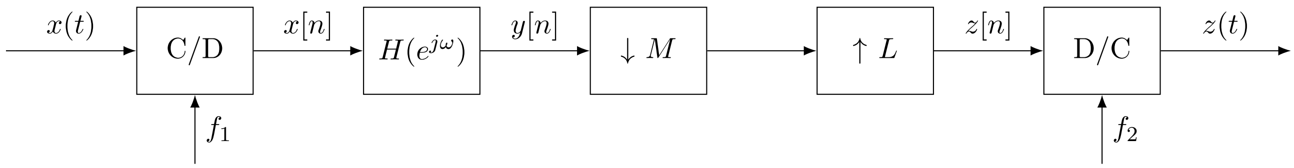
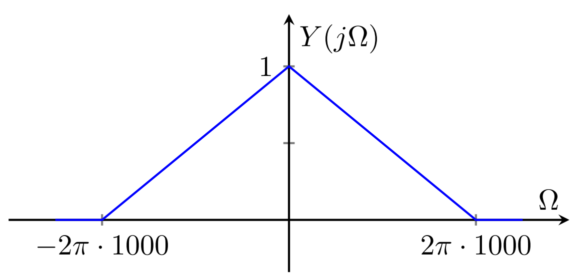
ECE 5210 hw05

1. DT processing of CT signals (Midterm 2 2025)

The discrete-time filtering system comprises a C/D converter sampling at rate $f_1 = 2\,\text{kHz}$, a filter with frequency response $H(e^{j\omega})$ (an ideal low-pass filter with gain of 1 and a cutoff frequency $\omega_c = \frac{\pi}{2}$), a resampler that resamples at a rate of $M : L$ (downsample by M immediately followed by upsample by L with no filtering in between the two operations) and an ideal D/C converter at rate f_2 or $T_2 = 1/f_2$. Ideal means that the D/C converter contains an ideal lowpass reconstruction filter with a cutoff frequency of $f_2/2$ (in Hertz, or $\Omega_c = \pi/T_2$ in radians per second) and a gain of $1/f_2$. The spectrum of the input, $X(j\Omega)$ is shown below.





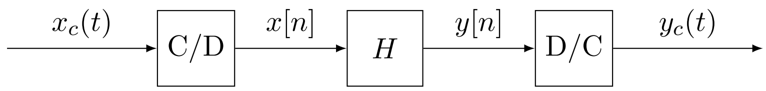
a) Plot the spectra $X(e^{j\omega})$ and $Y(e^{j\omega})$. Label all axes and relevant features. Plot the DTFTs from $\omega = -\pi$ to π.

b) Given $f_2 = 1\,\text{kHz}$, $M = 2$, and $L = 1$, plot the spectra $Z(e^{j\omega})$ and $Z(j\Omega)$. Label all axes and relevant features. Plot the continuous-time Fourier transform such that the entire signal bandwidth is represented. Plot the DTFT from $\omega = -\pi$ to π.

c) Given $f_2 = 4\,\text{kHz}$, $M = 1$, and $L = 2$, plot the spectra $Z(e^{j\omega})$ and $Z(j\Omega)$. Label all axes and relevant features. Plot the DTFT from $\omega = -\pi$ to π. Plot the continuous-time Fourier transform such that the entire signal bandwidth is represented. Plot the DTFT from $\omega = -\pi$ to π.

2. Sampling and reconstruction (Midterm 2 2024)

Consider the signal processing system below, where the C/D and D/C converters are ideal. Let the input $x_c(t) = 2\cos(30\pi t)$ and the sampling rate be $f_s = 40\,\mathrm{Hz}$.



The system $H(z)$ is described by the input/output relationship $y[n] = x^2[n]$.

a) Is the subsystem (and only the subsystem) H linear and/or time-invariant?

b) Sketch $X(e^{j\omega})$ and $Y(e^{j\omega})$ from $\omega = -\pi$ to π.

c) What is the output $y_c(t)$?

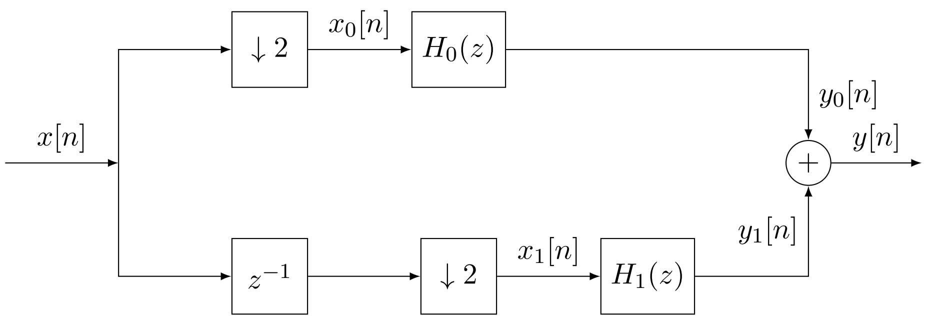
d) For which frequencies of Ω of $x_c(t) = 2\cos(\Omega t)$ will $y_c(t) = x_c^2(t)$?

3. Decimation filtering

Consider the system below where $y_0[n]$ and $y_1[n]$ and are generated by the difference equations

$$y_0[n] = \frac{1}{4}y_0[n-1] - \frac{1}{3}x_0[n] + \frac{1}{8}x_0[n-1]$$
$$y_1[n] = \frac{1}{4}y_1[n-1] - \frac{1}{12}x_1[n]$$

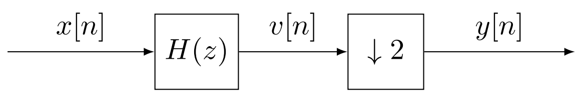
This total system is seen below.



a) Find an expression for $H_0(z)$.

b) Find an expression for $H_1(z)$.

The decimation filter can also be implemented as shown below. Find $H(z)$.



c) Find $H(z)$.

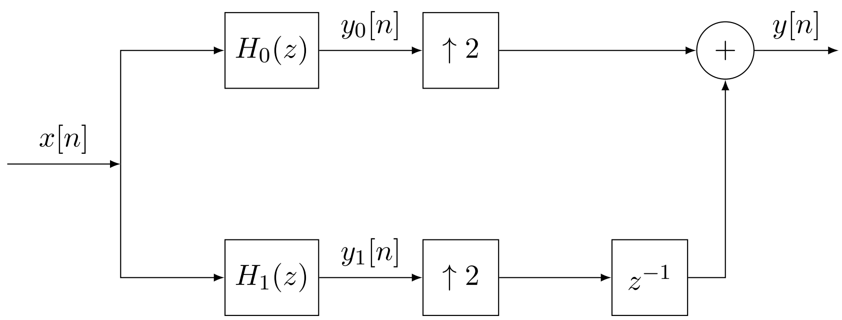
d) In the implementation above, $v[n] = av[n-1] + bx[n] + cx[n-1]$. Determine a, b, and c.

4. Interpolation filtering

Consider the system below where $y_0[n]$ and $y_1[n]$ are generated by the difference equations

$$y_0[n] = x[n] - 2x[n-1] + x[n-2]$$
$$y_1[n] = 1.5x[n] + 1.5x[n-1]$$

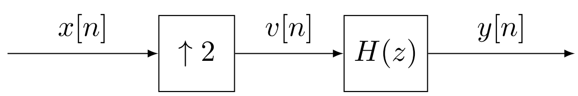
This total system is seen below.



a) Find an expression for $H_0(z)$.

b) Find an expression for $H_1(z)$.

The interpolation filter can also be implemented as shown below.

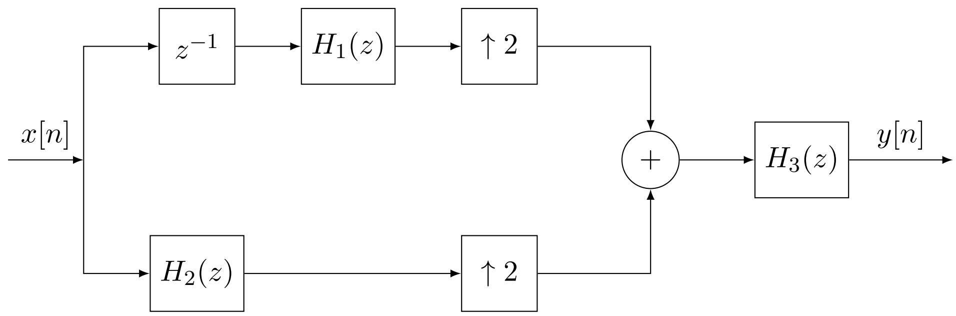


c) Find $H(z)$.

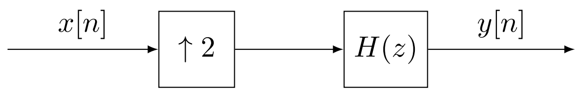
d) In the implementation above, $y[n] = a_0 v[n] + a_1 v[n-1] + a_2 v[n-2] + a_3 v[n-3] + a_4 v[n-4]$. Find a_k for $k = 0, 1, \ldots, 4$.

5. More interpolation

Consider the system below.



This system can also be implemented as a cascade as an expander followed by a simple FIR filter $H(z)$. This simplified system is shown below.



The filter $H(z)$ is given as

$$H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}.$$

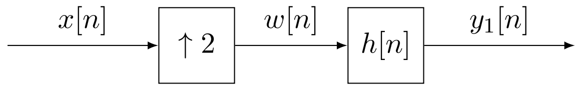
The filters $H_1(z)$, $H_2(z)$, and $H_3(z)$ each have two taps. Thus, we can represent

$$\begin{aligned} H_1(z) &= h_1[0] + h_1[1]z^{-1} \\ H_2(z) &= h_2[0] + h_2[1]z^{-1} \\ H_3(z) &= h_3[0] + h_3[1]z^{-1}. \end{aligned}$$

Find the values $h_1[0]$, $h_1[1]$, $h_2[0]$, $h_2[1]$, $h_3[0]$, and $h_3[1]$.

6. Decimation Interpolation (Midterm 2 2024)

We are interested in upsampling a sequence by a factor of 2, using a system similar to the one below.



The filter $h[n]$ is

$$h[n] = 2\delta[n] - \delta[n-1] + \delta[n-2] - 2\delta[n-3] + 2\delta[n-4].$$

A proposed implementation of the system is shown below. The three impulse functions $h_1[n]$, $h_2[n]$, and $h_3[n]$ are all restricted to be zero outside the range $0 \leq n \leq 2$. Find $h_1[n]$, $h_2[n]$, and $h_3[n]$ such that $y_1[n] = y_2[n]$ for any $x[n]$.