[n] - y[n-1] = x[n]$ is memoryless.

True	False

(j) The system $y[n] - y[n-1] = x[n+1]$ is causal.

True	False

2 Convolution

(a) Perform the convolution to find

$$y[n] = \left(\frac{1}{4}\right)^{n+1} u[n+1] * (u[n] - u[n-4])\,.$$

$y[n] =$

(b) Oftentimes we know some signal $y[n]$, which is the output of some LTI system H. We might even be lucky enough to know the system's impulse function $h[n]$. However, we are interested in the input signal $x[n]$, which is unknown. This process is called *de*convolution and is very common in image processing (e.g., removing camera shake blurring from photos).

Suppose we have a system with an impulse function

$$h[n] = 3\delta[n] + 2\delta[n-1] + \delta[n-2]$$

and we have some output

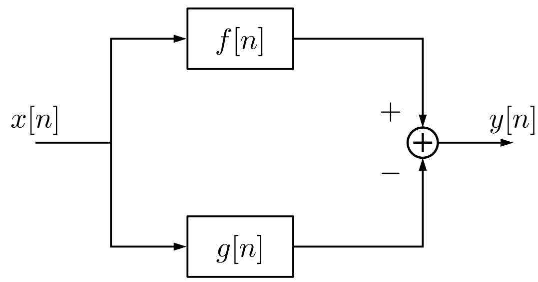
$$y[n] = 3\delta[n-1] - \delta[n-2] + 5\delta[n-3] + 3\delta[n-4] + 2\delta[n-5].$$

Please determine the input $x[n]$.

$x[n] =$

3 z-Transform

Suppose an input $x[n]$ is fitlered by a system comprising of two stable subsystems, with impulse responses $f[n]$ and $g[n]$, whose outputs are stracted to form the output $y[n]$, as shown below.



The difference equation for the entire system is

$$y[n] - \frac{1}{4}y[n-2] = \frac{1}{2}x[n] + \frac{3}{4}x[n-2].$$

(a) Find $H(z)$.

$H(z) =$

(b) Suppose you know that $F(z)$ has a single zero at $z = -3$, a single pole at $z = 0.5$, and that $F(0) = -3$. Find $F(z)$

$F(z) =$

(c) Find $f[n]$.

$f[n] =$

(d) Find $G(z)$.

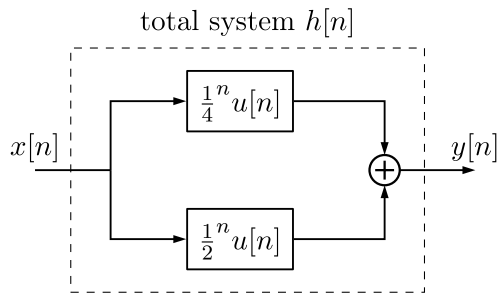
$G(z) =$

(e) Find $g[n]$.

$g[n] =$

4 DTFT

Consider the system below.



(a) Find the impulse response for the entire system $h[n]$.

$h[n] =$

(b) Find the DTFT of the entire system $H(e^{j\omega})$

$H(e^{j\omega}) =$

(c) Find the difference equation for this system.

(d) Suppose we were to send an input signal $x[n] = \cos(n)$ into this system. What is the output $y[n]$?

$y[n] =$

5 Sampling

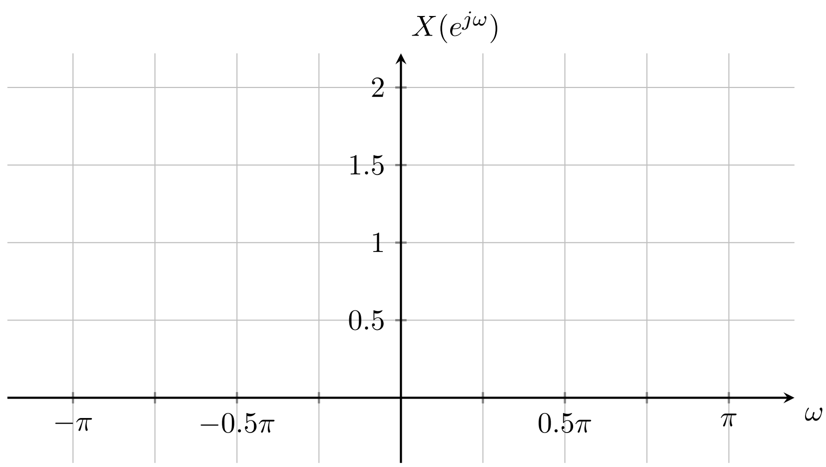
Consider the continuous-time signal $x_c(t) = \mathrm{sinc}^2(\pi t)$.

Recall the Fourier transform pairs $\frac{W}{\pi}\mathrm{sinc}\,(Wt) \Longleftrightarrow \mathrm{rect}\left(\frac{\Omega}{2W}\right)$ and $\frac{W}{2\pi}\mathrm{sinc}^2\left(\frac{Wt}{2}\right) \Longleftrightarrow \Delta\left(\frac{\Omega}{2W}\right)$.

(a) What is the bandwidth of this signal and the minimum sampling rate to satisfy Nyquist in hertz?

BW = $\qquad\qquad$ $f_s =$

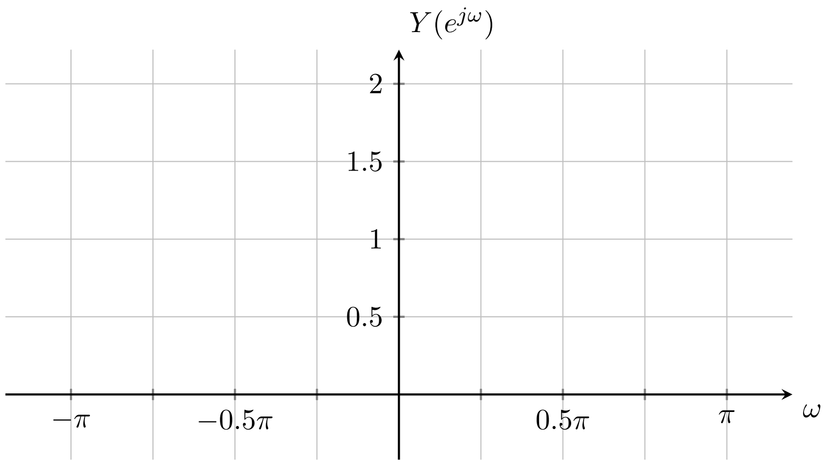
(b) Suppose we sampled this signal right at Nyquist, sketch the resulting DTFT $X(e^{j\omega})$ from $-\pi \leq \omega \leq \pi$.



(c) What if decide to discard half of the samples such that

$$y[n] = \begin{cases} x[n] & n \text{ even} \\ 0 & n \text{ odd.} \end{cases}$$

Sketch the resulting DTFT $Y(e^{j\omega})$. It might help to think of this system as $y[n] = \frac{1}{2}\left(x[n] + (-1)^n x[n]\right)$.



(d) Suppose we were able to reconstruct the continuous-time signal using an ideal reconstruction filter

$$H_r(j\Omega) = \begin{cases} T & |\Omega| < \pi/T \\ 0 & \text{else} \end{cases}.$$

Determine $y_c(t)$.

$y_c(t) =$