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Contents

1. Introduction

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At CERN, a new electron cooler is being commissioned for the AD experiment. This cooler shoots electrons into ion-beam path. These electrons then collide with the beam particles, and momentum is transferred from the beam particles to the electrons. The electrons are then steered away from the beam path, into an electron collector.

In the beam path drift of the cooler, a solenoid magnet is used to orient the electron path. This magnet comes with strict requirements on field quality, in the order of $\vec{B}_{\perp}/\vec{B}_{\parallel} < 10\mathrm{E}{-10}$. A new measurement system for solenoids has been proposed, using coils wound on a pcb. This pcb is then translated through the solenoid aperture, to obtain maps of the magnetic field. In this thesis, the metrological characterization of this system is presented, along with some post processing methods.

1.1 Project Purpose and Goal

1.2 Previous Work

2. Background

2.1 Electromagnetic Fields

The electromagnetic fields are a collection of closely linked fields. These fields govern the electric and magnetic interactions of charged particles and domains. These fields can be seen in table 2.1

Field	SI unit	Description
$\mathbf{H}$	$1Am^{-1}$	Magnetic Field
$\mathbf{E}$	$1Vm^{-1}$	Electric Field
$\mathbf{B}$	$1Vsm^{-2}$	Magnetic Flux Density
$\mathbf{D}$	$1Asm^{-2}$	Electric Flux Density
$\mathbf{J}$	$1Am^{-2}$	Electric Current Density
ρ	$1Asm^{-3}$	Electric Charge Density

These fields are described by Maxwells Equations. In differential form for the stationary case, these are as follows:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D} \tag{2.1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \tag{2.2}$$

$$\nabla \cdot \mathbf{B} = \mathbf{0} \tag{2.3}$$

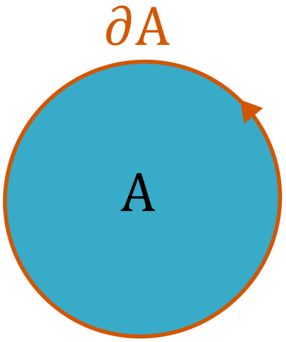
$$\nabla \cdot \mathbf{D} = \rho \tag{2.4}$$

Since we're dealing with measurement of magnetic fields in this thesis, equations 2.1 and 2.3 will naturally be of the most interest. In simple cases, the $\mathbf{H}, \mathbf{D}, \mathbf{E}$ and $\mathbf{B}$ field obey the easy relations

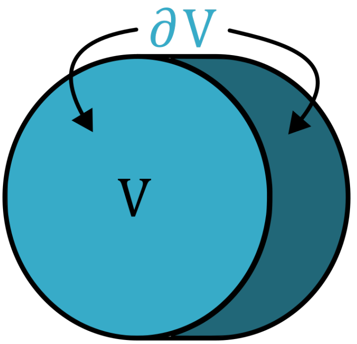
$$\mathbf{B} = \mu \mathbf{H} \tag{2.5}$$

$$\mathbf{D} = \varepsilon \mathbf{E} \tag{2.6}$$

where μ is the *magnetic permeability* and U is the *electric permittivity* in the domain of interest. Formally, simple cases are where the fields are located in a medium that is linear, homogenous across its domain, invariant depending on



(a) An area A and its boundary ∂A.



(b) A volume V and its surface boundary ∂V.

direction, and stationary. Since the magnetic measurements are made inside the empty aperture of the magnet, the domain is only made up of air. Thus, equation 2.5 holds, and the magnetic permeability is the one of free space, that is $\mu = \mu_0 = 4\pi \times 10^{-7} Hm^{-1}$. [1, Ch.4.1-4.4]

2.1.1 Magnetic Flux and Induction

Magnetic flux Φ is the surface integral of the $\mathbf{B}$ field along the normal vector to the surface. Mathematically, it is defined as:

$$\Phi(\mathrm{A}) = \iint_{\mathrm{A}} \mathbf{B} \cdot \hat{\mathbf{n}}\, d\mathrm{A} \tag{2.7}$$

where A is the surface, and $\hat{\mathbf{n}}$ is the normal vector to the surface. We then have the following governing laws of electromagnetism for objects at rest:

$$U(\partial\mathrm{A}) = -\frac{d}{dt}\Phi(\mathrm{A}) \tag{2.8}$$

$$\Phi(\partial\mathrm{V}) = 0 \tag{2.9}$$

Equation 2.8, also called faradays law, describes the voltage ε induced in a length of wire ∂A, enclosing an area A, when the magnetic flux Φ is changing with respect to time. The signs of U and Φ obey the right hand rule as indicated in figure 2.1a.

Equation 2.9 states that the total amount of flux flowing through the boundary ∂V of the volume V must equal 0. [1, Ch.4.1.1]

2.2 Series decompositions of the magnetic field

The magnetic field can be calculated in some different ways, either directly from Maxwells equations or using Biot-Savarts law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int\limits_{\mathrm{V}} \frac{\nabla \times \mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathrm{V} \tag{2.10}$$

where $\mathbf{B}(\mathbf{r})$ is the $\mathbf{B}$ field at coordinate $\mathbf{r}$ and $\mathbf{J}(\mathbf{r}')$ is the current distribution at coordinate $\mathbf{r}'$. [1, Ch.5.4] Except for very simple geometries, the magnetic field is rarely expressible using elementary functions. A common method is then to express it using fourier series solutions inside a specified domain. [1, Ch.6]

Inside the aperture of a magnet, the domain is free of currents and made up of air or vacuum. The current powering the magnet is constant, meaning we have a constant electric field. Equation 2.1 can then be rewritten as follows:

$$\begin{aligned} \nabla \times \mathbf{H} &= \mu_0 \nabla \times \mathbf{B} \\ \mu_0 \nabla \times \mathbf{B} &= \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D}\Bigg|_{\substack{\mathbf{J}=\mathbf{0} \\ \frac{\partial}{\partial t}\mathbf{D}=\mathbf{0}}} \\ &= \mathbf{0} \end{aligned} \tag{2.11}$$

This, along with equation 2.3 means that there exists a magnetic scalar potential $\Psi(\mathbf{r})$ of $\mathbf{B}$ that satisfies Laplace's equation

$$\nabla^2 \Psi = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = 0 \tag{2.12}$$

inside the domain, where the $\mathbf{B}$ field components are

$$B_x = \frac{\partial \Psi}{\partial x}, B_y = \frac{\partial \Psi}{\partial y}, B_z = \frac{\partial \Psi}{\partial z} \tag{2.13}$$

2.2.1 Cylindrical Coordinates

Since the aperture of our magnet is cylindrical, working in cylindrical coordinates (r, φ, z) is a natural choice. They are related to the cartesian system (x, y, z) through the relations:

$$\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \\ z &= z \end{aligned} \tag{2.14}$$

A vector $\mathbf{v}$ is defined by its distance r from the origin, its angle φ from the x-axis, and its offset in z as $\mathbf{v}(r,\varphi,z) = (r\cos\varphi, r\sin\varphi, z)$. Where in cartesian coordinates we have the basis vectors $\hat{\mathbf{e}}_\mathbf{x}, \hat{\mathbf{e}}_\mathbf{y}, \hat{\mathbf{e}}_\mathbf{z}$ along the x, y and z axis, in cylindrical coordinates we have $\hat{\mathbf{e}}_\mathbf{r}, \hat{\mathbf{e}}_\varphi, \hat{\mathbf{e}}_\mathbf{z}$ as illustrated in figure 2.2. Note that $\hat{\mathbf{e}}_\mathbf{r}$ and $\hat{\mathbf{e}}_\varphi$ change direction depending on the current value of φ.

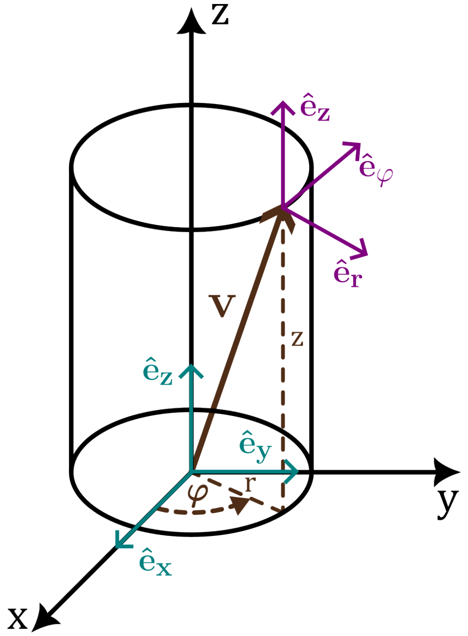


Figure 2.2. Cylindrical coordinates and their relationship to cartesian coordinates.

Scaling Factors

In cylindrical coordinates, scaling factors are needed for common differential operators. For a scalar field $\Psi(r,\varphi,z)$ the gradient is defined as

$$\nabla\Psi = \frac{\partial\Psi}{\partial r}\hat{\mathbf{e}}_\mathbf{r} + \frac{1}{r}\frac{\partial\Psi}{\partial\varphi}\hat{\mathbf{e}}_\varphi + \frac{\partial\Psi}{\partial z}\hat{\mathbf{e}}_\mathbf{z} \tag{2.15}$$

The divergence of a vector field $\mathbf{V}(r,\varphi,z)$ is

$$\nabla\cdot\mathbf{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\varphi}{\partial\varphi} + \frac{\partial V_z}{\partial z} \tag{2.16}$$

which gives the laplacian $\nabla^2\Psi = \nabla\cdot\nabla\Psi(r,\varphi,z)$

$$\nabla^2\Psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi}{\partial}\right) + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\varphi^2} + \frac{\partial^2\Psi}{\partial z^2} \tag{2.17}$$

[1, Ch.3.13]

The Laplace Equation in Cylindrical Coordinates

One way to solve the laplacian is to use separation of variables technique to find the set of potential solutions, and from that set choose the ones that make sense for our problem.

Firstly, we make the ansatz that the solutions can be written in the form

$$\Psi(r,\varphi,z) = R(r)\Phi(\varphi)Z(z) \tag{2.18}$$

Insertion of equation 2.18 into 2.17 then gives us

$$\nabla^2\Psi = \frac{1}{rR}\frac{d}{dr}\left(\frac{dR}{dr}\right) + \frac{1}{r^2\Phi}\frac{d^2\Phi}{d\varphi^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} \tag{2.19}$$

We know from equation 2.12 that this is equal to zero, and can therefore rewrite as

$$-\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2} = \frac{r}{R}\frac{d}{dr}\left(\frac{dR}{dr}\right) + \frac{r^2}{Z}\frac{d^2 Z}{dz^2} \tag{2.20}$$

Here, a contradiction emerges. A change in φ can only introduce a change in the left hand side of this equation. Likewise, this equality must still hold for a change in r or z. These conditions only hold under the assumption that both sides are constant, such that

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2} = \alpha_1 \tag{2.21}$$

where α_1 is constant. Using similar reasoning for $Z(z)$ and $R(r)$ we can reduce this partial differential equation to a system of ordinary differential equations.

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} = \left(\frac{\alpha_1}{r^2} + \alpha_2\right)R \tag{2.22}$$

$$\frac{d^2\Phi}{d\varphi^2} = \alpha_1\Phi \tag{2.23}$$

$$\frac{d^2Z}{dz^2} = \alpha_2 Z \tag{2.24}$$

2.2.2 Bessel Functions

2.2.3 Bessel-Fourier-Fourier Series

2.3 Signal Processing

2.3.1 Filters

2.3.2 Least Squares Fitting

3. The Translating Coil Magnetometer

3.1 PCB printed coils

3.2 Positional Encoder

3.3 Geometric Lidar Measurements

3.4 Fast Digital Integrators

3.5 The Measurement Assembly

4. Measurements

4.1 Solenoidal Field Maps

4.2 The Magnet-Magnetometer Yaw Angle Peak Shift

5. Post Processing

5.1 Lidar Scans

5.2 Coil Induction Analysis

5.3 Bessel-Fourier-Fourier Series Fitting

5.4 Estimating the Magnet-Magnetometer Yaw Angle

6. Discussion

6.1 Metrological Characterization

6.2 Future Design Considerations

Units

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References

[1] Stephan Russenschuck. *Field computation for accelerator magnets: analytical and numerical methods for electromagnetic design and optimization*. John Wiley & Sons, 2011.