550.420 Intro. to Probability - Spring 2017 **Homework #11**
Will NOT be collected.

From the textbook:
Chapter 6 / Problems: 6.42, 6.47, 6.52, 6.55*, 6.56, 6.57, 6.58; Theoretical exercises: 6.28, 6.35
* challenging: the answer to part (a) is $f_{U,V}(u,v) = \frac{1}{2u^2 v}$ for $u \geq 1$ and $\frac{1}{u} \leq v \leq u$, and for part (b) is $f_U(u) = \frac{\ln(u)}{u^2}$ for $u \geq 1$ and $f_V(v) = \begin{cases} 1/2 & \text{for } 0 < v < 1 \\ \frac{1}{2v^2} & \text{for } v \geq 1 \end{cases}$.

Additional problems:
A.11.1. In problem 6.52 in the textbook, the joint pdf is $f_{X,Y}(x,y) = \frac{1}{\pi}$ for $x^2 + y^2 \leq 1$. Compute $P(0 \leq Y \leq \frac{1}{2} | X = \frac{1}{2})$. That is, if we are told that the x-coordinate of the dart lands on the chord where $x = \frac{1}{2}$, compute the probability that the y-coordinate is between 0 and $\frac{1}{2}$.

11.2. Suppose Z_1 and Z_2 are independent standard normal random variables, and consider the random variables $U = \frac{1}{\sqrt{2}}(Z_1 + Z_2)$ and $V = \frac{1}{\sqrt{2}}(Z_1 - Z_2)$. Find the joint pdf of U, V. Is anything remarkable about this?

11.3. This problem is a re-phrasing of the textbook's theoretical exercise 6.21:
Suppose $W \sim \text{Gamma}(\alpha, \beta)$ and $N | W = w \sim \text{Poisson}(w)$.
(a) Find the joint distribution of W, N.
(b) Find the conditional distribution of W given $N = n$. Answer: $W | N = n \sim \text{Gamma}(n + \alpha, \frac{\beta}{1+\beta})$.