MIP School Scheduling

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1 Model Formulation

1.1 Variables

$$X_{s,t,c,p,d} \in \{0, 1\} \tag{1}$$

where $X_{s,t,c,p,d}$ is a binary decision variable indicating whether student $s \in S$ is assigned to teacher $t \in T$ for course $c \in C$ in period $p \in P$ on day $d \in D$ ($X_{s,t,c,p,d} = 1$ if student is assigned, 0 otherwise).

$$num\_students\_assigned_{t,c,p,d} \in \mathbb{Z} \tag{2}$$

where $num\_students\_assigned_{t,c,p,d}$ is an integer variable counting the number of students assigned to a given teacher $t \in T$ for course $c \in C$ in period $p \in P$ on day $d \in D$.

$$teacher\_scheduled\_indicator_{t,c,p,d} \in \{0, 1\} \tag{3}$$

where $teacher\_scheduled\_indicator_{t,c,p,d}$ is a binary variable indicating if a given teacher $t \in T$ is scheduled for course $c \in C$ in period $p \in P$ on day $d \in D$.

Table 1: The notation used in our formulation

S	Set of students
T	Set of teachers
C	Set of courses
P	Set of periods
D	Set of days

1.2 Constraints

$$\sum_{t=1}\sum_{c=1}\sum_{p=1} X_{s,t,c,p,d} = 7 \quad \forall s, d \tag{4}$$

Constraint 4 ensures that every student is fully scheduled (i.e. taking a full 7 periods each day).

$$num\_students\_assigned_{t,c,p,d} = \sum_{s=1} X_{s,t,c,p,d} \quad \forall t, c, p, d \tag{5}$$

Constraint 5 links the $num\_students\_assigned_{t,c,p,d}$ variable by counting the number of students assigned for each teacher $t \in T$, course $c \in C$, period $p \in P$, and day $d \in D$.

$$teacher\_scheduled\_indicator_{t,c,p,d} \cdot M \geq num\_students\_assigned_{t,c,p,d} \quad \forall t, c, p, d \tag{6}$$

Constraint 6 sets $teacher\_scheduled\_indicator_{t,c,p,d} = 1$ if $num\_students\_assigned_{t,c,p,d} > 0$ through the use of a sufficiently large 'Big M' variable M.

$$\sum_{c=1} teacher\_scheduled\_indicator_{t,c,p,d} \leq 1 \quad \forall t, p, d \tag{7}$$

Constraint 7 ensures every teacher is teaching no more than one course during each period on every day.

$$\sum_{t=1}\sum_{p=1}\sum_{d=1} X_{s,t,c,p,d} = 1 \quad \forall s, c \in S, C | P(s, c) \tag{8}$$

where $P(s, c)$ is the property in which student s is currently enrolled in course c and course c is a 'core course'. Constraint 8 requires a student to be enrolled in a course if it is designated as a 'core course' and they are currently enrolled in it. Note that the course does not need to be in the same period on the same day, or taught by the same teacher.

$$X_{s,t,c_{lunch},p_4,d} + X_{s,t,c_{lunch},p_5,d} = 1 \quad \forall s, t, d \tag{9}$$

Constraint 9 ensures every student and every teach is assigned one lunch period during either 4th or 5th period.