550.420 Probability - FALL 2015 **MIDTERM 2**

PRINT NAME HERE______________________________________

ATTENDING SECTION #______________

1 = Th@10:30 Mingyue, **2**=Th@12:00 Joshua, **3**=Th@1:30 Jen, **4**=Th@3:00 Addison

I agree to complete this examination without unauthorized assistance from any person, materials, or device.

Signature: ____

Instructions: This is a closed book examination. No notes are permitted. No cell phone use is permitted. Answers are to be written clearly and concisely, and clearly labeled (for example, with a box drawn around the intended answer). Please answer each question on the page on which it is stated, using the back of the page if necessary. It is important to show and explain your work; your answer must be properly justified (with at least key words and phrases) to receive full credit.

Problem 1. [20 pt]
Problem 2. [10 pt]
Problem 3. [15 pt]
Problem 4. [20 pt]
Problem 5. [10 pt]
Problem 6. [15 pt]
Problem 7. [10 pt]
Total = 100 pts.

1. X is continuous with pdf $f(x) = \frac{2}{x^3}$ for $x > 1$; $= 0$ otherwise.
(a) Compute the cdf $F(x)$ of X. *Be sure to show domains of definition.*
(b) The *median* of the continuous distribution is the value m such that $P(X \leq m) = \frac{1}{2}$. Find the median of this distribution.
(c) Compute $P(2 < X \leq 3)$.
(d) Compute $E(X)$.

2. Let X and Y be *independent* with respective means μ_X, μ_Y and respective variances σ_X^2, σ_Y^2. Show that $var(XY) = \sigma_X^2\sigma_Y^2 + \mu_Y^2\sigma_X^2 + \mu_X^2\sigma_Y^2$.

3. Recall the Gamma(α, β) distribution has pdf $f(x) = \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)}$ for $x > 0$ and has moment generating function $M(s) = (1-\beta s)^{-\alpha}$. Also, for positive integers n, when $\alpha = \frac{n}{2}$ and $\beta = 2$, the Gamma distribution is sometimes called the *chi-square distribution with n degrees of freedom.*

(a) If X has a chi-square distribution with 1 degree of freedom, show that $E(X) = 1$ and $var(X) = 2$ in any way you wish.

(b) If $X_1, X_2, \ldots, X_n$ are independent each distributed as a chi-square with 1 degree of freedom, identify the distribution of the sum $S = X_1 + X_2 + \cdots + X_n$.

(c) Find the mean and variance of S.

4. The joint pdf of X and Y is $f(x, y) = xe^{-x(1+y)}$ for $x > 0$ and $y > 0$.
(a) Find the marginal pdfs of X and Y. Clearly label each.
(b) Compute $P(Y > 1|X = \frac{1}{2})$.

5. There are two bank tellers waiting on customers. The time T_i (in minutes) it takes teller $\#i$ to service a customer is an exponential random variable with parameter $\lambda = i$ (for $i = 1, 2$). Assume the tellers operate independently so that T_1 and T_2 are independent. Two customers enter the bank simultaneously and are immediately serviced by tellers 1 and 2. Compute the probability that both customers are still being serviced after 1 minute. Hint: in the context of this problem what does the event $(\min\{T_1, T_2\} > t)$ mean?

6. A random rectangle is constructed as follows: the length X and the width Y are independent uniform$(0, 1)$ random variables, i.e., each have pdf $f(x) = 1$ for $0 < x < 1$. Find the pdf of the area $A = XY$ of this rectangle.

7. $X \sim \text{normal}(\mu, \sigma^2)$. Recall that the moment generating function of X is $M(s) = e^{\mu s + \frac{\sigma^2 s^2}{2}}$. Let $Y = e^X$ so that Y has the so-called *log-normal* distribution. Compute the mean and variance of Y.