AM1 - Zestaw 11

Wojciech Szlosek

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1 Zadanie 1, (h)

$$\int \frac{2x^4+5x^2-2}{2x^3-x-1}dx = \int xdx + \int \frac{6x^2+x-2}{2x^3-x-1}dx$$

Podzielmy sobie rozwiązanie na mniejsze składowe.

$$\int xdx = \frac{x^2}{2}$$

$$\int \frac{6x^2+x-2}{2x^3-x-1}dx = \int \frac{4x+3}{2x^2+2x+1}dx + \int \frac{1}{x-1}dx$$

$$\int \frac{1}{x-1}dx = ln|x-1|$$

$$\int \frac{4x+3}{2x^2+2x+1}dx = 2\int \frac{2x+1}{2x^2+2x+1}dx + \int \frac{1}{2x^2+2x+1}dx$$

Niech $t = 2x^2+2x+1$, $dx = \frac{1}{4x+2}dt$

$$\int \frac{2x+1}{2x^2+2x+1}dx = \frac{1}{2}\int \frac{1}{t}dt = \frac{lnt}{2} = \frac{ln(2x^2+2x+1}{2}$$

Niech $t = 2x+1$, $dx = \frac{1}{2}dt$

$$\frac{1}{2x^2+2x+1}dx = \int \frac{1}{t^2+1}dt = arctg(t) = arctg(2x+1)$$

Stąd:

$$\int \frac{4x+3}{2x^2+2x+1}dx = ln(2x^2+2x+1) + arctg(2x+1)$$

Wreszcie podsumujmy:

$$\int \frac{2x^4+5x^2-2}{2x^3-x-1}dx = \frac{x^2}{2} + ln|x-1| + ln(2x^2+2x+1) + arctg(2x+1)$$

2 Zadanie 2, (d)

Istnieje wzór: $\int cos^n(x)dx = \frac{n-1}{n}\int cos^{n-2}(x)dx + \frac{cos^{n-1}(x)sin(x)}{n}$, skorzystamy z niego.

$$\int cos^4(x)dx = \frac{3}{4}cos^2(x)dx + \frac{cos^3 x sinx}{4}$$
$$\int cos^2 xdx = \frac{cosxsinx}{2} + \frac{x}{2}$$

$$\int cos^4(x)dx = \frac{cos^3 xsinx}{4} + \frac{3cosxsinx}{8} + \frac{3x}{8} + C = \frac{3}{8}x + \frac{1}{4}sin(2x) + \frac{1}{32}sin(4x) + C$$

3 Zadanie 3, (c)

$$\int \sqrt{x^2-36}dx$$

Niech $x = 6ch(t)$, $dx = 6sh(t)dt$, $t = acosh(\frac{x}{6})$

$$\sqrt{x^2-36} = \sqrt{36ch^2(t)-36} = 6\sqrt{sh^2(t)} = 6|sh(t)| = 6sh(t)$$

$$\int \sqrt{x^2-36}dx = 36\int sh^2(t)dt = 18\int ch(2t-1)dt = -18\int 1dt + 18\int ch(2t)dt$$

$$= -18t + 9sh(2t) = \frac{x}{2}\sqrt{x-6}\sqrt{x+6} - 18\ln|x+\sqrt{x^2-36}| = \frac{x\sqrt{x^2-36}}{2} - 18\ln|x+\sqrt{x^2-36}| + C$$