

## Common discrete distributions; descriptions, pmfs, means and variances.

### The Bernoulli( $p$ ).

$X = 1$  if a success occurs,  $X = 0$  if a failure occurs.

$$p(x) = p^x(1-p)^{1-x} \text{ for } x = 0, 1.$$

$$E(X) = p, \text{Var}(X) = p(1-p).$$

### The binomial( $n, p$ ).

$X$  counts the number of successes in  $n$  independent Bernoulli( $p$ ) trials.

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ for } x = 0, 1, \dots, n.$$

$$E(X) = np, \text{Var}(X) = np(1-p).$$

NB: if  $X_1, X_2, \dots, X_n$  are independent Bernoulli( $p$ ) random variables, then

$X = \sum_{i=1}^n X_i$  has a binomial( $n, p$ ) distribution.

### The geometric( $p$ ).

$X$  returns the trial of the first success in repeated independent Bernoulli( $p$ ) trials.

$$p(x) = p(1-p)^{x-1} \text{ for } x = 1, 2, 3, \dots,$$

$$E(X) = 1/p, \text{Var}(X) = (1-p)/p^2.$$

### The negative binomial( $r, p$ ) a.k.a. Pascal( $r, p$ ).

$X$  returns the trial of the  $r$ -th success in repeated independent Bernoulli( $p$ ) trials.

$$p(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r} \text{ for } x = r, r+1, r+2, \dots,$$

$$E(X) = r/p, \text{Var}(X) = r(1-p)/p^2.$$

NB: If  $X_1, X_2, \dots, X_r$  are independent geometric( $p$ ) random variables, then

$X = \sum_{i=1}^r X_i$  has a neg.binom( $r, p$ ) distribution.

### The Poisson( $\lambda$ ).

$X$  counts the number of events that happen in a fixed amount of time, where we expect  $\lambda$  events to happen in the amount of time.

$$p(x) = e^{-\lambda} \lambda^x / x! \text{ for } x = 0, 1, 2, \dots,$$

$$E(X) = \lambda, \text{Var}(X) = \lambda.$$

### The hypergeometric( $n, M, N$ ).

$X$  counts the number of successes in a random sample of size  $n$  drawn (without replacement) from a population of  $N$  objects of which  $M$  are successes, and therefore,  $N - M$  are failures.

$$p(x) = \binom{M}{x} \binom{N-M}{n-x} / \binom{N}{n} \text{ for } x = 0, 1, \dots, n, x \leq M \text{ and } n-x \leq N-M,$$

$E(X) = nM/N, \text{Var}(X) = n \frac{M}{N} (1 - \frac{M}{N}) (\frac{N-n}{N-1})$ . NB: in sampling without replacement if we let  $X_i = 1$  if we draw a success and  $X_i = 0$  if we draw a failure, then  $X = \sum_{i=1}^n X_i$  has the hypergeometric( $n, M, N$ ) distribution...notice that unlike the binomial( $n, M/N$ ) distribution these Bernoulli( $M/N$ )'s are dependent.

Next page for continuous distributions

## Common continuous distrib.; descriptions, pdfs, means and variances.

### The uniform( $a, b$ ).

$(a, b)$  represents a (finite) interval of the real line with  $a < b$ ; this distribution is the generalization of equally-likely outcomes to a continuous interval of values.

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}.$$

$$E(X) = \frac{b+a}{2}, \text{Var}(X) = \frac{(b-a)^2}{12}.$$

### The exponential( $\lambda$ ).

$\lambda > 0$  represents a positive real parameter which is the reciprocal of the mean of  $X$ .

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}.$$

$$E(X) = \frac{1}{\lambda}, \text{Var}(X) = \frac{1}{\lambda^2}.$$

### The Gamma( $\alpha, \beta$ ).

$\alpha > 0$  represents the shape parameter,  $\beta > 0$  is the scale parameter.

$$f(x) = \begin{cases} \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}, \text{ where } \Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx \text{ is the Euler Gamma}$$

defined for  $\alpha > 0$ .

$$E(X) = \alpha\beta, \text{Var}(X) = \alpha\beta^2.$$

### The Normal( $\mu, \sigma^2$ ).

$\mu$  is a parameter that represents the mean,  $\sigma^2 > 0$  is a parameter that represents the variance.

$$f(x) = \frac{e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}}{\sigma\sqrt{2\pi}} \text{ for } -\infty < x < \infty.$$

$$E(X) = \mu, \text{Var}(X) = \sigma^2.$$

**NB:** When  $\mu = 0$  and  $\sigma^2 = 1$  we call this the **standard normal** distribution.

### The $\chi_\nu^2$ also called the chi-square distribution with $\nu$ degrees of freedom.

$\nu \geq 1$  represents the degrees of freedom.

$$f(x) = \begin{cases} \frac{x^{\frac{\nu}{2}-1} e^{-x/2}}{2^{\frac{\nu}{2}} \Gamma(\frac{\nu}{2})} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}.$$

$$E(X) = \nu, \text{Var}(X) = 2\nu.$$

**NB:** The  $\chi_\nu^2$  is just the Gamma( $\alpha, \beta$ ) with  $\alpha = \nu/2$  and  $\beta = 2$ .

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

**The standard normal table.** The entries in this table provide the numerical values of  $\Phi(y) = P(Y \leq y)$ , where  $Y$  is a standard normal random variable, for  $y$  between 0 and 3.49. For example, to find  $\Phi(1.71)$ , we look at the row corresponding to 1.7 and the column corresponding to 0.01, so that  $\Phi(1.71) = .9564$ . When  $y$  is negative, the value of  $\Phi(y)$  can be found using the formula  $\Phi(y) = 1 - \Phi(-y)$ .