HW #7 Additional problems solutions

$\boxed{A.7.1}$ The r.v.s X_n have binomial (n,p) distributions with $p = .3$.

Therefore,

$$P(X_m = x \mid X_n = k) = \frac{P(\{X_m = x\} \cap \{X_n = k\})}{P(X_n = k)}$$

$$= \frac{P(\{X_m = k\} \cap \{k-x \text{ red marbles in next } n-m \text{ draws}\})}{\binom{n}{k}(.3)^k(.7)^{n-k}}$$

$$= \frac{P(X_m = x)\, P(X_{n-m} = k-x)}{\binom{n}{k}(.3)^k(.7)^{n-k}}$$

$$= \frac{\binom{m}{x}(.3)^x(.7)^{m-x} \cdot \binom{n-m}{k-x}(.3)^{k-x}(.7)^{n-m-k+x}}{\binom{n}{k}(.3)^k(.7)^{n-k}}$$

$$= \frac{\binom{m}{x}\binom{n-m}{k-x}}{\binom{n}{k}}$$

$\boxed{A.7.2}$

$$E(W!) = \sum_{w=0}^{\infty} w! \, e^{-\lambda} \frac{\lambda^w}{w!}$$

$$= e^{-\lambda} \sum_{w=0}^{\infty} \lambda^w = e^{-\lambda} \cdot \frac{1}{1-\lambda}.$$

$$= \frac{e^{-\lambda}}{1-\lambda} = \frac{1}{e^{\lambda}(1-\lambda)}.$$