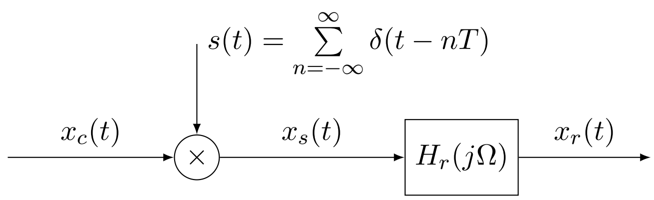
ECE 5210 hw05

1. Sampling and Reconstruction

Consider the representation of the process of sampling followed by reconstruction shown below.



Assume that the input signal is

$$x_c(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3)$$

The frequency response of the reconstruction filter is

$$H_r(j\Omega) = \begin{cases} T & |\Omega| \leq \pi/T \\ 0 & |\Omega| > \pi/T \end{cases}$$

a) Assume that $f_s = 1/T = 500\,\text{samples/sec}$, what is the output $x_r(t)$?

b) Assume that $f_s = 1/T = 250\,\text{samples/sec}$, what is the output $x_r(t)$?

c) What if you wanted the output to look like

$$x_r(t) = A + 2\cos(100\pi t - \pi/4)$$

where A is a constant. What is the sampling rate f_s and what is the numerical value of A?

2. Sampling and Reconstruction (Midterm 1 2024)

Consider the continuous-time signal $x_c(t) = \mathrm{sinc}^2(\pi t)$.
Recall the Fourier transform pairs $\frac{W}{\pi}\mathrm{sinc}\,(Wt) \Longleftrightarrow \mathrm{rect}\left(\frac{\Omega}{2W}\right)$ and $\frac{W}{2\pi}\mathrm{sinc}^2\left(\frac{Wt}{2}\right) \Longleftrightarrow \Delta\left(\frac{\Omega}{2W}\right)$.

a) What is the bandwidth of this signal and the minimum sampling rate to satisfy Nyquist in hertz?

b) Suppose we sampled this signal right at Nyquist, sketch the resulting DTFT $X(e^{j\omega})$ from $-2\pi \leq \omega \leq 2\pi$.

c) What if decide to discard half of the samples such that

$$y[n] = \begin{cases} x[n] & n\,\text{even} \\ 0 & n\,\text{odd.} \end{cases}$$

Sketch the resulting DTFT $Y(e^{j\omega})$.

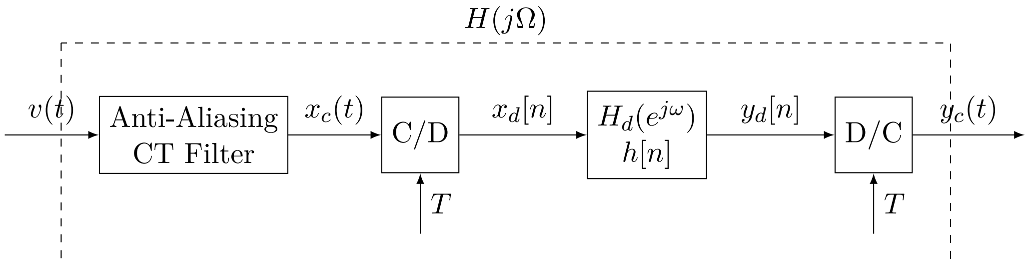
d) Suppose we were able to reconstruct the continuous-time signal using an ideal reconstruction filter

$$H_r(j\Omega) = \begin{cases} T & |\Omega| < \pi/T \\ 0 & \text{else} \end{cases}.$$

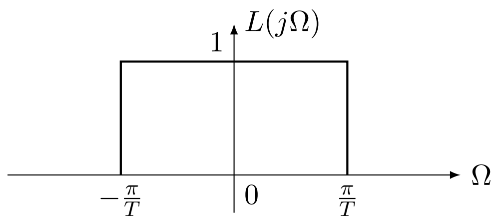
Determine $y_c(t)$.

3. Impulse Invariance

Consider the system shown below.



The frequency response of the anti-aliasing filter is seen below.



The frequency response of the LTI discrete-time system between the converters is given by

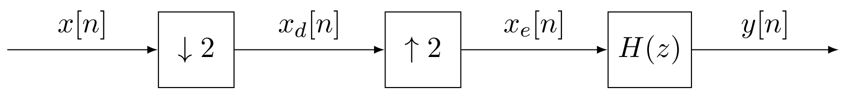
$$H_d(e^{j\omega}) = e^{-j\omega/3}, \qquad |\omega| < \pi$$

a) What is the effective continuous-time frequency response of the overall system $H(j\Omega)$?

b) Determine the impulse response $h[n]$ of the discrete-time LTI system.

4. Decimation and Interpolation 1

Consider the system below. For each of the following input signals $x[n]$ determine $y[n]$.



The system $H(z)$ has a frequency response from $-\pi < \omega \le \pi$

$$H(e^{j\omega}) = \begin{cases} 2 & |\omega| < \frac{\pi}{2} \\ 0 & \text{otherwise.} \end{cases}$$

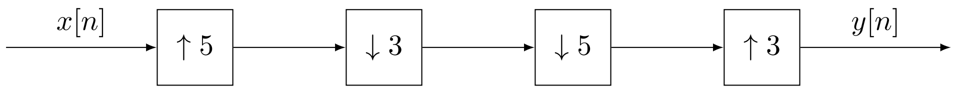
a) Find $y[n]$ given $x[n] = \cos\left(\frac{\pi n}{4}\right)$.

b) Find $y[n]$ given $x[n] = \cos\left(\frac{5\pi n}{4}\right)$.

c) Find $y[n]$ given $x[n] = \frac{\sin\left(\frac{\pi n}{3}\right)}{\pi n}$.

5. Decimation and Interpolation 2

Consider the system below.

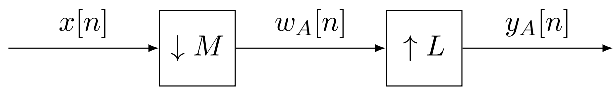


Suppose we had some input $x[n] = \cos(0.3n)$, what is $y[n]$?

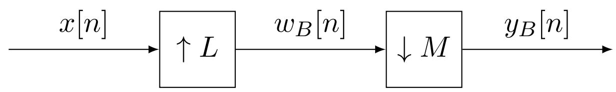
6. Decimation and Interpolation Similarities

Consider the two systems below.

System A:



System B:



a) For $M = 2$, $L = 3$, and any arbitrary $x[n]$, will $y_A[n] = y_B[n]$?

b) For $M = 4$, $L = 2$, and any arbitrary $x[n]$, will $y_A[n] = y_B[n]$?

c) For $M = 2$, $L = 4$, and any arbitrary $x[n]$, will $y_A[n] = y_B[n]$?