550.420 Probability - SPRING 2016 **MIDTERM 2**

PRINT NAME HERE____________________________________

ATTENDING SECTION #__________

1 = Th@10:30 William Hua, **2**=Th@12:00 William Hua, **3**=Th@9:00 Ting Chao

I agree to complete this examination without unauthorized assistance from any person, materials, or device.

Signature: ____

Instructions: This is a closed book examination. No notes are permitted. No cell phone use is permitted. Answers are to be written clearly and concisely, and clearly labeled (for example, with a box drawn around the intended answer). Please answer each question on the page on which it is stated, using the back of the page if necessary. It is important to show and explain your work; your answer must be properly justified (with at least key words and phrases) to receive full credit. You may use the result of an earlier part of a problem (even if you cannot do it) for later parts.

1. Suppose that X is a continuous random variable having pdf $f(x) = \begin{cases} \frac{3x^2}{2} & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$.

(a) Compute the cdf $F_X(x)$ of X. Recall that (just as a pdf) a cdf should be defined on the entire real line.

(b) Compute $P(0 < X \leq \frac{1}{2})$.

2. Suppose that X has a Gamma(α, β) distribution where $\alpha > 0$ is the shape parameter and $\beta > 0$ is the scale parameter.

(a) By performing an appropriate integration, *clearly* show why $E(X) = \alpha\beta$.

(b) If we further assume $\alpha > 1$, compute $E(\frac{1}{X})$. *For an extra bonus point*, why assume $\alpha > 1$?

3. Suppose that X and Y are jointly discrete random variables having the following joint pmf:

$p_{X,Y}(x,y)$	$y=1$	$y=2$	$y=3$
$x=1$	.30	.18	.12
$x=2$	.20	.12	.08

(a) Clearly *verify* whether or not X and Y are independent. Be sure to state your conclusion.
(b) Compute
  (i) $P(Y > X)$
  (ii) $P(Y = 1)$
  (iii) $P(X = 1 | Y <= 2)$
(c) Compute
  (i) $E(X)$
  (ii) $E(Y|X = 1)$.

4. (a) If Z is a standard normal random variable, compute $P(-2.1 < Z < 2.1)$.
(b) Suppose W represents the score on a certain exam. Assume that W is normally distributed having a mean $\mu = 50$ points and variance $\sigma^2 = 400$ points^2 (i.e., $\sigma = 20$ points). Compute the probability that W will be between 8 and 92 inclusive.

5. Suppose $U \sim \text{uniform}(0, \frac{1}{2})$, i.e., U is a continuous random variable with pdf $f(x) = 2$ for $0 < x < \frac{1}{2}$. Compute the n-th moment of U, i.e., $E(U^n)$, where $n \geq 0$ is an integer.

(*bonus question*) Show that $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n(n+1)} = 2\ln(2)$ by computing $E(\frac{1}{1-U})$ two different ways.

6. Suppose $X|Y = y \sim \text{uniform}(0, y)$ and $Y \sim \text{Gamma}(2, 1)$. That is,

$$f_{X|Y}(x|y) = \begin{cases} \frac{1}{y} & \text{if } 0 < x < y \\ 0 & \text{otherwise} \end{cases} \qquad \text{and} \qquad f_Y(y) = \begin{cases} ye^{-y} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Find the marginal pdf of X. *Be careful with your ranges of integration!*
(b) Compute *only one* of the following (you choose): $P(3 < X < 7|Y = 10)$ *or* $P(3 < X < 7|Y \leq 10)$.
Do not compute both!

7. Suppose X_1 and X_2 are independent random variables with $X_i \sim \text{Poisson}(\lambda_i)$ for $i = 1, 2$. Suppose we observe $X_1 + X_2 = n$, find the probability that $X_1 = x$, i.e., compute $P(X_1 = x | X_1 + X_2 = n)$ for appropriate x.

Feel free to use any results you may recall from homework about sums of independent Poisson random variables.