ECE 5210 Final Exam

Week of: April 28, 2021

Student's name: ______________________________

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You have 3 hours for 6 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

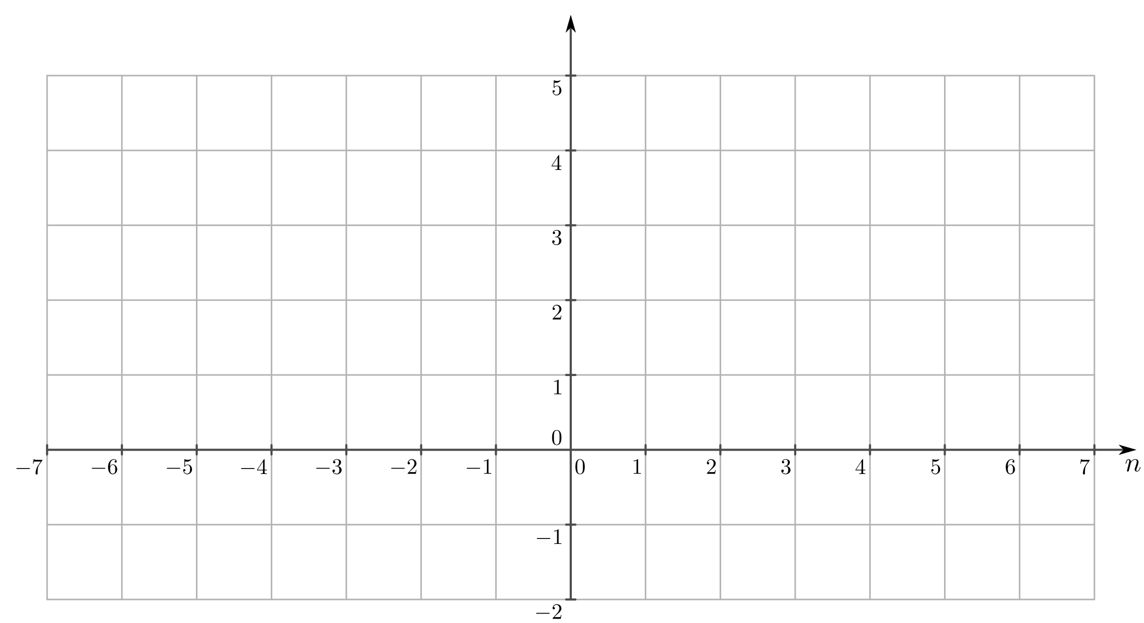
You are allowed THREE pages of notes (front and back) for this exam. You may use a graphing calculator of your choice. Consulting with any third party is considered cheating.

Problem	Score	Possible Points
1		25
2		20
3		20
4		20
5		20
6		20
Total score		125

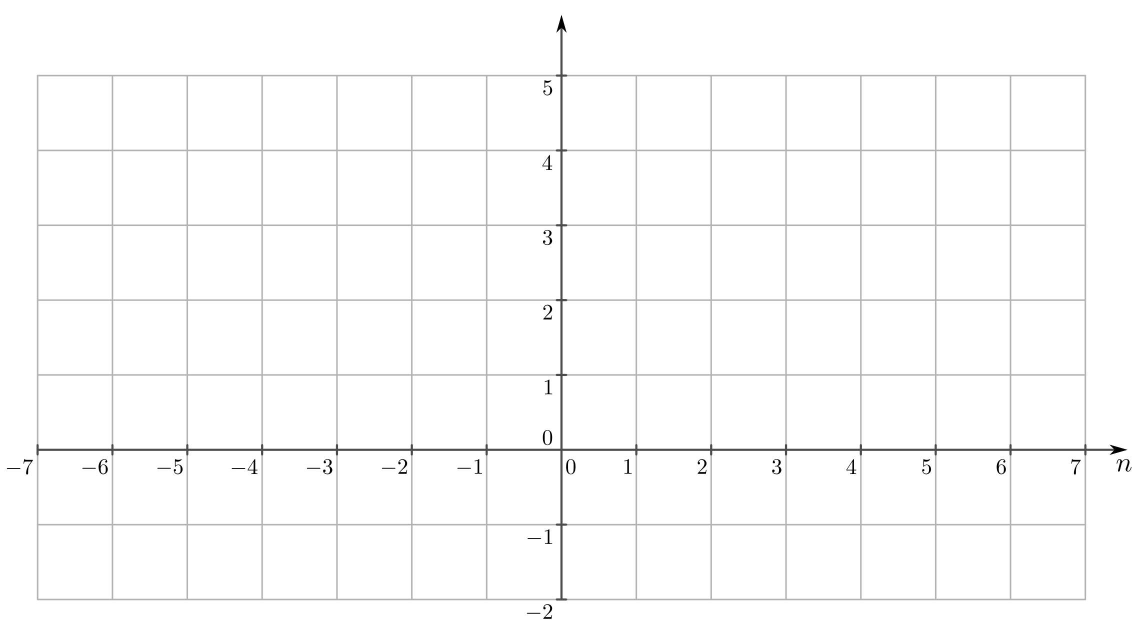
1 Convolution

Consider a system where there is a cascade connection of two LTI systems $h_0[n]$ and $h_1[n]$. Given some input $x[n]$, we will define an intermediate point $w[n] = h_0[n] * x[n]$. The overall output is $y[n] = h_1[n] * w[n]$. We define $h_0[n]$ as $h_0[n] = u[n] - u[n-4]$ and $h_1[n] = h_0[-n]$. Yes, $h_1[n]$ is noncausal.

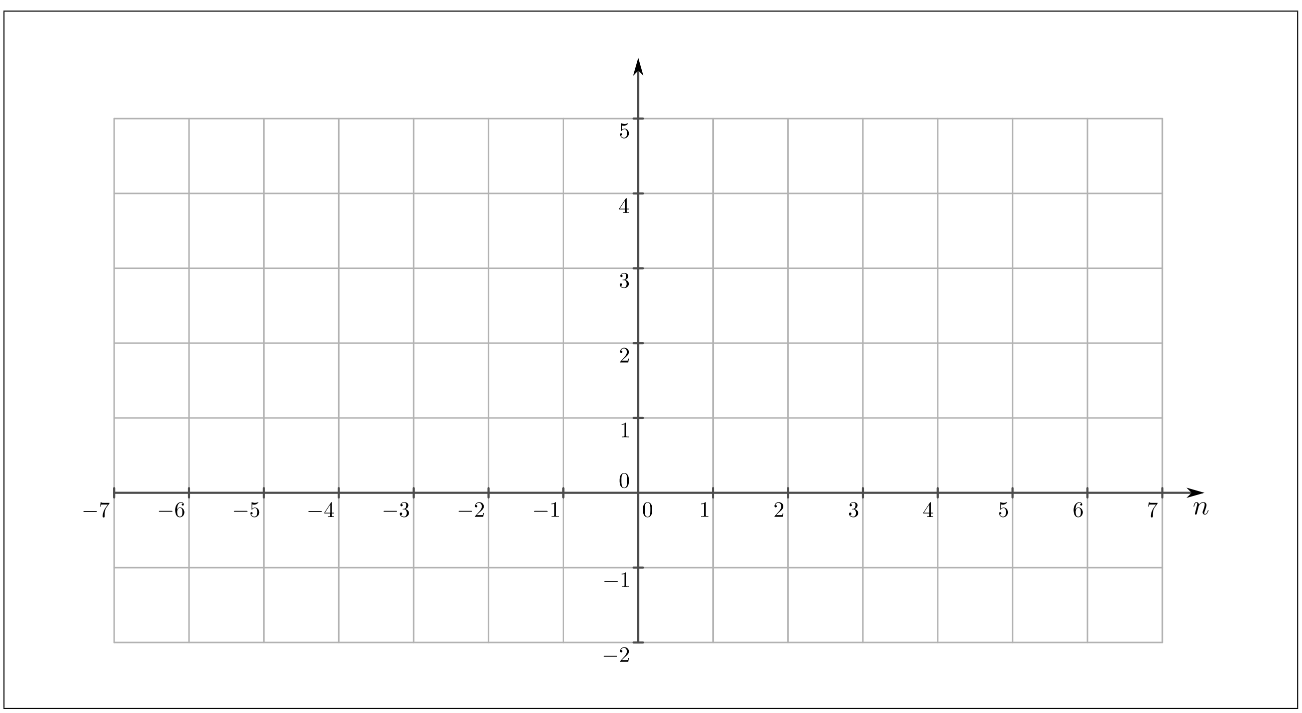
(a) Determine and sketch the overall impulse response of the cascade system; i.e., plot the output $y[n] = h[n]$ when $x[n] = \delta[n]$.



(b) Determine and sketch $w[n]$ if $x[n] = (-1)^n u[n]$.



(c) Determine and sketch overall output $y[n]$ if $x[n] = (-1)^n u[n]$.



2 Frequency Response

Consider a system that has an impulse response

$$h[n] = \delta[n] - \alpha\delta[n-1]$$

(a) Find the magnitude of the frequency response of this system.

$|H(e^{j\omega})| =$

(b) Find the phase of the frequency response of this system.

$\text{Arg}[H(e^{j\omega})] =$

(c) Find the group delay for this system. It might be helpful to remember that the following derivative

$$\frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2}\frac{du}{dx}$$

$\mathrm{grd}[H(e^{j\omega})] =$

3 DTFT

(a) Find the DTFT of $x[n] = (0.5)^n u[n+3]$.

$X(e^{j\omega})$

(b) Consider a 90° phase shifter which is defined as

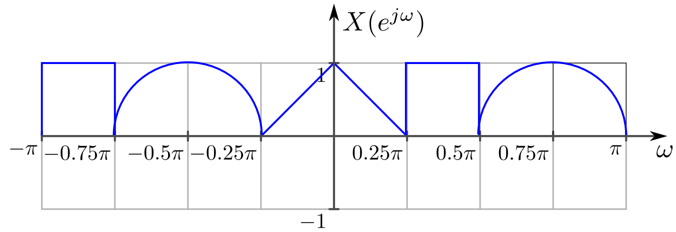
$$H(e^{j\omega}) = \begin{cases} -j & 0 < \omega < \pi \\ j & -\pi < \omega < 0. \end{cases}$$

Find the impulse response $h[n]$ for such as system.

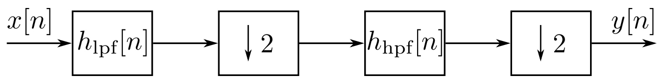
$h[n] =$

4 Sampling

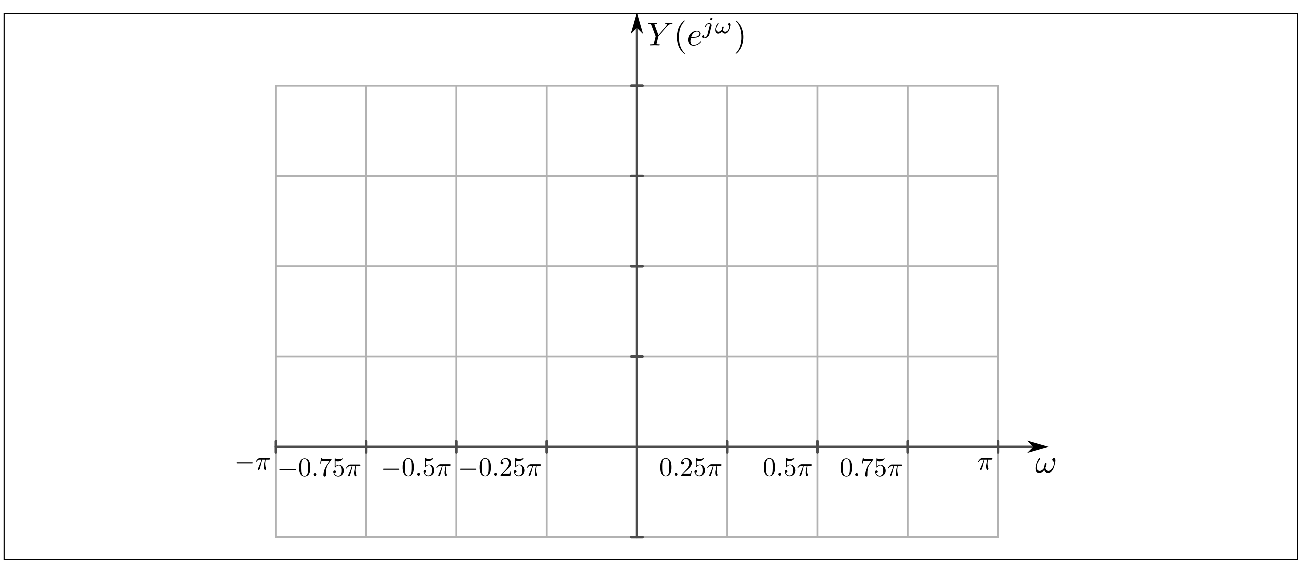
Consider the signal $x[n]$ with the DTFT $X(e^{j\omega})$, which is shown below.



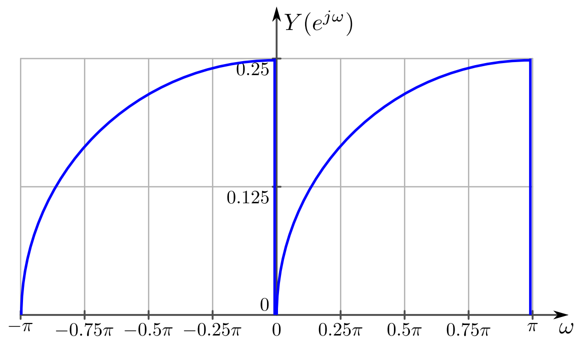
The following system has two downsampling stages as well as two filters. The filters $h_{\text{lpf}}[n]$ and $h_{\text{hpf}}[n]$ are ideal low-pass and high-pass filters, respectively, with cutoff frequencies $\omega_c = 0.5\pi$.



(a) Determine and sketch $Y(e^{j\omega})$, the frequency response of the system output $y[n]$ given some input $x[n]$. Make sure to label values on the y-axis.



(b) Now consider the frequency response $Y(e^{j\omega})$ which is the output of a *different* cascaded downsampling system given the same $x[n]$—and associated $X(e^{j\omega})$—as described above.



If you have the same components as described above—downsample by 2, the same $h_{\text{lpf}}[n]$, and the same $h_{\text{hpf}}[n]$—how would you design this cascaded system? Determine and sketch this system below.

5 DFT

(a) Let $h[n]$ be a finite-length sequence of length N with $h[n] = 0$ for $n < 0$ and $n \geq N$. The discrete-time Fourier transform of $h[n]$ is sampled at $3N$ equally spaced points

$$\omega_k = \frac{2\pi k}{3N} \quad k = 0, 1, \ldots, 3N - 1.$$

Find the sequence $g[n]$ that is the inverse DFT of the $3N$ samples of $H[k] = H(e^{j\omega_k})$. (*Hint:* write $g[n]$ in terms of $h[n]$.)

$g[n] =$

(b) Two finite-length sequences $x_1[n]$ and $x_2[n]$ are zero outside the interval of [0, 99] are circularly convolved to find a new sequence $y[n]$. If $x_1[n]$ is nonzero for only $10 \leq n \leq 39$, determine the values of n for which $y[n]$ is guaranteed to be equal to the *linear* convolution of $x_1[n]$ and $x_2[n]$.

Interval:

6 Filter design

If we are given a low-pass filter that has been designed and implemented, either in hardware or software, it may be of interest to try to improve the frequency response characteristics by repetitive use of the filter. Suppose that $h[n]$ is the unit sample response of a zero phase FIR filter (i.e., $H(e^{j\omega}) \in \mathbb{R}$ for all ω—the phase is 0 for all ω) with a frequency response that satisfies the following specifications

$$\begin{aligned} 1-\delta_p < H(e^{j\omega}) < 1+\delta_p \quad & 0 \le \omega \le \omega_p \\ -\delta_s < H(e^{j\omega}) < \delta_s \quad & \omega_s \le \omega \le \pi. \end{aligned}$$

(a) If a new filter is formed by cascading $h[n]$ with itself

$$g[n] = h[n] * h[n].$$

what is $G(e^{j\omega})$ in terms of $H(e^{j\omega})$?

$G(e^{j\omega}) =$

(b) If $G(e^{j\omega})$ satisfies a set of specifications of the form

$$\begin{aligned} A < G(e^{j\omega}) < B \quad & 0 \leq \omega \leq \omega_p \\ C < G(e^{j\omega}) < D \quad & \omega_s \leq \omega \leq \pi \end{aligned}$$

find A, B, C, and D in terms of δ_p and δ_s of the low-pass filter $h[n]$.

$A =$	$B =$	$C =$	$D =$

(c) Assuming $\delta_p \ll 1$ and $\delta_s \ll 1$, do we see improvement in either the passband or stopband ripple? If so, which one?