Engr 050: Amplifier Circuit Final Project Report

Due on December 15, 2021

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Contents

1. INTRODUCTION

Main Objective

This project uses C++ programming to solve the differential equation shown below using numerical differentiation and a given analytical solution for comparison. This ordinary differential equation models the equation of an amplifier circuit which we will solve to determine the output voltage V_0.

The Differential Equation

Amplifier Circuit ODE

$\frac{d^2V_0}{dt^2} + 2\frac{dV_0}{dt} + 5V_0 = 5V_s$

Initial Conditions

$t = 0$, $\frac{dV_0}{dt} = 0$, $V_0 = 0$, $V_S = 10$

Program Functionality

In addition to calculating the solution of the given ODE, the program is also able to compile the data into a table which can be printed to the console or into a file of the user's choice. Furthermore, this program can also create a graph of both the analytical and numerical solutions using DISLIN. This program is also able to calculate the minimum and maximum voltages with their respective times.

Equipment

Dev C++, Computer

2. PROCEDURE

Solving the Differential Equation

This ODE is solved numerically using central differencing. This method works by replacing differential parts of the ODE with the formulas for first and second derivatives.

$$\frac{dV_0}{dt} = \frac{V_0(t+\Delta t) - V_0(t-\Delta t)}{2\Delta t}, \ \frac{d^2V_0}{dt^2} = \frac{V_0(t+\Delta t) - 2V_0(t) + V_0(t-\Delta t)}{\Delta t^2}$$

After plugging these two into the original DE, we can factor out values so that we are left with an equation in terms of $V_0(t + \Delta t), V_0(t)$, and $V(t - \Delta t)$. Note that going forward we will be representing the coefficients of these terms such as $\frac{1}{\Delta t^2} + \frac{1}{\Delta t}$ by a, $-\frac{2}{\Delta t^2} + 5$ by b, and $\frac{1}{\Delta t^2} - \frac{1}{\Delta t}$ by c. Knowing this, we can solve for $V_0(t + \Delta t)$ to get our output voltages.

$$V_0(t + \Delta t) = \frac{50 - V_0(t)b - V_0(t-\Delta t)c}{a}$$

To solve this problem analytically, we use the given solution.

$$V_0 = 10 - e^{-t}[10 \cos 2t + 5 \sin 2t]$$

Into the Program

This section discusses how the problem is solved in programming. While there are many different class structures and functions involved, we will be ignoring these for the sake of a simpler breakdown.

To solve the problem, the program utilizes 11 different double variables, as well as a singular integer variable. The variables are as follows: a user-inputted delta value, a numerical voltage, an analytical voltage, a time value, a user-inputted maxTime value, constant a, constant b, constant c, a size variable, as well as 3 arrays to store the numerical voltages, analytical voltages, and times(volts, aVolts, times are their respective names).

After gathering the necessary inputs from the user, the program initializes the size value by dividing 1 + maxTime by the delta value. The program will then initialize the three arrays with this size value. Next, the initial values are set. The first two values in the volts and aVolts arrays are 0, while the first value of the times array is set to 0 and the second value of the times array is set to 0 + delta. The double time variable is also set to 0. Constants a, b, and c are set to the values stated in the previous section.

With this, the program will begin to calculate the actual voltage values with a for loop starting at index 2 and stopping at 1 less than the size value. Both the analytical and numerical voltages will be calculated here at times incrementing by delta and stored into their respective arrays.

3. EQUATIONS, ANALYSIS, FLOWCHART

Equations

Amplifier Circuit ODE

$\frac{d^2V_0}{dt^2} + 2\frac{dV_0}{dt} + 5V_0 = 5V_s$

Initial Conditions

$t = 0$, $\frac{dV_0}{dt} = 0$, $V_0 = 0$, $V_S = 10$

Analytical Solution

$V_0 = 10 - e^{-t}[10\cos 2t + 5\sin 2t]$

Numerical Solution

$V_0(t + \Delta t) = \frac{50 - V_0(t)b - V_0(t-\Delta t)c}{a}$

Flowchart and Class Structures

Classes

OutputVoltageModel

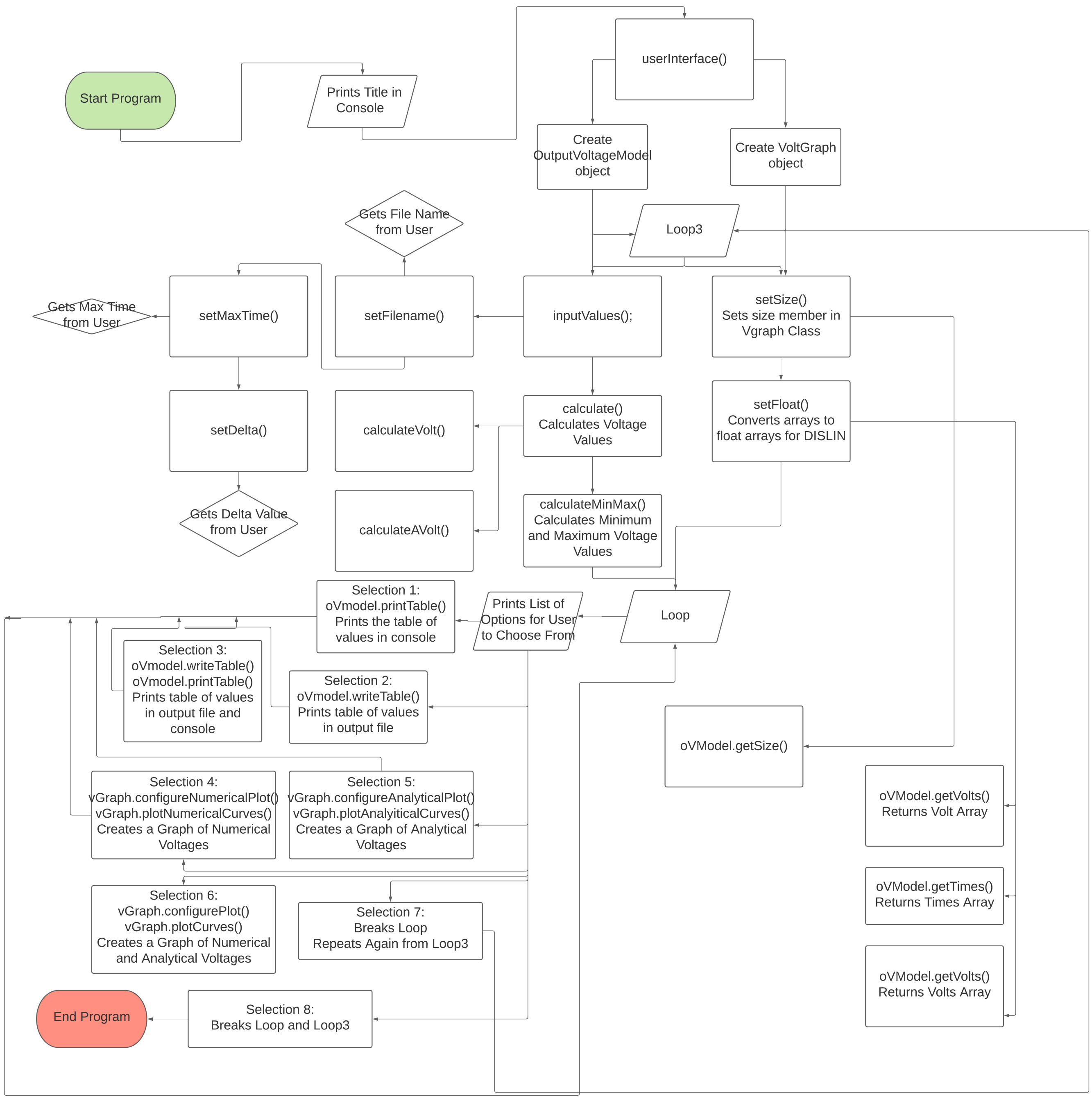
This class contains a function to grab user input(the output filename, the delta value, the maximum time value), a function to calculate the voltage values, a function to calculate the maximum/minimum voltage values and their respective times, functions to print to the console and the output file, and accessor functions to encapsulate private members.

VoltGraph

This class contains different functions to configure and graph the calculated values from the OutputVoltageModel using DISLIN.

Flowchart

Flowchart on the following page


Start Program
Prints Title in Console
userInterface()
Create OutputVoltageModel object
Create VoltGraph object
Gets File Name from User
Loop3
Gets Max Time from User
setMaxTime()
setFilename()
inputValues();
setSize() Sets size member in Vgraph Class
setDelta()
calculateVolt()
calculate() Calculates Voltage Values
setFloat() Converts arrays to float arrays for DISLIN
Gets Delta Value from User
calculateAVolt()
calculateMinMax() Calculates Minimum and Maximum Voltage Values
Selection 1: oVmodel.printTable() Prints the table of values in console
Prints List of Options for User to Choose From
Loop
Selection 3: oVmodel.writeTable() oVmodel.printTable() Prints table of values in output file and console
Selection 2: oVmodel.writeTable() Prints table of values in output file
oVModel.getSize()
Selection 4: vGraph.configureNumericalPlot() vGraph.plotNumericalCurves() Creates a Graph of Numerical Voltages
Selection 5: vGraph.configureAnalyticalPlot() vGraph.plotAnalyiticalCurves() Creates a Graph of Analytical Voltages
oVModel.getVolts() Returns Volt Array
oVModel.getTimes() Returns Times Array
Selection 6: vGraph.configurePlot() vGraph.plotCurves() Creates a Graph of Numerical and Analytical Voltages
Selection 7: Breaks Loop Repeats Again from Loop3
oVModel.getVolts() Returns Volts Array
End Program
Selection 8: Breaks Loop and Loop3

4. RESULTS (TABLES, GRAPHS, ETC)

Sample Calculation with Given Values

$\Delta t = 0.0001$, Maximum Time = 7.28

Sample Console UI

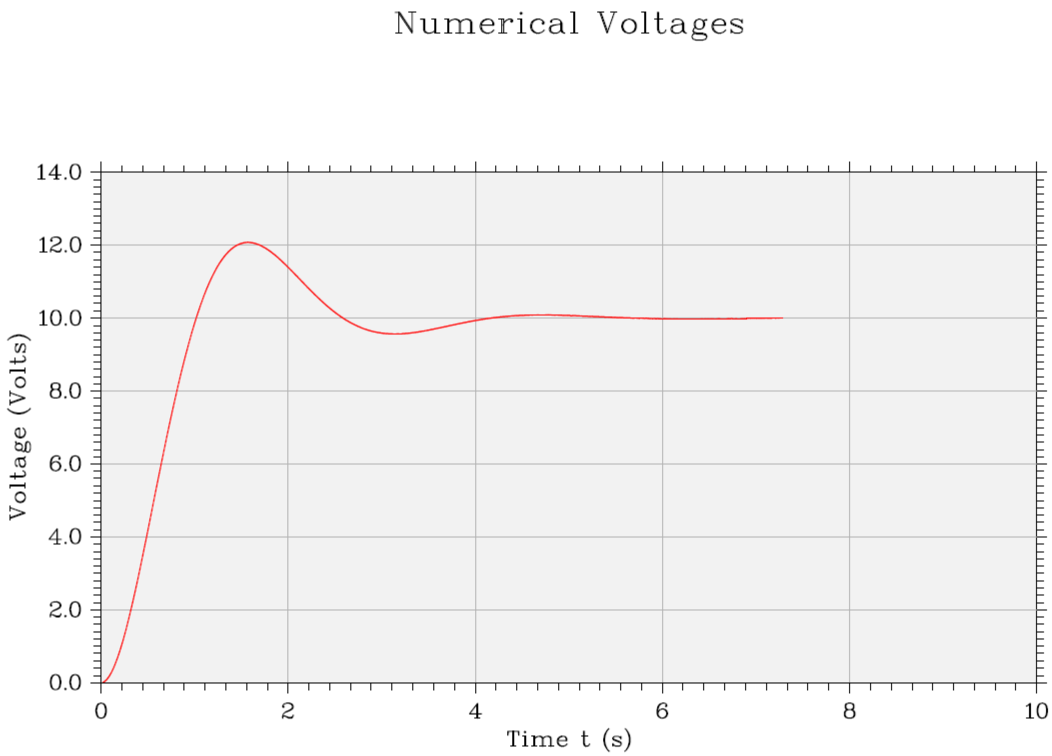
```
- Engr 050 Final Project: Amplifier Circuit -
This program solves the DE d^2Vo/dt^2 + 2dVo/dt + 5Vo = 5Vs numerically and analytically.

Initial Conditions:
dVo/dt = 0, Vo = 0, Vs = 10

Enter Output File Name(EX. filename.txt): test.txt
Enter Maximum Time(in seconds): 7.28
Enter Delta Value: 0.0001

SELECT AN ACTION TO PERFORM
Maximum Voltage: 12.0788 V at 1.5707 seconds.
Minimum Voltage: 0 V at 0 seconds.

1. Print Table in Console
2. Print Table in Output File
3. Print Table in Console and File
4. Create Graph of Numerical Solution
5. Create Graph of Analytical Solution
6. Create Graph of both Numerical and Analytical Solutions
7. Enter New Values
8. End the Program

Selection:
```

Sample Console Table (Same Table Will Be Printed in Output File)

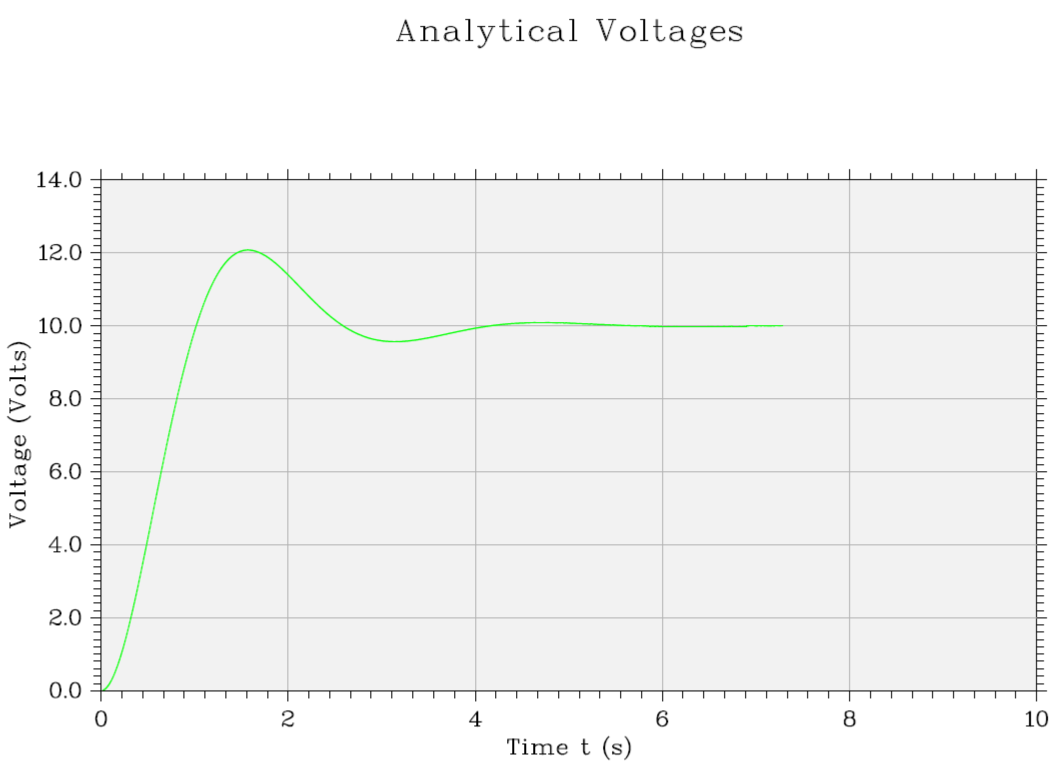
Table of Numerical and Analytical Voltages

Time(s)	Numerical Voltage(Volts)	Analytical Voltage(Volts)
0	0	0
0.0099	0.00245856	0.00243406
0.0199	0.00981735	0.0097686
0.0299	0.0219757	0.0219032
0.0399	0.0388325	0.0387368
0.0499	0.0602865	0.060168
0.0599	0.0862359	0.0860952
0.0699	0.116579	0.116417
0.0799	0.151214	0.15103
0.0899	0.190038	0.189833
0.0999	0.232949	0.232724
0.1099	0.279845	0.279601
0.1199	0.330625	0.330362
0.1299	0.385185	0.384903
0.1399	0.443425	0.443125
0.1499	0.505242	0.504924
0.1599	0.570535	0.5702
0.1699	0.639202	0.638851
0.1799	0.711144	0.710776
0.1899	0.786259	0.785876
0.1999	0.864448	0.86405
0.2099	0.945611	0.945198
0.2199	1.02965	1.02922
0.2299	1.11646	1.11602
0.2399	1.20595	1.2055
0.2499	1.29803	1.29756
0.2599	1.39259	1.39211
0.2699	1.48953	1.48904
0.2799	1.58877	1.58827
0.2899	1.69021	1.6897
0.2999	1.79375	1.79323
0.3099	1.89931	1.89878
0.3199	2.00678	2.00624
0.3299	2.11609	2.11554
0.3399	2.22713	2.22657

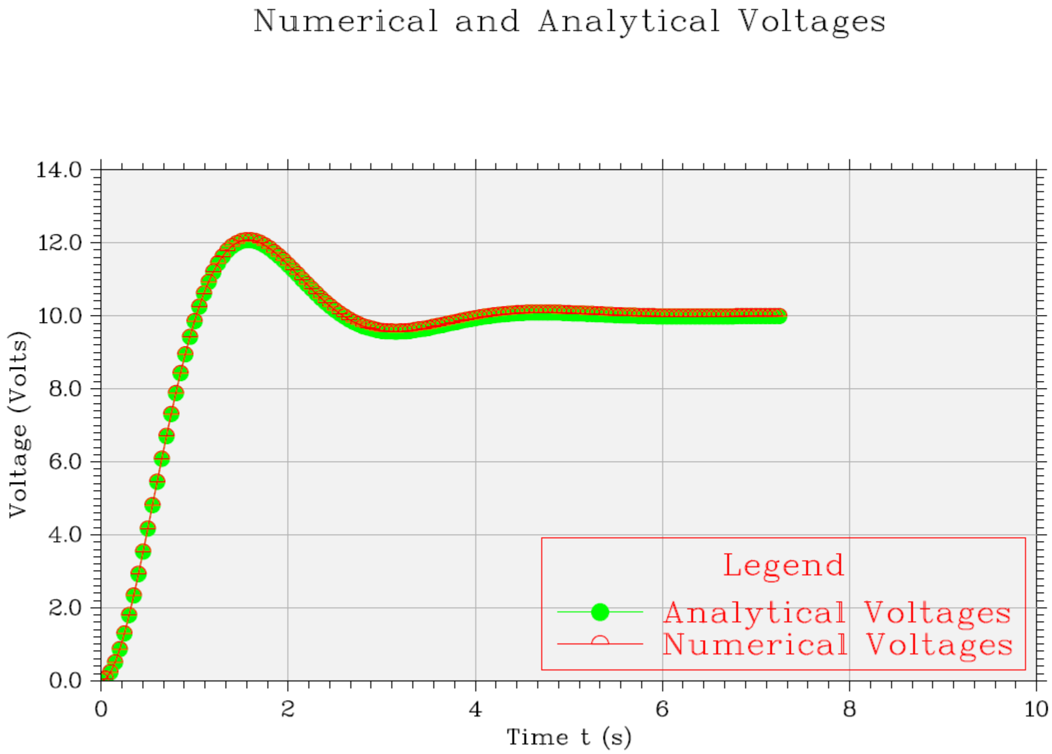
Sample Numerical Graph



Sample Analytical Graph



Sample Numerical and Analytical Graph



5. CONCLUSION

Differences Between Analytical and Numerical Solution

The greatest difference between the analytical and numerical solution is in the accuracy. The analytical solution will always be exact while the numerical solution's accuracy will depend largely on the Δt value. The smaller the Δt value, the more accurate the solution will be. The numerical solution will be the same as the analytical solution at around a 0.0001 Δt value.

Learning Objectives

With this project, I learned to work with OOP concepts such as classes and functions in C++. I also learned how to write user-friendly software that deals with distinct test cases on its own.