1. M F M F M F M F M F M F

or F M F M F M F M F M F M

Ans:

$2 \times (6!)^2$

2. $(7)_5 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$

1st person can get off at any one of 7 floors

2nd person must get off at one of 6 remaining floors

etc.

3. (a)

any of H or T

H	H	H	T	*	← 2 such sequences
T	H	H	H	T	← 1 such sequence
*	T	H	H	H	← 2 such sequences

5 total such sequence

probability is $\frac{5}{2^5} = \frac{5}{32}$

3. (b)

$\underline{H}\ \underline{H}\ \underline{H}\ \underline{T}\ \underline{*}\ \underline{*}$ ← 4 such sequences

$\underline{T}\ \underline{H}\ \underline{H}\ \underline{H}\ \underline{T}\ \underline{*}$ ← 2 such sequences

$\underline{*}\ \underline{T}\ \underline{H}\ \underline{H}\ \underline{H}\ \underline{T}$ ← 2 such sequences

$\underline{*}\ \underline{*}\ \underline{T}\ \underline{H}\ \underline{H}\ \underline{H}$ ← 4 such sequences

12 total such sequences

$$\text{probability} = \frac{12}{2^6} = \frac{12}{64} = \frac{3}{16}$$

4.

$$\underbrace{\underset{1}{\underline{A_1}}\ \underset{2}{\underline{A_2}}\ \underset{3}{\underline{A_3}}\ \underset{4}{\underline{A_4}}}_{4!\text{ ways to order aces}}\ \underset{5}{\underline{*}}\ \underset{6}{\underline{*}}\ \underset{7}{\underline{*}}\ \cdots\ \underset{48}{\underline{*}}\ \underset{49}{\underline{*}}\ \underset{50}{\underline{*}}\ \underset{51}{\underline{*}}\ \underset{52}{\underline{*}}$$

$\underline{*}\ \underline{A_1}\ \underline{A_2}\ \underline{A_3}\ \underline{A_4}\ \underline{*}\ \underline{*}\ \cdots\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}$

$\underline{*}\ \underline{*}\ \underline{A_1}\ \underline{A_2}\ \underline{A_3}\ \underline{A_4}\ \underline{*}\ \cdots\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}$

$\underline{*}\ \underline{*}\ \underline{\cancel{A_1}}\ \underline{A_4}\ \underline{A_2}\ \underline{A_3}\ \underline{A_4}\ \cdots\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}$

$\vdots$

$\underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \cdots\ \underline{A_1}\ \underline{A_2}\ \underline{A_3}\ \underline{A_4}\ \underline{*}$

$\underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \cdots\ \underline{*}\ \underline{A_1}\ \underline{A_2}\ \underline{A_3}\ \underline{A_4}$

$\underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{*}\ \cdots$

49 ways to situate the 4 aces next to each other.

There are a total of $4! \times 49 \times 48!$ ways that aces are next to each other

The probability is $\dfrac{4! \times 49 \times 48!}{52!} = \dfrac{4!}{52 \cdot 51 \cdot 50} \approx .000181$

5.* (tricky) $8!$ ways the 8 cards (4 ace, 4 kings) could appear in order. (ignoring the other cards)

there are $4!\,4!$ ways the Aces are first followed by the kings

The probability is $\dfrac{4!\,4!}{8!} \approx .014286$

6.

$$\_\ \_\ \_\ \underset{4}{K} \longleftarrow 4! \times 48!$$

$$\_\ \_\ \_\ \_\ \underset{5}{K} \longleftarrow \binom{4}{3} 4\cdot3\cdot2\cdot1\cdot48!$$

$$\_\ \_\ \_\ \_\ \_\ \underset{6}{K} \longleftarrow \binom{5}{3} 4\cdot3\cdot2\cdot1\cdot48!$$

$$\_\ \_\ \_\ \_\ \_\ \_\ \underset{7}{K} \longleftarrow \binom{6}{3} 4\cdot3\cdot2\cdot1\cdot48!$$

$$\_\ \_\ \_\ \_\ \_\ \_\ \_\ \underset{8}{K} \longleftarrow \binom{7}{3} 4\cdot3\cdot2\cdot1\cdot48!$$

$$\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \underset{9}{K} \longleftarrow \binom{8}{3} 4\cdot3\cdot2\cdot1\cdot48!$$

$$\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \underset{10}{K} \longleftarrow \binom{9}{3} 4\cdot3\cdot2\cdot1\cdot48!$$

The probability is

~~48! { 4!~~

$$\frac{48!\,4!\left\{1+\binom{4}{3}+\binom{5}{3}+\binom{6}{3}+\binom{7}{3}+\binom{8}{3}+\binom{9}{3}\right\}}{52!}$$

$\approx .000775695\ldots$

7. $\binom{n}{1}$ ways to select the one box to receive 2 balls.

there are $n-1$ ways to select one of the remaining $n-1$ boxes to receive No balls (so that all others receive exactly one each)

Thus, there are $n \times (n-1)$ ways.

8. The 47 remaining cards are comprised of 10 spades and 37 non-spades. The probability that two (randomly) selected cards from this reduced deck are both spades is

$$\frac{\binom{10}{2}}{\binom{47}{2}} \approx .041628$$

9. There are 10^3 possible drawings of three marbles each equally likely.

There are $5 \times \cancel{3} \times 2$ drawings that have a red marble drawn first, a blue marble drawn second, and a yellow marble drawn last. Each of the $3!$ permutations of the ordering of these colored marbles also yields one marble of each color. Therefore the desired probability is:

$$\frac{3!\; 5 \times 3 \times 2}{10^3} = .18$$

10. $P(A_i) = \frac{1}{6}$ and A_1, A_2, A_3 are independent

Therefore,

$$P(A_1 \cup A_2 \cup A_3) = 1 - P(A_1^c \cap A_2^c \cap A_3^c)$$

$$= 1 - P(A_1^c)P(A_2^c)P(A_3^c) \text{ by } \underline{\text{independence}}.$$

$$= 1 - \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = 1 - \frac{125}{216} = \frac{91}{216} \checkmark.$$

11. $\frac{\overset{\text{K spade}}{1} \cdot \overset{\text{one of the 3 other kings}}{\binom{3}{1}} \cdot \overset{\text{2 queens}}{\binom{4}{2}} \cdot \binom{44}{1}}{\binom{52}{5}} \approx .000305.$

one of the 44 remaining cards that are not a king or queen

12. Let F_1 be the event that component 1 fails
F_2 " " 2 fails

We are told

$$P(F_1 \cup F_2) = .28$$

But also $P(F_1) = p$; $P(F_2) = 2p$

and F_1 and F_2 are independent. So

$$.28 = P(F_1 \cup F_2) = P(F_1) + P(F_2) - P(F_1 \cap F_2)$$
$$= p + 2p - P(F_1)P(F_2) \quad \text{by independence}$$
$$.28 = 3p - 2p^2.$$

I.e.,

$2p^2 - 3p + .28 = 0$. by the Quadratic formula.

$$p = \frac{3 \pm \sqrt{9 - 4(2)(.28)}}{2(2)} = \frac{3 \pm \sqrt{6.76}}{4} = \frac{3 \pm 2.6}{4} = \begin{cases} .1 \\ \cancel{1.4} \end{cases}$$

probabilities cannot be greater than 1.

thus $P(F_1) = .1$. ✓