550.420 Intro to Probability - Spring 2017 **Homework #04**
Due: in lecture *Friday, Mar. 3.*

A.4.1. The genetic code specifies an amino acid by a sequence of three nucleotides. Each nucleotide can be one of four kinds T, A, C, or G, with repetition permitted. How many amino acids can be coded in this manner?

A.4.2. The Morse code consists of a sequence of dots and dashes with repetition permitted.
(a) How many letters can be coded with exactly n symbols?
(b) What is the number of letters that can be coded with n or fewer symbols?

A.4.3. A person has n keys exactly one of which fits the lock. The keys are tried one at a time, at each trial choosing at random from the keys that were not tried earlier. Find the probability that the rth key tried is the correct key.

A.4.4. A bus starts with 6 people and stops at 10 different stops. Assuming that passengers are equally likely to depart at any stop, find the probability that no two passengers leave at the same bus stop.

A.4.5. Suppose we have r boxes. Balls are placed at random one at a time into the boxes until, for the first time, some box has 2 balls. Find the probability that this occurs with the n ball.

A.4.6. A box has r balls labeled $1, 2, \ldots, r$. N balls (where $N \leq r$) are selected at random from the box, their numbers noted, and the N balls are then returned to the box. If this procedure is done r times, what is the probability that none of the original N balls are duplicated?

A.4.7. If Alice and Betty are among n women who are arranged at random in a line, what is the probability that exactly k women stand between them?

A.4.8. Consider the problem of matching n objects, and let i and r denote distinct specified positions.
(a) What is the probability that a match occurs at position i and no match occurs at position r?
(b) Given there is no match at position r, what is the probability of a match at position i?

A.4.9. Suppose n balls are distributed into n boxes.
(a) Show that the probability that box 1 is the only empty box is $\binom{n}{2}(n-1)!/n^n$.
(b) What is the probability that exactly one box is empty?
(c) Given that box 1 is empty, what is the probability that only one box is empty?
(d) Given that only one box is empty, what is the probability that box 1 is empty?

A.4.10. If n balls are distributed at random into r boxes, what is the probability that box 1 has exactly j balls, $0 \leq j \leq n$?

A.4.11. A box has b black balls and r red balls. Balls are drawn from the box one at a time without replacement. Find the probability that the first black ball selected is drawn at the n trial.

A.4.12. A box has 10 balls labeled $1, 2, \ldots, 10$. Suppose a random sample of size 3 is selected. Find the probability that balls 1 and 6 are among the three selected balls.

A.4.13. A box contains 40 good and 10 defective fuses. If 10 fuses are selected, what is the probability they will all be good? What is the probability there will be at most one defective?

A.4.14. Five cards are dealt to you from a well-shuffled standard deck of 52 cards. Compute the probabilities of the following:
(a) Royal flush (i.e., {10,J,Q,K,A} of the same suit)
(b) Straight flush (five cards of the same suit in sequence)
(c) Four of a kind (face values of the form $\{x, x, x, x, y\}$ where x and y are distinct)
(d) Full house (face values of the form $\{x, x, x, y, y\}$ where x and y are distinct)
(e) Flush (five cards of the same suit but not a straight flush nor a royal flush)
(f) Straight (five cards in sequence, regardless of suit - just not all the same suit)
(g) Three of a kind (face values of the form $\{x, x, x, y, z\}$ where x, y, and z are distinct)
(h) Two pair (face values of the form $\{x, x, y, y, z\}$ where x,y, and z are distinct)
(i) One pair (face values of the form $\{w, w, x, y, z\}$ where w, x, y, and z are distinct)

A.4.15. Cards are dealt from a standard deck of 52 one at a time until the first king appears. Find the probability that this occurs with the nth card dealt.

A.4.16. Roll 4 balanced 6-sided dice. Which event is more likely: Getting exactly one pair or getting distinct up-faces? Compute the probability of each event.

A.4.17. Two boxes each have r balls labeled $1, 2, \ldots, r$. A random sample of size $n \leq r$ balls is drawn without replacement from each box. Find the probability that the samples contain exactly k balls having the same numbers in common.

A.4.18. Suppose there are 100 people taking a course in probability and the professor decides to award letter grades at random to each student.
(a) In how many distinct ways can the professor award 30 A's, 45 B's, 15 C's, 8 D's, and 2 F's? Do not simplify to an integer. Please.
(b) Suppose each of the awardings in part (a) are equally likely. What is the probability that Allan and Anne receives A's and Betty and Bob receive B's? Do not simplify. ANS: $\approx .0183$

A.4.19. 40 people in a room are comprised of 20 married (female/male) couples.
(a) All 40 people are randomly paired. How many distinct pairings are possible?
(b) Fred and Carrie are a couple. How many pairing of the 40 people have Fred and Carrie paired?
(c) Assuming all pairing are equally likely, what is the probability that Fred and Carrie are paired? Could you have gotten this answer intuitively? How?