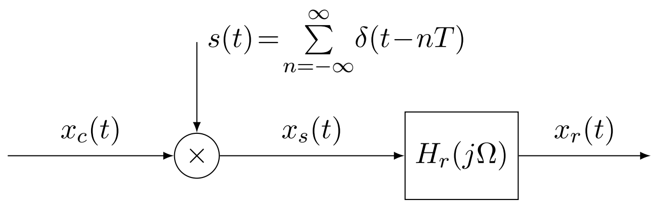
ECE 5210 quiz05

Name: SOLUTIONS

Consider the representation of the process of sampling followed by reconstruction shown below.



Assume that the input signal is

$$x_c(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3)$$

The frequency response of the reconstruction filter is

$$H_r(j\Omega) = \begin{cases} T & |\Omega| \le \pi/T \\ 0 & |\Omega| > \pi/T \end{cases}$$

Suppose you wanted the output to look like

$$x_r(t) = A + 2\cos(100\pi t - \pi/4)$$

where A is a constant. What is the sampling rate f_s and what is the numerical value of A?

Solution: We require the first term $2\cos(100\pi t - \pi/4)$ to be preserved, and the second term $\cos(300\pi t + \pi/3)$ to become a DC constant A. For the second term to alias to DC ($\Omega = 0$), we must have

$$300\pi = k\Omega_s = k(2\pi f_s)$$

for some integer k. Choosing $k = 1$, we get $\Omega_s = 300\pi$, so $f_s = 150$ Hz. Let's verify this sampling rate for the first term. With $f_s = 150$, $\Omega_s = 300\pi$ and $\Omega_N = 150\pi$. The first term has frequency 100π. Since $|100\pi| < 150\pi$, it is not aliased and is preserved. The second term samples as:

$$x_2[n] = \cos(300\pi(nT) + \pi/3) = \cos\left(300\pi n \frac{1}{150} + \pi/3\right) = \cos(2\pi n + \pi/3)$$

Since n is an integer, $2\pi n$ is a multiple of 2π, so

$$x_2[n] = \cos(\pi/3) = 0.5$$

The reconstruction filter $H_r(j\Omega)$ is an ideal lowpass filter with cutoff $\pi/T = \Omega_N = 150\pi$. The constant sequence 0.5 samples corresponds to a DC impulse at $\omega = 0$ in DTFT, which maps to analog DC. So the output is

$$x_r(t) = 2\cos(100\pi t - \pi/4) + 0.5$$

Thus,

$$f_s = 150 \text{ Hz}, \quad A = 0.5$$