550.420 Probability - SPRING 2016 **MIDTERM 1**

PRINT NAME HERE____________________________

ATTENDING SECTION #__________

1 = Th@10:30 William Hua, **2**=Th@12:00 William Hua, **3**=Th@9:00 Ting Chao

I agree to complete this examination without unauthorized assistance from any person, materials, or device.

Signature: ____________________________________

Instructions: This is a closed book examination. No notes are permitted. No cell phone use is permitted. Answers are to be written clearly and concisely, and clearly labeled (for example, with a box drawn around the intended answer). Please answer each question on the page on which it is stated, using the back of the page if necessary. It is important to show and explain your work; your answer must be properly justified (with at least key words and phrases) to receive full credit.

1. Let $P[A \cap B] = .2$, $P[A] = .6$ and $P[B] = .5$. Find $P[A^c \cap B^c]$ and $P[A^c \cup B^c]$. Clearly label each.

2. Eleven (11) girls on a soccer team are randomly grouped into 4 positions comprised of 2 forwards, 4 midfielders, 4 defenders, and 1 goalie. *There's no need to simplify your answers.*

(a) How many different groupings are possible?

(b) What is the probability that random grouping has Maddie as a midfielder and Denise as a defender?

3. A coin with head probability $\frac{2}{3}$ is tossed. If a head occurs, 1 balanced 6-sided die is rolled; if a tail occurs, 2 balanced 6-sided dice are rolled. Compute the probability the sum of the dice is 6.

4. Fred throws darts at a dartboard. On each trial, independently of other trials, he hits the bulls-eye with probability .05. How many darts should he throw so that the probability of hitting the bulls-eye (at least once) is .5?

5. Two marbles are drawn without replacement from a box containing 3 red and 3 blue marbles. Let R be the event that at least one red marble is drawn, S the event that both marbles are the same color. Are the events R and S independent? Justify your assertion. *In your calculations ignore the order in which marbles are selected.*

6. In 1992, Teen Talk Barbie dolls were introduced. Each Teen Talk Barbie doll could speak 4 of 270 possible sentences. One of the sentences was "Math class is tough!" – which caused a lot of protest, and production of dolls that had that sentence in their four-sentence repertoire were discontinued. Before the model was discontinued, what is the probability that a randomly-chosen Teen Talk Barbie doll could say the offending sentence? *Simplify completely.*

7. Suppose a box has 4 chips numbered 1 through 4. We draw chips one at a time without replacement and note its number. Let C_i be the event that there is a match at the i-th draw; i.e., the i-th chip drawn is numbered i. *Simplify completely.*

(a) Compute the probability of at least one match in the 4 draws, i.e., $P[\bigcup_{i=1}^{4} C_i]$.

(b) Given that there is at least one match, what is the probability there is a match on the 4th draw?

8. A balanced coin is tossed 4 times. Let R be the random variable that determines the length of the longest run of consecutive heads. For example, $R(\texttt{HHHH}) = 4$, $R(\texttt{THHH}) = 3$ and $R(\texttt{HTHT}) = 1$, etc. Here are all the possible outcomes:

$$\begin{array}{lllll} \Omega = \{ & \texttt{HHHH}, & \texttt{HHHT}, & \texttt{HHTH}, & \texttt{HHTT}, \\ & \texttt{HTHH}, & \texttt{HTHT}, & \texttt{HTTH}, & \texttt{HTTT}, \\ & \texttt{THHH}, & \texttt{THHT}, & \texttt{THTH}, & \texttt{THTT}, \\ & \texttt{TTHH}, & \texttt{TTHT}, & \texttt{TTTH}, & \texttt{TTTT} \quad \}. \end{array}$$

What are the possible of values of R? Also, construct the probability mass function of R.