550.420 Intro. to Probability - Spring 2017 **Homework #07**

Due by Friday, March 17 in lecture.**

** Can submit to your teaching assistant earlier if you will not be in lecture Friday.

From the textbook:

Chapter 4 / Problems 4.19, 4.22, 4.33, 4.38, 4.61

Chapter 4 / Theoretical exercises 4.10, 4.16†, 4.19, 4.36ß.

†: problems like this are very useful in statistics for finding maximum likelihood estimators.

ß: this problem is the precursor to *convolutions* which we will learn later in greater generality.

Additional problems:

A.7.1. A box has 3 red and 7 black marbles. A marble is randomly drawn, its color noted and then replaced in the box and the experiment is repeated indefinitely. For each $n \geq 1$ let X_n be the random variable that counts the number of red marbles drawn in the first n trials. Show that if $0 < k \leq m < n$, then

$$P(X_m = x | X_n = k) = \frac{\binom{m}{x}\binom{n-m}{k-x}}{\binom{n}{k}} \quad \text{for } x = 0, 1, \ldots, k.$$

This is one example of how the *hypergeometric* distribution arises in the context of the binomial distribution.

A.7.2. Suppose $W \sim \text{Poisson}(\lambda)$, where $0 < \lambda < 1$, i.e., W has a Poisson distribution with parameter λ and λ is positive but *less* than 1. Compute $E(W!)$, i.e., expected value of W-factorial.