ECE 5210 Final

Week of: April 23, 2024

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You have 3 hours for 6 problems.

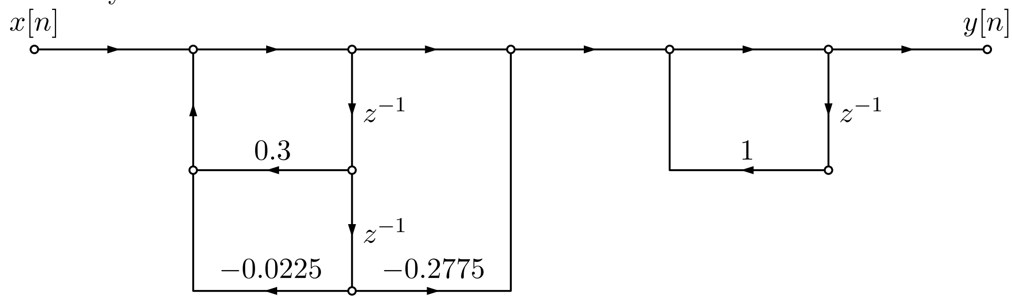
- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You are allowed THREE pages of notes (front and back) for this exam. You may use a graphing calculator of your choice. Consulting with any third party is considered cheating.

Problem	Score	Possible Points
1		20
2		20
3		20
4		25
5		20
6		20
Total score		125

1 Block Diagrams

Consider the system shown below.



(a) Determine the transfer function $H(z)$ for the system.

$H(z) =$

(b) Determine the difference equation for this system.

(c) Determine the impulse response $h[n]$ for the system. Assume this is a causal system.

$h[n] =$

(d) Draw a Direct Form II realization of this system.

2 Circular Convolution

Suppose we have two sequences

$$x_1[n] = \delta[n] + 2\delta[n-2] + \delta[n-3]$$
$$x_2[n] = \delta[n-1] + a\delta[n-3].$$

(a) Suppose $a = 2$. Find the 5-point circular convolution of $x_1[n]$ and $x_2[n]$. You can represent this $y[n]$ as a sum of shifted impulse functions from $0 \leq n \leq 4$.

$y[n] =$

(b) Suppose that a is unknown. The sequence $y[n]$ is a 4-point circular convolution of $x_1[n]$ and $x_2[n]$ and is given by

$$y[n] = \delta[n] - \delta[n-1] - \delta[n-2] + \delta[n-3].$$

Find a value of a that would satisfy this scenario.

$a =$

3 Difference Equation

An engineer is asked to evaluate a simple signal processing system with a single digital filter. The input $x[n]$ is obtained a continuous-time signal at a sampling rate of $1/T$. The goal for $H(e^{j\omega})$ is to be a linear-phase FIR filter, and ideally it should have the following amplitude response such that it acts as a bandlimited differentiator

$$|H_{id}(e^{j\omega})| = \begin{cases} -\omega/T & \omega < 0 \\ \omega/T & \omega > 0. \end{cases}$$

(a) For one implementation of $H(e^{j\omega})$, referred to as $H_(e^{j\omega})$, the designer, motivated by the definition

$$\frac{d[x(t)]}{dt} = \lim_{\Delta t \to 0} \frac{x(t) - x(t - \Delta t)}{\Delta t},$$

chooses the system impulse response $h_1[n]$ so the input-output relationship is

$$y[n] = \frac{x[n] - x[n-1]}{T}.$$

Find $H_1(e^{j\omega})$.

$H_1(e^{j\omega}) =$

(b) We are interested in how well this approximation matches against the ideal response $H_{id}(e^{j\omega})$. Find the difference in the squared magnitudes between the two responses, i.e.,

$$\text{error } = |H_1(e^{j\omega})|^2 - |H_{id}(e^{j\omega})|^2.$$

Represent your answer as 6^{th} order polynomial function of ω. It might be helpful to remember the Taylor series expansions centered around zero for the sine and cosine functions

$$\sin(\omega) = \omega - \frac{\omega^3}{3!} + \frac{\omega^5}{5!} - \cdots$$
$$\cos(\omega) = 1 - \frac{\omega^2}{2!} + \frac{\omega^4}{4!} - \cdots.$$

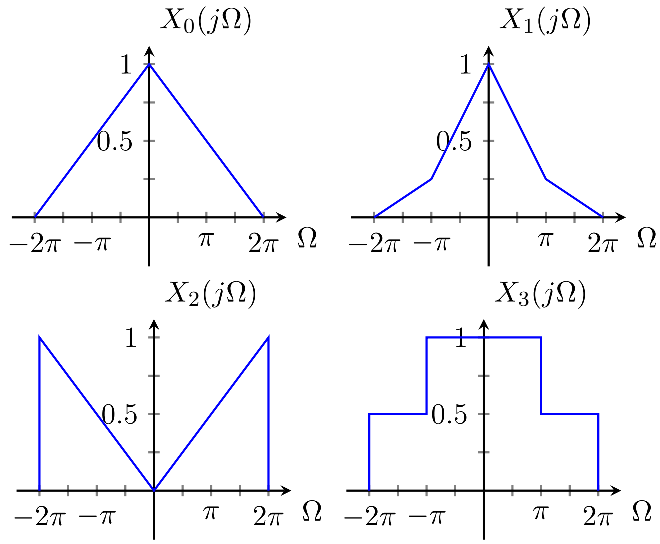
error =

(c) If we want to cascade $H_1(e^{j\omega})$ with another linear-phase FIR filter $G(e^{j\omega})$. If we want the combination of the two filters to have a group delay that is an integer number of samples, should the length of $g[n]$ have even or odd number of taps?

Circle one: even odd

4 Multi-rate Signal Processing

Consider the four different continuous-time signals with the continuous-time Fourier transforms plotted below.

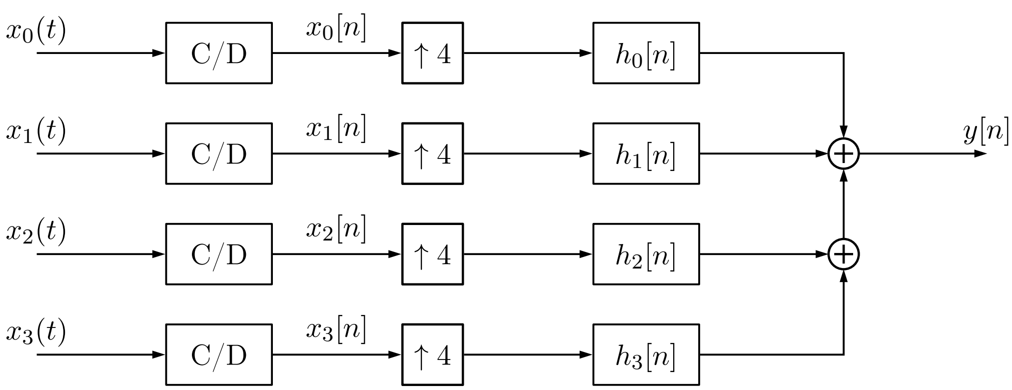


The signals are all sampled at the Nyquist rate and put into a digital subband system shown below. The impulse responses for the four filters

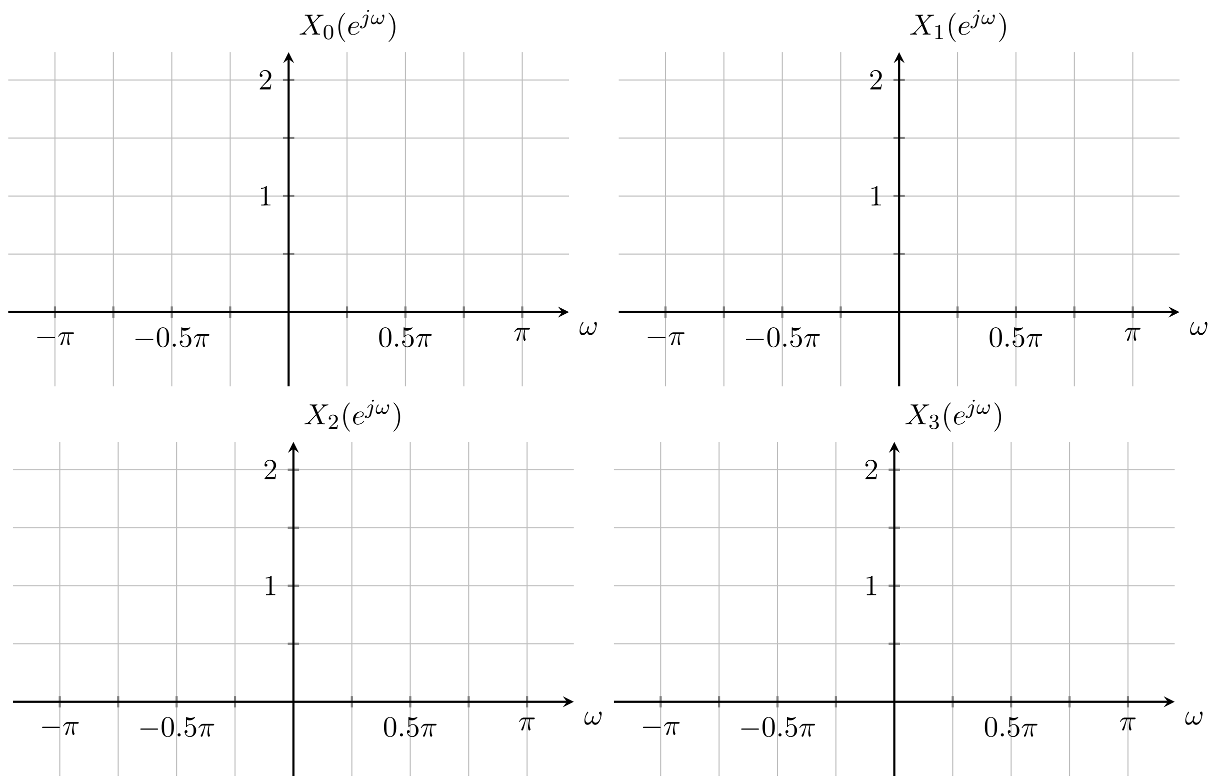
$$h_k[n] = e^{-j\left(\frac{2\pi}{4}k\right)n} h_{lp}[n] \quad \text{for} \quad k = 0, 1, 2, 3$$

are defined in terms of the ideal low pass filter given as

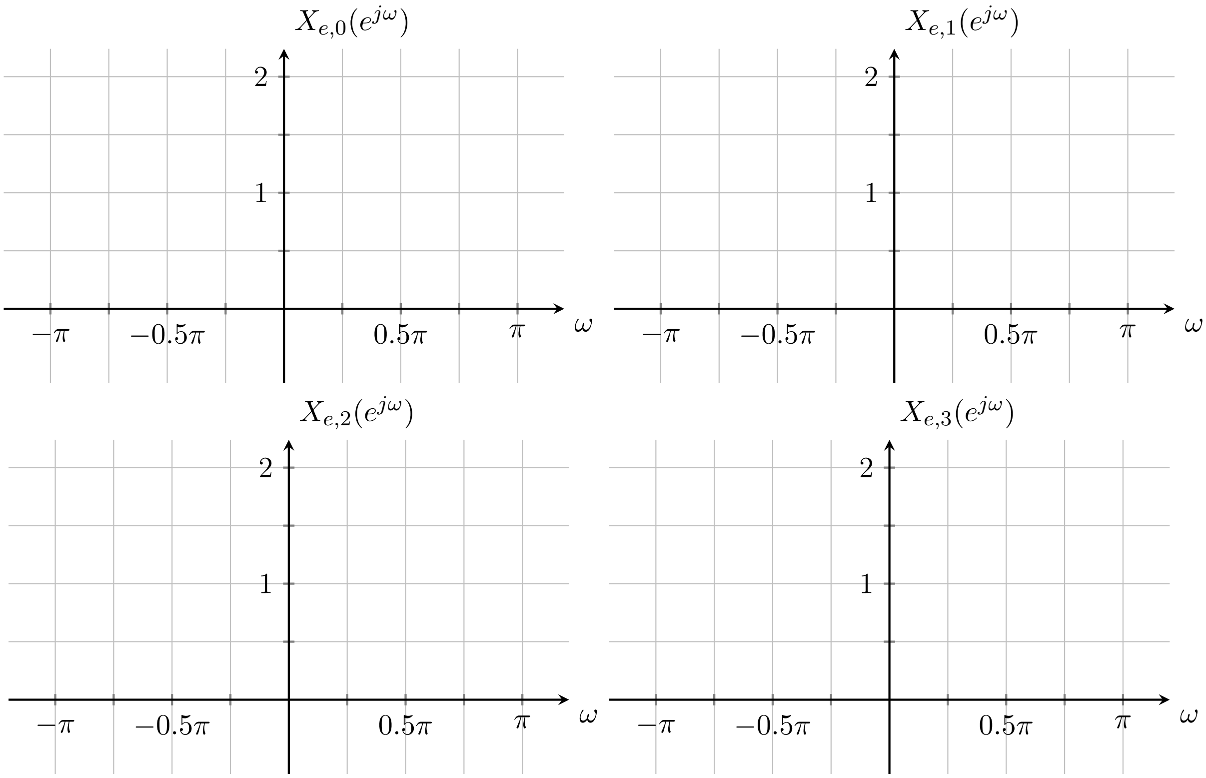
$$h_{lp}[n] = \frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}.$$



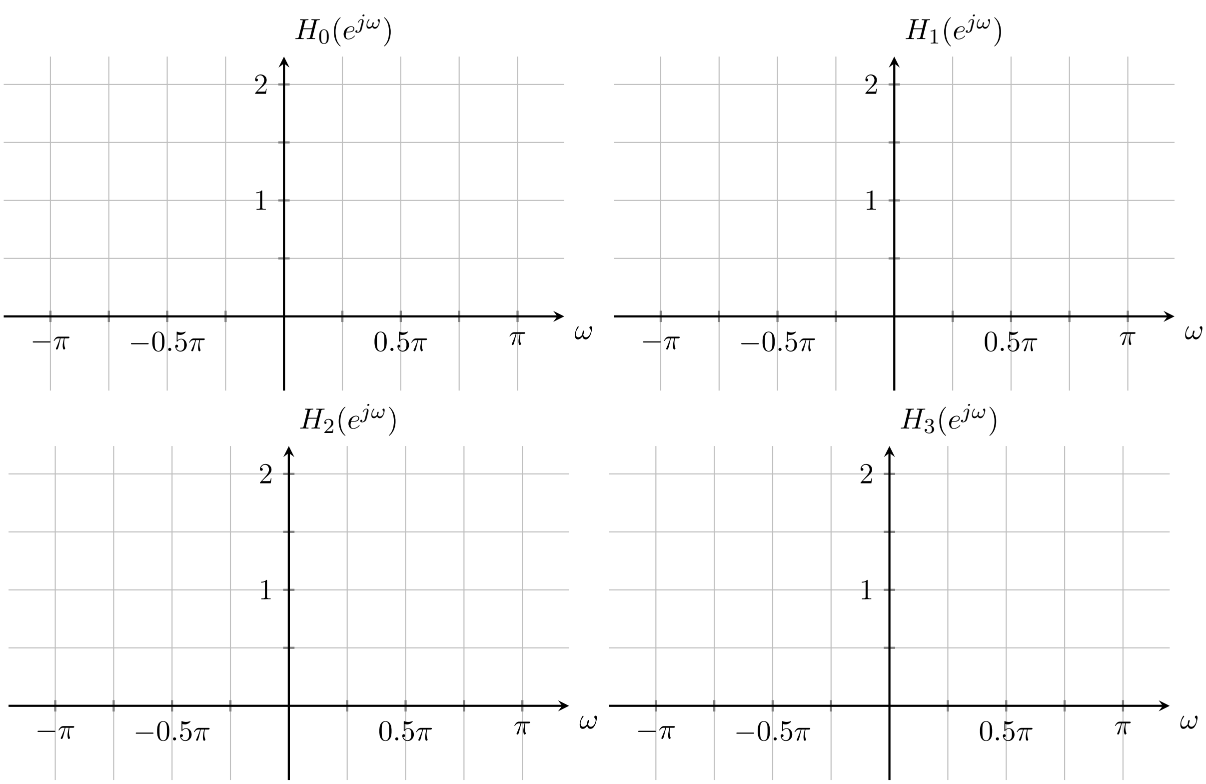
(a) Plot the magnitude of the DTFT of each of the four signals $x_k[n]$ for $k = 0, 1, 2, 3$.



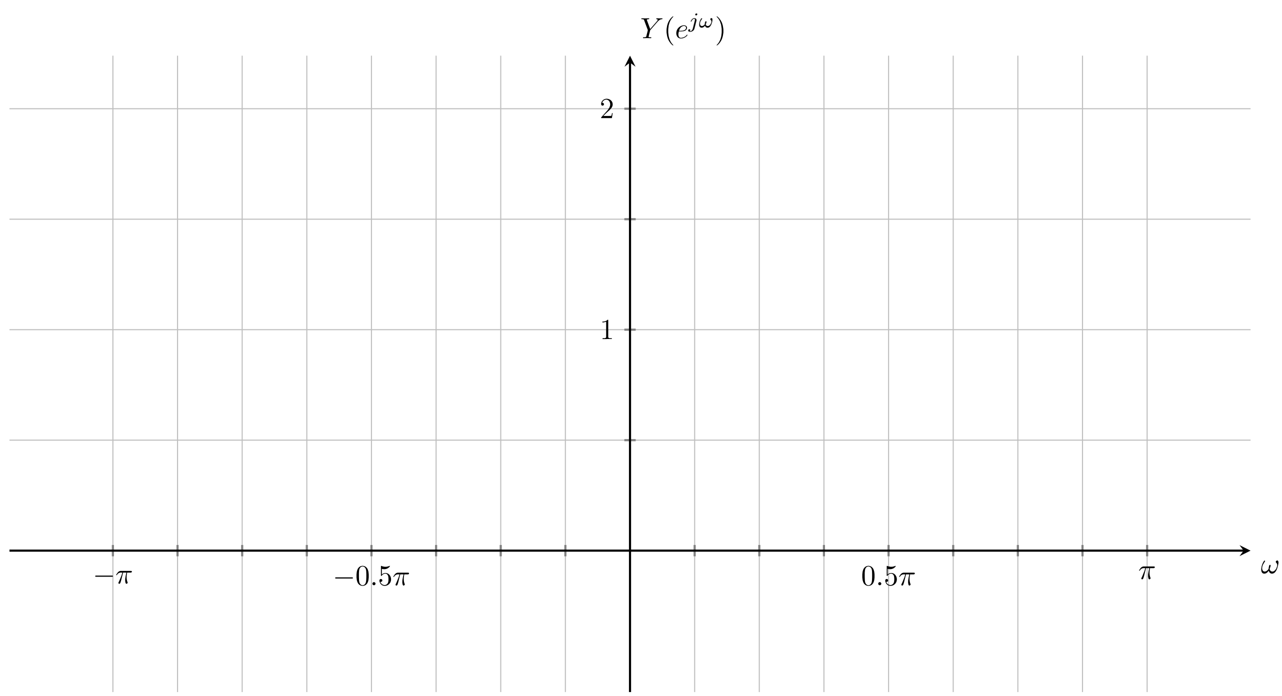
(b) Plot the magnitude of the DTFT of the output signal after upsampling by a factor of 4.



(c) Plot the magnitude of the frequency responses of each filter $H_k(e^{j\omega})$ for $k = 0, 1, 2, 3$.



(d) Plot the DTFT of the system output $Y(e^{j\omega})$.

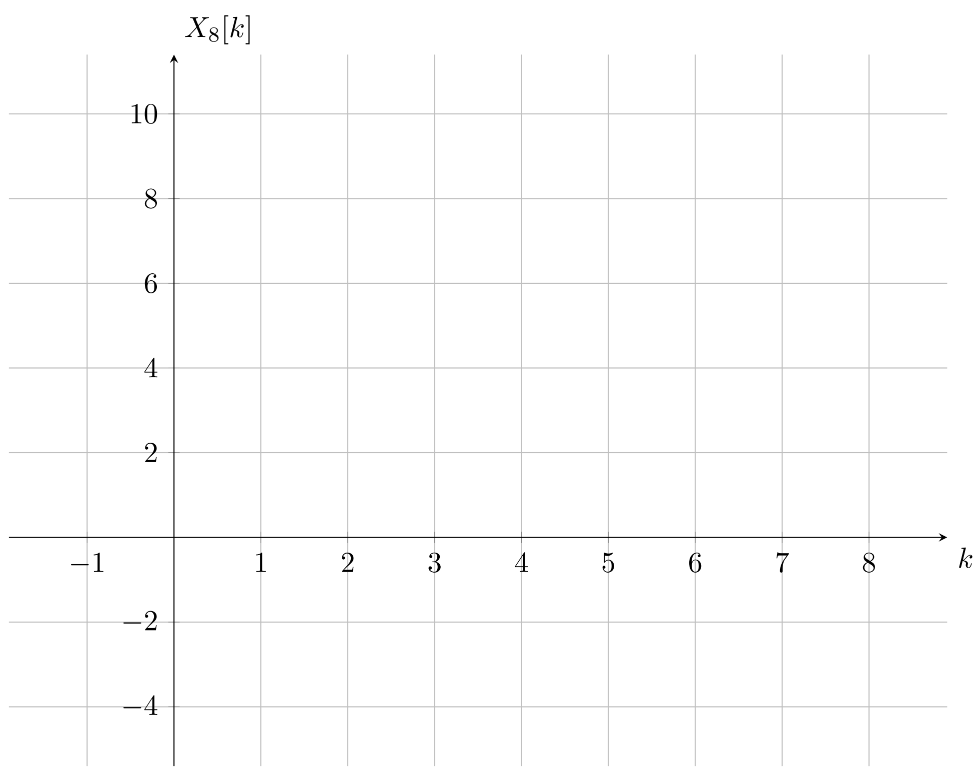


5 Discrete Fourier Transform

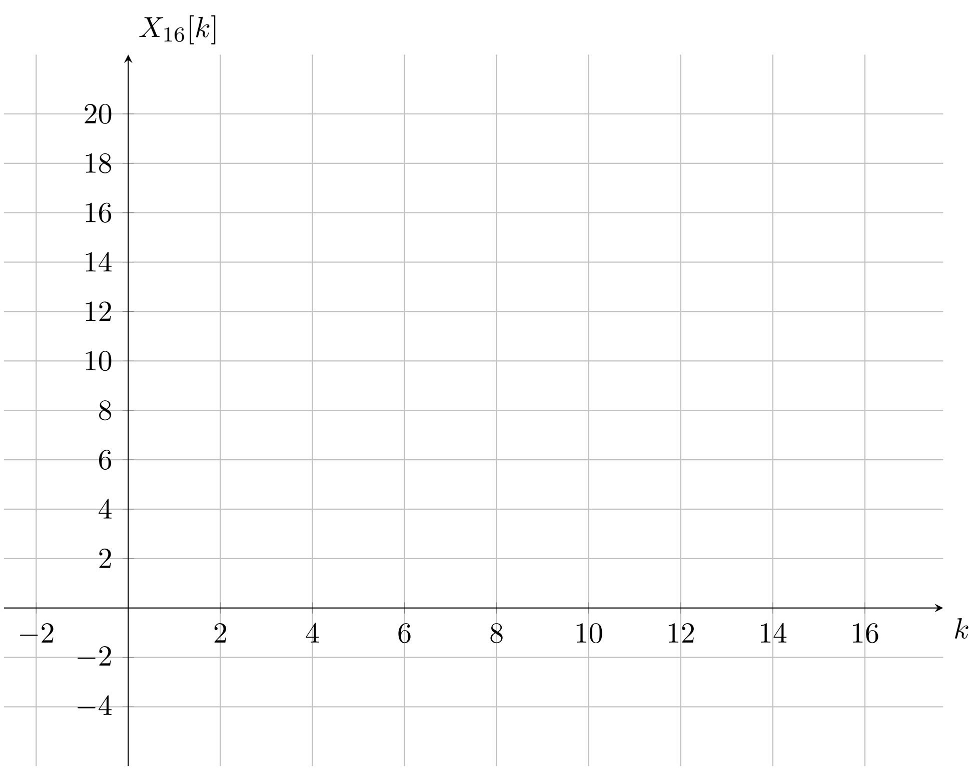
Consider the signal

$$x[n] = \begin{cases} 1 + \cos\left(\frac{\pi n}{4}\right) - 0.5\cos\left(\frac{3\pi}{4}\right) & 0 \le n \le 7 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Compute $X_8[k]$ which is the 8-point DFT of $x[n]$. Plot $X_8[k]$ for $0 \le k \le 7$.



(b) Compute $X_{16}[k]$ which is the 16-point DFT of $x[n]$. Plot $X_{16}[k]$ for $0 \leq k \leq 15$.



6 More Discrete Fourier Transform

(a) $X(e^{j\omega})$ is the DTFT of the discrete-time signal

$$x[n] = (1/2)^n u[n].$$

If we truncate $x[n]$ to 5 points to $x_5[n]$ for $0 \leq n \leq 4$, what is the 5-point DFT of $x_5[n]$?

DFT:

$X[0] =$

$X[1] =$

$X[2] =$

$X[3] =$

$X[4] =$

(b) Find a sequence $g[n]$ which has a length of 5 whose 5-point DFT $G[k]$ is identical to samples of the DTFT of $x[n]$ at $\omega_k = \frac{2\pi k}{5}$, i.e.,

$$g[n] = 0 \quad \text{for} \quad n < 0 \quad \text{and} \quad n \geq 5.$$

and

$$G[k] = X(e^{j2\pi k/5}) \quad \text{for} \quad k = 0, \dots, 4.$$

Remember that sampling in frequency will lead to time-domain aliasing.

$g[n] =$