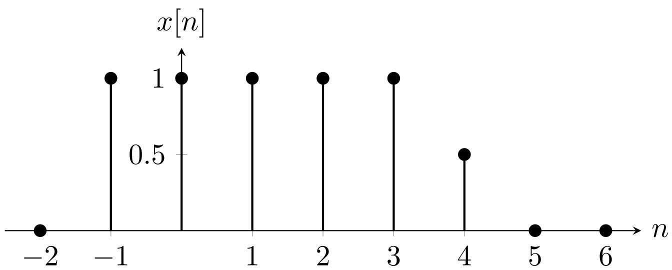
ECE 5210 hw01

1. Discrete-Time Signal Operations

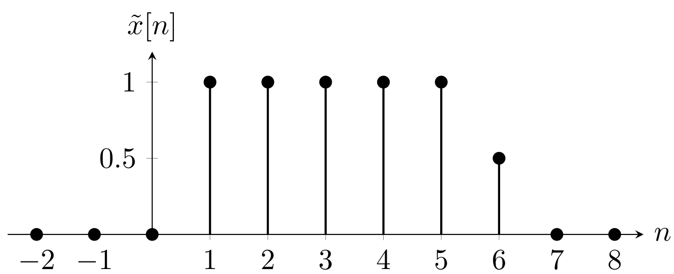
A discrete-time signal $x[n]$ is shown below:



Sketch the following signals $\tilde{x}[n]$:

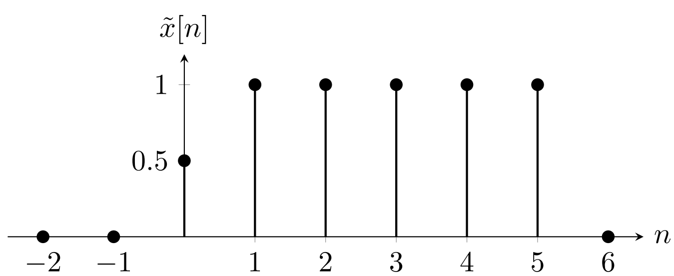
a) $\tilde{x}[n] = x[n-2]$

Solution:



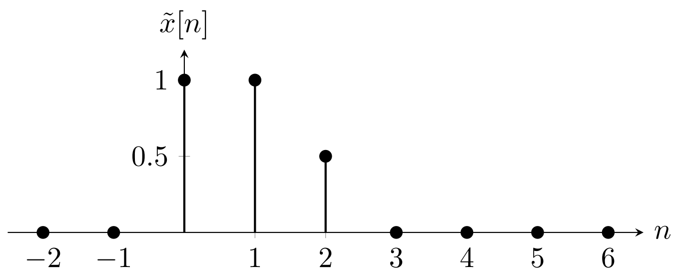
b) $\tilde{x}[n] = x[4-n]$

Solution:



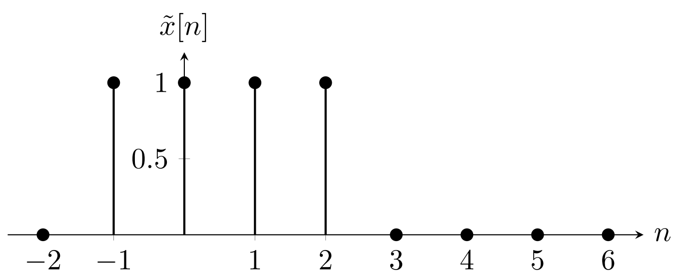
c) $\tilde{x}[n] = x[2n]$

Solution:



d) $\tilde{x}[n] = x[n]u[2-n]$

Solution:



2. Periodic Sinusoids

Determine whether each of the following signals is periodic. If the signal is periodic, state its period.

a) $x[n] = e^{j(\pi n/6)}$

> **Solution:** Yes. The fundamental period is $N = 12$ since
> $$e^{j(\pi(n+12)/6)} = e^{j(\pi n/6+2\pi)} = e^{j(\pi n/6)}.$$

b) $x[n] = e^{j(3\pi n/4)}$

> **Solution:** Yes. The fundamental period is $N = 8$ since
> $$e^{j(3\pi(n+8)/4)} = e^{j(3\pi n/4+6\pi)} = e^{j(3\pi n/4)}.$$

c) $x[n] = \frac{\sin(\pi n/5)}{\pi n}$

> **Solution:** No. The signal is not periodic since the denominator πn does not repeat for any integer N.

d) $x[n] = \cos(\sqrt{3}n)$

> **Solution:** No. The signal is not periodic since $\sqrt{3}$ is irrational, and thus there is no integer N such that $\sqrt{3}N$ is an integer multiple of 2π.

e) $x[n] = e^{j\pi n/\sqrt{5}}$

> **Solution:** No. The signal is not periodic since $\sqrt{5}$ is irrational, and thus there is no integer N such that $\pi N/\sqrt{5}$ is an integer multiple of 2π.

3. Sinusoid Sampling

The signal

$$x_c(t) = \cos(2\pi(600)t)$$

was sampled with a sampling period $T = 1/300$ to obtain a discrete-time signal $x[n]$. What is the resulting sequence $x[n]$?

Solution: The discrete-time signal $x[n]$ is given by

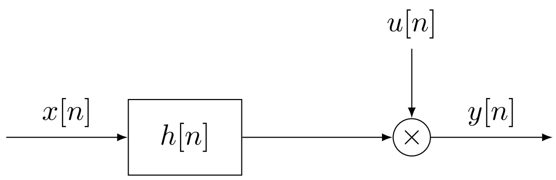
$$x[n] = x_c(nT) = \cos(2\pi(600)nT) = \cos\left(2\pi(600)n \cdot \frac{1}{300}\right) = \cos(2\pi(2)n) = \cos(4\pi n).$$

Since $\cos(4\pi n) = \cos(0 \cdot n)$ which is 1 for all integer n. Therefore

$$x[n] = 1 \quad \text{for all } n.$$

4. System Analysis

Consider the system illustrated below. The output of an LTI system with an impulse response $h[n] = (1/4)^n u[n+10]$ is multiplied by a unit step function $u[n]$ to yield the output of the overall system. Answer each of the following questions.



a) Is the overall system LTI?

> **Solution:** No. The multiplication by the unit step function $u[n]$ makes the system time-varying, since the output depends on the time index n explicitly through $u[n]$.

b) Is the overall system causal?

> **Solution:** No. The impulse response $h[n] = (1/4)^n u[n+10]$ is non-zero for $n < 0$ (specifically, for $n \geq -10$), indicating that the system responds to inputs before they occur, which violates the causality condition.
>
> Even though the multiplication by $u[n]$ at the output stage enforces causality on the output, the overall system is still non-causal due to the non-causal nature of the LTI component which will require inputs from the future to produce outputs at certain times.

c) Is the overall system stable in the BIBO sense?

> **Solution:** Yes. The impulse response $h[n] = (1/4)^n u[n+10]$ is absolutely summable since
>
> $$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-10}^{\infty} \left| \left(\frac{1}{4}\right)^n \right| = \sum_{n=-10}^{\infty} \left(\frac{1}{4}\right)^n = \left(\frac{1/4^{-10}}{1-1/4}\right) = \frac{4^{10}}{3} < \infty.$$
>
> The multiplication by $u[n]$ does not affect the boundedness of the output for bounded inputs, as it only zeroes out the output for $n < 0$. Therefore, the overall system is BIBO stable.

5. System Characterization

For each of the following systems, determine whether the system is stable, causal, linear, time-invariant, and memoryless.

a) $T\{x[n]\} = g[n]x[n]$ with $g[n]$ given (and is stable)

> **Solution:** The system is stable, causal, linear, time-variant, and memoryless.
>
> The multiplication by a given stable sequence $g[n]$ does not affect these properties.

b) $T\{x[n]\} = \sum_{k=n_0}^{n} x[k], \qquad n \neq 0$

> **Solution:** The system is unstable, non-causal, linear, time-variant, and has memory.
>
> The summation from a fixed index n_0 to the current index n ensures causality and stability for bounded inputs. The system is linear due to the summation operation. However, it is time-variant because the limits of summation depend on n. The system has memory since the output at time n depends on past input values.

c) $T\{x[n]\} = ax[n] + b$

> **Solution:** The system is stable, causal, time-invariant, and is memoryless. It is linear only if $b = 0$.
>
> The addition of a constant b makes the system non-linear unless $b = 0$. The system is causal since the output at time n depends only on the input at time n. It is time-invariant because shifting the input signal results in an equivalent shift in the output signal. The system is memoryless since the output depends only on the input value at time n.

d) $T\{x[n]\} = x[n] + 3u[n + 1]$

> **Solution:** The system is stable, causal, time-invariant, and is memoryless.
>
> It is non-linear due to the addition of the step function $3u[n+1]$. The system is causal since the output at time n depends only on the input at time n and the step function which is non-zero for $n \geq -1$. It is time-invariant because shifting the input signal results in an equivalent shift in the output signal. The system is memoryless since the output depends only on the input value at time n.

e) $T\{x[n]\} = (\cos(\pi n))x[n]$

> **Solution:** The system is stable, causal, linear, time-variant, and memoryless.
>
> The multiplication by the time-varying sequence $\cos(\pi n)$ makes the system time-variant. The other properties remain unaffected.