HW #2 Additional problems

A.2.1

$P(A) = \frac{1}{2}$ $\quad P(B) = 1 - P(B^c) = 1 - \frac{1}{4} = \frac{3}{4}$.

$$P(C|A) = \frac{P(A \cap C)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \boxed{\frac{1}{2}}.$$

$$P(C|B) = \frac{P(B \cap C)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \boxed{\frac{1}{3}}.$$

A.2.2

(a) $P(H \cap M) = P(H|M)P(M) = (.40)(.30) = .12$.

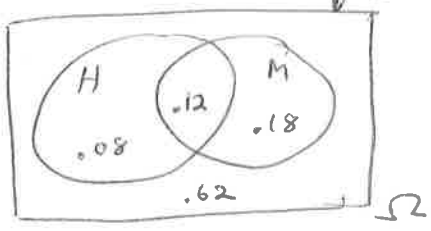
So, 12% of college freshman are both homesick and attend school more than 500 miles from home.

(b) $$P(M|H) = \frac{P(H \cap M)}{P(H)} = \frac{.12}{P(H|M)P(M) + P(H|M^c)P(M^c)}$$

$$= \frac{.12}{(.40)(.30) + \frac{.08}{.62}(\quad}$$

$$= \frac{.12}{.12 + .08} = \frac{.12}{.20} = \boxed{.60}$$

Created venn diagram from data given



(c) $$P(M|H^c) = \frac{P(M \cap H^c)}{P(H^c)} = \frac{.18}{.18 + .62} = \frac{.18}{.80} = \boxed{\frac{9}{40}}$$

A.2.3

(a) $P(B_1) = \frac{b}{b+g}$

$P(B_1^c) = \frac{g}{b+g}$

(b) $P(B_2 \mid B_1) = \frac{b-1}{b+g-1}$

$P(B_2 \mid B_1^c) = \frac{b}{b+g-1}$

(c) $P(B_2) = P(B_2 \mid B_1) P(B_1) + P(B_2 \mid B_1^c) P(B_1^c)$

$= \frac{b-1}{b+g-1} \cdot \frac{b}{b+g} + \frac{b}{b+g-1} \cdot \frac{g}{b+g}$

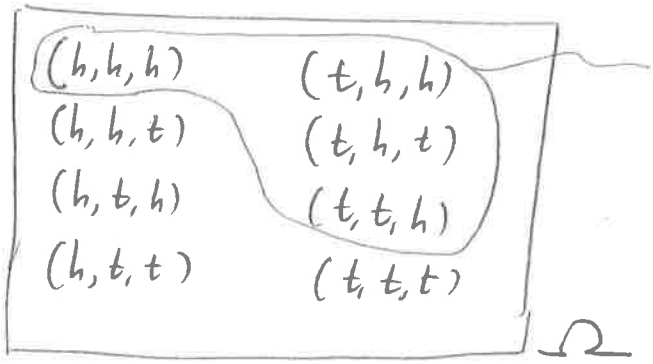
$= \frac{b^2 - b + bg}{(b+g-1)(b+g)} = \frac{b(b-1+g)}{(b+g-1)(b+g)} = \boxed{\frac{b}{b+g}}$

(d) $P(B_1 \mid B_2) = \frac{P(B_2 \mid B_1) P(B_1)}{P(B_2)} = \frac{\frac{b-1}{b+g-1} \cdot \frac{b}{b+g}}{\frac{b}{b+g}}$

$= \boxed{\frac{b-1}{b+g-1}}$

A.2.4

$$P(B \text{ wins}) = P(A \text{ tosses } 0 \text{ heads} \cap B \text{ tosses } 1 \text{ head})$$
$$+ P(A \text{ tosses } 0 \text{ heads} \cap B \text{ tosses } 2 \text{ heads})$$
$$+ P(A \text{ tosses } 1 \text{ head} \cap B \text{ tosses } 2 \text{ heads})$$



outcome (a,b,c) means A tossed a, B tossed b,c. Each outcome is equally likely.

the event the B tosses more heads (h) than person A is circled in the Venn diagram!

$$= \frac{4}{8} = \frac{1}{2}.$$ It's a fair game!

A.2.5

(a) $P(1^{st} \text{ head on even toss})$

$= P(1^{st} \text{ head on toss } 2) + P(1^{st} \text{ head on toss } 4)$

$+ P(1^{st} \text{ head on toss } 6) + \dots \text{ etc}$

$= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \frac{1}{2^8} + \dots$

$= \frac{\frac{1}{2^2}}{1 - \frac{1}{2^2}} = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \boxed{\frac{1}{3}}$.

(b) $P(1^{st} \text{ head on toss } 2 \mid 1^{st} \text{ head on even \#'d toss})$

$= \frac{P(1^{st} \text{ head on toss } 2 \cap 1^{st} \text{ head on even \#'d toss})}{P(1^{st} \text{ head on even \#'d toss})}$

$= \frac{P(1^{st} \text{ head on toss } 2)}{\frac{1}{3}} = \frac{\frac{1}{4}}{\frac{1}{3}} = \boxed{\frac{3}{4}}$.