550.420 Introduction to Probability - Spring 2017 **Homework #03**
Due Friday, Feb. 24 in lecture.

A.3.1. Consider an experiment where you draw two cards from a standard deck of 52 cards. Let H_i be the event you draw a *heart* on the ith draw and let F_i be the event you draw a *face card* on the ith draw ($i = 1, 2$). Verify whether or not the events H_1 and F_2 are independent. *This is a little challenging!*

A.3.2. Assume that in a ceratin population 10% of people are left-handed (more specifically have *left tendencies*). 95% of people with left-tendencies have counter-clockwise cowlicks (spiral behavior of a person's hair on the top of their head), while 50% of people without left tendencies have counter-clockwise cowlicks.
(a) What percentage of people have counter-clockwise cowlicks?
(b) If you have a child born with a counter-clockwise cowlick, what's the chance they will have left tendencies?

A.3.3. Imagine we toss a (possibly biased) coin 5 times in such a way that the outcomes on any trial are statistically independent of the outcomes on any other trial. The experiment observes the sequence of heads (`H`) and tails (`T`). Here are the outcomes of the experiment listed for your convenience:

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            HTTTH  THHHT
            HTTHT  THHTH
            HTHTT  THTHH
            HHTTT  TTHHH
            THTTH  HTHHT
     HTTTT  THTHT  HTHTH  THHHH
     THTTT  THHTT  HTTHH  HTHHH
     TTHTT  TTHHT  HHTTH  HHTHH
     TTTHT  TTHTH  HHTHT  HHHTH
TTTTT  TTTTH  TTTHH  HHHTT  HHHHT  HHHHH
```

If we define H_i to be the event that a head occurs on the ith trial (and, therefore, H_i^c a tail on ith trial), then each sample point is the intersection of five such events, eg., $\{HHTHT\} = H_1 \cap H_2 \cap H_3^c \cap H_4 \cap H_5^c$. Assume the probability the coin comes up heads on any trial is a constant p (and, therefore, tails is $1 - p$).
(a) Assign a probability to each sample point in Ω. Why was independence of the H_i's important to create this assignment? Note that if $p \neq \frac{1}{2}$, then this discrete sample space is *not* an equally likely sample space.
(b) If you did the assignment correctly in part (a) you will notice that all the sample points with the same total number of heads will be assigned the same probability, and therefore, the probability of observing a given number of heads is simply this common probability times the number of sample points in Ω with that given number of heads. Compute the probability of exactly two heads.

A.3.4. Five students decide to take a weekend hiking trip. Each student may or may not have an accident over the weekend *independently* of other students on the trip, and each student has a constant probability of having an accident over the weekend of .05.
(a) What is the probability that exactly two students will have an accident over the weekend?
(b) What is the probability that none of the students have an accident over the weekend?
(c) What is the probability that all the students have an accident over the weekend?
(d) What is the probability that at least one student will have an accident over the weekend?

A.3.5. An unprepared student is to sit for a 5-question multiple choice quiz. Each question has 5 possible answers of which only one is correct. If the student guesses the answer to each of the five questions, what is the probability...
(a) the student gets exactly two questions correct?
(b) student gets none of the questions correct?

A.3.6. Two letters have fallen off a sign that is supposed to spell `MIAMI`. A friendly drunk puts the two letters back into the two empty places in a random order. What is the probability the sign spells `MIAMI` when the drunk is finished? Hint: let S be the event that the two letters that have fallen are the same.

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A.3.7. You ask your neighbor to water a sickly plant while you are on vacation. Without water it will die with probability 0.8. With water it will die with probability 0.15. You are 90% certain that your neighbor will remember to water the plant.
(a) What is the probability that the plant will be alive when you return?
(b) If the plant is dead when you return, what is the probability that your neighbor forgot to water it?

A.3.8. You roll a balanced 4-sided die once. If you roll a 1 or a 2 you get to roll the die one more time. Compute the probability that the sum total of your rolls is 4 or more.

A.3.9. Let A and B be independent events. Show that A and B^c are independent.

A.3.10. (*How unlucky is it?*) It has been noted that a certain H.S. football team lost all its coin flips in the entire season of 12 games.
(a) Assuming the outcomes are independent what is the probability that a team incorrectly guesses the toss 12 times in a row? ANSWER:$\approx .000244$.
(b) In order to get a feeling for how rare (or not rare) this situation is we imagine that there are 1000 H.S. football conferences in the U.S. and within each conference there are 13 teams. Within a particular conference, what is the probability there exists a team that loses every coin flip of the 12 game season? Hint: Let A_j be the event that team j loses all its coin flips for the season, then $A = \bigcup_{j=1}^{13} A_j$ is the event that at least one team in a conference with 13 teams loses all its coin flips. ANSWER: $\approx .00317$.
(c) Now, let B_i be the event that at least one team in conference i loses all its coin flips ($i = 1, 2, \ldots, 1000$). From part (b) we note $P(B_i) \approx .00317$ for each i. Furthermore, intuitively the events $B_1, B_2, \ldots, B_{1000}$ are independent (do you see why?). What is the probability that at least one team in the U.S. will lose all its coin flips in the season, i.e., compute $P(\bigcup_{i=1}^{1000} B_i)$. In context, is a team losing 12 coin flips in a row *nationally* new-worthy?