

# Algebra 2 - Zestaw 9

Wojciech Szlosek

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## 1 Zadanie 1

### 1.1 (a)

Oznaczmy wektor przez  $x$ .

$$x = (-1, 1, 2, -3)$$

$$|x| = \sqrt{(x, x)} = \sqrt{(-1)^2 + 1^2 + 2^2 + (-3)^2} = \sqrt{15}$$

### 1.2 (b)

$x, y$  - wektory

$$x = (1, 4, -1, 2); y = (3, -1, 2, -1)$$

$$(x, y) = 3 - 4 - 2 - 2 = -5 \neq 0$$

(odp.) Wektory nie są więc ortogonalne.

## 2 Zadanie 1, (c)

$$x, y \text{ - wektory } |x|^2 = 1 + 9 + 1 = 11; |y|^2 = 9 + 1 + 1 = 11$$

$$\cos \angle(x, y) = \frac{(x, y)}{|x||y|} = \frac{6}{11}$$

### 2.1 (e)

$x, y$  - wektory

$$y = (1, 2, 0, -2); \cos \frac{2\pi}{3} = -\frac{1}{2}$$

Niech  $x = (a, b, c, d)$  będzie szukany wektorem. Wówczas mamy:  $a^2 + b^2 + c^2 + d^2 = 1$  (1) oraz:

$$\cos \angle(x, y) = \frac{a + 2b - 2d}{3} = -\frac{1}{2}$$

Przykładowo można przyjąć zatem, że  $a = -\frac{1}{2}$ ,  $b = -\frac{1}{4}$ ,  $d = \frac{1}{4}$ , oraz z warunku

$$(1): c = \sqrt{\frac{5}{8}}$$

$$(\text{Odp.}) x = (-\frac{1}{2}, -\frac{1}{4}, \sqrt{\frac{5}{8}}, \frac{1}{4})$$

### 3 Zadanie 2, (a)

$$p_0 = x + 1; q_0 = x - 2$$

$$(p, q) = p(1)q(1) + p(2)q(2) + p(3)q(3)$$

$$|p_0|^2 = 2^2 + 3^2 + 4^2 = 29$$

$$|q_0|^2 = (-1)^2 + 0 + 1^2 = 2$$

$$\cos \angle(p_0, q_0) = \frac{-2 + 0 + 4}{\sqrt{29}\sqrt{2}} = \sqrt{\frac{2}{29}}$$

$$(\text{Odp.}) \arccos(\sqrt{\frac{2}{29}})$$

### 4 Zadanie 2, (b)

$$p_0 = x + 1; q_0 = x - 2$$

$$(p, q) = p(0)q(0) + p'(0)q'(0) + p''(0)q''(0)$$

$$|p_0|^2 = 1^2 + 1^2 + 0 = 2$$

$$|q_0|^2 = (-2)^2 + 1^2 + 0 = 5$$

$$\cos \angle(p_0, q_0) = \frac{-2 + 1 + 0}{\sqrt{2}\sqrt{5}} = -\sqrt{\frac{1}{10}}$$

$$(\text{Odp.}) \arccos(-\sqrt{\frac{1}{10}})$$