ECE 5210 Midterm 1

Week of: February 20, 2025

Student's name: ______________________________

Instructor:
Eric Gibbons
ericgibbons@weber.edu
801-626-6861

You have 2.5 hours for 5 problems.

- Show enough (neat) work in the clear spaces on this exam to convince us that you derived, not guessed, your answers.
- Put your final answers in the boxes at the bottom of the page.

You are allowed ONE page of notes (front and back) for this exam. You may use a graphing calculator of your choice. Consulting with any third party is considered cheating.

Problem	Score	Possible Points
1		20
2		20
3		20
4		20
5		20
Total score		100

1 Short Answer

(a) Consider the system described by the equation

$$y[n] = |x[2n-2]|.$$

(i) Is this system linear?

Linear   Nonlinear

(ii) Is this system time-invariant?

Time-invariant   Time-varying

(iii) Is this system causal?

Causal   Non-causal

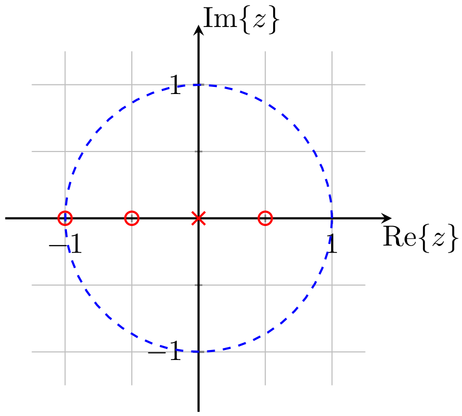
(iv) Is this system memoryless?

Memoryless   Not memoryless

(v) Is this system stable?

Stable   Unstable

(b) Consider the pole-zero plot, where the poles are marked with an "X" and the zeros are marked with an "O". Note: there are three poles, repeated, at the origin.



(i) Is this system stable?

Stable	Unstable

(ii) What is the transfer function $H(z)$?

$H(z) =$

(iii) What is the impulse response $h[n]$?

$h[n] =$

(iv) Is this system FIR or IIR?

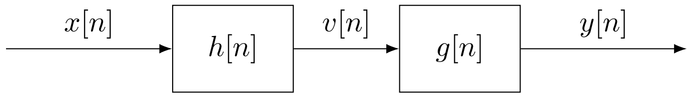
FIR	IIR

(v) What is the region of convergence?

ROC:

2 Z-transform

A stable and causal system is shown below, consisting of a cascade of two LTI discrete-time filters such that the overall system response will yield $y[n] = x[n]$. The first filter has an unknown impulse response



$h[n]$. The second filter is defined by the difference equation

$$y[n] = v[n] - \frac{3}{4}v[n-1] + \frac{1}{8}v[n-2].$$

(a) Find the transfer function $G(z)$.

$G(z) =$

(b) Find the impulse response $g[n]$.

$g[n] =$

(c) Find the transfer function $H(z)$.

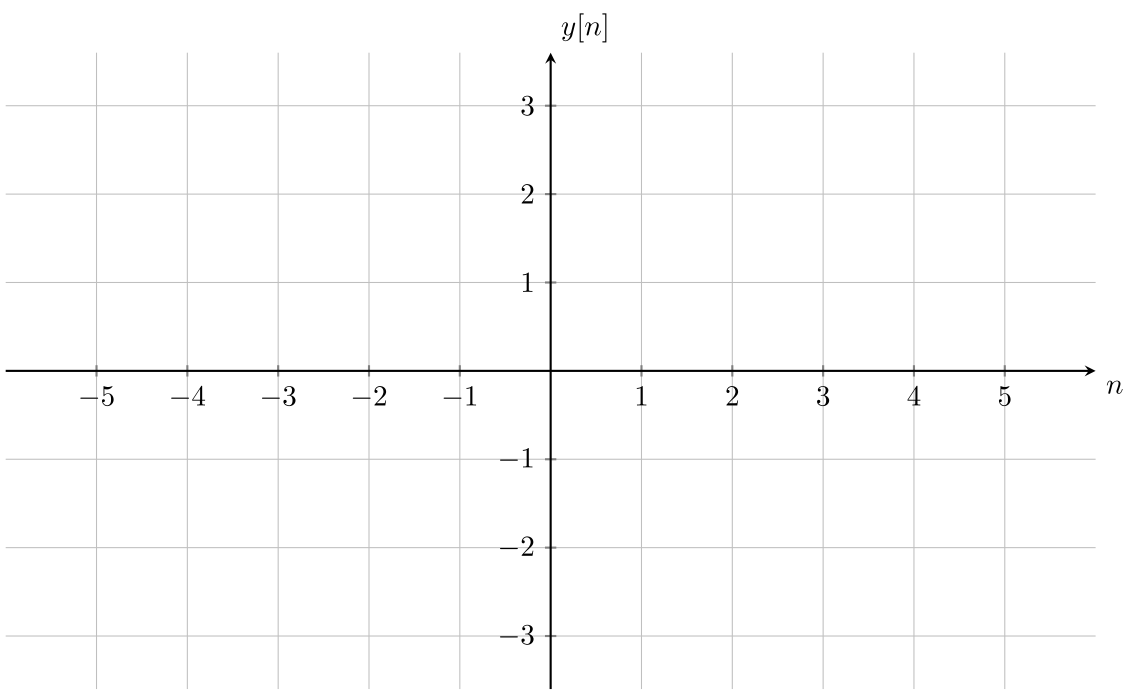
$H(z) =$

(d) Find the impulse response $h[n]$.

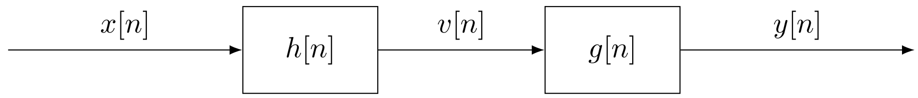
$h[n] =$

3 System response

(a) Suppose we have some LTI system with known impulse response $h[n] = u[n-1] - u[n-4]$. What is the system's output to input $x[n] = \cos(\pi n)u[n+1]$? Plot the result $y[n]$ as a stem plot on the axes below from $n = -5$ to $n = 5$.



(b) Consider the system below with input $x[n]$ and output $y[n]$.



Subsystems $h[n]$ and $g[n]$ are known to be LTI. The output of $h[n]$ is $v[n]$ and can be described by the difference equation

$$v[n] = x[n] - x[n-1].$$

Additionally, subsystem $g[n]$ is described by

$$y[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}.$$

(i) Find the impulse response $h[n]$.

$h[n] =$

(ii) Consider some input

$$x[n] = \cos(0.4\pi n) + \sin(0.6\pi n) + u[n].$$

What is the output $y[n]$? Note: The cosine and sine terms are everlasting in that they are not multiplied by $u[n]$.

$y[n] =$

4 DTFT

(a) (i) Suppose you were to implement a three sample moving average filter. The impulse response is

$$h[n] = \frac{1}{3}\left(\delta[n] + \delta[n-1] + \delta[n-2]\right).$$

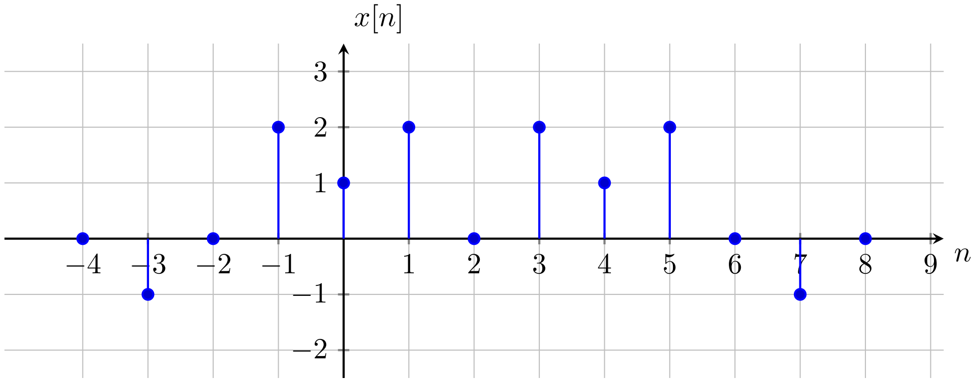
What is the frequency response of this filter?

$H(e^{j\omega}) =$

(ii) Suppose we were to send a signal $x[n]$ into this filter that is some cosine $x[n] = \cos\left(\frac{2\pi}{3}n\right)$. What is the output $y[n]$?

$y[n] =$

(b) Consider the signal plotted below.



(i) Evaluate $X(e^{j\omega})\big|_{\omega=0}$.

$X(e^{j\omega})\big|_{\omega=0} =$

(ii) Evaluate $X(e^{j\omega})\big|_{\omega=\pi}$.

$X(e^{j\omega})\big|_{\omega=\pi} =$

(iii) Find $\angle X(e^{j\omega})$.

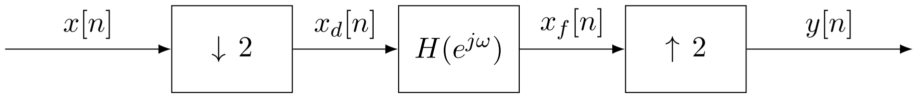
$\angle X(e^{j\omega}) =$

(iv) Find $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$.

$\int_{-\pi}^{\pi} X(e^{j\omega})d\omega =$

5 Decimation and Interpolation

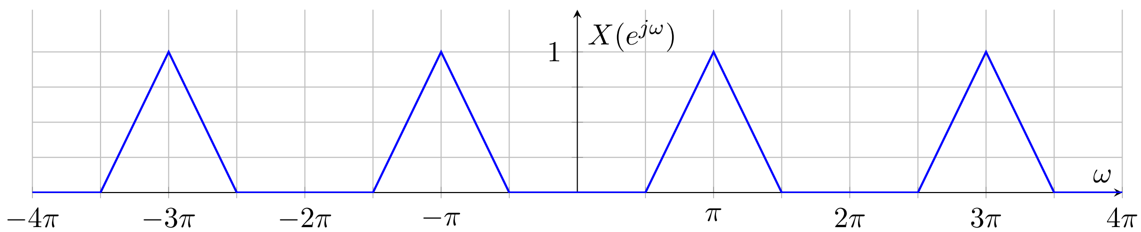
Consider the system below.



The system $H(z)$ has a frequency response from $-\pi < \omega \leq \pi$

$$H(e^{j\omega}) = \begin{cases} 2 & |\omega| < \frac{\pi}{2} \\ 0 & \text{else.} \end{cases}$$

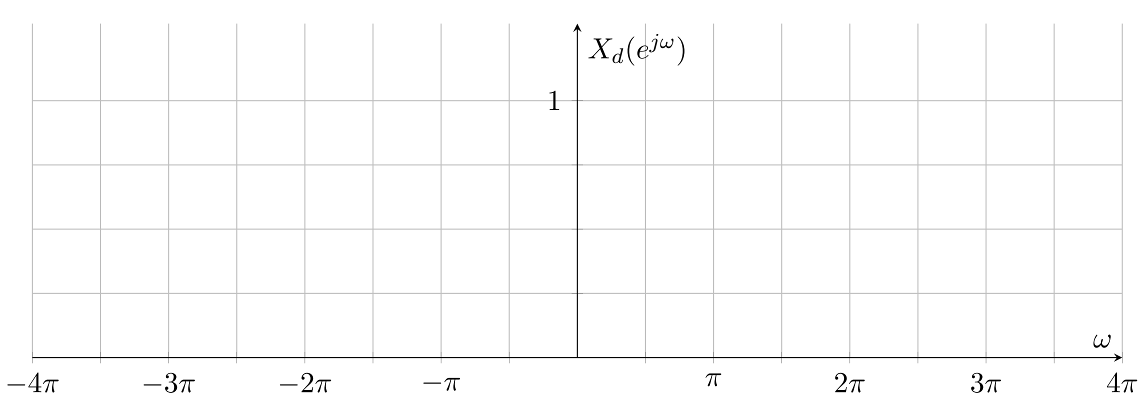
Suppose the input $x[n]$ has a DTFT $X(e^{j\omega})$ as shown below.



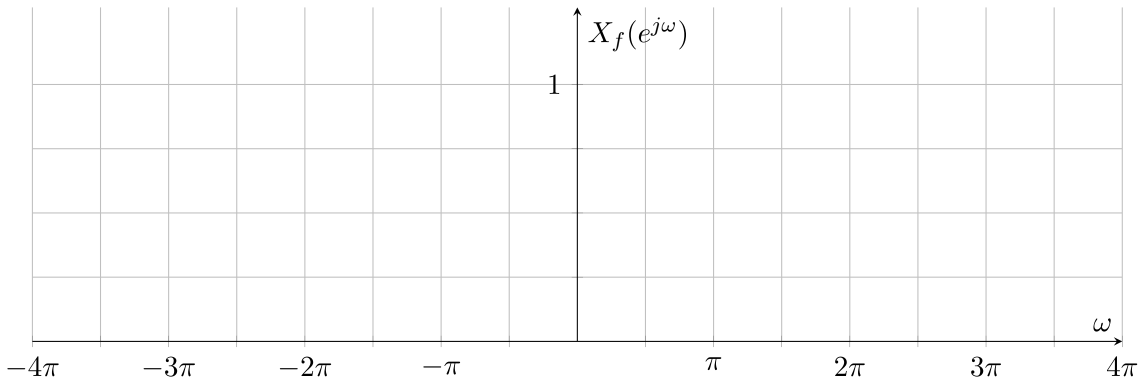
(a) What is the input $x[n]$? It might be helpful to remember the DTFT pair $\frac{\omega_c}{2\pi}\text{sinc}^2\left(\frac{\omega_c n}{2}\right) \Longleftrightarrow \Delta\left(\frac{\omega}{2\omega_c}\right)$ (defined for $-\pi \leq \omega \leq \pi$).

$x[n] =$

(b) Sketch the DTFT $X_e(e^{j\omega})$, the output of the downsampler.



(c) Sketch the DTFT $X_f(e^{j\omega})$, the output of the filter.



(d) Sketch the DTFT $Y(e^{j\omega})$, the output of the upsampler.