HW#3

A.3.1

$$P(H_1) = \frac{13}{52} = \frac{1}{4}$$

$$P(F_2) = P(F_2 | F_1) P(F_1) + P(F_2 | F_1^c) P(F_1^c)$$

$$= \frac{11}{51} \cdot \frac{12}{52} + \frac{12}{51} \cdot \frac{40}{52} = \frac{132 + 480}{52(51)}$$

$$= \frac{612}{52(51)} = \frac{12\cancel{(51)}}{52\cancel{(51)}} = \frac{12}{52} = P(F_1)$$

notice

$$P(H_1 \cap F_2) = P(H_1 \cap F_2 | H_1 \cap F_1) P(H_1 \cap F_1)$$

$$+ P(H_1 \cap F_2 | H_1 \cap F_1^c) P(H_1 \cap F_1^c).$$

$$= \frac{11}{51} \cdot \frac{3}{52} + \frac{12}{51} \cdot \frac{10}{52} = \frac{33 + 120}{52(51)}$$

$$= \frac{153}{52(51)} = \frac{3\cancel{(51)}}{52\cancel{(51)}} = \frac{3}{52}$$

Notice that

$$\frac{3}{52} = P(H_1 \cap F_2) = P(H_1) P(F_2) = \frac{1}{4} \cdot \frac{12}{52} \checkmark$$

Thus, H_1 and F_2 <u>are</u> independent.

$\boxed{A.3.2}$ We are told

$P(L) = .10$

$P(C|L) = .95$

$P(C|L^c) = .50$

(a) $P(C) = P(C|L)P(L) + P(C|L^c)P(L^c)$

$= .95 \cdot .10 + .50 \cdot .90$

$= .095 + .45$

$= .545$ or 54.5% of people have Counter-clockwise Cowlicks.

(b)

$$P(L|C) \overset{\text{Baye's rule}}{=} \frac{P(C|L)P(L)}{P(C)} = \frac{.95 \cdot (.10)}{.545} = \frac{.095}{.545}$$

$\approx .1743$ or about a 17.43% chance our newborn will have Left tendencies.

A.3.3

(a)

TTTTT $(1-p)^5$

TTTTH $p(1-p)^4$
TTTHT $p(1-p)^4$
TTHTT $p(1-p)^4$
THTTT $p(1-p)^4$
HTTTT $p(1-p)^4$

TTTHH $p^2(1-p)^3$
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TTHHH $p^3(1-p)^2$
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HHHHT $p^4(1-p)$

HHHHH p^5

The above shows the assignment of probability to each sample point of the experiment.

(b) $P(\text{exactly two heads}) = 10\,p^2(1-p)^3$.

$\boxed{A.3.4}$ refer to previous problem (Head = accident)

(a) $P(\text{exactly two students}) = 10\,(.05)^2(.95)^3$

$= .02143375 \approx .0214$

(b) $P(\text{none of students have accident}) = P(\{TTTTT\}) = (.95)^5 \approx .774$

(c) $P(\text{all students have accident}) = P(\{HHHHH\}) = (.05)^5 \approx 3.125\times 10^{-7}$

$\approx .0000003125$

(d) $P(\text{at least one student has accident})$

$= 1 - P(\text{no accidents}) \approx 1 - .774 \approx .246.$

$\boxed{A.3.5}$ see previous two problems (Head = guess correctly)
prob. guess correctly on a question $= \frac{1}{5} = .20$

(a) $P(\text{exactly 2 correct}) = 10\,(.2)^2(.8)^3 = .2048$

(b) $P(\text{no answer correct}) = (.8)^5 \approx .32768$

$\boxed{A.3.6}$ $P(S) = \frac{2}{10} = .2 \qquad P(S^c) = \frac{8}{10} = .8$

$P(\text{spell MIAMI} \mid S) = 1 \qquad P(\text{spell MIAMI} \mid S^c) = .5$

$P(\text{spell MIAMI}) = P(\text{spell MIAMI} \mid S)P(S) + P(\text{spell MIAMI} \mid S^c)P(S^c)$

$= 1 \cdot (.20) + .5(.8) = .2 + .4 = \boxed{.6}$

$\boxed{A.3.7}$ Let W be the event that your neighbor waters your sickly plant

Told $P(W) = .9$, $P(W^c) = .1$

we are also told

$$P(D|W) = .15 \quad \text{and} \quad P(D|W^c) = .80.$$

where D is the event our sickly plant dies.

(a) $P(D^c) = P(D^c|W)P(W) + P(D^c|W^c)P(W^c)$

$= .85(.90) + .20(.10)$

$= .785$

(b) $P(W^c|D) = \dfrac{P(D|W^c)P(W^c)}{P(D)} = \dfrac{(.80)(.10)}{1 - P(D^c)} = \dfrac{.08}{.215}$

$\approx .372$

$\boxed{A.3.8}$ Let F be the event that the sum total of your dice roll(s) is 4 or more, Let E_1 be the event you initially roll a 1

$P(F) = \underbrace{P(F|E_1)}P(E_1) + P(F|E_2)P(E_2) + P(F|E_1^c \cap E_2^c)P(E_1^c \cap E_2^c)$

$= P(3 \text{ or } 4) \cdot \frac{1}{4} + P(2 \text{ or } 3 \text{ or } 4) \cdot \frac{1}{4} + \dfrac{P(\text{getting a } 4)}{P(\text{getting } 3 \text{ or } 4)} \cdot \frac{1}{2}$

$= \frac{1}{8} + \frac{3}{16} + \frac{1}{4} = \frac{2+3+4}{16} = \frac{9}{16}.$

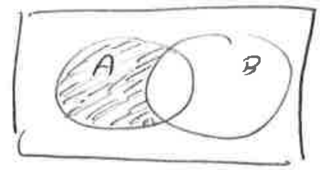
$\boxed{A.3.9}$ A and B are given independent

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$= P(A) - P(A)P(B)$$

$$= P(A)[1 - P(B)]$$

$$= P(A)P(B^c).$$



$A \cap B^c$ shaded

and therefore A and B^c are independent as well.

$\boxed{A.3.10}$

(a) $\left(\frac{1}{2}\right)^{12} = \frac{1}{4096} \approx .000244.$

(b) Since $A_1, A_2, \dots, A_{13}$ are mutually exclusive (we cannot have more than one team lose <u>all</u> its coin tosses)

$$P\left(\bigcup_{i=1}^{13} A_i\right) = P(A_1) + P(A_2) + \cdots + P(A_{13}) = 13\left(\tfrac{1}{2}\right)^{12} \approx .00317.$$

(c) Since whether a team from a different conference wins/loses a coin flip shouldn't influence coin flip from other conferences $B_1, B_2, \dots, B_{1000}$ are independent. Moreover,

$$P\left(\bigcup_{i=1}^{1000} B_i\right) = 1 - P\left(\bigcap_{i=1}^{1000} B_i^c\right) = 1 - \prod_{i=1}^{1000} P(B_i^c) = 1 - (1 - P(B_i))^{1000}$$

(This would <u>NOT</u> be nationally Newsworthy) $\star$ $= 1 - (1 - .00317)^{1000} \approx .958368\ldots$