550.420 Introduction to Probability - Spring 2017 **Homework #02**
Due Friday, Feb. 17 in lecture.

Do the following exercises from the course textbook:
Chapter 3 problems: 3.1, 3.5*, 3.11, 3.12*, 3.15, 3.30
* these problems may require the use of the extended multiplicative rule of conditional probability:
$P(A_1A_2A_3\cdots A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1A_2)\cdots P(A_n|\cap_{i=1}^{n-1} A_i)$.

Do the following additional problems:
A.2.1. An experiment is to toss a balanced coin 2 times and note the sequence of heads and tails that are tossed. Let A be the event that the first toss is a head, B the event that there is at least one head, and C the event of two heads. Compute $P(C|A)$ and $P(C|B)$. Clearly label each.

A.2.2. Suppose we are told that 20% of college freshmen get homesick and 30% of college freshmen attend school more than 500 miles from home, and of the the college freshmen that attend school more than 500 miles from home, 40% get homesick.
(a) What percentage of college freshmen are both homesick and attend school more than 500 miles from home?
(b) Of the college freshmen that get homesick, what percentage attend school more than 500 miles from home?
(c) If a college freshmen does not get homesick, what is the chance they attend school more than 500 miles from home?

A.2.3. A box has $b > 2$ blue marbles and $g > 2$ green marbles. An experiment is designed to draw 2 marbles without replacing after each draw. Let B_1 be the event that a blue marble is drawn on the first draw; B_2 the event that a blue marble is drawn on the second draw.
(a) Compute $P(B_1)$ and $P(B_1^c)$.
(b) Compute $P(B_2|B_1)$ and $P(B_2|B_1^c)$. Hint: these should be easy to compute by thinking of what the conditioning event has done to the experiment.
(c) Use the information in parts (a) and (b) and the Law of total probability, to compute $P(B_2)$.
(d) Compute $P(B_1|B_2)$, i.e., if you drew a blue marble on the second draw, what is the probability that the first was also blue?

A.2.4. Two people A and B play the following game. Person A tosses a balanced coin *once*; person B tosses a balanced coin *twice*. Person B wins if they toss (strictly) more heads than person A; otherwise, person A wins. Compute the probability that person B wins.

A.2.5. A balanced coin is tossed repeatedly until a head appears for the first time. The sample space for this experiment is $\Omega = \{H, TH, TTH, TTTH, \dots\}$ and it is natural to declare $P(\{\underbrace{TT\cdots T}_{n\ times}H\}) = \frac{1}{2^{n+1}}$.
(a) Compute the probability that the first head occurs on an even toss. Hint: this may require the use of the geometric series formula.
(b) If the first head occurs on an even numbered toss, compute the probability that it occurred on the second toss.