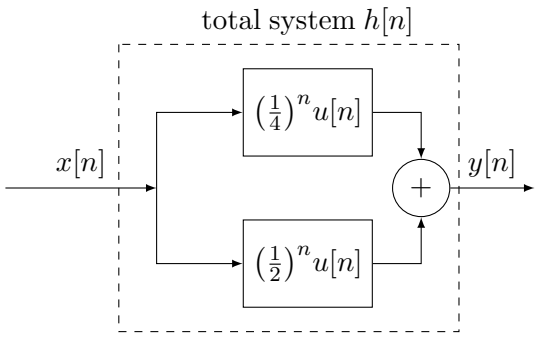
ECE 5210 quiz03

Name: SOLUTIONS

Consider the system below.



a) Find the DTFT of the entire system $H(e^{j\omega})$.

Solution: Using the linearity property and the known transform pair $a^n u[n] \leftrightarrow \frac{1}{1-ae^{-j\omega}}$:

$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}} + \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

Combining the terms:

$$H(e^{j\omega}) = \frac{(1-\frac{1}{2}e^{-j\omega})+(1-\frac{1}{4}e^{-j\omega})}{(1-\frac{1}{4}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})} = \frac{2-\frac{3}{4}e^{-j\omega}}{1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-j2\omega}}$$

b) Suppose we were to send an input signal $x[n] = \cos(n)$ into this system. What is the output $y[n]$?

Solution: For an input $x[n] = \cos(\omega_0 n)$, the output is $y[n] = |H(e^{j\omega_0})|\cos(\omega_0 n + \angle H(e^{j\omega_0}))$. Here $\omega_0 = 1$ rad/sample.

$$H(e^{j1}) = \frac{2-0.75e^{-j1}}{1-0.75e^{-j1}+0.125e^{-j2}}$$

Let $A = |H(e^{j1})|$ and $\phi = \angle H(e^{j1})$.

$$y[n] = 2.287\cos(n - 22.041^\circ)$$