550.420 Probability - FALL 2015 **MIDTERM 1**

PRINT NAME HERE______________________________

ATTENDING SECTION #__________

1 = Th@10:30 Mingyue, **2**=Th@12:00 Joshua, **3**=Th@1:30 Jen, **4**=Th@3:00 Addison

I agree to complete this examination without unauthorized assistance from any person, materials, or device.

Signature: ____

Instructions: This is a closed book examination. No notes are permitted. No cell phone use is permitted. Answers are to be written clearly and concisely, and clearly labeled (for example, with a box drawn around the intended answer). Please answer each question on the page on which it is stated, using the back of the page if necessary. It is important to show and explain your work; your answer must be properly justified (with at least key words and phrases) to receive full credit.

Problem 1. [20 pt]
Problem 2. [10 pt]
Problem 3. [10 pt]
Problem 4. [10 pt]
Problem 5. [10 pt]
Problem 6. [20 pt]
Problem 7. [10 pt]
Problem 8. [10 pt]
Total = 100 pts.

1. A box contains 2 red, 4 green marbles. You randomly select a marble, note its color, and put it back, and repeat.

You do not need to simplify your answers below.

(a) Find the probability that in 4 draws you will see 2 red marbles.

(b) Find the probability that the first red marble occurs on the 4th draw.

(c) Find the probability that the third red marble occurs on the 8th draw.

(d) If the first red marble occurred on the 4th draw, find the probability that the third red marble occurs on the 8th draw.

2. Nine children are randomly seated in three rows of three desks each.

Let A be the event that Alan and Betty are in the same row.

Let B be the event that Alan and Betty are each seated in one of the (four) corner desks.

Are A and B independent? *Use the definition of independence to justify your assertion.*

3. Each day you buy one lottery ticket even though the probability you will win money is only 1/1000 independent from day-to-day. *Do not simplify your answers below.*
(a) You play for 500 straight days. Find the probability you win at least once.
(b) *Approximate* the probability in part (a).

4. Box A has 2 red and 1 green marble in it, Box B has 1 red and 1 green marble in it. A marble is selected at random from box A and transferred to box B. Then a marble is drawn from box B. If a red marble is drawn from box B, find the probability a red marble was transferred.
To fix notation: let R be the event a red marble is drawn from box B and let R_T be the event that a red marble is transferred.

5. Consider the sample space of all possible 30-long sequences of **1**'s, **2**'s, and **3**'s. Assume that each possible sequence is equally-likely. We define A_i to be the event that there are exactly 10 i's for $i = 1, 2, 3$. Compute the probability that at least one of the numbers **1**, **2**, or **3** appears exactly 10 times in a sequence. *You do not need to simplify.*

Be sure to write down the event of interest in terms of the A_i.

6. (a) The facts $P(A \cup B) = P(A) + P(A^c \cap B)$ and $P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C)$ can be easily verified by a Venn diagram. Do not verify these facts but use them to show that

$$P(A \cup B \cup C \cup D) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C) + P(A^c \cap B^c \cap C^c \cap D).$$

(b) Compute the probability of at least 1 spade in a hand of 3 cards in *two* ways - one of which uses part (a). *Do not simplify either.*

7. This problem is a slight modification of the game YAHTZEE©. We wish to compute the probability of 3-of-a-kind rolling 3 dice in at most two stages: in the first stage, you roll the 3 dice. If you roll 3-of-a-kind, your done! Otherwise, we move to the next stage, where you get to re-roll at most 2 of the dice to get your 3-of-a-kind as follows: if you rolled a pair in stage 1, then re-roll the one die that is different; if you rolled 3 distinct numbers in stage 1, then grab any two and re-roll to try to match the other number. What is the probability of getting 3-of-a-kind in this scenario?

You do not have to simplify.

8. A room contains m married couples. The $2m$ people are randomly into m pairs. Let M be the random variable that counts the number of married couples paired. Compute $E(M)$. *Simplify completely.*