Assignment #2
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Chapter 2 Exercise 1, 5

Ex 1

(a)

n^2: $\frac{(2n)^2}{n^2} = 4$, running time 4 times slower.

n^3: $\frac{(2n)^3}{n^3} = 8$, running time 8 times slower.

$100n^2$: $\frac{100(2n)^2}{100n^2} = 4$, running time 4 times slower.

$n \log n$: $\frac{(2n)\log(2n)}{n \log n} = 2 \cdot \frac{\log(2n)}{\log n}$, running time $2 \cdot \frac{\log(2n)}{\log n}$ times slower;

Also $(2n)\log(2n) - n \log n = n \log n + 2n \log 2$, running time increases $n \log n + 2n \log 2$.

2^n: $\frac{2^{2n}}{2^n} = 2^n$, running time 2^n times slower.

(b)

n^2: $(n+1)^2 - n^2 = 2n + 1$, running time increases $2n + 1$.

n^3: $(n+1)^3 - n^3 = 3n^2 + 3n + 1$, running time increases $3n^2 + 3n + 1$.

$100n^2$: $100(n+1)^2 - 100n^2 = 200n + 100$, running time increases $200n + 100$.

$n \log n$: $(n+1)\log(n+1) - n \log n = \log(n+1) + n \log \frac{n+1}{n}$, therefore running time increases $\log(n+1) + n \log \frac{n+1}{n}$.

2^n: $\frac{2^{n+1}}{2^n} = 2$, running time 2 times slower.

Ex 5

(a) False

Let $f(n) = 2$ and $g(n) = 1$, then $f(n) = O(g(n))$ by definition. However, $\log f(n) = \log 2$ and $\log g(n) = 0$. It is clear that $\log f(n) = \log 2 \neq O(0) = O(\log g(n))$ and it is a counterexample.

(b) False

Let $f(n) = 2n$ and $g(n) = n$, then $f(n) = O(g(n))$ by definition. However $\lim_{n\to\infty} \frac{2^{2n}}{2^n} = +\infty$, therefore $\forall c \in \mathbb{R}$, $\neg\exists N$ s.t. $\forall n > N$, $2^{f(n)} \leq c \cdot 2^{g(n)}$, and $2^{f(n)} \neq O(2^{g(n)})$.

(c) True

Suppose $\exists c \in \mathbb{R}, N \in \mathbb{N}$ s.t. $\forall n > N$, $|f(n)| \leq c|g(n)|$, then $|f(n)|^2 \leq (c|g(n)|)^2$, $|f(n)^2| \leq c^2|g(n)^2|$. Choose $c' = c^2 \in \mathbb{R}$, it is equivalent to $f(n)^2 = O(g(n)^2)$.