

# ECE 5210 hw02

## 1. Convolution

Compute the following convolutions.

- a) Compute the convolution  $y[n] = n(u[n-1] - u[n-3]) * u[n-1]$  for  $n = -2, \dots, 5$ .
- b) Compute the convolution  $y[n] = (u[n-1] - u[n-4]) * u[n+1]$  for  $n = -2, \dots, 5$ .
- c) Compute the convolution  $y[n] = \sin\left(\frac{\pi n}{2}\right) u[n] * u[n+2]$  for  $n = -2, \dots, 5$ .
- d) Compute the convolution  $y[n] = (u[n-1] - u[n-5]) * 0.5^n(u[n] - u[n-8])$ . Express your answer as a piecewise function  $y[n]$ .

## 2. Difference Equation

An engineer is asked to evaluate a simple signal processing system with a single digital filter. The input  $x[n]$  is obtained a continuous-time signal at a sampling rate of  $1/T$ . The goal for  $H(e^{j\omega})$  is to be a linear-phase FIR filter, and ideally it should have the following amplitude response such that it acts as a bandlimited differentiator

$$|H_{id}(e^{j\omega})| = \begin{cases} -\omega/T & \omega < 0 \\ \omega/T & \omega > 0. \end{cases}$$

For one implementation of  $H(e^{j\omega})$ , referred to as  $H_1(e^{j\omega})$ , the designer, motivated by the definition

$$\frac{d[x(t)]}{dt} = \lim_{\Delta t \rightarrow 0} \frac{x(t) - x(t - \Delta t)}{\Delta t},$$

chooses the system impulse response  $h_1[n]$  so the input-output relationship is

$$y[n] = \frac{x[n] - x[n-1]}{T}.$$

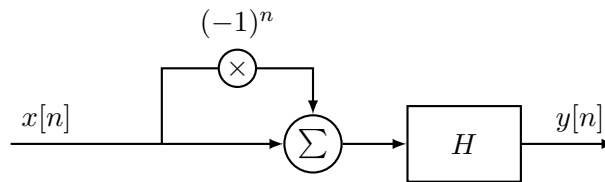
- a) Find  $H_1(e^{j\omega})$ .
- b) We are interested in how well this approximation matches against the ideal response  $H_{id}(e^{j\omega})$ . Find the difference in the squared magnitudes between the two responses, i.e.,

$$\text{error} = |H_1(e^{j\omega})|^2 - |H_{id}(e^{j\omega})|^2.$$

Represent your answer as 6th order polynomial function of  $\omega$ .

### 3. DTFT Systems

Consider the discrete-time system below where the subsystem  $H$  is an ideal low-pass filter with a passband gain of 1 and a cutoff frequency of  $\omega_c = \frac{\pi}{4}$ .



If we have an input  $x[n]$  that has a DTFT of

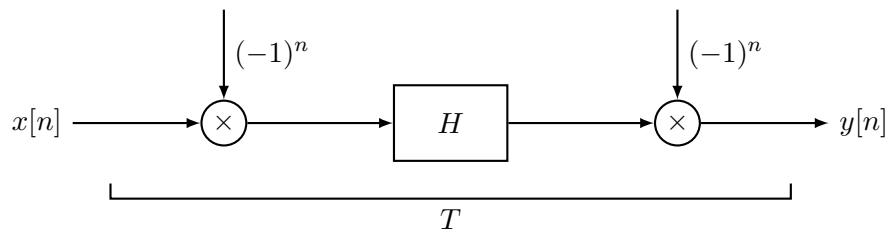
$$X(e^{j\omega}) = 2\Delta\left(\frac{\omega}{2\pi}\right)$$

from  $-\pi < \omega \leq \pi$ .

- Find an expression for the output  $Y(e^{j\omega})$  for  $-\pi < \omega \leq \pi$ .
- Find  $y[n]$  in the time domain.

## 4. More DTFT Systems

Consider the system below. The entire system  $y[n] = T\{x[n]\}$  will have the effect of being an ideal high-pass filter with a cutoff frequency of  $\omega_c = \frac{\pi}{2}$ .  $H$  is a subsystem within the larger system  $T$ .



- Derive an expression for  $H(e^{j\omega})$  such that the entire system will behave as an ideal high-pass filter as described above.
- Find  $h[n]$  that would satisfy such a system.

## 5. DTFT Pairs

The DTFT pair

$$a^n u[n] \Longleftrightarrow \frac{1}{1 - ae^{-j\omega}} \quad |a| < 1$$

is given.

- a) Determine the DTFT,  $X(e^{j\omega})$ , of the sequence

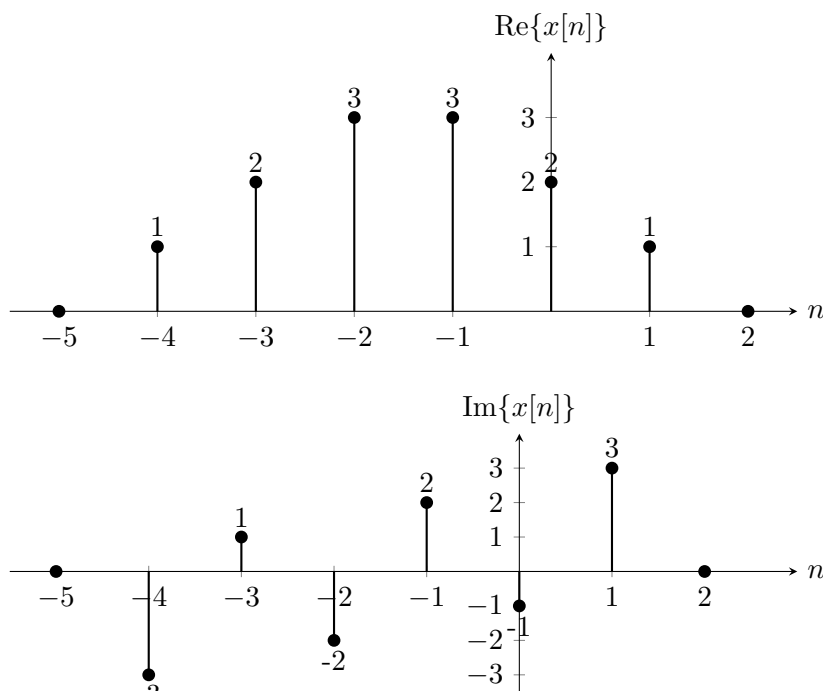
$$x[n] = -b^n u[-n - 1] = \begin{cases} -b^n & n \leq -1 \\ 0 & n \geq 0 \end{cases}.$$

- b) What restrictions must you put on  $b$  to make this a valid DTFT?
- c) Determine the sequence  $y[n]$  whose DTFT is

$$Y(e^{j\omega}) = \frac{2e^{-j\omega}}{1 + 2e^{-j\omega}}.$$

## 6. DTFT Values

$X(e^{j\omega})$  denotes the Fourier transform of the complex-valued signal  $x[n]$ , where the real and imaginary parts of  $x[n]$  are given below. (The sequence is zero outside the interval shown.)



- Evaluate  $X(e^{j\omega})|_{\omega=0}$ .
- Evaluate  $X(e^{j\omega})|_{\omega=\pi}$ .
- Evaluate  $\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$ .

## 7. Determining $h[n]$

Consider some input signal  $x[n] = -\delta[n] + \delta[n - 1]$  which passes through some causal system  $H$  which gives some output signal

$$y[n] = \delta[n] + \delta[n - 1] - \delta[n - 2] - \delta[n - 3].$$

- a) Find the impulse response  $h[n]$  for this LTI system.
- b) Find  $y_2[n]$ , which is the system response to the signal  $x_2[n] = \delta[n] - \delta[n - 5]$ .