**550.420 Intro. to Probability - Spring 2017** **Homework #09**
**Due by Friday, April 7 in lecture.**

**From the textbook:**
Chapter 5 / Problems 5.15, 5.34, 5.37, 5.40
Chapter 5 / Theoretical Exercises 5.13, 5.19, 5.21, 5.26, 5.30
Chapter 6 / Problems 6.1(c)*, 6.2(a).
* also, compute $P(Y \geq 2X)$.

**Additional problems:**
**A.9.1.** Compute the values of the following integrals *without the use of any computing device*:
(a) $\int_0^\infty u^3 e^{-2u}\,du$
(b) $\int_0^\infty \sqrt{x} e^{-x/2}\,dx$
(c) $\int_{-\infty}^\infty e^{-\frac{1}{2}(x-1)^2}\,dx$
(d) $\int_{-\infty}^\infty e^{-(x^2-2x)/2}\,dx$
(e) $\int_0^1 \sqrt{x}(1-x)^{3/2}\,dx$

**A.9.2.** (a) Verify the following about the product of the first $n$ positive even integers:

$$2n \cdot (2n-2) \cdot (2n-4) \cdots 6 \cdot 4 \cdot 2 = 2^n n!.$$

(b) Simplify $(2n-1)(2n-3)(2n-5)\cdots(5)(3)(1)$.

**A.9.3.** In class we defined when $n$ is a positive integer the *chi-square distribution with $n$ degrees of freedom* (abbreviated $\chi^2_n$) to be the Gamma$(\frac{n}{2}, 2)$ distribution. So that the pdf of a $\chi^2_n$ is given by

$$f(x) = \frac{x^{\frac{n}{2}-1} e^{-x/2}}{2^{n/2}\Gamma(\frac{n}{2})} \quad \text{for } x > 0.$$

Find the mean and variance of the $\chi^2_n$.

**A.9.4.** Suppose $X$ is a unit exponential, i.e., $X \sim \exp(1)$ where the pdf of $X$ is $f(x) = e^{-x}$ for $x > 0$. For any real constant $\nu$ and $\alpha > 0, \beta > 0$, define $Y = \nu + \alpha X^{1/\beta}$. Find the pdf of $Y$.
*The random variable $Y$ in this problem will have the so-called Weibull distribution.*

**A.9.5.** A machine makes perfect squares, however, the edge length $L$ of a square is a continuous random variable uniformly distributed on the interval $(1-\frac{h}{2}, 1+\frac{h}{2})$ where $0 < h < 1$, i.e., the pdf of $L$ is $f_L(x) = \frac{1}{h}$ for $1-\frac{h}{2} < x < 1+\frac{h}{2}$.
(a) Find the pdf of the area $A$ of a square produced by this machine.
(b) Compute $E(A)$.

**A.9.6.** The length of a certain insect is a continuous random variable that is normally distributed with mean $\mu = 10$mm and standard deviation $\sigma = 2$mm.
(a) Compute the probability that a randomly selected such insect is greater than 12mm in length.
(b) If 5 such insects were randomly selected, what is the probability that at most one of them greater than 12mm in length?
(c) A study requires such an insect exceeding 12mm in length. We randomly capture insects one at a time and measure them. What is the probability that you will have captured an insect exceeding 12mm in length by the second capture?
(d) What is the probability you will need to capture *more than* 4 (i.e., 5 or more) to find the first insect exceeding 12mm in length?