An Analytical Approach to the Music of Iannis Xenakis: Studies of Recent Works

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Abstract

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The music of Greek composer Iannis Xenakis is of considerable interest to composers and scholars of music. The analysis of his music, however, presents considerable difficulties due to the singular nature of Xenakis's compositional process. Not the least of these problems stems from the fact that much of the music is based, at least in part, on mathematical models of indeterminacy.

This study attempts in part to explain what these mathematical models are and how they relate to the nature of the musical surface. Close examination of the surface of several works by Xenakis reveals much of the music to be based upon a relatively small repertoire of basic textural types. These textural types prove to be adaptable to a wide variety of musical contexts and function differently in each of the works examined.

After specific aspects of the compositional process have been introduced in some detail, analyses of six solo instrumental works and one electroacoustic work are presented. Each of these works was composed in the 1970s or 1980s, a period in Xenakis's work about which comparatively little has been written in contrast to the earlier period, from 1955 to 1970, about which the composer himself has written extensively. The analytical approach

applied to these works includes their division into structural units, from smallest to largest, the identification of principal textural types, and the examination of palpable relations between these structural units. The relations between the structural units are considered both with respect to the precise settings of the units in finished compositions (i.e., inside-time) and between the units treated as analytically separable elements of finished compositions (i.e., outside-time). Where appropriate, relations among pitch (*not* pitch-class) sets are examined for evidence of the large-scale structuring of pitch.

A conclusion summarizes the general findings of the study and presents suggestions for further research into the structure of Xenakis's music. Some general thoughts are also offered about the applicability of the analytical approach developed here to the music of other avant-garde composers from the second half of the twentieth century.

Contents

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Introduction

Indeterminacy has been an important issue in musical composition and performance since the 1950s.[1] Among the composers who have made use of indeterminacy in their works are the Americans John Cage, Christian Wolff, and Earle Brown and the Europeans Karlheinz Stockhausen, Pierre Boulez, and Iannis Xenakis. Each of these composers has employed indeterminacy in particular ways, but Xenakis stands apart from the others for his direct application of scientific models of indeterminacy to the compositional process. The result is his stochastic music, which makes use of mathematical formulas derived from probability theory. These formulas are used to produce pseudo-random successions of pitch and time-point intervals, and may be applied to other aspects of structure as well. The musical works that result from this process respond well to quantitative methods of structural analysis because the composition of stochastic music is based on standard mathematical models whose statistical properties are well known. Structural analysis is further facilitated by the fact that the vast majority of Xenakis's compositions exist in a single, definitive version that is scored in conventional musical notation.[2] Thus the difficulties that arise

[1] For a discussion of problems in the definition of determinacy versus indeterminacy, see Horacio Vaggione, "Determinism and the False Collective: About Models of Time in Early Computer-Aided Composition," *Contemporary Music Review* 7 (1993): 91-104. See also Charles Ames, "Automated Composition in Retrospect: 1956-1986," *Leonardo* 20/2 (1987): 169-85, and Charles Ames, "Statistics and Compositional Balance," *Perspectives of New Music* 28 (1990): 80-111.

[2] To date, Xenakis has published nearly 150 compositions. Of these, only three—*Duel* for two orchestras (1959), *Stratégie* for two orchestras (1962), and *Linaia-Agon* for brass trio (1972)—are variable in their form, depending upon choices made by the conductors or players during

from multiple versions of a work, or dependence upon particular performances in order for the structure of a work to be determined, need not enter into the analysis of Xenakis's music. For these reasons, methods of structural analysis that have proven successful in the examination of works from the atonal and serial repertoires may also be applied to Xenakis's music, given certain necessary adaptations to the peculiarities of his compositional practice. Analysis of Xenakis's music, therefore, represents a cautious step in the direction of confronting problems in the analysis of music in which indeterminacy plays a significant structural role.

There is considerable variety in the types and degrees of indeterminacy in Xenakis's music. The most thoroughly formalized applications of indeterminacy in his music involve calculations, either by hand or with the aid of a computer for, according to Xenakis's original formulation of the theory of stochastic music, "chance needs to be calculated."[3] More informal applications of indeterminacy, inspired in part by complex mathematical models of random processes found in nature, make use of graphic images whose forms are then transcribed into musical notation.

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performance. The form of a fourth work, *Atrées* for chamber ensemble (1956-62) is variable in a more limited sense, since the order of its five movements may be decided upon by the conductor prior to each performance. All of Xenakis's instrumental and vocal works are written in some sort of staff notation, even where conventional notation has been adapted for special purposes, as in the simple grid notation used in *Psappha* for solo percussion (1975). The only works that are not scored in conventional musical notation (or in some adaptation thereof) are the electroacoustic works, of which *Diamorphoses* (1957), *Concret PH* (1958), *Analogique B* (1959), *Orient-Occident* (1960), *Bohor* (1962), *Hibiki Hana Ma* (1949-70), *Persepolis* (1971), *Polytope de Cluny* (1972), and *La Légende d'Eer* (1977) exist only on tape (or on other recording media), while graphic scores exist for *Mycenae-Alpha* (1978), *Pour la paix* (1981), *Taurhiphanie* (1987), *Voyage absolu des Unari vers Andromède* (1989), and *Gendy3* (1991), because of technology developed by Xenakis for this purpose. *Mycenae-Alpha* is analyzed in chapter 5 of this dissertation.

[3]Iannis Xenakis, *Formalized Music: Thought and Mathematics in Composition*, rev. ed. (Stuyvesant, NY: Pendragon Press, 1992), p. 38.

An additional element of indeterminacy in composition results from Xenakis's intuitive revisions of the results obtained by the models he uses, whether these be mathematical or graphic. Deviations from what would otherwise constitute a direct transcription of a calculated or visual model of a musical structure, then, may be regarded as a personal interpretation of indeterminacy that is independent of the formal methods upon which a particular compositional process may be based.

In practice, the analysis of Xenakis's music does not concern itself exclusively, or even primarily, with the nature and extent of the processes of indeterminacy used in its composition. Instead, analysis of his music properly concerns itself with the various factors that contribute to the structural coherence of specific works. The notion of coherence may at first appear to be at odds with the use of indeterminacy in the compositional process, but this is not necessarily the case. Close observation of a number of works reveals that indeterminacy is frequently confined to the succession of events on the surface of the music, while its application to more general levels of structure varies considerably from work to work. Various forms of indeterminacy were applied to the succession of musical segments, as well as to the succession of events within segments, in several works written during the formative phase of Xenakis's career, which lasted roughly from 1956 to 1962.[4] Thereafter Xenakis turned to other methods organizing the large-scale structure of his works, including group theory, which was applied in a series of works written between 1964 and 1969.[5] The large-scale structure of most of

[4]See *Formalized Music*, chapters 1-5.

[5]See *Formalized Music*, chapter 8. Group-theoretical operations have also been applied to the ordering of events on the musical surface , as in *Psappha* for percussion solo (1975). See Ellen Rennie Flint, "An Investigation of Real Time as Evidenced by the Structural and Formal Multiplicities in Iannis Xenakis' *Psappha*," (Ph.D. diss., University of Maryland at College Park, 1989) and E. R. Flint, "Metabolae, Arborescences and the Reconstruction of Time in Iannis

Xenakis's other works was composed without the direct application of formalized methods, whether stochastic or group-theoretical, and therefore these works are not so easily categorized. There are, however, several characteristics that appear frequently in these works. These characteristics include the derivation of the durations of segments of music from particular types of statistical distributions. Rather than being sequenced randomly, as in the more thoroughly stochastic works, however, the values drawn from these distributions tend to be organized into larger durational units whose relations reflect proportions that are found elsewhere within the same work. Another characteristic of these works is that the boundaries between segments tend to be articulated by means of sudden changes in one or more global structural features, including texture, pitch contents, registral position, intensity (i.e., dynamics), density (i.e., mean number of events per unit time), instrumentation and articulation. Larger structural units tend to be built up either from successions of similar segments or from regular patterns of contrasting segments. Analysis of the structure of such a composition, therefore, involves the consideration of the characteristics, on various levels of structure, that establish a context within which the details on the surface of the music are free to exhibit varying types and degrees of indeterminacy without seriously undermining the structural coherence of the whole.

At the heart of this dissertation are analyses of several complete works by Xenakis. Each analysis has been conducted according to a set of general principles that are outlined in chapter 2, where several important aspects of Xenakis's compositional technique are introduced. Thus, the perceptible structural characteristics of particular works are not considered apart from the

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Xenakis' *Psappha*," *Contemporary Music Review* 7 (1993): 221-48.

general principles underlying Xenakis's compositional process, and vice versa. Xenakis's music then is approached both from a *poietic* and an *esthesic* perspective in this study since, given the unusual nature of his compositional process and of the results obtained thereby, relatively few assumptions of a general nature may be made about the music in advance of a confrontation with its particular characteristics.[6] A broader perspective on his music is therefore offered here than is possible in studies where his compositional theories are examined apart from the structure of complete, finished compositions, or where only individual works are examined.[7] One

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[6]On the poietic and esthesic aspects of musical works and of musical analysis, see Jean-Jacques Nattiez, *Music and Discourse,* trans. Carolyn Abbate (Princeton: Princeton University Press, 1990), especially chapters 1 and 8. Briefly, the poietic aspects of musical works are those that are involved in their making (i.e., by composers, performers, or both together), while the esthesic aspects are those that pertain to the music's perception and subsequent interpretation (i.e., by listeners, some of whom may also be scholars of music).

[7]General studies of the theoretical aspects of Xenakis's music include most of *Formalized Music;* Rosalie La Grow Sward, "A Comparison of the Techniques of Stochastic and Serial Composition Based on a Study of the Theories and Selected Compositions of Iannis Xenakis and Milton Babbitt," (Ph.D. diss., Northwestern University, 1981); Nouritza Matossian, *Xenakis* (London: Kahn and Averill, 1986); Peter Hoffmann, *Amalgam aus Kunst and Wissenschaft: Naturwissenschaftliches Denken im Werk von Iannis Xenakis* (Frankfurt am Main: Peter Lang, 1994); and Christoph Schmidt, *Komposition und Spiel. Zu Iannis Xenakis* (Köln: Studio, 1995). Schmidt's book is a study of the compositional theory and aesthetics of Xenakis's open-form works based on game theory. Analyses of individual works include Fernand Vandenbogaerde, "Analyse de Nomos-Alpha de Iannis Xenakis," *Mathématiques et Sciences Humaines* 24 (1968): 35-50; Thomas DeLio, "Iannis Xenakis' *Nomos Alpha*: The Dialectics of Structure and Materials," *Journal of Music Theory* 24/1 (1980): 63-95; Jan Vriend, "'Nomos Alpha' for Violoncello Solo (Xenakis 1966): Analysis and Comments," *Interface* 10 (1981): 15-82; T. DeLio, "Structure and Strategy: Iannis Xenakis' *Linaia-Agon,*" *Interface* 16 (1987): 143-64; Flint, "An Investigation of Real Time" and "Metabolae, Arborescences, and the Reconstruction of Time"; L. Scott McCoy, "Duration, Pitch/Space, and Density in Iannis Xenakis's *Mists,*" (Master's thesis, University of Maryland at College Park, 1993); Yayoi Uno, "Avant-garde Music circa 1950-60: Analysis of Works by Boulez, Cage, Babbitt, and Xenakis," (Ph.D. diss., Eastman School of Music, University of Rochester, 1993); Yayoi Uno and Roland Hübscher, "Temporal-Gestalt Segmentation: Polyphonic Extensions and Applications to Works by Boulez, Cage, Xenakis, Ligeti, and Babbitt," *Computers in Music Research* 5 (1995): 1-38; and James Harley, "Sonic and Parametrical Entities in *Tetras*: An Analytical Approach to the Music of Iannis Xenakis," *Canadian University Music Review* 16/2 (1996): 72-99. Less detailed analyses, that demonstrate only a very limited understanding of the theoretical foundations of Xenakis's compositional practice, include Pierre Castanet, "*Mists*, oeuvre pour piano de Iannis Xenakis," *Analyse musicale* 4 (1986): 65-75; Christine Prost, "*Nuits*: Première transposition de la démarche de Iannis Xenakis du domaine instrumental au domaine vocal," *Analyse musicale* 15

advantage of presenting analyses of several works, all approached from the same set of general guidelines, is that it allows for the presentation, at least in a preliminary way, of some of the general principles underlying the structure of his music.

In choosing repertoire for analysis I have followed Xenakis's lead in *Formalized Music*, where the most detailed explanations are reserved for solo and chamber works, including *Achorripsis* for chamber orchestra (1956-7), *Analogique A* for nine strings (1958), *Herma* for piano (1960-1) and *Nomos Alpha* for violoncello (1966).[8] The choice to focus mainly on works for solo instruments makes sense, given the tremendous amount of detail that would have to be accounted for in a thorough analysis of any of the works for full orchestra. Xenakis's comments on *Achorripsis* and *Herma* are extended somewhat in chapter 2 in order to illustrate certain theoretical issues, but the repertoire analyzed in chapters 3-5 has been chosen from among works about which Xenakis has written only in passing, if at all. Chapter 3 contains analyses of three works for solo piano: *Evryali* (1973), *Mists* (1981), and *à r.* (1987); chapter 4 contains analyses of three works for solo strings: *Mikka* (1971) and *Mikka "S"* (1976) for violin and *Theraps* (1975-6) for double bass; and chapter 5 contains an analysis of *Mycenae-Alpha* (1978), the first electroacoustic work to be composed entirely on the UPIC system.[9] I chose

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(1989): 64-70; and articles by various authors in *Regards sur Xenakis* (Paris: Stock, 1981) and *Musik-Konzepte* 54/55 (1987).

[8]These works are discussed in chapters 1, 2-3, 6, and 8 of *Formalized Music*, respectively. The analyses of *Nomos Alpha* cited in note 8 depend heavily upon Xenakis's explanation of that work's structure in *Formalized Music*, and should therefore be regarded more as commentaries than as original analyses.

[9]The UPIC (Unité Polyagogigue Informatique de CEMAMu) is a system for the graphic composition of electroacoustic music. It was developed by Xenakis, along with a team of engineers and computer scientists, at CEMAMu (Centre d'Etudes Mathématiques et Automatiques Musicales), Xenakis's research facility located in the outskirts of Paris, in the 1970s.

these works in part because I found them aesthetically appealing, but also because they provide an opportunity to examine the application of Xenakis's compositional principles to various media. In addition, recordings of all of these works are currently available on compact disc.[10] One important facet of Xenakis's output that is not touched upon here is music for percussion, but I chose to limit my selection to works in which definite pitch plays a structural role.[11]

Xenakis's written works, particularly *Formalized Music,* are an invaluable source of information on his compositional practices. Significant gaps exist in his account of these practices, however, both in terms of the repertoire he discusses and in terms of connections between his theoretical propositions and the details of their application to actual compositional situations. The gaps in the repertoire discussed by Xenakis in his writings is the principal reason for the emphasis on "recent" works, i.e. works composted since 1970, in this study. Xenakis is virtually silent, both in his writings and in interviews, about the technical aspects of the works written after 1968.[12] Analyses of works composed in the 1970s and 1980s, such as those that are presented in this study, may help to fill in some of the gaps that remain in the general understanding of Xenakis's music. The analysis of

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[10]The works for piano and for violin are found on Disques Montaigne 2CD 782005 with Claude Helffer, piano, and members of the Arditti String Quartet. *Theraps,* performed by Robert Black, double bass, is in Neuma CD 450-71 and *Mycenae-Alpha* is on Neuma CD 450-74.

[11]*Psappha* for percussion solo has been analyzed in great detail in Flint, "An Investigation of Real Time." Although the instrumentation for this work is not precisely defined, Xenakis specifies categories of timbre and relative pitch level that are to be observed in choosing instruments. From this example it is clear that relative pitch levels may play a structural role where the instrumentation forbids the articulation of definite pitches.

[12]In *Formalized Music,* chapter 8, he discusses the composition of *Nomos Alpha* for violoncello solo (1966) and of a portion of *Nomos Gamma* for orchestra (1967-8). The technical aspects of specific works are, in general, not discussed after that point.

recent works can also provide a solid basis for comparison with those earlier works about which Xenakis and others have already written.

Apart from the information that he has chosen to reveal in his writings, Xenakis has not often answered direct questions pertaining to his compositional technique. Even the classroom presentations in the composition and analysis courses he has taught have not been as informative as some composers and scholars might have hoped. The Mexican composer Julio Estrada has reported his own experience in this regard:

> At the end of 1983, I attended Xenakis' courses at the Université de Paris to inquisitively interrogate him on the concrete way in which he put into practice in a score his theories and composition methods. This was the occasion in which I most arduously tried to know "how the method actually works," yet I did not achieve my goal. I wanted to know his answers with precision because in the Spring of 1984, I was going to give the first seminar ever on the formal analysis of his work to the doctoral students of composition at the University of California, San Diego. I must say that the solution to the problem was to approach it from the angle formed by my knowledge of his formal propositions and my own intuitions.[13]

Given Xenakis's apparent reluctance to answer direct questions relevant to his compositional technique, it is not altogether surprising that he never responded to the specific technical questions I addressed in correspondence to him.

Numerous requests for copies of the compositional sketches for the works discussed here likewise did not meet with the desired response. This is somewhat surprising, since Xenakis has sometimes released copies of

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[13]Julio Estrada, "Iannis Xenakis: A Vision That Listens," trans. Rafael Liñán, in the booklet accompanying *Iannis Xenakis*, Neuma CD 450-86. Estrada teaches composition at the University of Mexico and also at Ateliers UPIC, the research and teaching organization dedicated to the UPIC tool for electroacoustic composition. (See note 10 above.)

sketches, or portions thereof, of individual works to interested scholars. Xenakis's agent at Editions Salabert, who handles requests for sketches, eventually turned down my request by stating that Xenakis prefers to remain focused on the present and future, and therefore chooses not to spend time looking through the files of his previous works in order to extract sketches to be photocopied.

I did have the opportunity, however, to examine a copy of the sketches to one of the works analyzed here—*Mists* for piano (1980)—which Scott McCoy kindly lent me after he had obtained them for use in his Master's thesis on that work.[14] Examination of the sketches did help to clarify some questions I had about the work's pitch structure and provided some clues as to how to categorize the different types of texture that appear in it. The remainder of the analyses in this dissertation refer only to the published versions of the works, and do not deal with aspects of their genesis that may be inferred from careful study of the sketches.

Now in his mid-70s, Xenakis remains prolific and is therefore understandably conservative in the use of his time. Nonetheless, it is disappointing that no better provision has yet been made for the archiving and distribution of copies of his compositional sketches, in which there appears to be growing scholarly interest. Given the limitations on the information available to me in the preparation of this study, therefore, the analyses contained here, like the contents of Estrada's course on Xenakis in the 1980s, are of necessity based upon "my knowledge of [Xenakis's] formal propositions and my own intuitions."

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[14]I was unaware of McCoy's work on *Mists* until after I had begun my own. Unfortunately, by the time the sketches were sent to him, it was too late for him to include a detailed study of them in his Master's thesis. A photocopy of the sketches, however, is included as an appendix to the thesis. Ellen Flint, however, has made productive use of compositional sketches in her work on *Psappha* .

The dissertation is presented in two volumes. Volume one consists of text and volume two contains figures and tables. Chapter 1 presents some background on Xenakis's development as a composer. It contains some general biographical information along with some speculations regarding relations between his formative influences and the style of his music. Chapter 2 presents an introduction to Xenakis's compositional technique and its relationship to the analytical procedures used in the later chapters. Special emphasis is placed on those facets of his compositional technique that are most relevant to the repertoire analyzed in chapters 3-5, including two topics that have not been given sufficient attention in the literature on Xenakis so far: 1) the specifics of the technique of stochastic composition; and 2) the relationship between graphically designed musical structures and their realization in standard musical notation. Chapters 3 and 4 contain analyses of works for piano and for strings, respectively. Chapter 5 presents an analysis of Mycenae-Alpha (1978), composed on the UPIC system. Chapter 6 offers some provisional conclusions about the findings of this study and some thoughts about its possible relationship to research that remains to be done into the music of Xenakis and his contemporaries.

Chapter 1

The Formation of Xenakis's Compositional Style

Many of the structural features that are most characteristic of Xenakis's music are closely tied to the somewhat unusual circumstances of his education and early adult life. After the completion of a degree program in engineering, he came into contact with some of the most innovative artistic figures of the time, including Olivier Messiaen, Edgard Varèse, Le Corbusier, and Pierre Schaeffer. Each of these men influenced, to a greater or lesser degree, the course of Xenakis's development as a composer. In order to trace this development, this chapter contains a brief biographical sketch of Xenakis, with particular emphasis on those aspects of his biography that have had a direct impact on the formation of his distinctive compositional style.

Childhood, Education, and War

Iannis Xenakis was born in 1922 in Romania to Greek parents. His mother was an accomplished pianist who provided him with an early exposure to music. She died, however, when he was only five years old. At the age of ten he was sent to a boarding school in Greece, where he studied Greek philosophy, European literature, mathematics, science, and music. He developed a strong rapport with the school's British headmaster, Noël Paton, who introduced him to the music of Beethoven and other European composers via gramophone recordings.[1] Xenakis sang Renaissance polyphony in the school's choir, which also performed Byzantine liturgical music for the Orthodox Church. He also studied elementary harmony, somewhat unsuccessfully, and took piano lessons. In these studies, however, he was "dogged by unsatisfactory relations with his music teachers, a tendency which would continue all his life and affect his music."[2] When he was sixteen he went to Athens to prepare for the entrance exam in engineering at Athens Polytechnic. At that time he also studied harmony and counterpoint with a Russian teacher named Aristotle Kondourov who "impressed on the young man the necessity of absolute rigor and discipline in the pursuit of composition, a lesson Xenakis never forgot."[3]

Meanwhile, World War II had begun. Italian troops invaded Greece in 1940, followed by the Germans in 1941. Xenakis's studies at Athens Polytechnic were interrupted repeatedly by forced closings during and after the invasions. He, along with several fellow students, became involved in leftist Resistance activities, for which he was imprisoned several times. The Italians withdrew from Greece in 1943 and the Germans in 1944. British "liberation" forces arrived, with orders from Churchill to undermine the influence of political groups, and of the Leftist groups in particular. In December 1944, during a struggle against the British, Xenakis was hit by a mortar explosion and narrowly escaped death. The explosion cost him his left eye and resulted in permanent damage to the left side of his face. Despite all the interruptions in his schooling, in 1946 he completed his degree at

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[1]Nouritza Matossian, *Xenakis* (London: Kahn & Averill, 1986), p. 16.

[2]Matossian, *Xenakis*, p. 16.

[3]Matossian, *Xenakis*, p. 18.

Athens Polytechnic. Because of his continuing political activities, however, he was blacklisted and was forced to go underground. In 1947 his father succeeded in making arrangements for him to emigrate illegally to Italy. From there he continued on to France, eventually settling in Paris.

Shortly after his arrival in Paris, Xenakis visited Noël Paton in London. The two had remained in contact through correspondence after Xenakis had left boarding school and after Paton had had to flee Greece upon the arrival of the German troops in 1941. It was during this visit that Xenakis learned of Paton's political activities in Greece, which included espionage on behalf of the British government. The sense of betrayal that he had felt at the hands of the British now took on a more personal dimension. This, combined with the early loss of his mother, the political chaos of the war years, including his imprisonment and wounding, and the subsequent separation from his family, led to a profound disillusionment with politics and with social institutions in general. Several years later, Xenakis succinctly described the state of his mind during this period of his life: "Either I committed suicide. Or I started out on a new foot."[4]

This need to move forward, decisively and uncompromisingly, that Xenakis felt at this time would eventually impact the formation of his distinctive compositional style. At the same time, significant episodes from his past would also find expression in his earliest compositions. Xenakis has described the impetus behind *Metastaseis* for orchestra (1953-4) in the following way:

> Athens—an anti-Nazi demonstration—hundreds of thousands of people chanting a slogan which reproduces itself like a

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[4]Iannis Xenakis, "Xenakis on Xenakis," *Perspectives of New Music* 25/1 (1987): 19.

> gigantic rhythm. Then combat with the enemy. The rhythm bursts into an enormous chaos of sharp sounds; the whistling of bullets; the crackling of machine guns. The sounds begin to disperse. Slowly silence falls back on the town. Taken uniquely from an aural point of view and detached from any other aspect these sound events made out of a large number of individual sounds are not separately perceptible, but reunite them again and a new sound is formed which may be perceived in its entirety. It is the same case with the song of the cicadas or the sound of hail or rain, the crashing of waves on the cliffs, the hiss of waves on shingle [sic].[5]

Thus, the concepts of sound masses and of the Law of Large Numbers[6] that dominated Xenakis's first important compositions had their counterparts in actual experiences, which his chosen formalized methods reflected in musically convincing ways.

Le Corbusier

Soon after his arrival in France, Xenakis found an opportunity to make use of his training in engineering. He began work at the atelier of the famous architect Le Corbusier in 1948 and remained with the firm until 1959. Le Corbusier allowed his associates a fair amount of artistic freedom, but Xenakis's struggle to maintain that freedom and to receive recognition for his efforts caused their relationship to deteriorate over time. While working for Le Corbusier, however, Xenakis collaborated on several important projects, including the Convent of la Tourette, located outside Lyons (1954-7) and the Philips Pavilion at the Brussels World's Fair (1956-8). Both men shared an

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[5]Matossian, *Xenakis*, p. 58, from an interview with Xenakis conducted in 1972.

[6]A basic principle of probability theory that will be explained in chapter 2. See also Iannis Xenakis, *Formalized Music*, rev. ed. (Stuyvesant, NY: Pendragon Press, 1992), pp. 1-10.

interest in the spatial proportions of ancient architecture. Le Corbusier gave concrete expression to this interest through the invention of the Modulor, which is a system of architectural measurements based on the golden section and intended to reflect the proportions of the human body. His hope was that the Modulor would become an international standard and would improve the state of architectural design, which he felt to be unduly constrained by its dependence on the rectilinear measurements common to the English and metric systems.[7]

Matossian has described Le Corbusier's working method and its relevance to Xenakis's approach to composition:

> The process by which Le Corbusier assembled plans—many of the parts designed by different architects—into a single project was to be of great relevance to Xenakis's musical development. Le Corbusier always kept the overall concept in mind while he examined the several parts to see how they could function within that whole. Particularly adept at synthesizing and balancing many independent elements, he was never afraid to introduce a variety of forms and plastic ideas.[8]

Le Corbusier's use of precise systems of spatial proportions was one factor that allowed for the synthesis of diverse elements into a unified whole. In Xenakis's musical compositions Le Corbusier's spatial proportions were converted into systems of durational proportions and the diverse elements used in the firm's architectural projects were converted into starkly contrasting, and therefore immediately perceptible, changes in texture and timbre. Le Corbusier was intrigued by Xenakis's application of his

[7]See Le Corbusier, *The Modulor*, trans. P. de Francia and A. Bostock (Cambridge: Harvard University Press, 1954). The date of the original French publication is 1948.

[8]Matossian, *Xenakis*, p. 42.

proportional ideas to musical composition, especially since he was fond of any evidence that would support his claims for the universality of the Modulor. Consequently, in the appendix of his second volume on the Modulor, Le Corbusier included a statement from Xenakis regarding systems of proportions in architecture and music, illustrated with an excerpt from the plans for the Convent of la Tourette and a reproduction of two pages of the score to *Metastaseis*.[9]

Messiaen

While he was employed at Le Corbusier's firm Xenakis continued to pursue his musical interests. He studied briefly with Honegger and Milhaud, but the conventional exercises in harmony and counterpoint that they were teaching did not suit his needs at that time. He also approached Nadia Boulanger but she was reluctant to take him on as a student, for she felt that someone of his age would have difficulty starting at the beginning. Finally, at the suggestion of Le Corbusier, Xenakis introduced himself to Messiaen, from whom he received a quite different response from those he had had from the other teachers. Several years later, Messiaen recalled:

> I did something horrible which I should do with no other student, for I think one should study harmony and counterpoint. But this was a man so much out of the ordinary that I said, "No, you are almost thirty, you have the good fortune of being Greek, of being an architect and having studied special mathematics. Take advantage of these things. Do them in your music."[10]

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[9]Le Corbusier, *Modulor 2*, trans. P. de Francia and A. Bostock (Cambridge: MIT Press, 1958), pp. 326-30. The date of the original French publication is 1955.

[10]Matossian, *Xenakis*, p. 48. Excerpt of an interview with Messiaen conducted in 1977.

Messiaen's attitude toward Xenakis seemed to be the key that would unlock his latent musical instincts. Moreover, he was the first respected musical figure to encourage Xenakis's desire to connect his extra-musical expertise and experience with the compositional process.

Xenakis attended Messiaen's class at the Paris Conservatoire, whenever his schedule permitted, between 1951 and 1953. This was the environment in which he became acquainted with Messiaen's experiments in composition with "modes" not only of pitch, but also of duration, intensity, and articulation. Messiaen's work in this vein began with the piano étude *Mode de valeurs et d'intensités* (1949) and continued with other works in the 1950s.[11] Xenakis also saw these experiments amplified and transformed according to serial principles in the works of Messiaen's students, including Goeyvärts, Stockhausen, and Boulez. The most famous work of this type is Boulez's *Structures Ia* for two pianos (1951), whose 12-tone row is adapted from one of the pitch modes used in *Mode de valeurs et d'intensités.*[12] The use of pitch-class, register, intensity, and temporal factors in *Structures Ia* is markedly different from the treatment of these structural elements in *Mode de valeurs et d'intensités,* however, with the result that the two works, though derived from similar basic materials, sound quite different from one another in performance.

Messiaen took an extremely broad view of music history in his search for compositional resources. As Xenakis says of him, "It was he that made me

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[11]On this period in Messiaen's work, see Robert Sherlaw Johnson, *Messiaen* (Berkeley and Los Angeles: University of California Press, 1989), chapter 10, "The Experimental Period 1949-1951."

[12]On the structural principles of this work see György Ligeti, "Pierre Boulez: Decision and Automatism in *Structure Ia,*" *Die Reihe* 4 (1960): 36-62.

discover the possibilities of abstraction starting with Beethoven and Stravinsky."[13] Messiaen focused special attention on the rhythmic aspect of Stravinsky's music, which he related intuitively to certain rhythmic features of Indian music as revealed in a table of 120 *deçi-tâlas*, or Hindu rhythms, listed by the medieval theorist Sharngadeva in his treatise *Samgîta-ratnakâra*.[14] It is significant for the compositional practices that Messiaen would derive from the rhythms in Sharngadeva's table that few are periodic in metrical units of 2, 3, or 4 beats, as most traditional Western rhythms are. In fact, the majority of the rhythms are much less regular, sometimes achieving periodicity in units of 60 or more beats, e.g. *simhanandana* (rhythm #35), whose period is 64 beats long, or *miçra varna* (rhythm #26), with a period of 71 beats. Messiaen's search also extended beyond the bounds of music history per se, into the realm of nature, where he found fresh material in birdsong, which he transcribed intuitively into musical notation on his frequent ornithological expeditions.

Messiaen's compositional practice was characterized by peculiar associations between stylistic elements of the musical past and present, mediated through a compositional technique that was deliberately formalized in several important respects. He had a special affinity for certain harmonies (or pitch-class sets) found in the music of Debussy and Stravinsky, especially their use of the whole-tone and octatonic collections. Messiaen generalized

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[13]"Xenakis on Xenakis," 20.

[14]Olivier Messiaen, *Technique of My Musical Language*, tr. J. Satterfield (Paris: Leduc, 1956), v. 1, pp. 14-5, and v. 2, exx. 1-5. Messiaen's source for Sharngadeva's table was the *Encyclopédie de la musique et dictionnaire du conservatoire*, ed. Albert Lavignac and Lionel de la Laurencie (Paris: Delagrave, 1913-31), part 1, v. 1, pp. 301 ff. The table of rhythms is reproduced in Johnson, pp. 206-10. The transliteration of Sanskrit terms used here is the one used by Messiaen and reproduced in Johnson. It differs somewhat from the standard English transliteration.

the non-diatonic collections used by these and other composers into a system of "modes of limited transposition."[15] In addition, the prevalence of retrograde transformations in the serial music of the Viennese composers was reflected in the use of "non-retrogradable rhythms," which are symmetrical, palindromic rhythmic patterns.[16] Messiaen was also a professional church musician, which meant that he was familiar with the practices of medieval Latin chant and improvisation at the organ. Items from the chant repertoire were sometimes quoted directly in his works, but could also be transformed according to his own system of modes and rhythmic patterns.[17] In its reinterpretation of elements of the remote musical past through modern, formalized compositional procedures, Messiaen's compositional practice is comparable, in a very a general sense, to Le Corbusier's use of the Modulor, which represents a reinterpretation of certain structural features of ancient architecture by means of a thoroughly modern, deliberately formalized tool for the calculation of architectural proportions.

The breadth and eclecticism of Messiaen's approach to composition must have appealed to Xenakis, even though his approach is more sharply focused and systematic than his mentor's. During the ideological struggles of the 1950s, when many composers were taking strong positions for or against serialism—partly at the instigation of Boulez's polemical and inflammatory statements on the subject[18]—Messiaen adopted a middle position for, while

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[15]Messiaen, *Technique*, v. 1, pp. 58-70, and v. 2, exx. 312-82.

[16]Messiaen, *Technique*, v. 1, pp. 20-1, and v. 2, exx. 28-34.

[17]Messiaen, *Technique*, v. 1, p. 33, and v. 2, exx. 102-10.

[18]The following remark is representative: "I ... assert that any musician who has not experienced—I do not say understood, but, in all exactness, experienced—the necessity for he dodecaphonic language is *useless*. For his whole work is irrelevant to the needs of his epoch." Pierre Boulez, *Notes of an Apprenticeship*, ed. Paule Thévenin, tr. H. Weinstock (New York:

not against serialism, he maintained only a minimal and temporary involvement with it. In time, his interest in 12-tone serialism became absorbed into a more general interest in the permutation of ordered sequences of pitches (or pitch-classes) and durations.[19] Xenakis, however, took a decidedly negative position toward serialism, which he expressed in his article, "The Crisis of Serial Music."[20] This argument against serialism was accompanied by a musical response in the form of stochastic music.

Perhaps because of their mutual distance from strict serialism, there is a greater sense of continuity between subsequent developments in Xenakis's and Messiaen's compositional practice than there is between those of Messiaen and Boulez or Stockhausen. For example, it is significant that, with the exception of clearly experimental works such as *Mode de valeurs et d'intensités*, passages of music based upon modes (of pitches, durations, etc.), with or without the application of specific permutational operations, tend to be juxtaposed with non-modal passages in Messiaen's music. The resulting contrasts in texture articulate the boundaries between successive passages of music and also create associations between non-contiguous passages that are similar in texture.[21] The juxtaposition of contrasting textures operates similarly in much of Xenakis's music, particularly in the works written since

Knopf, 1968), p. 148.

[19]See, for example, the chart of durational permutations for Messiaen's *Chronochromie* for orchestra (1960) in Johnson, *Messiaen*, p. 177.

[20]Iannis Xenakis, "La crise de la musique sérielle," *Gravesaner Blätter* 1 (1955): 2-4. This article is quoted in *Formalized Music*, p. 8.

[21]On a similar structural characteristic in Stravinsky's *Symphonies of Wind Instruments*, see Edward T. Cone, "Stravinsky: The Progress of a Method," in *Perspectives on Schoenberg and Stravinsky*, ed. Benjamin Boretz and Edward T. Cone (New York: Norton, 1972): 155-64; Christopher F. Hasty, "On the Problem of Succession and Continuity in Twentieth-Century Music," *Music Theory Spectrum* 8 (1986): 58-74; and Jonathan D. Kramer, *The Time of Music* (New York: Schirmer, 1988), chapter 9.

the late 1960s. But where Messiaen might juxtapose passages of "modal" music with passages of birdsong or of chordal textures, Xenakis is likely to juxtapose passages of stochastic music with passages consisting of arborescences, glissandi, or some other contrasting type of texture. Thus, in the work of both composers, textures resulting from formalized processes tend to alternate with textures that result from less formalized (or non-formalized) procedures.

There are also structural correspondences in the theoretical basis of both composers' music. For example, in the "sieve theory" that he formulated in the mid-1960s, Xenakis generalizes Messiaen's practice of creating pitch collections from interval cycles and extends the periodicity of the collections beyond the limit of the octave. In this theory he also proposes the generation of rhythmic patterns from interval cycles, and thus formalizes the procedure for creating the long, complex rhythmic patterns favored by Messiaen in his use of the Indian *deçi-tâlas*. It is telling that, in chapter 7 of *Formalized Music*, which is the first of three separate expositions of sieve theory, Xenakis mentions specifically one of the modes of limited transposition, mode 1 (the whole-tone scale, which he associates with the music of Debussy), and also an Indian raga, along with several Greek and Byzantine modes. The similarity to Messiaen's *Technique of My Musical Language*, which illustrates several links between ancient and modern musical practices, could hardly be clearer.[22] Xenakis's application of mathematical group theory[23] likewise represents a generalization and

[22]Matossian reveals that, among the books on the shelf in Xenakis's study, the two volumes of Messiaen's *Technique of My Musical Language* were "especially well used with notes in the margins" (*Xenakis*, p. 83).

[23]See *Formalized Music*, chapter 8.

extension of Messiaen's use of order permutations for sequences of pitches (or pitch-classes) and durations.

On the basis of the evidence presented here, it is apparent that Messiaen has continued to exert an influence on the development of Xenakis's music that extends well beyond what he may have learned in the few years he spent attending Messiaen's classes at the Paris Conservatoire. It should also be clear that Xenakis's stochastic music, with its renunciation of a specific, pre-determined order of musical events, is more closely related to Messiaen's early (pre-permutational) modal technique than to the experiments in integral serialism that followed from it. Stochastic music, therefore, should not be viewed only as a reaction against serialism—even if that is how Xenakis may have presented it at the time[24]—for it has a significant pre-history in Messiaen's non-serial, formalized methods of composition. Messiaen's general approach to composition would provide further stimulation for Xenakis's compositional theory and practices in the 1960s and beyond. Most recently, especially since the mid-1980s, Xenakis has become increasingly interested in the use of simultaneities for the generation of a harmonically rich texture. As a result, the sonic resemblance between his music and Messiaen's has become stronger than ever before.[25]

Pierre Schaeffer, Pierre Henry, Hermann Scherchen, and Edgard Varèse

The emerging practice of electroacoustic composition caught Xenakis's attention in the early 1950s. He started to attend concerts of Pierre Schaeffer's

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[24]See "La crise de la musique sérielle."

[25]Two examples are *à r.* for piano (1987), which is analyzed in chapter 3, and *Echange* for bass clarinet and chamber ensemble (1989).

Groupe de Recherche de Musique Concrète (later known as the Groupe de Recherches Musicales) in 1950. *Musique concrète* is the name that Schaeffer gave to the composition of musical works from samples of recorded sounds. In his opinion this process provided the composer with materials that were more interesting than those currently available through electronic sound synthesis. Xenakis wanted to become involved in Schaeffer's studio, but Schaeffer did not respond to his correspondence until Xenakis sent him the score to one of his works. Schaeffer replied that, since he could not read music, he would have Pierre Henry examine the score if Messiaen were willing to write a letter of reference on his behalf.[26] Thus Xenakis came into contact with Henry, who was then involved in the preparation of recordings for Varèse's *Déserts* for orchestra and tape (1950-4). Henry passed on Xenakis's score to Hermann Scherchen, who was rehearsing the orchestral portions of *Déserts* for the first live stereophonic radio broadcast, to take place in December 1954. Xenakis attended the rehearsals for this event and gradually developed a personal rapport with Scherchen, who eventually agreed to examine the score to Xenakis's latest work, *Metastaseis*. Varèse had travelled from the United States to Paris to participate in the preparations for the performance of *Déserts*, but unfortunately the work was not well received by the public. Xenakis had made a recording of the broadcast and, despite the fact that the recording was marred by the vocal disapproval of some of the audience members, Varèse was interested in hearing the work a second time. Upon hearing the recording of *Déserts*, however, Varèse is reported to have wept in response to the audience's hostile reaction to his work.[27]

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[26]Matossian, *Xenakis*, p. 76.

[27]Matossian, *Xenakis*, p. 80.

When they met to listen to the recording of *Déserts*, Varèse also took time to examine the score to *Metastaseis*, and expressed his approval of the new work. At this time Xenakis was beginning to receive a measure of recognition from one major figure in contemporary music after another, even though not a note of any of his major works had yet been heard in public. The public's reaction to *Déserts*, and Varèse's response to this reaction, however, served to warn Xenakis of the risks involved in the public performance of innovative new works, and served as a premonition of the hostile audience reactions that he would have to endure on several occasions in the future.

In 1955 Scherchen invited Xenakis to participate in various activities at his electronic music studio and conference center in Gravesano, Switzerland. These activities included writing for Scherchen's new periodical, *Gravesaner Blätter*, to which Xenakis contributed his article, "The Crisis of Serial Music."[28] Scherchen's encouragement of Xenakis's self-expression through writing as well as through composition led to the series of articles in various periodicals that eventually became compiled and expanded to form the successive editions of *Formalized Music*.[29] Scherchen also agreed to conduct two of Xenakis's important orchestral scores, *Pithoprakta* (1955-6) and

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[28]See "La crise de la musique sérielle."

[29]The original edition, *Musiques formelles* (Paris: Richard-Masse, 1963; reprint, Paris: Editions Stock, 1981), was followed by *Formalized Music* (Bloomington: University of Indiana Press, 1971) and *Formalized Music*, rev. ed. (Stuyvesant, NY: Pendragon Press, 1992). Xenakis has also written *Musique Architecture* (Tournai, Belgium: Casterman, 1971; rev. ed., Casterman, 1976; Eng. ed., tr. S. Kanach, Stuyvesant: Pendragon Press, 1996); *Xenakis/les Polytopes*, with Olivier Revault d'Allones (Paris: Balland, 1975); *Arts/Sciences, Alliages* (Tournai: Casterman, 1979; Eng. ed., tr. S. Kanach, Stuyvesant: Pendragon Press, 1985); and *Kéleütha (Ecrits)* (Paris: L'Arche, 1994); in addition to a large number of separate articles. For a comprehensive bibliography, see *Formalized Music*, rev. ed., pp. 335-64. The most important source for information on Xenakis's compositional techniques remains *Formalized Music*, rev. ed., which is referred to in the notes simply as *Formalized Music*.

Achorripsis (1956-7). *Pithoprakta* received its première in Munich in 1957, followed by the première of *Achorripsis* in Buenos Aires in 1958. The Paris première of *Achorripsis*, effectively Xenakis's Paris debut, took place in 1959. *Metastaseis* had been accepted, at Xenakis's own initiative, at the Donaueschingen festival in 1955, where it received its première under Hans Rosbaud.

Meanwhile, Xenakis had become active at the Groupe de Recherches Musicales (GRM), where he learned the basics of electroacoustic composition from Pierre Henry. Finally, in 1957, he was permitted to realize an original work, *Diamorphoses*. Over the next five years he realized four more original works, either wholly or in part, at GRM: *Concret PH* (1958), *Analogique B* (1959), *Orient-Occident* (1960), and *Bohor* (1962). *Concret PH* was composed for performance in the Philips Pavilion at the Brussels World's Fair of 1958. The pavilion, which Xenakis helped Le Corbusier to design, was also the site for the performance of Varèse's *Poème Electronique* (1957-8).[30] *Analogique B* was designed to be performed simultaneously with *Analogique A* for nine strings (1958). This work is significant in part for the reaction it elicited from Schaeffer:

> You look firstly by instinct for sound materials amenable to numerical measurement, then you try to find processes which can take charge of the organization and where the intervention of the arbitrary or the unfolding of indeterminism, even aleatory are clearly separated, and finally, once certain sonic results are obtained through intensive work, you intervene in the results to render them as interesting as possible by editing.[31]

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[30]For illustrations and further information on the Philips Pavilion, see Matossian, *Xenakis*, chapter 5; Le Corbusier, *Poème électronique* (Paris: Editions Minuit, 1958); *Xenakis/les Polytopes*; *Musique Architecture*; and *Formalized Music*, chapter 1.

[31]Letter from Schaeffer to Xenakis, November 29, 1959, quoted in Matossian, *Xenakis*, p. 136.

Schaeffer offers here a fairly accurate description of Xenakis's working method at that time,[32] although it is articulated within the context of a general criticism of the formalized procedures in Xenakis's compositional process. Characteristically, Schaeffer urged Xenakis to forgo formalized methods in favor of purely empirical ones, but the one-sidedness of his attitude ultimately had the effect of alienating Xenakis who, along with Boulez, Stockhausen, and even Pierre Henry, became disillusioned with Schaeffer's vision for GRM and sought to move on.

By the end of 1959, Xenakis was beginning to achieve a sense of independence from the initial round of contacts he had made both in and out of the musical world. He had had a falling out with Le Corbusier, largely because of conflicts over the authorship of the Philips Pavilion, and thereafter decided to concentrate almost entirely on musical composition. His time with GRM would soon draw to a close, but he would eventually establish an electronic music studio of his own. Scherchen, however, remained a loyal supporter and continued to conduct new works by Xenakis until his death in 1966.

Further Mathematical Studies, Computer-Assisted Composition, and CEMAMu

Xenakis had studied various topics in mathematics at Athens Polytechnic. Thereafter he undertook the study of probability theory largely on his own, making use of the work of some of the founders of modern

[32]See the "Fundamental Phases of Musical Work" outlined by Xenakis in *Formalized Music*, p. 22.

probability theory, including Lévy, Borel, and Feller.[33] In 1960 he undertook further studies in algebra and logic with Georges Guilbaud at the University of Paris. Evidently he was seeking a more general basis to his understanding of mathematics and of its possible applications to composition. He may also have been aware of the possibility of a creative impasse if he were to try to extend stochastic methods indefinitely without introducing some new ideas into his compositional process. Xenakis experimented briefly with the application of game theory to the composition of open-form works,[34] but then turned his attention to the creation of "symbolic music"—the application of principles of symbolic logic to musical composition—in the formulation of his sieve theory, and in his compositional applications of group theory.[35] Xenakis also applied symbolic logic in the form of complex set-theoretic operations on pitch collections, in two works: *Herma* for piano (1960-1) and *Eonta* for piano and brass quintet (1963). In addition, simpler applications of the basic set-theoretic operations of union, intersection, and complementation, may be found in several of Xenakis's other works.[36] Sieve

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[33]P. Lévy, *Calcul des probabilités* (Paris: Gauthier-Villars, 1925); E. Borel, *Principes et formules classiques du calcul des probabilités* (Paris: Gauthier-Villars, 1947); Borel, *Eléments de la théorie des probabilités* (Paris: Albin Michel, 1950); William Feller, *An Introduction to Probability Theory and Its Applications* (New York: Wiley, 1950 and subsequent editions).

[34]The mathematical theory of games was applied to the composition of *Duel* for two orchestras (1959), *Stratégie* for two orchestras (1962), and *Linaia-Agon* for brass trio (1972). See *Formalized Music*, chapter 4; Thomas DeLio, "Structure and Strategy: Iannis Xenakis' *Linaia-Agon*," *Interface* 16 (1987): 143-64; and Christoph Schmidt, *Komposition und Spiel. Zu Iannis Xenakis* (Köln: Studio, 1995).

[35]See *Formalized Music*, chapters 6, 7, 11, and 12.

[36]Xenakis's use of set-theoretic operations in this music is distinct, however, from pitch-class set theory, which developed in the United States at around this time. In fact, an important early article in pitch-class set theory, Allen Forte, "Theory of Set-Complexes for Music," *Journal of Music Theory* 8/2 (1964): 136-83, is nearly contemporaneous with the composition of *Eonta*. Standard texts in pitch-class set theory include Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973); John Rahn, *Basic Atonal Theory* (New York: Longman, 1980); and Robert Morris, *Composition with Pitch-Classes* (New Haven: Yale

theory is a branch of number theory in which the principles of set theory are applied to modular arithmetic, resulting in operations on moduli (cycles) of intervals to produce sets of pitches or sequences of time-points.[37] Sieves appear in many of Xenakis's works, including several of those that are analyzed in chapters 3 and 4. Xenakis has made compositional use of the principles of group theory by performing systematic permutations upon ordered sets of elements. These elements typically consist of general structural features such as segment duration, density, intensity and texture. Works in which group-theoretical operations play a prominent structural role include *Akrata* for sixteen winds (1964-5), *Nomos Alpha* for violoncello solo (1966), *Nomos Gamma* for orchestra (1967-8), and *Persephassa* for percussion sextet (1969).[38] Significantly, Xenakis does not apply permutational operations to ordered sets of pitch-classes. His use of order permutations, therefore, is distinct from the practices of twelve-tone serialism, although the two are conceptually related.[39]

University Press, 1987). In general, Xenakis performs operations on elements in pitch space without applying the principle of octave equivalence that is fundamental to the concept of pitch-class space. Morris, despite the title of his text, defines operations in both pitch and pitch-class spaces.

[37]The analogy between these structures and Messiaen's modes of limited transposition and characteristic rhythmic patterns has been pointed out above.

[38]Group-theoretical operations have been applied in some later works as well, as in *Psappha* for percussion solo (1975), where they are used to determine the order of events on the surface of the music. See Ellen Rennie Flint, "An Investigation of Real Time as Evidenced by the Structural and Formal Multiplicities in Iannis Xenakis' *Psappha*," (Ph.D. diss., University of Maryland at College Park, 1989) and E. R. Flint, "Metabolae, Arborescences and the Reconstruction of Time in Iannis Xenakis' *Psappha*," *Contemporary Music Review* 7 (1993): 221-48.

[39]See the references to group theory in Milton Babbitt, "Twelve-Tone Invariants as Compositional Determinants," *Musical Quarterly* 46 (1960): 245-59; and M. Babbitt, "Set Structure as a Compositional Determinant," *Journal of Music Theory* 5/2 (1961): 72-94. Group theory is also significant in Morris, *Composition with Pitch-Classes* and in David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987).

It may appear that by the mid-1960s, when the systematic ordering of compositional events became a primary concern of his, that Xenakis had arrived at a position completely opposed to his original thesis, in which the order of events was to be established entirely according to the principles of indeterminacy. From a broader perspective, however, the new developments need not be seen as a contradiction of the earlier ones, but as an enrichment of the compositional resources available to him through the application of mathematical principles to the formation of musical structures. Xenakis has explained the integration of the extremes of determinacy and indeterminacy in his work in the following way:

> The two poles, one of pure chance, the other of pure determinacy, are dialectically blended in man's mind (and perhaps in nature as well, as Epicurus or Heisenberg wished it). The mind of man should be able to travel back and forth constantly, with ease and elegance, through the fantastic wall, of disarray caused by irrationality, that separates determinacy from indeterminacy.[40]

For the most part, the style of Xenakis's music from this time forward results from various combinations of stochastic and non-stochastic elements. In the music written since 1970, however, the implementation of these elements seems to be less directly tied to specific mathematical procedures than in the music of the 1950s and 1960s. One important exception to this generalization is the formation of stochastic textures, which has depended on calculations throughout the various periods of stylistic change in Xenakis's oeuvre. The technical aspects of stochastic composition, set theory and sieve theory will be explored in some detail in chapter 2. Group theory will also be touched upon

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[40]*Formalized Music*, pp. 237-8.

there in a general way, to the extent that it has influenced related procedures in Xenakis's recent music. These topics are treated at some length in chapter 2 so that the characteristics of actual compositional structures, and not the mathematical foundations of the compositional processes used to obtain them, may be the primary focus of the analyses in chapters 3-5.

Xenakis quickly found that the calculation of complex mathematical functions for compositional purposes by hand is laborious, time-consuming, and prone to error. Not surprisingly, he became interested in the possibility of performing calculations by computer almost as soon as he began to compose stochastic music. At that time, however, computers tended to belong only to corporations and large institutions for scientific research. Thanks to Scherchen's intervention at IBM in Paris, Xenakis was able to gain access to the 7090 computer beginning in 1961. One of the first projects he undertook was the calculation of a complex game matrix used in the composition of *Stratégie* for two orchestras (1962). He also used the computer to implement a program he had designed for the computer-assisted composition of stochastic music. The program included stochastic functions for calculating the duration and sequence of segments, their density, instrumentation, intensity, and articulation, as well as the succession of pitches (where appropriate) and time-points within each segment.[41] By varying the input in each case, Xenakis used the program to produce a series of works: *ST/48-1, 24 01 62* for orchestra; *ST/10-1, 08 02 62* for chamber ensemble; *ST/4-1, 08 02 62* for string quartet; *Morsima-Amorsima (ST/4-1, 03 07 62)* for piano and string trio, and *Atrées (ST/10-3, 06 09 62)* for chamber ensemble. The dates given for the works as a group are 1956-62, indicating the

[41]See *Formalized Music*, chapter 5.

time from their initial conception until their realization on the IBM 7090. The subtitle of each work indicates that it was produced with the stochastic composition program for an ensemble of a specified number of instruments, from data generated on a particular run of the program on a given date. For example, *Atrées (ST/10-3, 06 09 62)* was produced from the third run of the program for an ensemble of ten instruments on September 6, 1962. The subtitle of this work reveals that Xenakis generated more data with the program than was used in the composition of the five works listed above. Further, the titles of *ST/10-1, 08 02 62* and *ST/4-1, 08 02 62* indicate that the data for both works were generated on the same day. In fact, *ST/4*, perhaps the most frequently performed work of the series, is a string quartet based on the solo string parts of *ST/10*, with the occasional addition of parts transcribed from other instruments to fill out the texture.[42] The finished works from the series are made up of segments that have been selected, and slightly reordered, from those produced by the program.[43] The sole exception is *Morsima-Amorsima*, which consists of seventeen segments in their original order. In *Atrées*, on the other hand, the chosen segments have been organized into five separate movements that may be ordered freely according to the will of the conductor. In general, however, the transcription of the events within the segments into musical notation is faithful to the data produced by the program, except in the case of *Atrées*, in which the data has been considerably embellished with freely composed material.[44]

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[42]The Arditti String Quartet has recorded *ST/4* on Disques Montaigne 2 CD 782005.

[43]In the score of each of the works, the original order of the segments produced by the program is indicated by the letters JW followed by a number.

[44]Based on a conversation with Xenakis, Matossian estimates that about 75% percent of the material in *Atrées* was produced by the program, the remainder being freely composed. Matossian, *Xenakis*, p. 161.

The works produced with the stochastic composition program represent one of the first comprehensive projects in computer-assisted composition.[45] Overall, the project should be regarded as a success, even though the composer did not always find the succession of segments produced by the program to be aesthetically convincing. In any event, the stochastic composition program marks the end of Xenakis's attempts to compose thoroughly stochastic instrumental music. Further advances in stochastic composition would have to wait for the development of technology capable of producing electroacoustic sounds through stochastic synthesis. Such technology has been developed recently, allowing Xenakis to combine stochastic synthesis with the principles of stochastic composition in a single computer program. This new program was used to compose the electroacoustic work *Gendy3* (1991).[46]

In 1966, with the help of Guilbaud and others, Xenakis was finally able to found an electronic music studio of his own. At first it was called L'Equipe de Mathématique et Automatique Musicales, but the name was later changed to Centre d'Etudes Mathématiques et Automatiques Musicales (CEMAMu). One of the first projects of CEMAMu was the application of computer technology to the coordination of electroacoustic sounds with laser displays in a genre that Xenakis called the "polytope." Polytopes are spectacles of music and light, performed at a specific location or in a specially designed structure.[47] In some respects, the original polytope was the Philips Pavilion

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[45]See Charles Ames, "Automated Composition in Retrospect: 1956-1986," *Leonardo* 20/2 (1987): 169-85.

[46]See *Formalized Music*, chapters 9, 13, and 14; Marie-Hélène Serra, "Stochastic Composition and Stochastic Timbre: *Gendy3* by Iannis Xenakis," *Perspectives of New Music* 31/1 (1993): 236-57; and Peter Hoffmann, "Implementing the Dynamic Stochastic Synthesis," *Les cahiers du GREYC* 4 (1996): 341-7. A recording of *Gendy3* has been released on Neuma CD 450-86.

[47]See *Xenakis/Les Polytopes*.

at the Brussels World's Fair in 1958, in which Varèse's *Poème Electronique* was performed concurrently with a series of projected images designed by Le Corbusier. Xenakis's Concret PH was composed as a prelude and postlude to round out the 10-minute spectacle. The next polytope was *Polytope de Montréal* for four orchestras accompanied by a display of flashing lights suspended from a steel structure designed by Xenakis, performed at the Montréal Expo in 1967. This was followed by two electroacoustic works, *Hibiki Hana Ma*, performed at the Osaka World's Fair in 1970 in conjunction with a laser show designed by Keiji Usami, and *Persepolis* (1971), which was performed along with a spectacle of electric lights and flaming torches at the ruins of the palace of Apadana near Persepolis, Iran. The first polytope to involve CEMAMu was *Polytope de Cluny* for electroacoustic sounds and computer-driven laser display, which was performed in the Roman baths at the Cluny Museum in Paris from 1972 to 1974. This was followed by the most ambitious polytope of all, La *Légende d'Eer* for electroacoustic sounds and computer-driven laser display (1977), which was performed in the Diatope, an enclosed structure designed by Xenakis and installed on the grounds of the Pompidou Center in Paris in 1978 and again in Bonn in 1979.[48]

The electroacoustic music produced for the polytopes was still patterned after Xenakis's earlier work at GRM, relying mainly upon the techniques of *musique concrète,* but the next project completed by CEMAMu introduced a new element into his electroacoustic music. The Unité Polyagogique Informatique du CEMAMu (UPIC), first produced in 1977, is a system designed for the graphic composition of electroacoustic music based

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[48]See Matossian, *Xenakis*, chapter 11, and Iannis Xenakis, "Music Composition Treks," in *Composers and the Computer*, ed. Curtis Roads (Los Altos, CA: William Kaufmann, 1985), pp. 171-92.

on synthesized or digitally sampled sounds. In the original system compositions were designed in separate sections, or "pages." The pages were created one at a time on a digital drawing board with the aid of an electromagnetic pen. Once completed, a page could be heard by entering it into a converter.[49] Later developments include the introduction of real-time technology, allowing for the playback of a page as it is being composed, and the creation of software for greater portability, making it possible for the UPIC to run on a personal computer, with the computer's monitor and mouse replacing the drawing board and electromagnetic pen of the original version.[50] Works composed on the UPIC include portions of *La Légende d'Eer* (1977), *Mycenae-Alpha* (1978), *Pour la Paix* for chorus, narrators, and electroacoustic sounds (1981), *Taurhiphanie* (1987), and *Voyage absolu des Unari vers Andromède* (1989).[51] An analysis of *Mycenae-Alpha* is given in chapter 5.

Xenakis has made significant contributions to twentieth-century music through his applications of mathematics and computer technology. In the 1950s and 1960s, through the application of stochastic functions, as well as game theory, set theory, sieve theory, and group theory, he created a distinctive style of composition for instruments, voices, and electroacoustic media (limited mainly to the techniques of *musique concrète*). He is also a significant pioneer in the field of computer-assisted composition. Finally,

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[49]See Henning Lohner, "The UPIC System: A User's Report," *Computer Music Journal* 10/4 (1986): 42-9.

[50]See *Formalized Music*, Appendix III, and Gérard Marino, Marie-Hélène Serra, and Jean-Michel Raczinski, "The UPIC System: Origins and Innovations," *Perspectives of New Music* 31/1 (1993): 258-69.

[51]*Mycenae-Alpha* may be heard on Neuma CD 450-74, *Taurhiphanie* on Neuma CD 450-86, and *Voyage absolu des Unari vers Andromède* on the CD accompanying *Perspectives of New Music* 28 (1990).

with the development of the UPIC system and the stochastic synthesis program, Xenakis has combined computer technology and electronic sound synthesis with, respectively, his interest in graphic and stochastic composition.

General Stylistic and Structural Orientation

The preceding sections of this chapter are intended to provide an outline of some of the circumstances surrounding the development of Xenakis's compositional style. It should be evident from what has been written above that some of the basic assumptions and techniques that are normative within the context of his work are different from those of many of his predecessors and contemporaries. For example, the idea of complex masses of sound, derived in part from his experiences of mass rallies in Greece and also from his exposure to the technique and aesthetics of *musique concrète*, has continued to affect much of his subsequent work and carries with it definite consequences for the segmentation of his music during the analytical process. In most cases it is senseless to seek for motives, cells, or sets containing only a handful of elements, since the basic units responsible for structural articulation in Xenakis's music are segments that typically contain a large number of individual elements and generally have a duration of several seconds or more. The boundaries between these segments are usually fairly obvious, since they are articulated by changes in global characteristics, such as texture, density, intensity, articulation, and instrumentation. Such means of articulating differences between segments help to simplify perception of the large-scale structure of the music, thereby complementing the complexity of the musical surface within the segments.

In the absence of intrinsic criteria, such as the establishment of harmonic syntax or of processes of completion (such as the formation of twelve-tone aggregates), the boundaries separating collections of individual elements must be imposed extrinsically, through the careful management of changes in the global characteristics of the segments. It is the formation of structural frameworks by means of such extrinsic criteria, which allows the successions of individual elements to follow relatively indeterminate courses without undermining the coherence of the overall structure, that will be the main focus of attention in the analytical portions of the dissertation.

Among the general properties of segments, duration plays an especially important role in shaping musical materials into the structure that is characteristic of a given composition. Frequently in Xenakis's music, a definite structure is given to the temporal aspect of a composition through the establishment of a system of proportions that governs global as well as local divisions within the work. This procedure has a clear antecedent in Le Corbusier's use of nested proportional systems in his architectural projects, including some projects on which Xenakis collaborated. Pitch is another aspect of structure that is basic in much of Xenakis's music. In general, he tends to conceive of individual pitches as specific points along a continuous frequency spectrum, rather than as representatives of classes within the octave-based model of pitch that is standard in most theories of tonal, atonal, and serial music.[52] Xenakis's model of pitch is closer to Varèse's, which was highly unusual for its time. The probability of direct influence from Varèse is somewhat unlikely, however, for Xenakis had already made use of his spatial model of pitch before he had had an opportunity to meet the elder composer.

[52]For representative texts in atonal and serial theory, see the items mentioned in notes 36 and 39.

Moreover, there are important differences in the kinds of operations both of these composers perform in their respective pitch spaces.[53] Nonetheless, several of the distinctive structural features of Xenakis's music relate in striking ways to the unusual circumstances of his early development as a composer. In the following chapter, attention is directed away from the biographical circumstances surrounding the development of his unusual compositional style and is turned instead toward specific technical features of his compositional process that are directly responsible for the music's distinctive sonic qualities.

[53]See Jonathan Bernard, *The Music of Edgard Varèse* (New Haven: Yale University Press, 1987) and J. Bernard, "Pitch/Register in the Music of Edgard Varèse," *Music Theory Spectrum* 3 (1981): 1-25.

Chapter 2

The Structural Elements of Xenakis's Music

The stylistic characteristics of Xenakis's music described in the previous chapter are realized by means of particular structural elements. Several of these elements, if not absolutely unique to his music, are certainly different from those that form the basis of the music of most other composers, including composers of Xenakis's own generation. This chapter provides an outline of the structural elements that are found most frequently in his music, beginning with the characteristics of individual elements and progressing to descriptions of larger and more complex structural units. The chapter ends with a discussion of aspects of large-scale structure in some early works by Xenakis, in order to establish a context from which to approach the detailed structural analyses of recent works found in chapters 3-5.

In compiling material for this outline I have relied principally upon the descriptions of musical structure and compositional technique given by Xenakis in *Formalized Music.*[1] Where the descriptions are incomplete, or where Xenakis's terminology is unclear or otherwise open to misunderstanding, I have incorporated information from other sources, including the composer's sketches and writings by others on the topics of probability theory, stochastic composition, and general compositional and

[1]Iannis Xenakis, *Formalized Music*, rev. ed. (Stuyvesant, NY: Pendragon Press, 1992).

analytical theory.[2] The relationship between the structural elements described here and the methods used to make analytical statements about them in the succeeding chapters will be considered at appropriate points as the discussion proceeds.

2.1 Basic Concepts

2.1.1 Sonic Events

Individual musical sounds may be thought of as *sonic events*[3] located in an abstract musical space.[4] The various attributes of of sonic events, such as pitch, duration, and intensity, may be thought of as so many *dimensions* within musical space. The various degrees of a particular attribute—e.g., low, high, or soft, loud—may be imagined as relative positions along an axis in the dimension associated with that attribute. Further, the degrees of an attribute may be measured precisely in terms of multiples of a unit value appropriate to the attribute. In this way it is possible to express the degree of an attribute as a specific position along the axis of its associated dimension. If positions

[2]Two works in the latter category that have been particularly helpful in developing a vocabulary with which to identify specific structural features of Xenakis's music are Robert Morris, *Composition with Pitch-Classes* (New Haven: Yale University Press, 1987) and James Tenney, *Meta/Hodos and Meta-Meta/Hodos* (Hanover, NH: Frog Peak Press, 1989).

[3]Terms introduced in italics are also found in the Glossary.

[4]Some problematic aspects of spatial metaphors for musical structures are discussed in Christopher Hasty, "Succession and Continuity in Twentieth-Century Music," *Music Theory Spectrum* 8 (1986): 58-74. Despite their limitations (as pointed out by Hasty and others), spatial metaphors are adopted freely in the present context because they are fundamental to the thought underlying Xenakis's compositional technique, which relies heavily on numerical and graphic representations of musical structures. The practical utility of spatial metaphors is evident in the work of other theorists and composers as well, e.g. in Morris, *Composition with Pitch-Classes* and in Robert Cogan and Pozzi Escot, *Sonic Design* (Englewood Cliffs, NJ: Prentice-Hall, 1976).

are specified along the axes of several dimensions simultaneously, it is possible to describe the *state* of a sonic event quantitatively in terms of the *coordinates* of the axes. The result is a coordinate model of musical space in which the states of sonic events may be represented with relative precision in several dimensions at once.

Several relevant attributes of sonic events are listed below, along with names for the dimensions associated with them, relative degrees along the axes of the dimensions, and examples of appropriate unit values.[5]

		relative	*position*	
attribute	*dimension*	*low*	*high*	*unit(s)*
pitch	p-space	low	high	semitone, quarter-tone
sequential time	st-space	before	after	ordinal number
time-point	tp-space	early	late	beat, second
duration	d-space	short	long	beat, second
intensity	i-space	soft	loud	level (*pp, p, ..., f, ff*)

The number of dimensions in a coordinate model need not express all conceivable attributes of a sonic event in order for the model to be useful in composition and analysis. In many analytical situations it will be sufficient to express the states of sonic events in p-space and tp-space, and frequently also in i-space, d-space. It should also be pointed out that the temporal aspects of a sonic event have been differentiated into three dimensions in the model proposed above: d-space for the duration of the event; st-space for the temporal order of the event in relation to other events; and tp-space for the instant at which the event is initiated. The distinction between these three dimensions will be discussed further below, in connection with Xenakis's model of musical time.

[5]The table below is based loosely on one found in Morris, *Composition with Pitch-Classes*, p. 282. Definitions of the attributes and dimensions are given in the Glossary.

2.1.2 A Coordinate Model of Musical Space

A brief musical example will serve to illustrate the representation of sonic events in coordinate notation. Figure 2.1 shows the first three measures of Xenakis's *Herma* for piano (1960-1). The states of the fifteen sonic events contained within these measures are represented in coordinate notation below. The four dimensions represented by the coordinates are, in order, tp-space, p-space, d-space, and i-space. The unit for tp-space is one beat, where each beat is equivalent to a quarter-note at 104 MM (as indicated in the score). The origin, or zero-value, along the axis in tp-space is set at the downbeat of measure 1. The unit for p-space is one semitone, and the origin of the p-space axis is set at C4 (middle C).[6] The unit for d-space, as for tp-space, is one beat, and the origin in d-space represents a duration of zero beats. The unit for i-space is one level of intensity, where a level of *ppp* is represented by the number 1. The origin of i-space represents silence. The intensity in *Herma* increases steadily from a level of *ppp* in m. 1 to *fff* in m. 27. For the purposes of the illustration below, the crescendo is represented by multiples of the increment Δi added to the original level of 1 with the appearance of each successive event. It is unlikely that in an actual performance of the work, the intensity would increase by a steady increment with the sounding of each successive event. The notation indicated at the end of this paragraph,

[6]The system for labelling pitches used here is the one proposed by the Acoustical Society of America. In this system, pitches are denoted by the letter (and symbol) associated with the mod 12 pitch-class to which they belong, followed by a number indicating the octave in which they appear. The lowest C on the standard piano keyboard is denoted C1 and the highest C as C8. In each of the musical examples presented, the origin of the p-space will be assumed to be 0 = C4 unless otherwise noted.

therefore, is merely intended to indicate that the intensity increases gradually over the course of the passage by an amount that need not, indeed cannot, be determined with precision on the basis of the information given in the musical score.[7] Subdivisions of the unit beat in tp-space and d-space are represented by decimal approximations.

$$s = (tp, p, d, i)$$

$s_0 = (0, -20, 2, 1)$	$s_8 = (7.33, -19, 2.67, 1 + 8\Delta i)$
$s_1 = (1, 18, 2, 1 + \Delta i)$	$s_9 = (8, 8, 0.67, 1 + 9\Delta i)$
$s_2 = (2, -14, 1, 1 + 2\Delta i)$	$s_{10} = (8.67, 33, 1.16, 1 + 10\Delta i)$
$s_3 = (3, -37, 2, 1 + 3\Delta i)$	$s_{11} = (9.83, -10, 1.16, 1 + 11\Delta i)$
$s_4 = (4, 19, 1, 1 + 4\Delta i)$	$s_{12} = (10, -11, 2, 1 + 12\Delta i)$
$s_5 = (5, 39, 3, 1 + 5\Delta i)$	$s_{13} = (11, 40, 0.67, 1 + 13\Delta i)$
$s_6 = (5.33, -24, 1, 1 + 6\Delta i)$	$s_{14} = (11.67, 23, 3.33, 1 + 14\Delta i)$
$s_7 = (6.33, -35, 0.67, 1 + 7\Delta i)$	

2.1.3 A Vector Model of Musical Space

The coordinate model of musical space is similar to the model proposed by Xenakis in chapter 6 of *Formalized Music*, with one important difference: the coordinate model focuses on the states of sonic events while the vector model proposed by Xenakis focuses on the relations between the states of the sonic events. Before illustrating Xenakis's model, it is necessary to introduce some basic concepts and notational symbols appropriate to vector spaces.[8] A *vector* is a measure of the distance between two points in one or

[7]It would also be possible to define the dimension of intensity as an ordinal, but unmeasured space. The use of increments is somewhat problematic, but is also more consistent with the coordinate model of musical space proposed here.

[8]The coordinate space model was introduced first because it is assumed that the reader is familiar with coordinate notation from elementary studies in mathematics, whereas vector notation is used in more specialized mathematical studies and may, therefore, present greater

more dimensions. A vector indicates both the magnitude of the distance between the points and its direction. In this respect a vector is similar to a directed (or ordered) interval between two events in some dimension. Ordinarily, however, directed intervals refer to only a single dimension, whereas vectors may refer to directed distances in several dimensions simultaneously. A vector is usually identified by a boldface letter associated with an ordered tuple of real numbers that are separated by a comma and enclosed in angle brackets. Each number in the ordered tuple represents a directed distance in a particular dimension. For example, the ordered pair $\mathbf{v} = \langle a, b \rangle$ signifies a vector in two dimensions. Each member of the ordered pair $\langle a, b \rangle$ is a *component* of the vector. Two representations of $\mathbf{v} = \langle a, b \rangle$ are shown in Figure 2.2. Both representations are equivalent since $\langle (x + a) - x, (y + b) - y \rangle = \langle a - 0, b - 0 \rangle = \langle a, b \rangle = \mathbf{v}$. The representation of vector $\mathbf{v}$ that extends from from (0, 0) to (a, b) is known as a *position vector* because its initial point is located at the origin of the coordinate space.

Vectors may be represented in a coordinate space or in a *vector space*. In a vector space, distances along the axes are measured in terms of *basis vectors* (also called *unit vectors*) instead of the points that are used to indicate positions along the axes in a coordinate space. The *basis* of a three-dimensional vector space is shown in Figure 2.3. Vector spaces are usually symbolized by the letter V followed by a subscript indicating the number of dimensions in the space. The vector space in Figure 2.3, therefore, may be denoted V_3. The dimensions of V_3 are d-space, p-space, and i-space. Its basis vectors are $\mathbf{d} = \langle 1, 0, 0 \rangle$, $\mathbf{p} = \langle 0, 1, 0 \rangle$, and $\mathbf{i} = \langle 0, 0, 1 \rangle$. Distances along the axes of

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conceptual difficulties. The relationship between the coordinate and vector models will become clearer as the discussion proceeds.

V_3 are measured in multiples of these basis vectors. For example, $\mathbf{v} = \langle 2, -10, 3\rangle$ represents an increase of 2 units in duration, a decrease of 10 units in pitch, and an increase of 3 units in intensity between the states of two sonic events. The details of the graphic representation and notation of vector spaces need not detain us any further.[9] For our purposes it is sufficient to remember that vectors represent distance and direction, while coordinate points represent location.

Let us turn now to a vectorial representation of the relations between the successive sonic events in the first three measures of *Herma* (Figure 2.1). Like the coordinate space used to represent the states of the events, the vector space used below to represent the relations between their states contains four dimensions. The order of the dimensions, the units used in each one, and the attributes associated with the origin of each axes are all the same as before.

$$V_4 = \langle tp, p, d, i\rangle$$

$\mathbf{v}_0 = \langle 0, -20, 2, 1\rangle$	$\mathbf{v}_8 = \langle 1, 16, 2, \Delta i\rangle$
$\mathbf{v}_1 = \langle 1, 38, 0, \Delta i\rangle$	$\mathbf{v}_9 = \langle 0.67, 27, -2, \Delta i\rangle$
$\mathbf{v}_2 = \langle 1, -32, -1, \Delta i\rangle$	$\mathbf{v}_{10} = \langle 0.67, 25, 0.49, \Delta i\rangle$
$\mathbf{v}_3 = \langle 1, -23, 1, \Delta i\rangle$	$\mathbf{v}_{11} = \langle 1.16, -43, 0, \Delta i\rangle$
$\mathbf{v}_4 = \langle 1, 56, -1, \Delta i\rangle$	$\mathbf{v}_{12} = \langle 0.17, -1, 0.84, \Delta i\rangle$
$\mathbf{v}_5 = \langle 1, 20, 2, \Delta i\rangle$	$\mathbf{v}_{13} = \langle 1, 51, -1.33, \Delta i\rangle$
$\mathbf{v}_6 = \langle 0.33, -63, -2, \Delta i\rangle$	$\mathbf{v}_{14} = \langle 0.67, -17, 2.66, \Delta i\rangle$
$\mathbf{v}_7 = \langle 1, -11, -0, \Delta i\rangle$	

Vector $\mathbf{v}_0$ is a position vector, since its initial point is (0, 0, 0, 0). Its components, therefore, are identical to the coordinates of its terminal point,

[9]An explanation of vector spaces is given in *Formalized Music*, pp. 161-6. Readers interested in further aspects of vectors may wish to consult a standard mathematical text, such as James Stewart, *Multivariable Calculus* (Pacific Grove, CA: Brooks/Cole, 1991).

(0, -20, 2, 1), which are also identical to the coordinates of s_0. By the principles of vectorial addition it is possible to sum the components of vectors. When the sums of vectors $\mathbf{v}_0 - \mathbf{v}_{14}$ are taken cumulatively the results are as follows:

$$\mathbf{v}_0 = \langle 0, -20, 2, 1\rangle$$
$$\mathbf{v}_0 + \mathbf{v}_1 = \langle 1, 18, 2, 1 + \Delta i\rangle$$
$$\mathbf{v}_0 + \mathbf{v}_1 + \mathbf{v}_2 = \langle 2, -14, 1, 1 + 2\Delta i\rangle$$
$$\ldots$$
$$\mathbf{v}_0 + \mathbf{v}_1 + \ldots + \mathbf{v}_{14} = \langle 11.67, 23, 3.33, 1 + 14\Delta i\rangle$$

Clearly, the cumulative addition of vectors $\mathbf{v}_0 - \mathbf{v}_{14}$ yields components that are identical with the coordinates of $s_0 - s_{14}$. This fact has relevance for the process of stochastic composition and, for that matter, for any compositional process involving the calculation of directed distances between values associated with the attributes of sonic events. Xenakis's stochastic composition computer program, for example, relies upon the random selection of components in several dimensions, followed by their cumulative addition. The result is a *notelist* that contains numerical data representing the states of sonic events in several dimensions. It is possible for all four dimensions represented above to have been calculated with the aid of mathematical functions, either by hand or with a computer.[10] Based on the description of the compositional process of *Herma* in chapter 6 of *Formalized Music* and on an analysis of the entire passage from which the excerpt in Figure 2.1 was drawn, however, it appears that the components for tp-space

[10]The dates of *Herma* are 1960-1, so it is possible that Xenakis may have had access to the IBM 7090 during the time of its composition, but the composer has not revealed whether the calculations for this work were performed by computer or by hand.

and p-space were produced with the aid of stochastic functions, while those for d-space and i-space were freely composed.[11]

2.1.4 Attributes of Collections of Sonic Events

The discussion so far has centered around the attributes of individual sonic events. There are, in addition to these, attributes appropriate to collections of sonic events. A representative list is given below.

attribute	*dimension*	*relative* *low*	*position* *high*	*unit(s)*
pitch	p-space	min	max	semitone, quarter-tone
registral span	r-space	small	large	semitone, quarter-tone
sequential time	st-space	before	after	ordinal number
time-point	tp-space	early	late	beat, second
duration	d-space	short	long	beat, second
intensity	i-space	soft	loud	level (*pp, p, ..., f, ff*)
density	∂-space	low	high	sounds/second (s/s)

This list features two significant additions and one important change from the list given above of the attributes of individual sonic events. In making observations on collections of sonic events contained within individual segments of music, it is often useful to note the minimum, maximum, and mean pitch levels that occur within the segments. Another useful measure of the use of p-space within a segment is its registral span, which is the size of the interval between the minimum and maximum pitches in the segment. The most significant addition to the list, however, is *density,* which is a numerical measure of the textural density of segments of music determined as the mean number of sonic events per unit time. The measurement of

[11] The process of stochastic composition will be explained in detail in section 2.2.1 below.

density depends both upon the number of sonic events in a segment and the segment's duration. The most frequently used unit for density is sounds per second (s/s).[12]

In chapter 6 of *Formalized Music* Xenakis proposes a model of the structure of sonic events that is relevant both to the analysis of individual sonic events and collections of sonic events. Here is Xenakis's summary of that model:

> Let there be three events *a, b, c* emitted successively.
> *First stage*: Three events are distinguished, and that is all.
> *Second stage*: A "temporal succession" is distinguished, i.e., a correspondence between events and moments. There results from this
>
> $$a \text{ before } b \neq b \text{ before } a. \quad \ldots$$
>
> *Third stage*: Three sonic events are distinguished which divide time into two sections within the events. These two sections may be compared and then expressed in multiples of a unit. Time becomes metric and the sections constitute generic elements of set *T* [i.e., distances along a temporal axis]....
> According to Piaget, the concept of time among children passes through these three phases....
> Fourth stage: Three sonic events are distinguished; the time intervals are distinguished; and independence between the sonic events and the time intervals is recognized. An algebra *outside-time* is thus admitted for sonic events, and a secondary *temporal algebra* exists for temporal intervals; the two algebras are otherwise identical.... Finally, one-to-one correspondences are admitted between algebraic functions outside-time and temporal algebraic functions. They may constitute an algebra in-time.
> In conclusion, most musical analysis and construction may be based on: 1. the study of an entity, the sonic event, which, according to our temporary assumption groups three

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[12] The dimension associated with density is identified as ∂-space to differentiate it from d-space (duration). The letter ∂ (small delta) is frequently used in mathematical formulas to indicate the parameter for density, as in the probability formula used by Xenakis for the random distribution of time-point intervals: $P_x = \partial e^{-\partial x} dx$. (See *Formalized Music*, p. 12.)

> characteristics, pitch, intensity, and duration, and which possesses a structure *outside-time*; 2. the study of another simpler entity, time, which possesses a *temporal structure*; and 3. the correspondence between the structure outside-time and the temporal structure: the structure *in-time*.[13]

This presentation of the model, though concise, is somewhat unclear since it contains four stages but only three "structures." The outside-time structure of sonic events refers only to those characteristics that can be determined independently of their temporal succession and therefore also of their precise locations in time, i.e. to stage one of the four-stage model. The temporal structure refers to the locations of points in time and to the relations between them, prior to the assignment of sonic events with specific characteristics to these points. This structure corresponds to stage three of the four-stage model. This leaves stage two, temporal succession, without an association to any of the three structures. From Xenakis's presentation of the model it is unclear whether temporal succession should be considered prior to, and therefore independent of, the temporal structure, or whether it is to be subsumed within that structure. Since there are analytical situations in which it is useful to distinguish between temporal succession and temporal structure, the former will be regarded here as an independent, fourth element alongside the three structures. The inside-time[14] structure, on the other hand, corresponds unambiguously to stage four. This structure represents the

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[13]*Formalized Music*, pp. 160-1. References to the commutativity or non-commutativity of the sets in which the temporal and outside-time structures exist have been deleted, since they are not directly relevant to the present discussion. The work by Piaget to which Xenakis refers is Jean Piaget, *The Child's Conception of Time*, trans. A.J. Pomerans (New York: Ballantine Books, 1969).

[14]I prefer the term "inside-time" to "in-time," since it is a more obvious antonym to "outside-time."

coordination of the outside-time and temporal structures, and thereby the completion of the process by which abstract (or potential) sonic events are realized in actual compositional situations.

The model may be clarified further by assigning the three structures, plus temporal succession, to specific dimensions in musical space. The outside-time structure, as stated in Xenakis's summary of the model, includes such dimensions as pitch, intensity, and duration (p-space, i-space, and d-space). Note that duration, even though it represents a temporal quality, may be determined independently of the order or location of an event in time. Temporal succession corresponds to the dimension of sequential time (st-space). Temporal structure corresponds to the time-point dimension (tp-space). The inside-time structure of sonic events corresponds to the multi-dimensional models of the states and relations of sonic events which were expressed above in terms of coordinates and vectors, respectively.[15]

2.1.5 A Demonstration of the Applicability of Xenakis's Temporal Model to the Analysis of His Music

The analysis of a work necessarily begins from the perspective of its inside-time structure, as represented in its performance and as symbolized in its notated score. From close observation of a work's inside-time structure, it is possible to reconstruct the elements of the underlying temporal structure, temporal succession, and outside-time structure and to examine the relations within and between them. This is best demonstrated in connection with

[15] The summary cited in note 12 immediately precedes Xenakis's discussion of the vector model of musical space. The order of presentation has been reversed here in the interests of greater clarity.

actual musical examples. The demonstration presented here is based upon the structure of *à r.* for piano (1987), which is analyzed in greater detail in chapter 3.[16]

The succession of letters shown below represents a temporal succession of thirty-nine segments of music. The segments are divided into two types, a and b, on the basis of their general textural characteristics. For the purposes of this demonstration, let us assume that the letter a represents stochastic texture and that b represents some type of non-stochastic texture.

ababababab bbbaaaababab bbbaaabbbbaaaabab

Based on the patterns observable in this succession of segment types, the long sequence shown above may be broken down into shorter sequences. In order to do this consistently it is important that some criteria be established for subdividing the long sequence. Our criteria will be the following: 1) where an a is followed by a single b or a b is followed by a single a, successive segments of opposing types are grouped together; 2) where an a is followed by more than one b the succession of b's is grouped together, followed by the succession of a's that comes after it (but note the exception indicated in the following criterion); 3) each succession of opposing types must begin with an a. The first criterion indicates that a subdivision occurs between the ninth and tenth segments in the sequence above. The second criterion implies that the tenth through seventeenth segments should be grouped together, but the third criterion revises this grouping by stating that the seventeenth segment

[16]Some of the details of this work have been simplified for the purposes of the present discussion. The reading presented here, therefore, does not correspond in every respect to the more faithful, and necessarily more complex, analysis presented in chapter 3. The general points made above, however, are valid with respect to the structure of the actual work.

belongs to the group of opposing types that extends through the twenty-first segment.[17] Repeated applications of these criteria result in the groups of sections indicated below, which divide the work into six sections.

section	succession
1	abababababa
2	bbbbaaa
3	ababa
4	bbbbaaa
5	bbbbaaa
6	abab

It is evident from this grouping that there are two basic patterns in the succession of segment types. The first consists of an alternation of contrasting types, and the second of a succession of b's followed by a succession of a's. The two basic types of patterns may be labelled X and Y. Thus sections 1, 3, and 6 may be classified according to pattern type X and sections 2, 4, and 5 according to pattern type Y. Further, it should be noted that each of the X-type sections begins with a segment of type a, and also that sections 3 and 6 represent progressively abbreviated versions of the X-type succession initially found in section 1. The Y sections, on the other hand, all begin with segments of type b, and all contain the same number of segments.

The duration of an individual sonic event is generally equated with the time value of the notehead and/or flag or brace with which it is symbolized. The duration of a segment, however, may often be measured as the distance between the time-point of the first event in the segment and the

[17]The third criterion may appear arbitrary if it is interpreted as a general principle. Within the context of this work, however, it serves to differentiate segments that contain alternations of contrasting configuration types throughout from those that contain direct, extended successions of similar configuration types.

time-point of the first event in the following segment.[18] Thus, the duration of a collection of sonic events is dependent upon the temporal succession and temporal structure of the events it contains. The duration of a section, in turn, is equivalent to the sum of the durations of the segments it contains. The durations of the sections in *à r.*, measured according to the tempo indications in the score, are shown below.

section	pattern type	duration (seconds)
1	X	28.00
2	Y	20.50
3	X	21.67
4	Y	16.67
5	Y	17.50
6	X	22.50

When the six sections are grouped into pairs by pattern type, it is apparent that the X-type sections are longer than the Y-type sections with which they are associated. Not only is there a consistent difference in their durations, but this difference appears to be based upon a constant proportion, as demonstrated in the following table.

sections	durations	proportion
1/2	28.00/20.50	1.36
3/4	21.67/16.67	1.30
6/5	22.50/17.50	1.29

No single proportion is reproduced precisely between pairs of sections, but each of them lies reasonably close to the mean, which is 1.32. Thus, the

[18]If segments overlap or if no segment follows immediately, the duration may be measured as the distance between the time-point of the first event in the segment and the time-point at the expiration of the duration of the last event. When segments follow one another immediately, this measure is equivalent to the simpler one specified in the text.

temporal structure of *à r.* is characterized by remarkably consistent relations among the durations of pairs of sections. This temporal structure has been revealed through the observation of patterns in the succession of textures within the segments followed by calculation of the durations of the resulting sections. Because temporal succession was taken into account in the determination of sections and in the manner of their pairing, this interpretation of the temporal structure suggests one possible way of perceptually grouping sections within the work's inside-time structure.

The proportions discovered in the temporal structure of *à r.*, however, may also be found when the temporal succession of sections and segments is disregarded. The total duration of the X-type sections in relation to the total duration of the Y-type sections is 72.17"/54.67" = 1.32, which is the mean of the proportions listed above for the three pairs of sections. Furthermore, the relation between the total duration of the a-type segments and the total duration of the b-type segments is 71.9"/54.9" = 1.31, which is very close to the mean of 1.32. These proportions indicate the existence of a conceptual link between the inside-time and outside-time aspects of the temporal structure of *à r.*

From this example, which will be elaborated and supported more fully in the more detailed analysis of *à r.* in chapter 3, it is evident that Xenakis has implemented compositionally some of the principles that are introduced abstractly in chapter 6 of *Formalized Music.*[19] Remarkably, in his writings he never mentions the use of common proportions to create associations between the inside-time and outside-time structures of his compositions. Perhaps this special use of temporal proportions is among the "trade secrets"

[19]See the extended quote from chapter 6 of *Formalized Music* given in section 2.1.4 above.

that he has chosen not to disclose publicly, but analysis of his music reveals numerous instances of the proportional organization of temporal structures, and of proportional correspondences between inside- and outside-time structures.

At this point questions may legitimately be raised regarding the perceptibility of these proportionally organized structures. I do not believe that this is a question that can be answered objectively, but I can report on the basis of my own listening experience that Xenakis's music seems to be designed in a way that facilitates the perception of structural coherence despite the absence of traditional means for generating structural coherence, such as tonal or serial organization, or "contextual" atonality. Jonathan Kramer refers to a similar phenomenon in his description of listening to proportionally organized music:

> The farther along we are in listening to a composition, the more durational information we have acquired (we have experienced and remembered a greater number of structurally significant lengths) and therefore the more nearly complete is our knowledge of and feeling for the work's proportional system (or lack thereof).[20]

Kramer also refers to a process of "cumulative" listening, whereby one perceives a sense of structural completeness when coming to the end of a section, or of a whole work, in which a previously articulated proportional relationship is replicated.[21] This general principle may shed some light on the way in which proportions in the outside-time structure may, at least subliminally, reinforce perception of the inside-time proportions and thereby

[20]Jonathan D. Kramer, *The Time of Music* (New York: Schirmer Books, 1988), p. 326.

[21]See Kramer, *The Time of Music*, especially chapters 2 and 10.

contribute to the sense of structural completeness that may be felt at the conclusion of many of Xenakis's works. Whatever the value of such speculations may be, the fact remains that temporal proportions are a recurrent feature in the compositional organization of Xenakis's music and they therefore play a prominent role in the analysis of the music's structure.

2.1.6 Sets and Sequences

Sets provide a convenient way of representing the attributes of collections of sonic events in a single dimension. A *set* is an unordered collection of elements. By convention, the elements of a set are listed in ascending order, with duplicate elements represented only once each. For example,

$$A = \{-5\ -2\ 1\ 3\ 7\ 11\ 13\ 16\ 17\}$$

is a set containing ten elements. The curly brackets surrounding the list of elements indicate that order is not a property of the set. Because order is not a property of sets, two listings that contain the same elements in different orders, with or without duplicates, are equivalent.[22] Therefore,

$$A = \{-5\ -2\ 1\ 3\ 7\ 11\ 13\ 16\ 17\} = \{1\ 16\ -5\ 3\ 1\ 11\ -2\ 17\ 7\ 3\ 13\}.$$

Both are representations of the set A containing ten distinct elements, but the former representation lists the elements in ascending order for convenience.

22 A set with duplicates is called a multiset. A proper set has no duplicate elements by definition. (This clarification was provided in a personal communication from Robert Morris.)

A *sequence*, abbreviated *seq*, is an ordered collection of elements.[23] The elements in a seq are listed in the order in which they appear in some dimension, such as st-space or tp-space. The elements in a seq are surrounded by angle brackets,[24] for example

B = <8 -3 4 6 4 19 23 -7>.

Because order is a property of seqs, duplicate elements are preserved in the listing. Moreover, seqs containing the same elements, but in different orders, are not equivalent. Thus,

B = <8 -3 4 6 4 19 23 -7> ≠ C = <4 6 19 4 23 -3 -7 8>.

The *spacing* of a set, abbreviated SP, is an ordered listing of the intervals between successive elements in the set listing. For example,

SP(A) = <3 3 2 4 4 2 3 1>.

The spacing of a set contains positive integers only, because the elements in a set are conventionally listed in ascending order and duplicate elements are listed only once each. The spacing of a set contains one element fewer than the number of elements in the set.

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[23]The term "sequence" as used here is equivalent to "segment" (abbreviated "seg") as used in Morris, *Composition with Pitch-Classes*. Because the term segment is used in this dissertation to refer to a collection of sonic events inside-time, i.e. in several dimensions, a slight change of terminology seems warranted in order to reduce the possibility of confusion.

[24]Note that these angle brackets (<>) are not the same angle brackets used to indicate vectors (⟨⟩). The former are used to enclose an ordered listing of elements while the latter are used to enclose the values of components in a multi-dimensional space.

The *interval succession* of a seq, abbreviated INT, is an ordered listing of the intervals between successive elements in the seq. For example,

INT(B) = <-11 7 2 -2 15 4 -30>, and

INT(C) = <2 13 -15 19 -26 -4 15>.

Interval successions contain negative as well as positive integers because they list directed intervals between the successive elements of a seq.

Sets and seqs may be labelled according to the types of elements they contain. For example, a set of elements in p-space is called a *pset,* and a seq of elements in tp-space is a *tpseq.* These are the two labels for specific types of sets or seqs most commonly used in this dissertation. In the event that other special names need to be used, their derivations will be explained when they are first introduced.

If the spacing of a set or the INT of a seq displays a repeated pattern of intervals, the set or seq is described as *modular.*[25] For example, let

SP(D) = <1 1 3 2 3 4 2 1 1 3 2 3 4 2>

and let the initial element of set D be 0. Then

D = {0 1 2 5 7 10 14 16 17 18 21 23 26 30 32}.

The repeated interval pattern in this case is <1 1 3 2 3 4 2>. When the intervals in this pattern are summed, the result is

[25]A modular set or seq is similar to a pcyc as defined in Morris, *Composition with Pitch-Classes,* chapter 2. The presentation of these concepts here is intended as a preparation for the discussion of sieve theory in section 2.1.7. Any inconsistencies or confusion that result from the conflation of notations and concepts from Morris and Xenakis in this section remains my responsibility alone.

$$1 + 1 + 3 + 2 + 3 + 4 + 2 = 16.$$

The sum of the intervals in the pattern is known as the *period,* or *modulus,* of the modular set. Observation of the contents of set D reveals that the interval pattern begins on element 0 and once again on element 16. Elements that lie at a distance from one another equal to the value of the modulus m, or some multiple of the modulus m, are said to be *congruent modulo m.* (The expression modulo m is often abbreviated mod m.) For example, in set D, where m = 16,

$$\begin{array}{rlr} 16 & \bmod 16 \equiv & 0 \\ 17 & \bmod 16 \equiv & 1 \\ 18 & \bmod 16 \equiv & 2 \\ 21 & \bmod 16 \equiv & 5 \\ 23 & \bmod 16 \equiv & 7 \\ 26 & \bmod 16 \equiv & 10 \\ 30 & \bmod 16 \equiv & 14 \\ 32 & \bmod 16 \equiv & 0. \end{array}$$

(The symbol ≡ indicates congruence.) Thus, set D contains two complete periods, plus the first element of a third. The formula for congruence modulo m is $x = mk + r$, where x is a value, m is the modulus, k is a multiple (positive, negative, or zero) of m, and r is the value mod m with which x is congruent. The value r is known as the *residue class* mod m. Thus, in set D, $17 = 16(1) + 1$ and $32 = 16(2) = 0$. Both of these statements are consistent with the congruence relations listed above.

The term "modular set" may be abbreviated *m-set*. This term may be combined with the label representing the type of elements in the set, as in m-pset, which is an abbreviation for "modular set of pitches." Similarly, a modular seq may be indicated by the abbreviation m-seq, and an m-seq of time-points may be called an m-tpseq. Since any m-set contains one or more periods from a potentially endless series of periodic cycles, the class of m-sets from which a particular m-set is drawn may be represented concisely by indicated only one of its periods. For example,

$$M_D = \{0\ 1\ 2\ 5\ 7\ 10\ 14\ (16/0)\}$$

represents the class of m-sets of which set D is a member. The final element—actually a pair of elements—is enclosed in parentheses in order to indicate the completion of one period at this point. The larger of the pair of elements enclosed in parentheses indicates the identity of the modulus. In this case the modulus is 16 and therefore 16 mod 16 ≡ 0. If one desires to represent a single period of an m-set and the size of the modulus is known, it is possible to omit the pair of elements in parentheses, as in

$$D = \{0\ 1\ 2\ 5\ 7\ 10\ 14\}.$$

The structure and representation of m-seqs are similar to those of m-sets, except that angle brackets are used to enclose the elements of m-seqs.

Set-theoretic operations may be performed on sets, seqs, m-sets, and m-seqs. The three basic operations are *union* (∪ or +), *intersection* (∩ or ·) and *complementation* ( ). In the interests of brevity, examples will be confined to sets only. The union of two sets is the set that contains the elements that belong to one or the other of the sets. For example,

let A = {-5 -2 1 3 7 11 13 16 17}, as above,

and E = {-10 -4 -2 0 5 7 12 16 19}.

Then

$$A \cup E = \{-10\ -5\ -4\ -2\ 0\ 1\ 3\ 5\ 7\ 11\ 12\ 13\ 16\ 17\ 19\}.$$

The intersection of two sets is the set that contains the elements that belong to both sets.[26] With sets A and E as above,

$$A \cap E = \{-2\ 7\ 16\}.$$

The complement of a set is the set that contains the elements that do not belong to the original set but that do belong to a reference collection that includes the original set. For example, if the reference collection U is a set that contains all of the integers from -5 to 20, inclusive, the complement of A with respect to U is

$$\overline{A} = \{-4\ -3\ -1\ 0\ 2\ 4\ 5\ 6\ 8\ 9\ 10\ 12\ 14\ 15\ 18\ 19\ 20\}.$$

Somewhat akin to the set-theoretic operations are the inclusion relations involving sets. A set is included in, i.e. is a *subset* of another, if all of its elements are members of the other set. The subset relation is indicated by the symbol $\subset$. Thus,

$$\text{if } A = \{-5\ -2\ 1\ 3\ 7\ 11\ 13\ 16\ 17\}$$
$$\text{and } F = \{1\ 7\ 13\ 16\},$$

$F \subset A$. A set includes, i.e. is a *superset* of another, if it contains all of the elements in that set plus some additional elements of its own. The superset

[26]For special issues related to the intersection of ordered collections, see Morris, *Composition with Pitch-Classes*, pp. 51-2.

relation is indicated by the symbol $\supset$. In the example above, A $\supset$ F. Both relations play an important role in the pitch structure of some of Xenakis's works.

Several types of transformational operations may be performed on sets, seqs, m-sets, and m-seqs. The operations of *transposition* and *inversion* may be performed on all four types of collections. Transposition of a set by n, symbolized T_n, is performed when a quantity n is added to each element in the set. Thus,

$$\text{if } G = \{0\ 2\ 3\ 5\ 7\ 10\ 12\ 15\ 17\ 19\ 20\} \text{ and } n = 10,$$
$$T_{10}(G) = \{10\ 12\ 13\ 15\ 17\ 20\ 22\ 25\ 27\ 29\ 30\}.$$

The spacing of the transposition of a set is the same as that of the original set. For example,

$$SP(G) = \langle 2\ 1\ 2\ 2\ 3\ 2\ 3\ 2\ 2\ 1 \rangle,$$
$$\text{and } SP(T_{10}(G)) = \langle 2\ 1\ 2\ 2\ 3\ 2\ 3\ 2\ 2\ 1 \rangle.$$

The operation of inversion on a set, symbolized I, is performed by reversing the direction of the intervals between its consecutive elements. For example,

$$\text{for } G = \{0\ 2\ 3\ 5\ 7\ 10\ 12\ 15\ 17\ 19\ 20\},$$
$$I(G) = \{0\ {-2}\ {-3}\ {-5}\ {-7}\ {-10}\ {-12}\ {-15}\ {-17}\ {-19}\ {-20}\}.$$

The standard form of I(G) is obtained by arranging its elements in ascending order:

$$I(G) = \{-20\ {-19}\ {-17}\ {-15}\ {-12}\ {-10}\ {-7}\ {-5}\ {-3}\ {-2}\ 0\}.$$

When the inversion of a set is in standard form, the order of the intervals in its spacing is reversed with respect to that of the original set, for example

$$SP(I(G)) = \langle 1\ 2\ 2\ 3\ 2\ 3\ 2\ 2\ 1\ 2 \rangle.$$

The operations of transposition and inversion may be combined, as in

$$T_{-5}I(G) = \{-25\ -24\ -21\ -19\ -17\ -15\ -12\ -10\ -8\ -7\ -5\}.$$

The elements, reading from left to right, of a set, and from right to left of T_nI of the set, sum to n. For example, in set G and $T_{-5}I(G)$, $0 + -5 = -5$, $2 + -7 = -5$, etc. Note: In general, $T_nI \neq IT_n$.

When performing transposition on m-sets, the operation T_n is often combined with mod m. Consider, for example, the class of m-sets

$$M_H = \{0\ 3\ 5\ 7\ 11\ 16\ 19\ 22\ 24\ (27/0)\}.$$

This is a class of m-sets where m = 27 and

$$SP(M_H) = \langle 3\ 2\ 2\ 4\ 5\ 3\ 3\ 2\ 3 \rangle.$$

If we take a set representing one period of this class of m-sets, i.e.

$$H = \{0\ 3\ 5\ 7\ 11\ 16\ 19\ 22\ 24\}$$

and transpose it by 8 units, we obtain

$$T_8(H) = \{8\ 11\ 13\ 15\ 19\ 24\ 27\ 30\ 32\}.$$

But since this is an m-set with m = 27, all elements whose values are 27 and above may be replaced by their mod 27 equivalents so that they remain within the compass of a single period. Thus,

$$T_8(H)(\text{mod } 27) = \{8\ 11\ 13\ 15\ 19\ 24\ 0\ 3\ 5\}.$$

In ascending order, this is

$$T_8(H)(\text{mod } 27) = \{0\ 3\ 5\ 8\ 11\ 13\ 15\ 19\ 24\}.$$

The spacing of this m-set is

$$SP(T_8(H)(\text{mod } 27)) = \langle 3\ 2\ 3\ 3\ 2\ 2\ 4\ 5\ (3)\rangle,$$

which shows a rotation in the ordering of the intervals three places to the right with respect to $SP(M_H)$. The last interval is enclosed in parentheses since it is the interval that would result were the period to repeat itself.

The necessity for the composite operation T_n(mod m) may not immediately be apparent, but there are cases in Xenakis's music where, for an example, an m-pset spans the entire range of the piano keyboard. If such an m-pset is transposed, the application of mod m in addition to T_n is necessary in order for all of the elements of the transposed m-pset to be included within the range of the keyboard. T_n(mod m) is thus a formal label for the operation that Xenakis identifies as "cyclic transposition."[27] When cyclic transposition is performed under such conditions, the elements of the m-pset may be thought to exist on an endless loop that "wraps around" from the top to the bottom of the keyboard—or vice versa, if $n < 0$—whenever the operation of

[27]See, for example, the Preface to *Mists* for piano (Paris: Editions Salabert, 1981).

transposition alone would cause some elements to exceed the instrument's range.

The reader may have noticed that H and $T_8(H)$ hold several elements in common. Specifically,

$$H \cap T_8(H) = \{11\ 19\ 24\}.$$

The number of elements in this intersection is 3. In general, #A = the cardinality of A. The cardinality of this intersection, therefore, may be expressed as

$$\#(H \cap T_8(H)) = 3.$$

The number of elements in the intersections of specific transpositions of H may be determined by constructing a *T-matrix* for it, as shown below.

T	0	3	5	7	11	16	19	22	24
0	0	3	5	7	11	16	19	22	24
3	-3	0	2	4	8	13	16	19	21
5	-5	-2	0	2	6	11	14	17	19
7	-7	-4	-2	0	4	9	12	15	17
11	-11	-8	-6	-4	0	5	8	11	13
16	-16	-13	-11	-9	-5	0	3	6	8
19	-19	-16	-14	-12	-8	-3	0	3	5
22	-22	-19	-17	-15	-11	-6	-3	0	2
24	-24	-21	-19	-17	-13	-8	-6	-4	0

The numbers in the center of the matrix are determined by subtracting the m-set element at the beginning of each row from the element at the top of each column. The number of elements in the intersection of m-set X and $T_n(X)$, i.e. $\#(X \cap T_n(X))$, is equal to the number of occurrences of n within the matrix. In the T-matrix of m-set H, the number 8 appears three times, indicating that $\#(H \cap T_8(H)) = 3$, which has already been demonstrated above. The use of T-matrices is not restricted to m-sets. They may also be used to determine the number of elements in the intersections of sets and their transpositions.[28]

T-matrices may also be used to determine the value of $\#(X \cap T_n(X)(\text{mod } m))$. For an m-set containing a single period, this is done by performing mod m upon all of the values within the matrix. For example, the T-matrix used to determine $\#(H \cap T_n(H)(\text{mod } 27))$ is

T	0	3	5	7	11	16	19	22	24
0	0	3	5	7	11	16	19	22	24
3	24	0	2	4	8	13	16	19	21
5	22	25	0	2	6	11	14	17	19
7	20	23	25	0	4	9	12	15	17
11	16	19	21	23	0	5	8	11	13
16	11	14	16	18	22	0	3	6	8
19	8	11	13	15	19	24	0	3	5
22	5	8	10	12	16	11	24	0	2
24	3	6	8	10	14	19	21	23	0

The matrix shows that, for $n = 8$, $\#(H \cap T_n(H)(\text{mod } 27)) = 6$. This is true, since

$$H \cap T_8(H)(\text{mod } 27) = \{0\ 3\ 5\ 11\ 19\ 24\}.$$

In general, $X \cap T_n(X)(\text{mod } m)$ will contain as many or more elements as $X \cap T_n(X)$. The reason for this is that that the transposition of an m-set effectively relocates the set within the space occupied by its elements. As the m-set is relocated at greater distances, whether up or down (positive or negative), the number of elements that may intersect between the original m-set and its

[28]On the general properties of T-matrices in p-space see Morris, *Composition with Pitch-Classes*, pp. 49-51. See also Robert Morris, "Some Compositional and Analytic Applications of T-Matrices," *Intégral* 3 (1990).

transformations correspondingly decreases. When the composite operation T_n(mod m) is performed, however, the elements of the original set and its transformation circulate around in the same space, thus increasing the likelihood of intersection between X and its transformations. In the case of both T_n and T_n(mod m), maximal intersection occurs when $n = 0$, as the matrices above demonstrate. Depending upon the structure of the m-set, maximal intersection may occur for other values of n when T_n(mod m) is applied.

The operations and relations that are applied most frequently in Xenakis's music are union and intersection, the inclusion relations, and T_n(mod m). Intersections between m-psets and their T_n(mod m) transformations, in particular, can help to establish degrees of similarity between different areas in the inside-time pitch structure of a work. Thus, specific operations and relations may aid in the articulation of form. Inversion occurs much less frequently in Xenakis's music than does transposition. Similarly, operations specific to ordered collections also appear rarely, if at all, in his music.[29] The formation of complex m-sets through the performance of set-theoretic operations on simple m-sets is the basis of sieve theory, which is the next topic to be introduced.

[29]Operations specific to ordered collections, such as retrogression and other forms of permutation, were not introduced above. For information on these operations in p-space, see Morris, *Composition with Pitch-Classes*, chapter 2.

2.1.7 Sieve Theory

Xenakis developed his *sieve theory* in the mid-1960s.[30] Since then it has provided him with a way of constructing modular pitch sets (m-psets) and modular time-point sequences (m-tpseqs). Briefly, a *sieve* is a selection from among the points available along the axis of some dimension, such as p-space or tp-space. The simplest type of sieve is called a *module*. A module is symbolized by the ordered pair (m, r) which indicates a modulus and a residue class within that modulus. When m = 3 and r = 0, for example,

$$(3, 0) = \{\ldots -12\ -9\ -6\ -3\ 0\ 3\ 6\ 9\ 12 \ldots\}.$$

A module, then, is a very simple m-set that contains one element per period for an unspecified number of periods. More complex sieves may be formed by taking unions or intersections of two or more modules. Thus, the prototypes for sieves are

$$(m_0, r_0) + (m_1, r_1) + \ldots + (m_{n-1}, r_{n-1}) + (m_n, r_n)$$

for unions, and

$$(m_0, r_0)\cdot(m_1, r_1)\cdot\ldots\cdot(m_{n-1}, r_{n-1})\cdot(m_n, r_n)$$

[30]The first published explanation of sieve theory appeared in Xenakis's article, "Vers un métamusique," *Le Nef* 29 (1967). This article was reprinted as chapter 7 of *Formalized Music*. Sieves had already been used, however, in the composition of *Nomos Alpha* for violoncello solo (1966) (see *Formalized Music*, chapter 8). A second article on sieves appeared in *Perspectives of New Music* 28/1 (1990): 58-78. This article contains computer programs, in C, for the synthesis and analysis of sieves. The article and the programs, respectively, appear as chapters 11 and 12 of *Formalized Music*. Unfortunately, the versions of the programs in *Formalized Music* contain numerous typographical errors that prevent them from functioning properly. Corrected versions appear in Appendix I of this dissertation.

for intersections, where + stands for union and · stands for intersection. The period of a sieve is equal to the lowest common multiple (LCM) of the moduli (i.e., the values for m).

A few simple illustrations will serve as an introduction to the formation of sieves. Consider two modules, (3, 0) and (4, 0). Their LCM is 12, which is therefore the period of the sieves that can be formed with them. Starting at 0, a single period of the union of these modules is

$$(3, 0) + (4, 0) = \{0\ 3\ 4\ 6\ 8\ 9\}.$$

This represents the union of

$$(3, 0) = \{\ldots\ 0\ 3\ 6\ 9\ \ldots\} \text{ and}$$
$$(4, 0) = \{\ldots\ 0\ 4\ 8\ \ldots\}$$

for values from 0 to 11 (the residue classes mod 12), inclusive. If we let

$$J = \{0\ 3\ 4\ 6\ 8\ 9\},$$

then it is evident that

$$SP(J) = <3\ 1\ 2\ 2\ 1\ (3)>.$$

The final interval in SP(J) is shown in parentheses because it connects the final element of m-set J with the first element of the next period. Note the symmetrical arrangement of the intervals in SP(J): the intervals are symmetrically disposed around an axis situated between the pair of interval 2s at the center. Symmetrical spacings are a general characteristic of sieves formed from the union of two or more modules. If the elements in J were interpreted as time-points, thus forming an m-tpseq, the result, in Messiaen's terminology, would be a "non-retrogradable rhythm."[31]

Now let us consider two *metabolae*, i.e. transformations, of this simple sieve.[32] If the values for r in each of the modules are increased by 2, the new the new sieve produced by their union is

$$(3, 2) + (4, 2) = \{2\ 5\ 6\ 8\ 10\ 11\},$$

which is equivalent to $T_2(J)(\text{mod } 12)$. The spacing of the transformed m-set is

$$SP(T_2(J)(\text{mod } 12)) = <3\ 1\ 2\ 2\ 1\ (3)>,$$

which is the same as SP(J) above. If the values for r are changed independently, e.g. (3, 1) and (4, 3), the result is

$$(3, 1) + (4, 3) = \{1\ 3\ 4\ 7\ 10\ 11\},$$

which is equivalent to $T_7(J)(\text{mod } 12)$.

$$SP(T_7(J)(\text{mod } 12) = <2\ 1\ 3\ 3\ 1\ (2)>,$$

which represents a rotation of SP(J) (and $SP(T_2(J)(\text{mod } 12))$) three places to the right. In this sieve the starting point, relative to the starting point of J, has shifted upward by 7 units, causing the m-set to wrap around within the range of 0-11. The spacing of the transformed m-set rotates accordingly.

[31] See the section on Messiaen in chapter 1.

[32] Xenakis uses "metabolae" as a general term for transformations on sieves. As the examples illustrate, the most common metabola is $T_n(\text{mod m})$, but changes in the m-values of the modules are also possible.

The intersections of modules produce quite different results. The intersections of the modules introduced above, within the period whose range is 0-11, are

$$(3, 0)\cdot(4, 0) = \{0\},$$
$$(3, 2)\cdot(4, 2) = \{2\},$$
$$\text{and } (3, 1)\cdot(4, 3) = \{7\}.$$

Each intersection results in an m-set that contains only one element per period. Furthermore, the element in each intersection is equivalent to n in the corresponding expressions $T_n(J)(\text{mod } 12)$ above.

In summary, sieves formed from unions result in m-sets that have symmetrical spacings. Transpositions mod m of these m-sets, which may be performed by changing the values for r in the modules, either preserve their spacings or cause the intervals within the spacings to undergo order rotations. Intersections of modules result in m-sets that contain one element per period. Simple unions and intersections produce m-sets whose structural regularity conflicts with Xenakis's general aesthetic orientation, which tends toward complex, highly irregular structures. It is not surprising, therefore, that the sieves found in Xenakis's compositions are based on complex formulas in which the operations of union and intersection are combined. The formula given as an example below produces one of the m-psets used in *à r.* Because of the length of this formula, the notation of the modules has been simplified so that the values of the moduli appear as full-size numbers and the values for the residue classes appear in subscript. For example, the module that would formerly have been notated as (8, 0) is shown below as 8_0. Two moduli, 8 and 11, are represented in the formula.

$(8_0\cdot(11_0 + 11_4 + 11_5 + 11_6 + 11_{10})) + (8_1\cdot(11_2 + 11_3 + 11_6 + 11_7 + 11_9)) + (8_2\cdot(11_0 + 11_1 + 11_2 + 11_3 + 11_5 + 11_{10})) + (8_3\cdot(11_1 + 11_2 + 11_3 + 11_4 + 11_{10})) + (8_4\cdot(11_0 + 11_4 + 11_8)) + (8_5\cdot(11_0 + 11_2 + 11_3 + 11_7 + 11_9 + 11_{10})) + (8_6\cdot(11_1 + 11_3 + 11_5 + 11_7 + 11_8 + 11_9)) + (8_7\cdot(11_1 + 11_3 + 11_6 + 11_7 + 11_8 + 11_{10}))$.

By using the distributive property the complex modules in this formula may be broken down and expressed as a union of forty-two simple intersections: $(8_0\cdot 11_0) + (8_0\cdot 11_4) + \ldots + (8_7\cdot 11_8) + (8_7\cdot 11_{10})$. The m-set generated by this formula is

K = {0 2 3 4 7 9 10 13 14 16 17 21 23 25 29 30 32 34 35 38 39 43 44 47
48 52 53 57 58 59 62 63 66 67 69 72 73 77 78 82 86 87}

and its spacing is

SP(K) = <2 1 1 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2
3 1 4 1 4 4 1 (1)>.

This spacing does not exhibit the symmetrical quality observed in the spacings of m-set J and its transformations above.

The formula for m-set K can be transformed in a number of ways. The simplest type of transformation, equivalent to T_n(mod m), can be performed by adding some value n to every value for r in the formula. For example, if n = 10 is added to 8_0, r becomes (0 + 10)(mod 8), which is 2. Similarly, if n = 10 is added to 11_0, r becomes (0 + 10)(mod 11), which is 10. A formula is given below in which 10 has been added to every value for r. The modules have been arranged in ascending order as before.

$(8_0 \cdot (11_0 + 11_2 + 11_4 + 11_6 + 11_7 + 11_8)) + (8_1 \cdot (11_0 + 11_2 + 11_5 + 11_6 + 11_7 + 11_9)) + (8_2 \cdot (11_3 + 11_4 + 11_5 + 11_9 + 11_{10}) + (8_3 \cdot (11_1 + 11_2 + 11_5 + 11_6 + 11_8)) + (8_4 \cdot (11_0 + 11_1 + 11_2 + 11_4 + 11_9 + 11_{10}) + (8_5 \cdot (11_0 + 11_1 + 11_2 + 11_3 + 11_9)) + (8_6 \cdot (11_3 + 11_7 + 11_{10})) + (8_7 \cdot (11_1 + 11_2 + 11_6 + 11_8 + 11_9 + 11_{10}))$

which is equivalent to

T_{10}(K)(mod 88) = {0 4 8 9 10 12 13 14 17 19 20 23 24 26 27 31 33 35 39 40 42 44 45 48 49 53 54 57 58 62 63 67 68 69 72 73 76 77 79 82 83 87}.

Thus, transformation by T_{10}(mod m) of each module, whether m = 8 or m = 11, results in transformation by T_{10}(mod 88) of the entire m-set. The spacing of this m-set is

SP(T_{10}(K)(mod 88)) = <4 4 1 1 2 1 1 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 4 (1)>,

which represents a rotation of the intervals in SP(K) four places to the right.

A different type of transformation may be performed by changing the values of r separately for the different values of m. Beginning with the formula for m-set J, if 7 mod 8 is added to all of the values for r in 8_r and if first 1, then 2, then 3, all the way up to 8 mod 11 are added to the values for r in each union of modules 11_r, the result is the following formula:

$(8_0 \cdot (11_0 + 11_4 + 11_5 + 11_8 + 11_9)) + (8_1 \cdot (11_1 + 11_3 + 11_4 + 11_5 + 11_6 + 11_8)) + (8_2 \cdot (11_3 + 11_5 + 11_6 + 11_7 + 11_8)) + (8_3 \cdot (11_2 + 11_5 + 11_9)) + (8_4 \cdot (11_2 + 11_4 + 11_5 + 11_6 + 11_8 + 11_9)) + (8_5 \cdot (11_0 + 11_1 + 11_3 + 11_4 + 11_5 + 11_8)) + (8_6 \cdot (11_0 + 11_3 + 11_4 + 11_5 + 11_7 + 11_9)) + (8_7 \cdot (11_0 + 11_1 + 11_5 + 11_6 + 11_7)$.

The m-set produced by this formula is

L = {0 1 4 5 7 8 14 16 17 18 20 22 23 25 27 28 35 37 38 39 41 45 48 49 50
52 55 58 60 62 64 68 69 70 71 74 75 77 81 82 85 86}

and its spacing is

SP(L) = <1 3 1 2 1 6 2 1 1 2 2 1 2 2 1 7 2 1 1 2 4 3 1 1 2 3 3 2 2 2 4 1 1 1
3 1 2 4 1 3 1 (2)>,

which is obviously quite different from SP(K). Because the spacings of m-psets K and L are not identical and cannot be related by rotation, they belong to different classes of m-sets. M-set L is not used in *à r.*, but was constructed here in order to illustrate an additional possibility for the transformation of a sieve besides the standard operation T_n(mod m). Sieves may also be transformed by changes in the values for m. This transformation, like the one above, generates m-sets that do not belong the same m-set class as the original. Finally, changes in the realization of a sieve may be made by changing the size of the unit, e.g. semitone or quarter-tone, on which the elements are based.

Since complex sieve formulas may be broken down into unions of simple intersections, and since each intersection produces only one element per period, the necessity for the formulas is not entirely clear.[33] If values for m and r are chosen according to a system, however, the use of formulas facilitates the application of systematic transformations, whether or not these transformation result in m-sets of the same class. This is the basis of the method used in the derivation of the pitch sieves in *Nomos Alpha* for

[33]By definition, the moduli in a sieve produce only one intersecting element at each occurrence of the lowest common multiple (LCM).

violoncello solo (1966), for example.[34] The sketches to *Mists* for piano (1980), however, suggest that the elements of the sieve were chosen a priori, and that a formula was developed for it afterward. On one page of the sketches a list of elements appears above several attempts to derive a formula for the sieve. The formula that Xenakis finally settled on has 2, 5, and 9 as values for m, and therefore the period of the sieve is 90, which exceeds the range of the piano keyboard by two semitones.[35] Other possible formulas considered by Xenakis for this sieve made use of 3, 4, and 7; 3, 5, and 7; and 3, 5, and 8 for values of m, which would have resulted in periods of 84, 105, and 120, respectively. In this case, then, the contents of the sieve seem to have been determined prior to the construction of its formula.[36] This is not to suggest, however, that Xenakis's choice of elements is either completely systematic or completely arbitrary. On the contrary, it stands to reason that some kind of criteria regarding the intervallic structure of sieves underlies the choice of elements, even when this choice is not strictly systematic. For example, SP(K) above shows intervals whose size varies only from 1 to 4, whereas SP(L), which results from a relatively arbitrary transformation of modules from the formula for K, shows intervals that vary in size from 1 to 7. Clearly, the spacing of m-set K, which was produced by Xenakis, demonstrates greater

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[34]See *Formalized Music*, pp. 230-2 and 234. For a critique of the consistency with which the systematic derivation and transformation of sieves is applied, see Jan Vriend, "'Nomos Alpha' for Violoncello Solo (Xenakis 1966): Analysis and Comments," *Interface* 10 (1981): 15-82. The comments on the sieves are found in sections 3.0 and 3.1, pp. 54-65, of Vriend's article.

[35]This sieve, its transformations, and their compositional manifestations will be discussed in chapter 3.

[36]For a simple method of deriving a formula for a sieve by hand, given the elements of the sieve, see Appendix II.

intervallic homogeneity, despite the relative irregularity of its interval pattern, than the spacing of m-set L, which was not produced by him.

A rhythmic application will serve as a final example of the uses of sieve theory. Figure 2.4 shows the opening four measures of *Evryali* for piano (1973). This passage shows a set of seven pitches, {0 2 3 4 5 7 9} (0 = C4), each of which is activated by a separate time-point sequence. The pset is a subset of a larger pitch sieve and the tpseqs associated with each of its elements are rhythmic sieves in which the unit value is the sixteenth-note. The time-points in this passage fall within a range from 0 to 60. The tpseqs are shown below, each identified by the pitch with which it is associated. The formulas for the tpseqs are not shown.

C4 = <19 24 27 31 36 41 46 50 53 57 60>

D4 = <3 7 9 14 16 18 21 24 28 31 35 38 43 46 51 54 58 60>

D#4 = <29 32 48 52 55 57>

E4 = <12 16 21 26 29 34 39 44 48 52 55 60>

F4 = <0 4 6 9 13 18 21 23 26 30 34 36 38 41 45 47 49 53 56 58
60>

G4 = <6 11 15 20 23 34 40 43 48 52 55 58 60>

A4 = <10 14 18 24 28 30 33 38 42 47 51 56 60>.

The tpseqs differ in the number of their elements, from 6 (D#4) to 21 (F4). The sole exceptions are G4 and A4, which have thirteen elements each. These sieves do not belong to the same tpseq class, and therefore there are no clear subset/superset relations among them. Their INTs are

INT(C4) = <5 3 4 5 5 5 4 3 4 3>

INT(D4) = <4 2 5 2 2 3 3 4 3 4 3 5 3 5 3 4 2>

INT(D#4) = <3 16 4 3 2>

INT(E4) = <4 5 5 3 5 5 5 4 4 3 5>

INT(F4) = <4 2 3 4 5 3 2 3 4 4 2 2 3 4 2 2 4 3 2 2>

INT(G4) = <5 4 5 3 11 6 3 5 4 3 3 2>

INT(A4) = <4 4 6 4 2 3 5 4 5 4 5 4>.

The INTs consist mainly of intervals between 2 and 5, with larger intervals appearing only in INT(D#4), INT(G4) and INT(A4).

In this rhythmic application the operations of intersection and union have a concrete effect on the structure of the musical surface. Intersections between the separate tpseqs result in simultaneous attacks. The greatest number of simultaneous attacks to occur on a given time-point is six, which occurs at the final time-point, 60. The union of all of the tpseqs results in an aggregate tpseq whose elements are

P = <0 3 4 6 7 9 10 11 12 13 14 15 16 18 19 20 21 23 24 26 27 28 29 30
31 32 33 34 35 36 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52
53 54 55 56 57 58 60>

and whose INT is

INT(P) = <3 1 2 1 2 1 1 1 1 1 1 1 2 1 1 1 2 1 2 1 1 1 1 1 1 1 1 1 1 2 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2>.

INT(P) shows that the passage gains rhythmic impetus gradually, and then features attacks on virtually every time-point until just before the end, where the six simultaneous attacks on time-point 60 are prepared slightly by an increase in the time-point interval from 1 to 2.

Regarding the analysis of structures produced according to sieve theory, some aspects of the theory are more significant than others. It is useful to note that m-psets and m-tpseqs may be designed with particular periods, and therefore particular spacings or INTs, in mind. In fact, sieves belonging to a particular class are usually identified indirectly, on the basis of identical or rotated spacings or INTs. On the compositional surface, however, m-psets are often represented by their subsets, with relatively few appearances of complete m-psets occurring in a given work. In analytical situations, therefore, it is possible, on the basis of what is known about sieves and their transformations, to propose a background model of the pitch structure of a work that generates a context for the collections of pitches that occur on the musical surface. Examples of this approach appear in chapters 3 and 4.

2.2 Sonic Configurations

Sonic configurations are collections of sonic events in their inside-time contexts within compositions. The boundaries of sonic configurations are coextensive with the boundaries of the segments in which they are situated. Normally, a segment contains only one sonic configuration. Sonic configurations are the basic units of inside-time structure at the level immediate above that of individual sonic events, just as segments are the basic units of temporal structure at the level immediate above that of individual time-points. Two facets of sonic configurations are of particular analytical interest: their textural density, and the shape articulated by the aggregate of events in a configuration in p-space and tp-space as the configuration unfolds in time. Textural density may be measured quantitatively as the mean number of sounds per second, as indicated above.

The shapes, or *morphological* characteristics, of sonic configurations may be described verbally and may also be represented graphically in graphic transcriptions of the scores in which they are found. In the case of works composed on the UPIC system, the morphological characteristics of sonic configurations are indicated directly by the score's graphic notation.

Specific morphological types have been favored by Xenakis at various points in his career. Stochastic configurations were predominant in his music from the mid-1950s until the late 1960s, at which point arborescences (branching structures) were introduced. Still more morphological types were introduced in the 1970s and 1980s.[37] In this section three morphological types will be introduced that are particularly relevant to the repertoire examined in chapters 3-5. The first type consists of the stochastic configurations. The most common variety of stochastic configuration may be represented graphically as a field—or, in Xenakis's terminology, a "cloud"—of points in p- and tp-space. A less common variety of stochastic configuration appears as a meandering stream of points in p- and tp-space. The general structural characteristics and the method of formation of both varieties will be described below.

The second morphological type is the *random walk*. This type is based on mathematical models that describe random changes in velocity with respect to time. Random walks are characterized by linear, "melodic" motion as opposed to the primarily spatial arrangement of isolated points that is typical of stochastic configurations.[38] The contours typical of certain types of

[37]For surveys of the morphological types used in Xenakis's compositions, see his "Music Composition Treks" in *Composers and the Computer*, ed. Curtis Roads (Los Altos, CA: William Kaufmann, 1985): 171-92, and Peter Hoffmann, *Amalgam aus Kunst und Wissenschaft: Naturwissenschaftliches Denken im Werk von Iannis Xenakis* (Frankfurt am Main: Peter Lang, 1994).

[38]For applications of random walks to the synthesis of wave forms in electroacoustic music, see *Formalized Music*, chapters 9, 13, and 14; Marie-Hélène Serra, "Stochastic Composition and

random walks are sometimes imitated graphically during the process of composition. Graphic images of this type are found in the sketches to *Mists* for piano (1980), providing evidence that random walks designed by hand play a part in the structuring of sonic configurations. Both calculated and graphically designed random walks will be considered below.

The third, and final, morphological type to be discussed here is the *arborescence,* or branching structure. This type has played a significant role in Xenakis's music since the late 1960s, when it was introduced in *Synaphaï* for piano and orchestra (1969). Arborescences generate quasi-polyphonic textures made up of lines that split off from nodal points along other lines. The individual lines are frequently differentiated rhythmically, giving the configuration a complex rhythmic and melodic profile. Arborescences have apparently been inspired by extremely complex mathematical models of branching processes, but the sketches of compositions that employ them suggest that they originate in graphic images that are then transcribed into musical notation.[39] Once formed, the structure of an arborescence may be subjected to transformational operations, including compression, elongation, and rotation. These operations will be illustrated below with examples from *Mists*.

Stochastic Timbre: *Gendy3* by Iannis Xenakis," *Perspectives of New Music* 31/1 (1993): 236-57; and Peter Hoffmann, "Implementing the Dynamic Stochastic Synthesis," *Les Cahiers du GREYC* 4 (1996): 341-7.

[39]See, for example, the sketches to *Erikhthon* for piano and orchestra (1974), reprinted in Nouritza Matossian, *Xenakis* (London: Kahn & Averill, 1986), p. 237, and in Iannis Xenakis, "Xenakis on Xenakis," *Perspectives of New Music* 25/1 (1987): 16-49, p. 24.

2.2.1 Stochastic Configurations

2.2.2.1 Probability Theory

In order to understand the process of stochastic composition it is first necessary to become familiar with some basic principles of *probability theory.* Probability theory is a branch mathematics that is designed to answer the basic question: in a particular situation, given a number of possible outcomes, what is the likelihood that each one of them will occur? For example, given a coin that is to be tossed, what is the likelihood that the toss will result in the head of the coin facing upward? Similarly, what is the likelihood that the toss will result in the tail of the coin facing upward? Intuitively, we might say that there is a fifty-fifty chance of the head or the tail facing upward after a toss of the coin. But how can this intuition be expressed mathematically?

First, the possible outcomes of the coin toss can be represented by the letters H for heads and T for tails. The possible outcomes can then be gathered together to form a set of values, known as the *outcome set*. The outcome set for the coin toss experiment, then, is

$$\{H, T\}.$$

Each element within this outcome set is known as an *event*. Each event, therefore, constitutes a subset of the outcome set. The probability of an event may be defined as the expected long-run relative frequency of its occurrence in a series of trials.[40] In this case, a trial consists of a toss of the coin. If we represent the outcome set by the symbol Ω, and a subset of Ω with the symbol

[40]This definition is given in Douglas G. Kelly, *Introduction to Probability* (New York: Macmillan, 1994), p. 3. Kelly's text provides the background for the introduction to probability theory given here, and is recommended to readers who wish to explore this topic further.

A, we can say that the probability of A is equal to the quotient of the number of outcomes in A and the number of outcomes in Ω, that is

$$P(A) = \frac{\text{number of outcomes in A.}}{\text{number of outcomes in } \Omega.}$$

The probability of any single outcome ω, therefore, is

$$P(\omega) = \frac{1}{\text{number of outcomes in } \Omega}.$$

In the case of the coin toss experiment

$$\Omega = \{H, T\}$$

and the number of outcomes in Ω is 2. For the possible subsets A of Ω, the probabilities are as follows:

when $A = \{\varnothing\}$ (the null set), $P(A) = 0$;

when $A = \{H\}$, $P(A) = \frac{1}{2}$;

when $A = \{T\}$, $P(A) = \frac{1}{2}$;

when $A = \{H, T\}$, $P(A) = 1$.

Note that the probability of the entire outcome set Ω is 1. It is a general rule of probability theory that the probability of any outcome set be equal to 1.

2.2.1.2 Discrete Probability Spaces

The mathematical apparatus used to describe the coin toss experiment constitutes a *discrete probability space.* A discrete probability space consists of a finite or countable set Ω called the outcome set and a function p from Ω to the set of real numbers such that

$$p(\omega) \geq 0 \text{ for all } \omega \in \Omega \quad \text{and} \quad \sum_{\omega \in \Omega} p(\omega) = 1.$$

This expression indicates that there is a probability for every outcome in the outcome set Ω and that the sum of the probabilities of all of the outcomes in Ω is equal to 1.[41] Both of these facts have been demonstrated with respect to the coin toss experiment. The function described above is a *probability mass function*. On the basis of this probability mass function, we can say that for every event (i.e., subset) A in Ω,

$$P(A) = \sum_{\omega \in A} p(\omega).$$

It is possible to generate from the probability mass function a *discrete distribution* of the probabilities of its events. A graph, or *histogram*, of the discrete distribution for the coin toss experiment is shown in Figure 2.5. The histogram demonstrates what we established previously, namely that the probability of H is 0.5 and the probability of T is also 0.5. The probabilities of the null set and of Ω are not represented explicitly in the histogram, but their values are implicit nonetheless, for $P(\emptyset)$ would be represented by an empty graph and $P(\Omega)$ is the sum of P(H) and P(T).

[41]This is a paraphrase of a definition given in Kelly, *Introduction to Probability*, p. 18. The expression $\omega \in \Omega$ means that outcome ω is an element of the outcome set Ω.

What we have demonstrated so far is theoretically sound, but this discrete probability space does not necessarily present an accurate picture any specific series of coin tosses in the real world. Recall that the probability of an event represents the expected long-run relative frequency of its occurrence in a series of trials. Let us compare the expected probabilities of heads and tails with the observed relative frequencies of two runs of the coin toss experiment. The first run contains 10 trials, and the second contains 100. The results of the runs are shown below, compared with the expected probabilities.

		observed frequencies	
event	probability	10 trials	100 trials
H	0.5	0.7	0.51
T	0.5	0.3	0.49

A comparative histogram of the expected probabilities of the events versus their observed frequencies is shown in Figure 2.6. The table and the histogram illustrate that, the greater the number of trials, the more closely the observed frequencies approximate the expected probabilities. This fact forms the basis for the Law of Large Numbers, a general principle of probability theory to which Xenakis refers in the first chapter of *Formalized Music.*[42] This principle will be demonstrated again when we analyze the statistical properties of passages of stochastic music, in which the frequencies of events—more specifically, of the intervals between sonic events—approximate, but do not precisely correspond to, the statistical properties of the distributions upon which they are based. These approximations are consistent with the nature of probability theory.

[42]*Formalized Music*, p. 4.

Up to this point we have been defining events and finding the probabilities associated with them. But what happens when we begin with a probability and try to find the event associated with it? In discrete probability spaces like the one defined for the coin toss experiment this question poses a problem, since the probability of both events is the same. In order that this question may be answered accurately, two steps are required. The first involves the derivation of a *cumulative distribution function* from the probability mass function. For the probability mass function of the coin toss experiment, let events H and T be expressed by the variable x, which takes the value 0 for H and 1 and for T. Then the probability mass function is

$$p(x) = \frac{1}{2} \text{ if } x = 0, 1,$$
$$0 \text{ otherwise.}$$

The cumulative distribution function (CDF) measures the sum of the probabilities for x as x increases from $-\infty$ to a specific value. The basic form of the CDF is

$$F(x) = P(-\infty, x],$$

where $P(-\infty, x]$ indicates the probabilities of events greater than $-\infty$ and less than or equal to x. For the coin toss experiment the CDF takes on the following values:

$$F(0) = P(-\infty, 0] = P\{0\} = \frac{1}{2} \text{ and}$$

$$F(1) = P(-\infty, 1] = P\{0, 1\} = 1.$$

Thus, when $F(x) = \frac{1}{2}$, $x = 0$, i.e. H, and when $F(x) = 1$, $x = 1$, i.e. T. The cumulative probability of each event is therefore unique. If we identify the values of F(x) with the variable y, such that

$$\text{when } F(x) = \frac{1}{2}, y = \frac{1}{2} \text{ and}$$

$$\text{when } F(x) = 1, y = 1,$$

it is possible to establish an inverse function, $F^{-1}(y)$, that, when given the value of F(x), produces the value of x. In this case, for example,

$$F^{-1}\left(\frac{1}{2}\right) = 0 \text{ and}$$

$$F^{-1}(1) = 1.$$

Further examples of CDFs and inverse CDFs will be given below, for they play an important role in the process of stochastic composition.

2.2.1.3 Continuous Probability Spaces

In addition to the discrete probability space there is a second type of space known as the *continuous probability space.* This is the probability space used most often in the composition of stochastic music. In a continuous probability space events are represented by real numbers that may be found anywhere along the real number line, as opposed to the discrete values used to represent events in a discrete probability space. This change in the representation of events immediately raises a technical problem, for the number of possible values between any two positions, no matter how close, along the real number line is infinite and dense. If the number of events in the outcome set is infinite, the probability of any single event is 0, for

$$P(\omega) = \frac{1}{\infty} = 0.$$

In order to get around this problem it is necessary to divide the real number line into discrete intervals and to calculate the probabilities of the intervals.

A simple example of a continuous probability space is the uniform distribution over the interval [0, 1).[43] In the uniform distribution the probability that event x will occur within the interval [a, b) is equivalent to the length of the line from a to b, i.e. b – a. Some examples follow:

$$P[0, \tfrac{1}{2}) = \frac{1}{2},$$

$$P[\tfrac{1}{2}, 1) = \frac{1}{2},$$

$$P[\tfrac{1}{3}, \tfrac{2}{3}) = \frac{1}{3}, \text{ and}$$

$$P[0, 1) = 1.$$

The examples above may be summarized by the expression

$$P[a, b) = b - a.$$

[43]The notation [0, 1) signifies that $0 \le x < 1$. The uniform distribution is unchanged whether it is over the interval (0, 1), i.e. $0 \le x \le 1$, [0, 1], i.e. $0 \le x \le 1$, or [0, 1). Use of the notation [a, b) will help to avoid confusion later when the probabilities of consecutive intervals are calculated. On the theoretical advantages of using this particular notation, see Kelly, *Introduction to Probability Theory*, pp. 30-5.

A continuous probability space consists of an interval Ω called the outcome set and a function f(x) from Ω to the set of real numbers such that

$$f(x) \geq 0 \quad \text{for each } x \in \Omega \quad \text{and} \quad \int_{\Omega} f(x)dx = 1.$$[44]

In the expression above, f(x) represents the *probability density function* of a probability space. The probability density function does not indicate the probabilities of events in the outcome set. Rather, the probability of an event is equal to the area bounded by the x axis and the graph of f(x) over the interval representing the event. In the histogram of the uniform distribution in Figure 2.7, for example, the probability that x lies within the interval [a, b) is represented by the area of the rectangle formed by the x axis, the graph of f(x), and the endpoints of the interval [a, b). The probability density function of the uniform distribution is f(x) = 1, and therefore the formula for finding probabilities for x is

$$P[a, b) = \int_a^b 1\, dx.$$

This integral has a simple algebraic solution, which is

$$\int_a^b 1 dx = b - a,$$

and therefore

$$P[a, b) = b - a,$$

[44]This definition is paraphrased from Kelly, *Introduction to Probability Theory,* p. 46.

as indicated above. For $\Omega = [0, 1)$, $a = 0$ and $b = 1$, and

$$\int_0^1 1 \, dx = 1,$$

as required by the definition of a continuous probability space.

The CDF of a continuous probability space is defined as the integral of the probability density function from the lowest possible value within the outcome set to some value x within the outcome set. In its general form, this is expressed as

$$F(x) = \int_{-\infty}^{x} f(x)dx \quad \text{for } x \in \Omega.$$

For the uniform distribution, $f(x) = 0$ for all values outside of the outcome set and zero is the lowest possible value within the outcome set, so

$$\int_0^x 1 \, dx = x - 0 = x,$$

therefore,

$$F(x) = \begin{cases} 0 & \text{if } x < 0, \\ x & \text{if } 0 \le x < 1, \\ 1 & \text{if } x \ge 1. \end{cases}$$

A histogram of the CDF of the uniform distribution is shown in Figure 2.8. The graph rises steadily over the interval [0, 1) but levels off at $F(x) = 1$ because $P[1, \infty) = 0$ and therefore nothing more can be added to $F(x)$ after x reaches 1.

The uniform distribution is unique because it is the only distribution in which $F(x) = x$. If we substitute the variable u for x, we can rewrite this expression as

$$F(u) = u \quad \text{for } 0 \leq u < 1.$$

Since, according to the basic principles of probability theory, values for P[a, b) for any distribution fall within the interval [0, 1), u may substitute for F(x) in any distribution. The general expression for the values taken by F(x), then, is

$$F(x) = u,$$

independently of the specific formula used to express F(x) in a given distribution. Given $F(x) = u$, the value for x associated with a particular value of u is expressed by the inverse of the CDF,

$$F^{-1}(u) = x.$$

The specific formula with which $F^{-1}(u)$ is associated varies from distribution to distribution, but once the formula is known, it remains only to introduce values for u into it in order to obtain appropriate values for x. This fact underlies the computer-assisted generation of events in probability spaces. A function known as a *pseudo-random number generator* is used to produce successions of numbers in the interval [0, 1) that exhibit properties similar to those of genuinely random number successions. These numbers serve as values for u which, when fed into $F^{-1}(u)$, produce values for x. This is one of the general principles underlying the composition of stochastic music by computer.[45]

2.2.1.4 Stochastic Modelling

Stochastic composition is a type of stochastic modelling. Stochastic modelling consists of the use of mathematical models to simulate the actions of stochastic processes that occur spontaneously in nature or in society. An example of a stochastic process in nature is radioactive decay, in which radioactive particles are emitted at random as a function of time. An example of a stochastic process in society is the arrival of telephone calls at a switchboard. When the arrivals are recorded over a specific period of time, both the average number of calls received per unit time and the average length of time between calls can be used to create a model that possesses a definite statistical structure. A stochastic process, then, contains both a temporal aspect and a non-temporal aspect, consisting of events that occur at specific points in time. The temporal and non-temporal aspects of stochastic processes both possess random structures, each of which may be described in terms of a specific probability distribution.

At this point it is may be useful to consider a tentative definition of randomness. According to physicist Charles Whitney,

> "Random" does not mean uniform on a small scale. It implies uniformity of structure on a large scale and a lack of regular structure on the small scale.[46]

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[45]This process is explained further in Denis Lorrain, "A Panoply of Stochastic 'Cannons'" in *The Music Machine*, ed. Curtis Roads (Cambridge, MA: MIT Press, 1989), pp. 351-79.

[46]Charles A. Whitney, *Random Processes in Physical Systems* (New York: Wiley, 1990), p. 9.

This general characterization of randomness serves quite well also as a general description of the structure of stochastic music. In a way that is consistent with the separation between temporal and outside-time structures introduced in section 2.1, Xenakis adapted the procedures of stochastic modelling to musical composition. By modelling the temporal structure of the music after one probability distribution and an outside-time dimension, such as pitch, after a different probability distribution, he was able to create a logically justifiable and musically useful analog of a stochastic process. Now that the necessary elements of probability theory have been introduced, we shall examine in the next three sections how the process of stochastic composition actually works.

2.2.1.5 The Exponential Distribution

Xenakis has modelled the succession of time-points in his stochastic music after the *exponential distribution,* which is the distribution most commonly associated with temporal processes. In the exponential distribution the probability of an interval of size x between two time-points decreases exponentially as x increases linearly. Thus, short time-point intervals occur with much greater frequency than long time-point intervals, which are relatively rare. The result in musical applications is a succession of irregularly spaced time-points that demonstrates, however, a relatively coherent structure overall. This is due to the fact that the events in an exponential distribution are organized around a mean value, which in musical applications is equivalent to the textural density, measured in sounds per second.

The probability density function for the exponential distribution is

$$f(x) = \partial e^{-\partial x},$$

where ∂, the density parameter, is equal to the number of sounds per second and e, the base of the natural logarithm, is approximated by the value 2.718.... The probabilities of events in the exponential distribution are determined by

$$P[a, b) = \int_a^b \partial e^{-\partial x}\, dx \quad \text{for } x \in \Omega, \Omega = [0, \infty).$$

Solving the integral in this formula gives

$$\int_a^b \partial e^{-\partial x}\, dx = e^{-\partial a} - e^{-\partial b}.$$

Both expressions are equivalent to the area bounded by the graph of f(x), the x axis, x = a and x = b in the histogram of the exponential distribution in Figure 2.9. For the exponential distribution in the figure, the value of ∂ is 1. In Appendix I of *Formalized Music*, Xenakis gives an even simpler formula for calculating the probabilities of events in the exponential distribution:

$$e^{-\partial iv}(1 - e^{-\partial v}) \quad \text{for } i = 0, 1, 2, \ldots.$$[47]

The variable v in Xenakis's formula represents the size of the intervals [a, b).

The table below shows values for x and f(x) along with corresponding values for the interval [a, b) and P[a, b), which is equivalent to $\int_a^b f(x)dx$.

[47]This formula is derived in *Formalized Music*, pp. 324-5. It is given here in simplified form, since Xenakis's variable c (my ∂) cancels out when his equation (5) (p. 325) is substituted for z in his equation (3) (p. 324).

x	f(x)	[a, b)	P[a, b)
0	1.000	[0, 1)	.632
1	.368	[1, 2)	.232
2	.135	[2, 3)	.085
3	.050	[3, 4)	.031
4	.018	[4, 5)	.012
5	.007	[5, 6)	.004
6	.002	[6, 7)	.002
7	.001	[7, 8)	.001
8	.000	[8, 9)	.000

The table demonstrates that the values for the probability density function, f(x), are not the same as those for P[a, b). The table also shows the reliability of Xenakis's simplified formula for the calculation of probabilities. The values for f(x) and for P[a, b) are compared graphically in Figure 2.10. Strictly speaking, the histogram of the exponential distribution should be represented as an area under the graph of f(x), as in Figure 2.9, but for our purposes it will be sufficient to substitute a graph of the values of P[a, b), as shown in Figure 2.10. Such a graph provides a convenient representation of expected probabilities for comparison with the observed frequencies of events in actual samples.

The CDF of the exponential distribution is

$$F(x) = \int_0^x \partial e^{-\partial x}\, dx = 1 - e^{-\partial x} \quad \text{for } x \in \Omega,$$

and the inverse of the CDF is

$$F^{-1}(u) = \frac{-\ln(u)}{\partial} = x \quad \text{for } 0 \le u < 1.^{48} \qquad (2.1)$$

This formula states that, given a value for P[a, b), here represented by u, and taking the natural logarithm of this value, multiplying by -1 and dividing by ∂, it is possible to find the value of the event x in Ω that corresponds to P[a, b). Each value for x actually represents the approximation of a very small interval around a real-number outcome, since the probability of any event containing a single outcome is theoretically 0, as explained in section 2.2.1.3.

This formula can be used to generate values for x by drawing values for u from the computer's pseudo-random number generator and calculating the results. After the values for x have been calculated, the relative frequency of their occurrence within intervals of a given size may be tested against the expected probability of events in the same intervals. A comparative histogram of the distribution of 100 generated values with the expected probabilities for the exponential distribution, with $\partial = 1$ and interval size 1, is shown in Figure 2.11. The close resemblance between the expected and observed distributions is evident in the figure.[49]

2.2.1.6 The Linear Distribution

One of the distributions used to model events in the non-temporal aspect of a stochastic process is the *linear distribution*. This distribution may be used to generate intervals between pitches, levels of intensity, or states in

[48]This formula is derived in Lorrain, "A Panoply," pp. 366-7, and also in *Formalized Music*, p. 142. In Lorrain's version, u substitutes for the more precise 1 - u, since both expressions represent values in the interval [0, 1).

[49]Program listings in C for the calculation of probabilities and for the generation of values for x in the exponential distribution are given in Appendix III.

other dimensions. In this distribution the probability of an interval of size x between two events decreases linearly as x increases linearly. The rate of the decrease, therefore, is less severe than it is in the exponential distribution. The probability density function for the linear distribution is

$$f(x) = \frac{2}{g}\left(1 - \frac{x}{g}\right)$$

where g represents the maximum interval size. The formula for P[a, b) is

$$P[a, b) = \int_a^b \frac{2}{g}\left(1 - \frac{x}{g}\right)dx \quad \text{for } x \in \Omega, \Omega = [0, \infty).$$

Solving the integral in this formula gives

$$P[a, b) = \int_a^b \frac{2}{g}\left(1 - \frac{x}{g}\right)dx = \frac{1}{g^2}\left(a^2 - 2ag - b^2 + 2bg\right).$$

Xenakis substitutes for P[a, b) the simpler expression

$$\frac{2}{n+1}\left(1 - \frac{iv}{g}\right) \quad \text{for } i = 0, 1, 2, \ldots, n,$$

in which v represents the size of the intervals [a, b), as above.[50] However, since vn = g,

$$v = \frac{g}{n}.$$

Substituting this value for v into the formula gives

[50]This formula is derived in *Formalized Music*, p. 326.

$$P[a, b) = \frac{2}{n+1}\left(1 - \frac{i}{n}\right)$$

If we decide arbitrarily that g = 20 and v = 5, then n = 4 and i = 0, 1, 2, 3, 4. Values for x and f(x), and corresponding values for [a, b) and P[a, b), are shown in the table below and are represented in Figure 2.12.

x	f(x)	[a, b)	P[a, b)
0	.100	[0, 5)	.4
5	.075	[5, 10)	.3
10	.050	[10, 15)	.2
15	.025	[15, 20)	.1
20	.000	[20, 25)	.0

The CDF of the linear distribution is

$$F(x) = \int_0^x \frac{2}{g}\left(1 - \frac{x}{g}\right)dx \quad \text{for } x \in \Omega.$$

The inverse of the CDF is

$$F^{-1}(u) = g\left(1 - \sqrt{u}\right) = x \quad \text{for } 0 \leq u < 1.^{51} \qquad (2.2)$$

This is the formula used to generate the values of events in the linear distribution. A comparative histogram of the distribution of 100 generated values with the expected probabilities for the linear distribution, with g = 20 and interval size 5, is shown in Figure 2.13.[52]

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[51]This formula is derived in Lorrain, "A Panoply," p. 366 and in *Formalized Music*, p. 327. Lorrain gives $\sqrt{u}$ instead of the $\sqrt{1 - u}$ found in *Formalized Music*, since both expressions represent values in the interval [0, 1).

[52]Program listings in C for the calculation of probabilities and for the generation of values for x in the linear distribution are given in Appendix III.

2.2.1.7 The Process of Stochastic Composition

Equipped with a basic understanding of the exponential and linear distributions, which Xenakis has used to model temporal and non-temporal processes, respectively, we are now ready to consider more closely the process of stochastic composition. A brief example of stochastic music is shown in Figure 2.14. It was composed with STMus1, a simple program that incorporates a few of the functions found in Xenakis's more extensive program, "Free Stochastic Music."[53] STMus1 prompts the user for information related to the general features of the passage of stochastic music to be composed and returns a notelist consisting of time-points and numbered pitch positions. The data entered for the passage of music in Figure 2.14 were:

Start time (in seconds):	0.00
End time:	32.00
Number of sounds per second:	3.00
Minimum pitch position:	-24
Maximum pitch position:	24

The number of sounds per second is the value of the variable ∂ in the exponential distribution, and the difference between the minimum and maximum pitch positions is the value of the variable g in the linear distribution. In this case $g = 48$. In the transcription of the notelist into musical notation, pitch position 0 was assigned to C4. In this case the pitch

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[53]The program listing, in C, for STMus1 may be found in Appendix IV. STMus1 is my own adaptation of several functions used in "Free Stochastic Music." The program listing for "Free Stochastic Music," in Fortran IV, is given in *Formalized Music*, pp. 145-51.

positions were spaced in uniform semitonal increments. There are other cases in which the pitch positions refer to the positions occupied by elements in psets (or m-psets) when they are arranged in ascending order. In such cases the size of the intervals between successive elements is not uniform and it is necessary, therefore, to first calculate the pitch positions and then match up the positions with their respective pitches. An example of this procedure will be given below, but in the present example there is no difference between the pitch-position labels and the numerical labels of the pitches themselves.

The beginning of the notelist for the passage in Figure 2.14 is given below, along with the sizes of the intervals used to produce the time-points and pitches. In preparation for the transcription of the notelist data into musical notation, the original time-point values were rounded to the nearest 0.25 second, which is equivalent to one sixteenth-note at ♩ = 60 MM. Pitch positions are given in integer values in the program itself, thereby eliminating the need for rounding the values during the transcription process.

time-point interval	time-point	rounded time-point	pitch interval	pitch
0.17	0.17	0.25	25	1
0.52	0.69	0.75	-5	-4
0.49	1.18	1.25	25	21
0.31	1.49	1.50	-7	38
0.55	2.04	2.00	-14	24
0.12	2.16	2.25	5	29
0.50	2.66	2.75	-28	1
0.47	3.13	3.25	-1	0
0.67	3.80	3.75	2	2
1.43	5.23	5.25	22	24
0.00	5.23	5.25	-10	14
0.09	5.32	5.25	2	16

In order to obtain the initial time-point, the program generates an interval using $F^{-1}(u)$ for the exponential distribution (equation 2.1 in section 2.2.1.5) and adds this value to the start time entered by the user. Subsequent time-points are generated by the successive addition of new time-point intervals until the specified time limit is reached. This process is related to the successive addition of vectors, beginning from a point of origin, illustrated in section 2.1.3. The size of each interval is determined by the value for u drawn from the pseudo-random number generator. In STMus1, as in "Free Stochastic Music," the pseudo-random number generator is connected to the computer's internal clock, so that a different sequence of random numbers is produced with each run of the program.[54]

The sizes of the pitch-position intervals are chosen somewhat differently. The initial pitch position is chosen according to the uniform distribution (see section 2.2.1.3). A value produced by the pseudo-random number generator is multiplied by the size of the interval between the maximum and minimum pitch positions and is added to the value of the minimum pitch position. This allows the initial pitch position to occur anywhere within the available range. The remaining pitch-position intervals are generated using $F^{-1}(u)$ for the linear distribution (equation 2.2 in section 2.2.1.6). An additional function determines whether each new interval is to be added to or subtracted from the previous pitch position. The result of this function, too, depends upon the value for u drawn from the pseudo-random number generator: following "Free Stochastic Music," if $u \geq 0.5$, the interval is subtracted from the previous pitch position; otherwise, the interval is

[54]This is not necessarily the case in programs in which the user enters a "seed" value for the pseudo-random number generator manually. So long as the seed value remains the same, the succession of numbers, though it exhibits properties of randomness overall, remains identical.

added to it. Finally, a test function is applied which determines whether the resulting pitch position falls within the prescribed range. If the test fails, the interval is rejected and the entire process repeats itself until an acceptable pitch position is obtained.

The relative frequencies of the time-point and pitch-position intervals used to generate a passage of stochastic music may be compared with the expected probabilities of the distributions upon which they were modelled. A comparative histogram of the observed distribution of time-point intervals with the exponential distribution, $\partial = 3$, is shown in Figure 2.15. The figure shows that both the expected and observed distributions are relatively close to one another. A comparative histogram of the observed distribution of pitch-position intervals with the linear distribution, $g = 48$, is shown in Figure 2.16. In determining the observed distribution of pitch-position intervals, only the magnitude of the intervals is taken into consideration. The effect of the signs—positive or negative—associated with the direction of the intervals is eliminated by taking their absolute values.[55] This is necessary because the linear distribution is defined only for values greater than or equal to 0. Because of the test function that rejects pitch-positions that fall outside of the prescribed range, observed pitch-position distributions tend to contain a greater number of small intervals, and consequently a lesser number of large intervals, than are indicated by the expected probabilities of the linear distribution.

The procedures described here produce a type of sonic configuration known as a *stochastic field*. When represented graphically, a stochastic field configuration appears as a quasi-random distribution of points, somewhat

[55]For $x \geq 0$, the absolute value of x, notated $|x|$, is equal to x; for $x < 0$, $|x| = -x$.

like a star map. A graphic representation of the passage of music in Figure 2.14 is shown in Figure 2.17. Graphs such as these are useful in categorizing the sonic configurations in Xenakis's music according to their morphological type. The stochastic field is the most common type of stochastic configuration found in his music.

2.2.1.8 Remarks on the Aesthetics of Stochastic Music

It was mentioned above that different runs of a stochastic composition program produce different results. This raises the question of the structural identity of passages of stochastic music that share the same general characteristics. In his writings on the theory of stochastic music, Xenakis has never discussed the criteria for choosing one realization of a passage of stochastic music over another. One may conjecture that if he were to produce more than one realization for a given passage, he might choose the one that corresponded most closely to his intuitive vision for that portion of the composition, or he might choose a realization that possesses some particularly unusual, and possibly unexpected, structural features. Whatever the criteria for his choice, he could certainly modify the results of the calculations either to tailor the passage more closely to his intuitive vision or to help situate it more convincingly within a larger structural context. The general impression given by his writings, however, is that one realization is sufficient—particularly in the early days when the calculations were done by hand!—and that any passage of stochastic music that exhibits, always approximately, the general characteristics desired by him is acceptable. In fact, in the composition of his stochastic music Xenakis seems to be concerned much more with varying the general characteristics of the sonic configurations than with generating alternative versions of configurations that share the same set of general characteristics. It appears that the stochastic composition of the details of sonic configurations serves mainly to actualize the general concepts that support the unfolding of the large-scale structure of the music.

Passages of stochastic music that share the same general characteristics tend to be similar with respect to their overall structure but distinct with respect to their details. An alternative realization of the stochastic field based on upon the exponential distribution, $\partial = 3$, and the linear distribution, $g = 48$, is shown in Figure 2.18. A graphic representation of the passage is shown in Figure 2.19. This graph should be compared with the graph of the first passage (Figure 2.17). Both are similar in appearance, although the graph in Figure 2.17 contains slightly more points. The differences in detail between the two passages are more evident in their scores. The passage in Figure 2.14, for example, contains what appear to be some distinctively "concluding" gestures, including a "rhetorical" pause over the barline between measures 7 and 8, followed by concentrated activity in the high register and an isolated event in the low register. These features of the music are, of course, completely fortuitous, since the events were transcribed directly from the notelist. The ending of the passage in Figure 2.18 is much more continuous, and therefore less likely to be interpreted as suggesting any kind of structural or gestural closure. The statistical characteristics of both passages, on the other hand, are quite similar, as demonstrated in the comparative histograms in Figures 2.20 and 2.21. In the legends of the graphs, the first passage is referred to as ex. 1 and the second as ex. 2. The frequencies of the time-point and pitch intervals in both passages conform reasonably well to the expected probabilities of the distributions upon which they were modelled.

The quotes around the descriptive terms used above to characterize some of the details in the passages of stochastic music should not be regarded as merely ironic or as implying that it is somehow incorrect to try to relate this music to other, possibly atonal or serial, music that we have heard previously. On the contrary, Xenakis is quite aware that listeners will try to make perceptual sense of stochastic music by discovering, or rather inventing, possible structural connections among its details. With regard to the perception of stochastic music, he has written:

> In fact, the data [upon which the structural details are based] will appear aleatory only at the first hearing. Then, during successive rehearings the relations between the events of the sample ordained by "chance" will form a network, which will take on a definite meaning in the mind of the listener, and will initiate a special "logic," a new cohesion capable of satisfying his intellect as well as his aesthetic sense; that is, if the artist has a certain flair.[56]

Xenakis is not specific about what he means by "flair," but presumably it implies a degree of artistry in deciding upon the general structural features of the music, and possibly also judicious editing of the results of the calculations used to generate the structural details. In a more general sense this quote makes clear that, once a passage of stochastic music has been removed from the ideal realm and has assumed a specific form in the temporal realm, it assumes the status of an artifact upon which various interpretations may be inscribed as an inevitable consequence of its history. Xenakis's aesthetic stance is not, therefore, one of radical indeterminacy, in which the specific form of a potential structure is renewed with each performance of a work, but

[56]*Formalized Music*, p. 37.

rather is closer to that of action painting in which the details, though they may be produced through a combination of planning and fortuitous occurrence, remain fixed once the paint has come in contact with the canvas. Like the action paintings of Jackson Pollack, the fixed structures of Xenakis's stochastic compositions not only preserve the memory of, but also reflect, the explosive energy of an artistic process that combines highly focussed concentration with a deliberate renunciation of control over the precise arrangement of details.

2.2.1.9 Analysis of a Passage of Stochastic Music by Xenakis

A representative example of Xenakis's stochastic music is shown in Figure 2.22, which shows mm. 29-62 of *Herma* for piano (1960-1). The portion of this excerpt that will be discussed begins in m. 30 and extends through m. 59. This is the passage in which the pset A is introduced. Pset A is one of the primary sets involved in the set-theoretic operations that are used to produce the work's large-scale pitch structure. The context of mm. 30-59 within the structural plan of *Herma* may be appreciated by examining the temporal flow chart that Xenakis prepared for the work, shown in Figure 2.23.[57] The chart shows that pset A is presented in three segments, the first of which starts at the beginning of the second system and continues past the beginning of the third system. Overlapping this long segment are two shorter segments, one which starts near the beginning of the second system and the other which starts at around the middle of the same system and ends simultaneously with the long segment. The superimposed segments are differentiated, both with

[57]The flow chart appears in *Formalized Music*, p. 177.

respect to their intensity and their density. The long segment is to be performed *ff* and has a density of 0.8 sounds per second (s/s). Both short segments are to be performed *ppp*. The first short segment has a density of 3.3 s/s and the second has a density of 5 s/s. The general features of these segments indicated in the chart correspond rather closely to the general features indicated in the score, but there are some slight differences as well. The long segment begins in m. 30 and continues through m. 59. Its intensity is *ff* and, when it appears alone, it is performed without pedal. The first short segment begins in m. 32 and continues through m. 38. Its intensity is *pp*—not *ppp* as indicated in the chart—and it is performed with pedal throughout. The second short segment begins in m. 46 and continues through m. 59. Its intensity is also *pp* and it is performed with pedal virtually throughout, with pedal changes as indicated in the score. The pedal from this segment sustains through the two measures of rest following it. As indicated in the chart, these measures represent a pause between this segment and the one that follows and do not belong, therefore, to either the short or long segments, both of which conclude at the end of m. 59. At the indicated tempo of ♩ = 180, the duration of the long segment is 60 seconds, the duration of the first short segment is 14 seconds and that of the second short segment is 28 seconds. These durations are represented fairly accurately in the chart. The densities of the segments are given in the score exactly as they appear in the chart. The descriptive terms "linear" and "cloud" found within the long segment and the first short segment in the chart appear in the score in French as "linéaire" and "nuage." According to the score, the second short segment (mm. 46-59) is also "nuage." These terms appear to designate a qualitative distinction between the events in the superimposed segments. This distinction is made concrete by the differences in density, intensity, and pedalling between the "linear" long segment and the "cloud"-like short segments.[58]

The contents of pset A are as follows:

A = {-31 -29 -27 -25 -22 -21 -20 -19 -13 -12 -11 -5 -4 -1 2 3 5 6 7 17 18 19 21 22 24 26 28 33 35 36 38}.

This set was constructed before Xenakis had composed with sieves. In contrast to the spacings of the m-psets in section 2.1.3, which showed limited ranges in the sizes of their intervals, SP(A) reveals clusters of closely packed pitches separated by some comparatively large gaps:

SP(A) = <2 2 2 3 1 1 1 6 1 1 6 1 3 3 1 2 1 1 10 1 1 2 1 2 2 2 5 2 1 2>.

These differences may have to do with the fact that the structural purpose of this pset is different from that of the m-psets, which were probably designed with the transformation T_n(mod m) in mind. Their more even spacing appears to be more compatible with the "wrap-around" nature of the T_n(mod m) transformation, where it allows for a relatively homogeneous dispersion of intervals in p-space. Pset A, on the other hand, is designed to enter into relations of union, intersection, and complementation with other sets. The structural characteristics of pset A help to differentiate it from the other primary psets, B and C, each of which has its own special characteristics.[59]

[58]The association of "linear" segments with low density, high intensity, and the absence of pedal, and of "cloud" segments with high density, low density, and the presence of pedal, breaks down, however, in the presentation of psets B and C. This is demonstrated both in the chart and in the score.

[59]The realization of the set-theoretic operations described in *Formalized Music*, pp. 173-6, is blurred somewhat by the inclusion of apparently stray elements in several of the psets. It is impossible, therefore, to determine from the score the precise contents of the psets Xenakis had in mind when he designed the large-scale pitch structure of *Herma*. Rosalie La Grow Sward, in "An Examination of the Mathematical Systems Used in Selected Compositions of Milton

Xenakis has been cited as claiming that, in the stochastic composition of the pitch succession in *Herma*, pitch-position intervals were distributed over the set of all pitches available on the piano. If the endpoint of an interval fell on a pitch that was not an element of the desired pset, the nearest pset element would be substituted for it.[60] If this account is indeed accurate, the rationale for this method is not entirely clear. It is true that in earlier stochastic works, such as *Achorripsis* (1956-7) and *Analogique A* (1958), Xenakis distributed pitch-position intervals over psets in which the elements were separated by uniform semitonal increments, i.e. "chromatic" psets. The distribution of pitch-position intervals psets in which the intervals between the elements are non-uniform was one of the structural innovations introduced in *Herma*. It is possible, therefore, that Xenakis's compositional technique took time to adjust to the full implications of this structural innovation. It is also true that the function in "Free Stochastic Music" that is designed for the generation of pitch-position intervals was applied to

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Babbitt and Iannis Xenakis" (Ph.D. diss., Northwestern University, 1981), pp. 573-4, tries to rectify this situation by publishing a list of errata for *Herma* supplied to her by pianist Claude Helffer. A "corrected" list of contents for psets A, B, and C, also supplied by Helffer, is given on p. 377 as part of Sward's discussion of *Herma* (pp. 374-400). The contents of pset A, according to Sward (via Helffer) is {-29 -27 -22 -21 -15 -13 -12 -11 -5 -4 2 5 6 7 17 19 21 22 26 28 33 35 36 38}, which is a subset of the contents for A given above. Rudolf Frisius, in "Konstruktion als chiffrierte Information," *Musik-Konzepte* 54/55 (1987): 91-160 gives, on p. 143, a chart of several of the psets in *Herma*. The contents of pset A, according to him, are {-29 -27 -22 -21 -15 -13 -12 -11 -7 -6 2 5 6 9 17 19 21 22 26 28 33 35 36 38}. The replacement of pitches -5 and -4 with -7 and -6, and 7 with 9 may be the result of errors made in drawing up the chart. The source for Frisius and Helffer is probably Xenakis, by means of the sketches to *Herma*, but this is not mentioned explicitly in either Sward or Frisius. From the point of view of performance, the recordings by Yuji Takahashi, to which the work is dedicated, and by Helffer both follow the published score. The structure of the finished work, therefore, is probably best regarded as an approximate realization of the original compositional plan. In view of the discrepancies with regard to the contents of the psets, my analysis of the passage in Figure 2.22 follows the published score, without proposing any "corrections" that have not been endorsed publicly by the composer.

[60] Sward, "An Examination," p. 397.

chromatic psets in the computer-assisted stochastic works, which were composed between 1956-62. It is possible that the pitch-position and time-point intervals used in the composition of *Herma* were generated with the aid of the computer, to which Xenakis had access beginning in 1961. If this were the case, it would make sense that raw material generated under conditions in which computer time was extremely limited may have been produced quickly and later adapted for other compositional purposes. For the purposes of assessing the statistical characteristics of the excerpt from *Herma*, however, the distribution of pitch-position intervals will be examined as if it had been generated according to the method described above in connection with the STMus1 examples. In this method, if the spacing of a pset is non-uniform, intervals are generated between the positions occupied by the elements in the pset, and not between the pitches themselves. If Xenakis did, in fact, use a different method in the composition of *Herma*, it is reasonable to expect that goodness-of-fit of the observed frequencies to the expected probabilities will be compromised. On the other hand, the method introduced above is more logically consistent than the one Xenakis is reported to have used in the composition of *Herma*. It is appropriate, therefore, to use the statistical models associated with the method proposed here as a standard with which to compare examples of actual stochastic music.

The maximum pitch-position interval size for pset A is 30, since A contains 31 elements which may be numbered from 0 to 30. A histogram comparing the expected probabilities of the linear distribution, $g = 30$, with the frequencies of pitch-position intervals in the three segments in the excerpt from *Herma* is shown in Figure 2.24. The segments are numbered in the legend according to the order in which they appear in the excerpt. The histogram demonstrates that the fit between the expected and observed distributions for segment 1 is not very close, but as the density of the segments increases, the fit improves.

The method for generating time-point intervals has remained fairly consistent throughout Xenakis's compositional practice. The statistical analysis of the time-point succession in the excerpt from *Herma*, therefore, is much less problematic than the analysis of the pitch-position intervals. It is normal for the observed densities of passages of stochastic music to deviate slightly from the value for ∂ used in the generation of the time-point intervals. In the excerpt from *Herma* the expected and observed densities are as follows:

	density	
segment	expected	observed
1	0.80	0.90
2	3.30	3.07
3	5.00	5.11

Histograms comparing the expected probabilities of exponential distributions (with densities as given in the score) and the observed frequencies of time-point intervals in each of the segments are shown in Figure 2.25. The expected probabilities and observed frequencies are very close in all three segments.

The transcription of the time-point values into musical notation is less straightforward in *Herma* than it is in the examples produced with STMus1 presented above, and therefore deserves some explanation. At the tempo of ♩ = 180 which is introduced in m. 30, along with the time signature of $\frac{12}{8}$, each measure has a duration of 2 seconds. Each half-measure is divided into straight eighth-note units or eighth-note quintuplet units (5 eighth-notes in the time of 6). The durations that appear on the surface of the music are multiples or subdivisions of either of these two basic units, which may appear separately or in combination. The use of two basic units of duration increases the number of possible approximations of the time-point values generated in the process of stochastic composition. As a result the rhythm is more complex than in the examples of stochastic music shown previously, but definite limits are placed on its complexity through the creation, in each half-measure, of rhythmic groups based on one or the other of the principal rhythmic units. The limit of complexity occurs when groups based on both units appear simultaneously, effectively creating a two-part rhythmic texture.

The passage from *Herma* was analyzed statistically by the same method that was used to analyze the passages composed with STMus1. Using the same general characteristics that were identified in connection with the *Herma* excerpt—temporal structure, differentiation between "linear" and "cloud" configurations on the basis of intensity, density, and pedalling, and contents of pset A—it is possible to recompose the passage with STMus1 and to compare the results to Xenakis's original. This will provide, in a preliminary way, a means of considering some of the stylistic features that tend to distinguish Xenakis's stochastic music from a virtually unretouched sample of computer-assisted stochastic composition. A recomposed version of the excerpt from *Herma* is shown in Figure 2.26. The general structural framework is such that the linear configuration has very little time to establish the character of the passage before the first cloud configuration enters. By no means did STMus1 provide as interesting a bit of music in these crucial measures as did Xenakis. (Cf. Figure 2.22, mm. 30-1.) Another crucial area in the structure of the passage is mm. 10-6, between the two appearances of the cloud configurations. Due to the fact the linear configuration appears without pedal in these measures, a gap or a succession of unwanted intervals (such as unisons or octaves) could easily mar the integrity of the larger passage. STMus1 does not fare quite as badly in this passage as in the earlier one. (Cf. Figure 2.22, mm. 39-45.) In the remainder of the passage, however, where linear and cloud configurations are superimposed once again, STMus1 did not produce raw material whose musical realization results in as interesting a rhythmic structure as that produced by Xenakis. (Cf. Figure 2.22, mm. 46-59.) By remaining as faithful as possible to the time-points generated by STMus1, a rhythmic structure was produced whose general level of activity is less evenly distributed than that in the Xenakis. The method of transcription is partly responsible for this, for there are fewer clef changes and staff crossings in the notation of Figure 2.26 than in the original score, but it is also true that the data produced by STMus1 resulted in more simultaneities and less variety in the registral disposition of the pitches than are found in the Xenakis.

Comparative histograms for the STMus1 example are shown in Figure 2.27 and 2.28. Figure 2.27 shows that the frequencies of the time-point intervals conform quite closely to the expected probabilities of their respective exponential distributions, but not as closely as in the excerpt from *Herma*. The expected and observed densities of the segments in the STMus1 example are shown in the following comparative table.

		density	
segment	expected	Xenakis	STMus1
1	0.80	0.90	0.85
2	3.30	3.07	3.43
3	5.00	5.11	4.11

The observed densities are closer to the expected values in segments 1 and 2 of the STMus1 example than they are in the excerpt from *Herma*, but the density of segment 3 is quite low. It is likely that, if the segment were longer, its mean density would approach the expected mean density more closely, in conformity with the Law of Large Numbers. Much more striking differences are found, however, in the comparative histograms for pitch-position intervals. The deviations from the expected probabilities in the Xenakis example show an increase toward the middle range of the interval values, whereas the deviations in the STMus1 example are greatest at the low end of the range. An increase in the frequency of values at the low end is to be expected in music composed with STMus1, given the fact that large intervals are more likely to result in pitch positions that lie outside the boundaries of the pset, and are therefore filtered out by the program.

The differences in the comparative histograms of the Xenakis and STMus1 examples raise once again the question of compositional technique. When Xenakis first began to compose stochastic music, he drew up tables of intervals from the formulas for the probability distributions he chose to use. This method is similar to the one used here to determine the expected probabilities in a histogram. Intervals would then be selected from the tables one at a time and each interval would be used only once. This method is known generally as *sampling without replacement*. Naturally, the frequencies of the intervals in a passage of music composed this way should conform quite closely to the probabilities of the distribution on which they are based. Computer-assisted stochastic composition, in which interval sizes are generated according to the inverse CDFs of the chosen distributions, produces results that are less predictable. In this method intervals of approximately the same size may be selected over and over again. Computer-assisted stochastic composition, then, approximates the method known generally as *sampling with replacement*.[61] If Xenakis were still calculating interval sizes by hand at the time *Herma* was composed, this might explain the closeness of the observed time-point interval frequencies to the expected probabilities of the exponential distribution that are found in the excerpt. It is also possible that, if he used a computer in the composition of *Herma*, he may have had access to a more sophisticated pseudo-random number generator than the one used by STMus1. In any event, the method of inquiry into the statistical structure of stochastic configurations presented here is relevant both to manual and automated methods of stochastic composition.

The deviations from the expected probabilities of the linear distribution in the frequencies of the pitch-position intervals, however, point in a different direction. These appear to be the result of revisions of the results of the calculations. In contrast to the passage composed with STMus1, the presence of pitch intervals commonly associated with tonal composition, such as unisons, octaves, and multiple octaves, is minimal in the *Herma* excerpt. If these pitch intervals are found in a pset, the techniques of stochastic composition—particularly computer-assisted stochastic composition—contain no provisions for avoiding or eliminating them if the composer finds them to be aesthetically undesirable. The appearance of these intervals, however, is more liberal in the passages where the linear and cloud configurations are superimposed, since the characteristics of the individual events in these areas are less exposed due to the application of the pedal, the

[61]For more on statistically-based composition methods, see Charles Ames, "Statistics and Compositional Balance," *Perspectives of New Music* 28/1 (1990): 80-111.

low intensity of the majority of the events, and the increase in textural density.

Another aspect of the pitch structure of the *Herma* excerpt that is suggestive of manipulation of the details is revealed in the graphic representation of the passage, shown in Figure 2.29. The events in the linear configuration are much more evenly distributed within p-space than are the events in the linear configuration of the STMus1 passage, whose graphic representation is shown in Figure 2.30. The contrast between areas in which events cluster together and then spread out as the passage unfolds in time is characteristic of music composed with STMus1. (Cf. Figures 2.17 and 2.19.) The evenness with which the events are distributed in the linear configuration of the Xenakis excerpt, though perhaps more aesthetically appealing to the eye and the ear, is not particularly characteristic of the linear distribution. This observation is corroborated by the deviations in the observed frequency of the pitch-position intervals from the linear configuration shown in the comparative histogram in Figure 2.24 (and also in Figure 2.28, top).

In summary, a reasonable statistical and (perhaps less successful) musical imitation of a passage of stochastic music by Xenakis was produced with STMus1. Missing from the imitation are certain aspects of the composer's "flair" which Xenakis mentions in the quote given above. (I am making the assumption here that, for Xenakis, "flair" refers not only to creative vision, but also to personal interpretation, and perhaps revision, of the results of formalized compositional processes.) On the basis of the comparison of the two passages, it appears that any revisions Xenakis might have made during the preparation of the finished version of the *Herma* excerpt were grounded in a working knowledge of the possibilities and limitations of statistically-based composition. The indirect evidence for this is that, on the whole, the statistical characteristics of the passage conform quite closely to the models upon which the time-point and pitch structures were based, according to Xenakis's compositional theory and personal testimony. Furthermore, the enduring importance of pitch relationships in this music, even though they are not defined by the intervallic criteria that are commonly observed in atonal and serial composition, is revealed by the tendency to avoid conspicuous use of intervals that have strong tonal associations. These apparent avoidances occur particularly in areas where the texture is thin and individual events are therefore more exposed. The importance of pitch relationships in this music is also revealed in Xenakis's apparent tendency to change (consciously or not) the details of the pitch structure more frequently than those of the time-point structure.

2.2.1.10 Stochastic Streams

The general procedure for the composition of stochastic field configurations, which has been demonstrated above, entails an equivalence between the maximum pitch-position interval size available within a given pset and the maximum size of the intervals that may be generated by the program. As we have seen, the maximum possible interval size is rarely used in composition since it is likely to result in pitch positions that lie outside the given pset, and any interval that does so is rejected automatically by the program. If the size of the maximum pitch-position interval allowed by the program is reduced significantly, a different type of stochastic configuration is produced. An example of such a configuration is shown in musical notation in Figure 2.31 and in a graphic representation in Figure 2.32. This example was composed with STMus1, with $\partial = 3$ and $g = 48$ as in the first two examples, but the maximum pitch-position interval size (here, once again, equivalent to pitch interval size) has been set to 6. Comparison with the scores of the earlier examples (Figures 2.14 and 2.18) and with their graphic representations (Figures 2.17 and 2.19) shows a marked difference in the morphology of this configuration. The new configuration, particularly in its graphic representation, resembles a stream of events that meanders its way through p- and tp-space. Because of the limitation on the maximum possible pitch-position interval size, the pitches in this configuration articulate a quasi-melodic contour that contrasts with the more spatial contour articulate by the events in stochastic field configurations. In their formation, *stochastic stream* configurations are very similar to stochastic fields, except that there is a marked difference between the size of the maximum pitch-position interval in their pset and the size of the maximum interval used in composition, a difference that does not figure into the composition of stochastic fields.

An example of a stochastic stream from one of Xenakis's compositions is given in Figure 2.33, which shows mm. 41-6 from *Mists* for piano (1980). The segment that contains the stochastic stream begins in m. 44 (second system, second measure) and ends before the fourth quarter-note beat of m. 45. This segment is differentiated from those that precede and follow it by changes in pedalling as well as by the changes in morphology from stochastic field to stochastic stream, and back to stochastic field. Time-points are notated in the score but the durations of the events are not precisely determined, as indicated by the composer's note, which is included in the figure. This method of notation provides for greater flexibility in the transcription of the calculated time-points into musical notation than does the method used in *Herma*. The eighth- and sixteenth-note beams form a temporal grid that assists the performer in the placement of the time-points. The pitches in this configuration form a subset of an m-pset that is introduced earlier in the work.[62] The maximum pitch-position interval size possible in the complete m-pset is 28, and the maximum pitch-position interval size in the subset represented in the configuration is 18. The maximum pitch-position interval size observed in the configuration, however, is 8, which occurs only once (between pitches -14 and -38 in m. 45, beat 2, ordering the simultaneously occurring pitches from low to high). Most of the other intervals range in size between 0 and 4, with a few going as high as 6. A graphic representation of the graphic stream configuration is shown in Figure 2.34.

One of the striking musical features of this passage is the frequency of unisons. Unisons result whenever a pitch-position interval of size 0 is generated. In the introduction to *Mists* Xenakis mentions that the exponential, Cauchy, and hyperbolic distributions were used in its composition. The exponential distribution was most likely used to determine the time-point intervals in the stochastic configurations. The Cauchy and hyperbolic cosine distributions, which have not been discussed above, were likely used to determine pitch-position intervals. Both of the these distributions include positive and negative values in their outcome sets, i.e. $\Omega = (-\infty, \infty)$. This eliminates the need for the sign-change function that had to be appended to the inverse CDF of the linear distribution previously in order

[62]The contents of the complete m-pset are {-39 -38 -33 -32 -28 -24 -22 -19 -18 -14 -11 -9 -5 -3 0 2 8 10 13 16 21 23 27 30 36 38 41 43 44} and the contents of the subset are {-39 -38 -33 -32 -28 -24 -22 -19 -18 -14 -11 -9 -5 -3 0 2 8 10 13 16}. If F3 (-7) in m. 44, beat 3, is included within the 8vb bracket that appears to begin with E1 (-32), it becomes F2 (-19), which is a member of the subset. Otherwise, it must be regarded as a stray element, for it does not appear consistently in the subsets of the original m-pset, whose contents are established without doubt earlier in the work. The change of F3 to F2 also lends greater consistency to the contour and keeps the size of the pitch-position intervals within the limits observed elsewhere in the configuration. The pitch of this event, therefore, is interpreted as -19 for the purposes of the analysis given above.

to produce positive and negative pitch-position intervals. Of the two distributions, the hyperbolic cosine features a larger cluster of values around 0 and therefore it was probably the one used in the composition of the stochastic stream configuration in Figure 2.33.[63] Stylistically, this passage is quite different from the music found in *Herma*, where intervals with tonal associations, such as unisons and octaves, were used rather sparingly. In the composition of *Mists*, Xenakis seems to have been more interested in exploring the possibilities of different stochastic distributions than in aiming for intervallic variety as a primary stylistic characteristic. The chronological proximity of Xenakis's early stochastic music to well-known examples of integral serialism, such as Boulez's *Structures Ia* for two pianos (1952), may—despite Xenakis's statements to the contrary—have had an effect upon certain aspects of its style, such as the use of extreme registral contrasts, which is characteristic both of some integral serial textures and stochastic field configurations. In any event, the variety of configuration types in Xenakis's music has increased as his interest in the strict formalization of nearly all aspects of composition has decreased. This general pattern in his work parallels developments in the work of other composers of Xenakis's generation, such as Stockhausen, Boulez, and Ligeti, that demonstrate in part a turn away from some of the more radical compositional techniques and stylistic characteristics found in their earlier music and the adoption of new techniques, which may or may not be as radical as the earlier ones.

[63]Histograms and formulas for several distributions are given in Lorrain, "A Panoply" and in Phil Winsor and Gene DeLisa, *Computer Music in C* (Blue Ridge Summit, PA: Windcrest, 1991).

2.2.1.11 Conclusion

This introduction to stochastic music has demonstrated some of the methods that may be used in its composition and analysis. More research remains to be done in order for the musical potential of various probability distributions to be better understood. Xenakis has revealed the general principles underlying stochastic composition, particularly in chapters 1-3 and 5 of *Formalized Music*, but important details of the implementation of these principles and of the criteria for rendering the results of calculations into musical structures have been left out of his account.[64] An attempt has been made here to identify, in a preliminary way, some of the stylistic traits of Xenakis's stochastic music that result from a compositional "flair" not evident in unretouched examples of computer-assisted stochastic composition. A comparison of a passage from *Herma* with one composed with STMus1 has pointed toward some of these traits. Comparative analysis of other passages from Xenakis's music will surely yield further insights. For the remainder of this study, however, the analysis of stochastic configurations in their compositional contexts will focus on their general structural features, such as pitch contents, intensity, density, duration, articulation (e.g., pedalling), and morphological characteristics. No further statistical analyses of the intervals between events within configurations will be undertaken, for it is assumed that by now the reader has at least a preliminary grasp of how these intervals are selected and of the role they play in the formation of stochastic configurations. Applications of probability distributions to the determination of the durations and densities of segments will be illustrated, however, in section 2.3 and in the analyses in chapters 3 and 4.

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[64]See the remarks to this effect by Julio Estrada cited in the Introduction, note 13.

2.2.2 Random Walks

There are many types of stochastic processes that can be described as random walks.[65] Xenakis has used the term rather loosely to refer to stochastically generated waveforms in his electroacoustic music, to long glissando lines in his music for strings, and even to stochastic configurations in his music for piano.[66] Use of the term here will be limited to configurations whose events are presented in a continuous manner, either in a single line or in two or more independent lines. Absolute continuity in the change of pitch with respect to time is possible on string instruments, which may be played glissando. Relative continuity, possible on any pitched instrument, consists of ordered sequences among adjacent elements in psets or m-psets. Random walks of both types feature unpredictable changes in direction and may also demonstrate differences in the speed at which they move through p-space. Changes in speed and in direction, i.e. in *velocity*, are characteristic of a class of random walks that are used to model the irregular motion of small particles in a liquid or gaseous medium. This motion, known as *Brownian motion*, will serve as the physical model to which the random walks described in this section refer.

In musical applications of the principles of Brownian motion, changes in the position of a particle may be interpreted as changes in pitch with respect to time. A simple model of Brownian motion is represented by the formula

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[65]Several mathematical models of random walks are given in William Feller, *An Introduction to Probability Theory and Its Applications* (New York: Wiley, 1968). Some physical applications are discussed in Whitney, *Random Processes*.

[66]For an example of each, see *Formalized Music*, chapter 14; "Music Composition Treks," p. 191; and the introduction to *Mists*.

$$\frac{d^2x}{dt^2} = -\alpha\frac{dx}{dt} + \varepsilon,$$

which is Newton's law of motion for an object of unit mass under the influence of friction and a random external force ε. This formula is also known as the Langevin equation.[67] This equation may be broken down into the following series of equations:

$$x_{i+1} = x_i + v_i dt,$$

$$v_i = v_{i-1} + a_{i-1}dt,$$

$$a_{i-1} = -\alpha v_{i-1} + \varepsilon_{i-1}.$$

In these equations, x represents the position of the particle, v represents its velocity, dt represents a uniform increment of time, a represents the particle's acceleration, α represents the friction constant for the medium in which the particle is moving, and ε represents the effect of individual collisions between the particle and other particles moving through the medium.[68] Values for ε may be made to fluctuate randomly between -0.5 and 0.5. Using a simple program based on these equations, it is possible to generate relatively long random walks in a matter of seconds.[69] Graphic representations of two random walks are shown in Figures 2.35 and 2.36. In both examples, pitch is calibrated in quarter-tones in a range suitable for performance on the violin. The continuous line in the graph, which contrasts with the points that have

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[67]Whitney, *Random Processes*, p. 75.

[68]Whitney, *Random Processes*, pp. 74-5.

[69]A listing, in C, for the program RWV (random walk according to changes in velocity) is given in Appendix IV.

been seen in previous graphs, represents glissando articulation, which is commonly found in Xenakis's music for strings. The contour of the first random walk (Figure 2.35) is smooth, indicating relatively slow changes in pitch with respect to time. The contour of the second random walk (Figure 2.36) is jagged, indicating relatively rapid changes in pitch with respect to time. The change in contour has been obtained by increasing the values for α and dt used in calculating the positions of the "particle" that produced each of the walks.

A graphic transcription of a portion of a work by Xenakis that is based on random walks is shown in Figure 2.37. The work from which the excerpt was drawn is *Mikka* for violin solo (1971), which is made up entirely of single-line random walks. As the graph indicates, both smooth and jagged random walks are used in *Mikka*. On its surface, this four-minute work appears to consist of a single, continuous glissando, but differences in the contour of the line articulate the boundaries of segments, and therefore define a temporal structure that is similar to those found in other works by Xenakis. The visual similarity between the smooth and jagged random walks in Figure 2.37 and the sample random walks in Figure 2.35 and 2.36 is striking, suggesting that the walks in *Mikka* may have been calculated using the Langevin equation or a similar model. An annotated score of *Mikka* is shown in Figure 4.1, and therefore the excerpt from the score that corresponds to Figure 2.37 is not duplicated here.

The next musical example demonstrates how a graphically designed random walk can serve as the basis for the structure of a passage of music. A graphic image of a random walk, from the sketches to *Mists* for piano (1980), is shown in Figure 2.38. The walk has a generally ascending contour, and changes in velocity are indicated in the sketch both graphically and verbally.

Changes in velocity are indicated graphically by variations in the size of the "steps" between positions in the walk. These steps represent discrete changes of position in p-space with respect to time, since the walk is to be performed on the piano where continuous changes in pitch are not possible. The Greek terms for "faster" and "slower" are indicated above and below the graphic image, with arrows pointing to the appropriate portions of the walk.[70] The graphic image in the sketches, therefore, represents an ascending random walk that begins slowly, then accelerates rapidly, and finally decelerates as it reaches its conclusion. This random walk prototype serves as the basis for the configurations in the first seven measures of *Mists*, shown in Figure 2.39. The score shows five random walks whose entrances are staggered, the first beginning in m. 1, the second at the end of m. 1, the third in m. 4, the fourth in m. 5, and the fifth in m. 6. Each of the walks begins in the lowest register of the piano, and all but the fifth ends in the highest register. In comparison to the others, the fifth walk appears to be cut off prematurely.[71] The changes in the velocity of these walks are manifested in musical terms as gradual changes in the durations of their elements accompanied by changes in the direction of the lines. Each walk is unique in terms of duration and contour, but all of them move in a generally ascending direction. This is in accordance with the prototype in the sketches. Also notable is the fact that the duration of each successive walk is shorter than that of the previous one.

A graphic representation of the five random walks is shown in Figure 2.40. The walks are represented as continuous lines in order that their

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[70]Greek notations in the sketches were translated by Johnnie Athena Konstantos.

[71]This may have to do with the work's temporal structure, which is discussed in detail in chapter 3.

contours may be more easily appreciated, but it should be remembered that their elements are articulated in discrete positions in p-space. Pitch in the graph is calibrated in semitones. The graph makes it appear that the first four walks divide into two pairs in which the changes in direction take place at about the same time. In their details, however, the changes of direction are different for each walk. The impression that the walks pair off is strengthened later on in the work—beginning at the end of m. 7, in fact—when walks are presented in pairs and in groups of four. In these later instances, the grouping of the walks is unambiguous, since each member of the group begins simultaneously. These structures will be examined further in the detailed discussion of *Mists* in chapter 3.

The m-pset in which the five random walks take place is

A = {-38 -36 -30 -28 -25 -22 -17 -15 -11 -8 -3 0 3 5 6 11 13 14 19 20 24
28 30 33 34 38 41 43 47}.

As in the case with stochastic configurations in which pitches in a pset or m-pset are distributed in p-space, the changes in pitch with respect to time that take place in these random walks occur as a result of changes in the size and direction of pitch-position intervals. One important difference is that, in a random walk, pitch positions normal change by an interval of ±1. Intervals between the elements of unevenly spaced sets may be measured in *contour space*, or *c-space*. In c-space, an interval of 1 signifies a move up one position in a set, regardless of the measurable size in units (such as semitones) between the the two elements in the set.[72] Graphically designed

[72]For more on c-space, see Morris, *Composition with Pitch-Classes*, pp. 23-33. The use of the term "contour" that is associated with c-space, which has to do with measurable intervals between the positions of elements within sets, is to be distinguished from the more informal use of the term, which refers to the general outlines of the path(s) traced through p- and tp-space

configurations in Xenakis's music are generally conceived in c-space, and so the intervals between pitch positions in the original design may undergo enlargement when the configuration is transcribed into p-space. Consequently, the morphology of the original design may appear somewhat distorted when it is transcribed back from musical notation into a graphic representation in p- and tp-space. As an example of this process of distortion, graphic representations of the five random walks from *Mists* are shown in both c-/tp-space and p-/tp-space in Figure 2.41. The events in the walks are represented as points so that the differences in interval spacing may be observed more easily.

In conclusion, a random walk is a sonic configuration in which events are arranged in a relatively or absolutely continuous manner such that changes in pitch with respect to time proceed with relative unpredictability. Whether changes in velocity are calculated mathematically or are designed graphically, considerable variety may be achieved in the contour of the walks. Similarities and differences in the contour of walks, in turn, may be used to articulate structural relationships among them in compositional contexts. In this respect random walks differ from stochastic configurations, in which the more radical unpredictability in the succession of events discourages the establishment of definite structural relationships among configurations. Random walks may be gathered into pairs or into larger groups in which each walk tends to retain its identity as an independent line. Examples of pairs of

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by events in (relatively or absolutely) continuous configurations, such as random walks or arborescences. Since contextual clues should make the significance of the term obvious in any particular instance, use of the term to denote a quantitative measure does not necessitate abandoning its use as an informal descriptive term.

random walks will be given in the discussions of *Mists* and *à r.* in chapter 3, and *Mikka "S"* and *Theraps* in chapter 4.

2.2.3 Arborescences

Arborescences are branching structures that are manifested musically as quasi-polyphonic configurations in which individual voices branch off into pairs from specific nodal points. Both the contours of the voices and the locations of the nodal points are patterned after models of random branching processes. The individual voices resemble random walks, but unlike configurations in which random walks are presented simultaneously, the voices in an arborescence are interdependent, since the initial point of each new voice is a point that it shares with a voice that has begun previously. Since arborescences are graphically designed, they may assume a great variety of forms according to the will of the composer. Once the prototype for an arborescence is created, Xenakis tends to transform its shape according to relatively simple processes of transformation, such as rotation and elongation. Thus, the audible relationship between two arborescences based on the same prototype is likely to be greater than that between two passages of stochastic music that are based on identical formulas.

The examples of arborescences in this section are taken from *Mists* for piano (1980). Two different types of arborescence are represented in the examples. Xenakis's graphic design for the first type of arborescence is shown in Figure 2.42. This arborescence appears three times in the finished composition. Its appearances are as follows:

measures	duration (sec)	m-pset
14-6	11.250	A
22-4	9.375	A
28-30	12.500	B

The arborescences are shown in graphic representation and in musical notation in Figures 2.43-5. Graphic representations are given both in c-/tp- and p-/tp-space. Continuous lines have been used in the graphs so that the identity of the voices with respect to Xenakis's graphic design may be appreciated more easily. (Note: the continuous lines do not indicate continuous motions within p-space. Neither is such continuity implied in Xenakis's sketches.) The contents of the m-psets in which the arborescences have been inscribed are

A = {-38 -36 -30 -28 -25 -22 -17 -15 -11 -8 -2 0 3 5 6 11 13 14 19 20 24 28 30 33 34 38 41 43 47} and
B = {-36 -32 -30 -27 -26 -22 -19 -17 -13 -11 -8 -6 0 2 5 8 13 15 19 22 28 30 33 35 36 41 43 44}.

The two m-psets are related by the operation T_n(mod m) such that B = T_{30}(mod 90)A.[73] The subsets of A and B that are used in the arborescences are

A_S = {-17 -15 -11 -8 -2 0 3 5 6 11 13 14 19 20} and
B_S = {-17 -13 -11 -8 -6 0 2 5 8 13 15 19 22 28}.

One of the elements in the intersection of A_S and B_S is 5, which serves as the starting point for all three configurations. The pitch position of 5 in A is 13 and, in B, 14. This accounts for the slight difference in the c-space locations of the arborescences in Figures 2.43a and 2.44a versus Figure 2.45a. The

[73]These m-psets are introduced in full in random-walk configurations prior to the appearance of the arborescences. M-pset A is used in the random walks from *Mists* discussed in section 2.2.2.

distortions in the contour of the arborescence in the p-/tp-space representation in Figure 2.45b with respect to the p-/tp-space representations in Figures 2.43b and 2.44b result from differences in the spacings of A and B, which are related by rotation under the operation T_n(mod m). The spacings of subsets A_S and B_S are

SP(A_S) = <2 4 3 6 2 3 2 1 5 2 1 5 1> and
SP(B_S) = <4 2 3 2 6 2 3 3 5 2 4 3 6>.

The sum of the intervals in SP(A_S) is 37, whereas in SP(B_S) it is 45. Thus, the third arborescence is more widely spaced overall than the first two. This difference in overall spacing is observable both in the musical notation of the arborescences and in their respective graphic representations.

Change of m-pset is one type of transformation that may be performed on arborescences. Another type is change of duration. As indicated in the table above, the duration of the first arborescence is 11.25". In the second arborescence the duration is shortened to 9.375" and in the third it is lengthened to 12.5".[74] These operations on the duration of the arborescences correspond visually to the horizontal compression and expansion, respectively, of the graphic image of the arborescence. The effects of the compression and expansion of the graphic image may be observed by comparing the lengths of the graphic representations of the arborescences in Figures 2.43-5. The transcription of the transformed arborescences into

[74]The durations of these segments have been measured according to the number of beats at the indicated tempo, in which the quarter-note equals 48. Fermatas have been ignored in the calculations, since their duration varies from performance to performance. The rests that follow the first and second arborescences were also not included since they are given the status of independent segments for reasons that are explained in the detailed discussion of *Mists* in chapter 3.

musical notation is approximate in that it does not reveal strictly proportional diminutions or augmentations of the notated durations found in the first arborescence. There are also slight differences in detail among the arborescences. For example, in the second arborescence, which in most respects is identical to the first in the content and order of its pitches, G5 in m. 23 is followed by B4 instead of the C#5 that appears in the corresponding position in the first arborescence (m. 15). Despite the slight differences in the notational realization of the durations and minor alterations in pitch-position structure, the transformation of the arborescences is relatively strict. The nature of the transformations is best understood, however, when the arborescences are viewed as complete structural units and not as collections of sonic events whose elements are to be compared one at a time. Such a reading would probably fail to uncover the global nature of the transformations in duration and in m-pset contents.

Graphic designs of several other arborescence prototypes appear in the sketches to *Mists*. One of these other prototypes also undergoes systematic transformations during the course of the composition, but the nature of these transformations is different from those that have been demonstrated above. This prototype and its transformations are shown in the detail from the sketches that is presented in Figure 2.46. The original prototype is shown in the upper right portion of the detail, directly beneath the numbers 93 and 94, which indicate the measures in which this particular arborescence appears in the score. This prototype differs significantly from the one examined previously, in that it does not exhibit the type of branching process that is so evident in the earlier example. Instead it consists of a continuous, partly curved and partly peaked main body which is surrounded by small curves that are not connected directly to it. In the verbal commentary included in the sketches, Xenakis refers to these arborescences as "bushes" or "shrubs." Thus, despite the lack of a well-defined branching structure, it is appropriate to describe these images as arborescences, even if the definition of the term must be stretched somewhat in order to include them. The musical manifestations of this prototype and its transformations are also different from those of the other arborescences, for these are inscribed into the (chromatic) p-/tp-space directly without the intermediary step of being inscribed first into an m-pset.

Located beneath the original prototype in Figure 2.46 is a mirror-inversion transformation which, as indicated in the sketches, appears in mm. 109-10 of the score. The remainder of the transformations appear in the lower left of the detail, enclosed in a border. As the sketch indicates, three of these transformations were retained in the finished composition, while the three that are crossed out do not appear in the score. These additional transformations result from rotations of the original prototype within p-/tp-space.[75] Rotation causes changes in the spatial orientation of the arborescences. Each of these changes of orientation may be compared to the orientation of the original prototype. Note the vertical peak in the main body of the original prototype, located near its horizontal midpoint, below and just to the left of the number 94. In the transformation that appears near the right side of the border, midway up from the bottom, this peak is shown pointing at an angle toward the lower right corner of the page. In the transformation that appears in the upper right portion of the border, this peak points upward at an angle toward the upper right corner of the page. In the transformation

[75]What is referred to here is rotation in the graphic sense, not in the sense of order rotation in which, for example, a twelve-tone row is rotated by shifting the order positions of its elements one or more places to the left or right.

that appears at the bottom of the bordered area—partly obscured because of the way the original sketch was photocopied—the peak points horizontally to the left side of the page. These transformations are found in the score in mm. 115-6, 129-30 (*not* 128-9 as indicated in the sketch), and 133-4, respectively. The last transformation occurs in the penultimate segment of the work, and is followed by a half-measure of notated silence. Were all the transformations in the sketch to be included in the finished composition, it would have ended with five arborescences in direct succession which, though making for a spectacular conclusion, might possibly have overtaxed the abilities of even the most capable of performers.

The original prototype and its four transformations are shown in Figures 2.47-51 in graphic representation and in musical notation. Because of the density of these configurations and the number of repeated pitches in their musical realization, they have been transcribed in discontinuous points rather than in continuous lines. The musical notation makes it clear that the component lines of these arborescences are distinguished through the use of non-coinciding subdivisions of the beat. The reader may also note that the approximate locations of the arborescences in p-space are indicated in the sketch by means of staff notations. (Compare Figure 2.46 with Figures 2.47-51). The locations of all five arborescences in p-space, when shown together in unmeasured sequential time (st-space) as in Figure 2.52, reveal a U-shaped pattern that gives a definite large-scale structure to the sequence of configurations, even though their appearances are spread over three-and-a-half minutes and are interspersed with contrasting material. The possible structural significance of the precise temporal locations of these arborescences will be considered in the detailed discussion of *Mists* in chapter 3.

Arborescences have been found in many of Xenakis's works since their initial appearance in *Synaphaï* (1969). Their morphological characteristics vary according to the requirements of each of the works in which they occur, and may also vary within works, as illustrated in the examples from *Mists* presented here. The structural functions of arborescences in *Mists* and in *Evryali* will be explored in detail in chapter 3.

2.3 Large-Scale Structure and the Articulation of Form

Several factors affect the design of large-scale structure in Xenakis's compositions. These factors include the degree of formalization in the compositional method, the choice of configuration types, and instrumentation. The composer's decisions with respect to any one of these factors will partly determine the feasibility of specific plans for the large-scale structure of a work. The articulation of form, which depends in part upon the projection of perceptible divisions between structural units and, in so doing, provides a basis for the potential perception of the temporal proportions that coincide with major structural divisions, also differs from work to work on the basis of the factors indicated above. This is because the articulation of form in music that is highly sectionalized in its structure, as Xenakis's music nearly always is, depends upon the establishment of immediately perceptible degrees of similarity and contrast among its materials as the music unfolds in time. In the remarks that follow, as well as in the analyses in chapters 3-5, descriptions of structure focus on arrangements of elements inside-time and outside-time. Temporal structure will also be regarded independently of the non-temporal characteristics of the elements that make up the structural units according to which time is measured. Descriptions of structure, therefore, include information that is closely related to local occurrences of events in the music and also information regarding analyzable features of the music that are relatively remote, conceptually and perceptually, from the local details. The distinctions among these various aspects of structure have been explained in section 2.1. The usefulness of these distinctions in uncovering evidence of compositional planning, and also in relating abstract features of the music to potentially perceptible aspects of its surface, will be tested below and in subsequent chapters. Discussions of the articulation of form in Xenakis's music, on the other hand, focus almost entirely on those aspects of structure that are assumed to be immediately perceptible, or potentially so. These discussions, therefore, are limited to the inside-time structure of the music and to the ways in which the play of similarities and contrasts on the musical surface give concrete expression to the often highly abstract large-scale structures favored by Xenakis. Since the topics of structure and form are so intimately related, a clear distinction between the two will not always be made in practice. Instead, when the discussion centers around outside-time and temporal aspects of the music, it may be assumed that abstract features of the structure are being referred to, and when inside-time and perceptual aspects form the main topic of discussion, reference is being made to the articulation of form and to the way that it both gives concrete expression to and is supported by the more abstract features of the structure.

It was mentioned at the end of the previous chapter that stochastic configurations dominated Xenakis's music from the time that stochastic music was introduced (ca. 1955) until the mid-to-late 1960s. Some works present exceptions to this general rule, such as *Eonta* for piano and brass quintet (1963) which, though mainly stochastic, features passages in which simultaneities and linear, random-walk-like configurations are prominent. Nonetheless, the notion of variety in the types of configurations was first given systematic treatment in *Nomos Alpha* for violoncello solo (1966) and is characteristic of most of Xenakis's music from the late 1960s to the present. Because this topic is so important to structure and form in Xenakis's recent music, it will be useful to examine briefly its application in *Nomos Alpha,* which is the work that is discussed more thoroughly than any other in *Formalized Music.*[76]

Nomos Alpha contains four basic configuration types, each of which may be articulated either discontinuously or continuously. Discontinuous styles of articulation include pizzicato, col legno, arco normale (non-legato, i.e., detached), and pizzicato glissando, while continuous styles include arco normale (legato), harmonics, and sul ponticello, each with or without tremolo.[77] The four configuration types and two categories of articulation are represented graphically in Figure 2.53 by prototypes S_{1-8}, where S indicates "sonic configuration." S_1 represents a stochastic field of discontinuous sounds and S_5 a stochastic field of glissandi. S_2 represents a relatively ordered, ascending or descending succession of discontinuous sounds. A possible manifestation of this configuration type would be an ascending or descending glissando involving a constant interval, activated by col legno articulations in a regular rhythmic pattern. S_6, on the other hand, represents a relatively ordered, ascending or descending succession of continuous sounds, such as a cluster of pitches played glissando. S_3 is similar to S_2, but without discernable movement through p-space, and S_7 is similar likewise to S_6. S_4 consists of a

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[76]*Nomos Alpha* is virtually the sole topic of chapter 8 of *Formalized Music.*

[77]*Formalized Music,* 228-9.

bowed simultaneity of closely related pitches, producing beats, combined with pizzicati. S_8 is like S_4, but without the pizzicati.[78]

The structure of *Nomos Alpha* is generated principally by the presentation of sequences of sonic configurations corresponding to the eight prototypes illustrated in Figure 2.53. The order of the configurations within the sequences is determined by ordered sets whose transformations belong to a group structure. The sequence of the eight sonic configurations is permuted by the members of the transformation group. This group is isomorphic to the (symmetric) group of permutations on four elements. The sets that result from the twenty-four distinct permutations in this transformation group are shown in the table below.[79]

I	1234*5678*	G	3241*7685*	Q_5	*6857*2413
A	2143*6587*	G^2	4213*8657*	Q_6	*6578*2134
B	3412*7856*	L	1342*5786*	Q_7	*8756*4312
C	4321*8765*	L^2	1423*5867*	Q_8	*7586*3142
D	2314*6758*	Q_1	*7865*3421	Q_9	*5876*1432
D^2	3124*7568*	Q_2	*7658*3421	Q_{10}	*5768*1324
E	2431*6875*	Q_3	*8675*4231	Q_{11}	*8567*4123
E^2	4132*8576*	Q_4	*6785*2341	Q_{12}	*5687*1243

Each ordered set in the table is divided into parallel halves, with the numbers 1 through 4 contained in one half and 5 through 8 in the other. The numbers 5 through 8 are shown in italics, while 1 through 4 are not. In the ordered sets labelled with the letter Q the numbers 5 through 8 appear first, followed by 1 through 4. In those labelled with letters other than Q, 1 through 4 appear first, followed by 5 through 8. Identical permutations are applied to the elements within the separate halves of each set, and thus only the twenty-

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78 *Formalized Music,* 222.

79 This table is based on the one found in *Formalized Music,* pp. 221, which unfortunately contains several errors in the labelling of the sets.

four distinct permutations applicable to an ordered set containing four elements are represented. The structure of the ordered sets corresponds to the configuration prototypes illustrated in Figure 2.53, in which four basic configuration types are represented in either of two contrasting types of articulation. The ordered sets in the table are involved in the sequencing of quantities associated with the densities, intensities, and durations of the configurations as well, but the present discussion will focus mainly on the association of their elements with the four configuration types and two categories of articulations.[80]

The ordered sets in the table above result from permutation operations that may be performed on the elements of other sets. For example, if the elements of I, the "identity" set, are permuted according to the order of elements in set D, the result is,

for I = 1234*5678* and
D = 2314*6758*,
ID = 2314*6758*.

That is, the second element of I is moved to the first position, the third element to the second position, the first element to the third position, and the fourth element remains in the fourth position. The same permutation is carried out on the elements in the fifth through eighth positions in I. From this example it is clear that ID = D. In general, the permutation of the elements of I by the elements in any other set yields the order of the elements in that set. Another example:

[80]For more detailed accounts of the compositional process of *Nomos Alpha* see *Formalized Music*, pp. 215-36; Thomas DeLio, "Iannis Xenakis' Nomos Alpha: The Dialectics of Structure and Materials," *Journal of Music Theory* 24 (1980): 63-95; and Vriend, "'Nomos Alpha'."

$$\begin{gathered}\text{for } D = 23146758 \text{ and}\\ D^2 = 31247568,\\ DD^2 = 12345678 = I \text{ and}\\ D^2D = 12345678 = I.\end{gathered}$$

In this case D and D^2 are inverses of one another. Either permutation performed on the elements of the other set produces I. The same is true for E and E^2, G and G^2 and L and L^2.

Chains of permutations may be constructed, such that ab = c, bc = d, cd = e, and so forth. An example:

$$\begin{gathered}\text{for } D = 23146758 \text{ and}\\ Q_{12} = 56871243,\\ DQ_{12} = 67852341 = Q_4.\end{gathered}$$

Then

$$Q_{12}Q_4 = 24316875 = E.$$

This chain, therefore, consists of D, Q_{12}, Q_4, and E. This is the beginning of the chain that Xenakis used to determine the sequences of sonic configurations in *Nomos Alpha*. In its entirety the chain consists of eighteen sets before it returns to its origin, D.

The eighteen sequences of sonic configurations included in the chain are shown in the chart in Figure 2.54. The sequences appear in three columns, reading from left to right and top to bottom, and there is a division between every group of three sequences. The numbers associated with the sonic configurations are shown in the first row of each sequence, where they appear as subscripts to the letter σ (sigma). The numbers associated with the general qualities of the sonic configurations appear in the second row, as subscripts to the letter k. The sets for both σ and k belong to different chains of permutations that begin with D and eventually return to D, although this return does not form part of the compositional plan. Both chains have D as their first set and D^2 as their tenth—see the second column, fourth row of the chart—but contain different sets everywhere else. Each group of three sequences is associated with a different sieve which determines the contents of the m-pset from which its pitches are drawn. Abbreviated formulas for these sieves are shown to the left of the third and fourth rows of the initial sequence in each group of three. A Greek letter—either α, β, or γ—occurs just above the abbreviated sieve formulas. These letters represent modifications in the ordering of the sonic configurations, different tempi, and different scales of densities. The scales of densities are shown in the upper left portion of the chart. The densities associated with the letters α, β, and γ affect the numbers of events that occur within the sonic configurations in a sequence. Calculations for the number of events associated with elements in the k sets are shown in the upper right portion of the chart. In the calculations density is followed by intensity and duration. The product of density and duration gives the number of events in a configuration. The number of events in each configuration, in the order in which these quantities occur in the composition, is shown in the fourth row of each sequence, directly below the configuration prototypes.

The succession of Greek letters in the six sequence groups is $\beta\gamma\alpha\beta\gamma\alpha$. In each β group, the tempo is ♩ = 75 and the succession of sonic configurations corresponds directly to the order indicated in the first row of each sequence. In each γ group, the tempo is ♩ = 84 and the identity set I is replaced by

$$\gamma = 15672348.$$

This permutation causes continuous and discontinuous configurations to cross over the dividing line that separates them in the original form of the ordered sets. The application of permutation E to this replacement of the identity set in the fourth sequence, for example, produces,

$$\text{for } \gamma = 15672348 \text{ and}$$
$$E = 24316875,$$
$$\gamma E = 57613842,$$

which is the order of sonic configurations in the third row of that sequence. In each α group, the tempo is ♩ = 62 and the order of elements in the identity set is replaced by

$$\alpha = 17345628.$$

Application of permutation E^2 to α, in the seventh sequence, produces,

$$\text{for } \alpha = 17345628 \text{ and}$$
$$E^2 = 41328576,$$
$$\alpha E^2 = 41378526.$$

The chart may be consulted for verification.

As elaborate as the mechanism in the chart may appear to be, it does not, in fact, represent the structure of the entire composition. Following each group of three sequences, an episode of contrasting material is inserted. The tempo of these contrasting episodes is invariably ♩ = 75 and the sieves that determine their pitch contents are different from those used in the sequences that precede them. The most notable difference, however, is that each contrasting episode consists of a single, complex configuration in which the extremes of the 'cello's register are explored, either through the use of harmonics, scordatura, or both. In addition, the intensity in these episodes varies within a full range from *pppp* to *fff*, unlike the sequences, in which the range of intensity is only from *mf* to *fff*.

If each sequence is represented by the letter X and each contrasting episode by the letter Y, the structure of the complete composition may be represented by the following schematic:

XXXY XXXY XXXY XXXY XXXY XXXY.

Counting sequences and episodes together, twenty-four structural units are represented in this schematic. Each sequence X contains eight segments, i.e. sonic configurations with their associated qualities of density, intensity, and duration, and each episode Y consists of a single segment. Thus, *Nomos Alpha* contains one hundred fifty segments. If each X or Y is considered to be a subsection, each group of four subsections XXXY may be regarded as a section. Finally, each group of three sections, which is coextensive with a single cycle of βγα, may be regarded as a part. The categories of structural units in *Nomos Alpha* and the number of units in each category are summarized in the following table:[81]

[81]The terms used here to label structural units are adapted from Tenney, *Meta/Hodos* and from James Tenney and Larry Polansky, "Temporal Gestalt Perception in Music," *Journal of Music Theory* 24/2 (1980): 205-42. The hierarchical progression of structural units given in Tenney and Polansky is: element, clang, sequence, segment, section. An element is roughly equivalent to a sonic event in Xenakis's terminology. Because the succession of elements within configurations in Xenakis's music is either determined stochastically or is a secondary result of the morphology of the configuration, which is designed prior to the selection of individual elements, there is a direct progression from sonic events to segments in the structural model presented here. In Xenakis's music, therefore, the formation of clangs (similar to "motives" in standard terminology) and sequences ("phrases"), is likely to be fortuitous unless these structures are generated deliberately for stylistic reasons (such as in his setting of Greek dramas and other works with text). The hierarchical progression of structural units used here, then, is: sonic event, segment, subsection (if needed), section, and part. For the most part, the analyses in the following chapters begin with the general properties of segments and work up from there.

structural unit	quantity
segment	150
subsection	24
section	6
part	2

The compositional process of *Nomos Alpha* is not typical of Xenakis's work as a whole. In its deterministic formalism it bears a conceptual resemblance to the early integral serialism that he repudiated ten years previously, but characteristically for Xenakis the deterministic aspects of the group structure begin at the level of the segments and do not affect the succession of sonic events within segments. The realization of the sonic configurations with respect to registral position and to interpretation of the quantities of events specified in the compositional plan is generally quite free. Despite the peculiarities of its compositional method, however, its large-scale structural organization provides a clear illustration of tendencies that are observable both in Xenakis's earlier and more recent music. For example, many of his works—including all of those discussed in chapters 3-5—divide quite easily into two or three parts and tend to divide further into four, five or six sections. The organization of the sections differs considerably from work to work, but in general the recent works show greater variety in the articulation of sections than that found in *Nomos Alpha*. In the simplified

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For analytical applications of Tenney and Polansky's model see their article and Yayoi Uno and Roland Hübscher, "Temporal-Gestalt Segmentation: Polyphonic Extensions and Applications to Works by Boulez, Cage, Xenakis, Ligeti, and Babbitt," *Computers in Music Research* 5 (1995): 1-37. Uno and Hübscher's article contains a large-scale segmentation of Xenakis's *Herma* which corresponds generally to the segmentation presented in the composer's flow chart (Figure 2.23). Their analysis of *Herma* leads them to conclude that, "In Xenakis's music ... it is not pitch or rhythmic relationships per se, but global changes in texture and temporal density that articulate formal boundaries" (p. 2). This supports the general assumptions underlying the segmentations presented here.

account of the structure of *à r.* given in section 2.1.5, for example, the work's six sections are articulated by means of differences in the presentation of contrasting configuration types. Sections of type "X" in *à r.* feature alternations of contrasting configuration types, while sections of type "Y" are divided into complementary halves in which first configurations of one type and then of another are presented. The order of section types also changes as the work unfolds, the succession of sections articulating the pattern XYXYYX. The structural organization of *à r.*, therefore, is more dynamic than that of *Nomos Alpha*, as are the structural plans of most of Xenakis's works.

The scheme of temporal proportions in *Nomos Alpha* is also uncharacteristically regular for Xenakis. According to the score the total duration of the work is 625.5" although, given its extreme difficulty, most performances are considerably longer.[82] The durations of parts 1 and 2, again according to the score, are 301.3" and 324.2", respectively. In relation to the duration of the total work, their proportions are 0.48 and 0.52, i.e. nearly equal. The durations of the individual X and Y subsections vary, but the total duration of the X subsections versus the Y subsections is 469.9"/155.6" = 3.02, which is almost exactly three times as much. This proportion reflects the number of X subsections in relation to Y subsections per section and also within the work as a whole.

A more typical scheme is one in which the proportions of the parts are unequal with respect to one another, and in which the same or similar proportions are reflected in the relations between sections and/or groups of segments containing similar types of materials. For example, *Metastaseis* for

[82] 625.5" is equivalent to 10'25.5". At the top of the score, Xenakis gives an approximate duration of 15'. Rohan de Saram plays it in 15'22" on Disques Montaigne 2 CD 782005 and Pierre Strauch plays it in 12'31" on Erato 2292-45770-2.

orchestra (1953-4) divides into two parts whose durations are, respectively, 242.4" and 172.8". The proportions of the durations of the parts in relation to the duration of the whole work are 0.58 and 0.42, which are quite close to the simple proportion 3:2 (i.e., 3/5 versus 2/5). The work further divides into four sections, of which the first and fourth are characterized mainly by massed glissandi in the strings, while the second and third sections feature a variety of articulations, including legato lines, pizzicati, harmonics, and short glissandi, but no prolonged massed glissandi. The boundary between sections 2 and 3 is articulated by changes in density and intensity, and also by the introduction of short glissandi, which are completely absent from section 2. The characteristics of the sections are summarized in the table below.

section	measures	duration (sec)	characteristics
1	1-104	124.8	massed glissandi
2	105-202	117.6	various, no glissandi
3	203-309	128.4	various, incl. short glissandi
4	310-46	44.4	massed glissandi

The proportion of the total duration of the outer sections with respect to the duration of the whole work is 169.2"/415.2" = 0.41 and the proportion of the total duration of the inner sections with respect to the duration of the whole is 246"/415.2" = 0.59. The proportions between inner and outer section durations are very close to the 3:2 that is approximated the division of the work into parts, but the order of the terms is reversed, making this the reciprocal, or inverse, of the original proportion. Reciprocal relationships of this sort are quite common in associations between the inside- and outside-time structures or between different levels of the inside-time structure of a given work. Both types of correspondences may be found in many of

Xenakis's works. This has already been illustrated with respect to *à r.* in section 2.1, and further examples are demonstrated in the following chapters.

With the development of stochastic music, other methods for generating large-scale temporal structures were introduced. Among these methods is the use of the exponential probability distribution to determine the durations of segments. As demonstrated in section 2.2.1, the exponential distribution can be used to determine the lengths of time-point intervals between events. In computer-assisted stochastic composition, both the sizes and the succession of the intervals are determined in a quasi-random manner. Although large-scale temporal structures can be generated in this way, as they were the works composed with "Free Stochastic Music," quantities for the durations of segments may also be derived according to the exponential distribution and placed in succession manually so as to articulate a more subjectively determined temporal structure. The latter appears to be the one preferred in the majority of Xenakis's works in which segment durations derive from the exponential distribution. Two advantages of this approach are: 1) that it allows the composer to work within a specifically proportioned temporal structure, and 2) it allows for differences in the durations of the segments to be used as a means for creating distinctions between sections or parts. The large-scale temporal structure of *Herma* will serve as an illustration of how this method may be applied.

The temporal flow chart for *Herma*, shown in Figure 2.23, contains forty-four segments. Some of the segments follow one another without interruption, some are separated by rests and some overlap, as in the excerpt discussed in section 2.2.1.9. In this segmentation the introduction (mm. 1-29), shown in the first system of the flow chart, is regarded as a single segment since its subsegments, though different in density, flow into one another imperceptibly. The entire segment is based on a single pset, R, which contains all of the pitches available on the piano keyboard, and the gradual increases in intensity and density found within it are accompanied by a progressive decrease in the duration of its subsegments, creating an overall effect of nearly continuous acceleration in the presentation of sonic events. The remaining segments differ significantly from one another with respect to density, intensity, pedalling, and/or pitch contents, and are therefore regarded as separate. The greatest separation of all occurs when silences occur between the segments, which happens fairly frequently. The sum of the durations of all forty-four segments, determined according to the tempi and the metric structure found in the score, is 442", or 7'22". The durations of the segments range from 1" to 60". The mean density of the segments is $44/442 \approx 0.1$. In order to determine whether the segment durations may have been calculated according to the exponential distribution, the observed frequency of the durations may be compared with the expected probabilities of the exponential distribution, $\partial = 0.1$. A comparative histogram of the expected probabilities and the observed frequency of segment durations is shown in Figure 2.55. From the histogram it would appear that the segment durations in *Herma* were indeed calculated according to the exponential distribution, $\partial = 0.1$.

Segment durations, when viewed apart from their temporal succession and their precise locations in tp-space, constitute an aspect of the outside-time structure of a work. Recall the distinction between duration and location in tp-space that was made in section 2.1.1 as part of the outline of Xenakis's general theory of musical structure. In this theory, duration is categorized as an outside-time characteristic of sonic events which is independent from temporal succession and temporal structure. The temporal structure consists of the positions in tp-space at which sonic events are initiated. By extending these concepts from individual events to segments, a clear distinction can be made between the durations of segments and the positions in tp-space at which they begin. The latter constitutes the temporal structure of a work at the level of its segments. A work's temporal structure may be determined by measuring the intervals between the time-points at which successive segments are initiated. The time-point of the barline at the conclusion of the final measure must be considered in these calculations so that the duration of the complete temporal structure may match the duration of the whole work. The inclusion of the final time-point also ensures that the number of segments inside-time is the same as the number of segments in the outside-time structure. The total duration of *Herma* inside-time, according to the score, is 404".[83] The mean density of the forty-four time-point intervals that make up the temporal structure of the work, therefore, is 44"/404" ≈ 0.11. Using this value as the density parameter ∂, the frequency of intervals in temporal structure may be compared with the expected probabilities for the exponential distribution, $\partial = 0.11$, to determine if the temporal structure, as well as the segment durations, may have been calculated according to the exponential distribution. A comparative histogram is shown in Figure 2.56. The fit between the expected probabilities and observed frequency is not as

[83]Note that the inside-time duration of 404" is 38" shorter than the total duration of the segments outside-time, which is 442". The inside-time duration nearly corresponds to the outside-time duration minus the 35" introduction, which would be 407". It may be that the introduction, which ultimately serves as a structural "upbeat " to the main body of the work, may have been added at a different stage of the compositional process from the plan that gave rise to the main body of the work. It may be that Xenakis began with a specific total duration in mind, divided it into segments that followed one another end-to-end, and then rearranged their positions so that some overlapped and some were separated by silences. If such a procedure were used, it would explain the close correspondence between the inside-time duration of the finished work and the total outside-time duration of the segments that make up the main body. Inclusion of the introduction would then make up the difference between the original outside-time duration (407") and the inside-time duration of the finished composition (404").

close as it is in Figure 2.55, but the histogram suggests nonetheless that Xenakis may have had the exponential distribution in mind when planning the temporal structure of *Herma* as well as when determining its segment durations.

Proportions also play a part in the temporal structure of *Herma*. After the presentation of the referential pset R, the basic psets A, B, and C and their complements, the psets used in the remainder of the work are derived from the basic psets through intersection, union, complementation, or various combinations of these operations. The appearance of the first intersection, AB, marks the division of the work into two parts. Pset AB occurs at 247" (m. 136), and is represented at the end of the fifth system of the flow chart in Figure 2.23. The division at this point, in relation to the total inside-time duration, is 247"/404" = 0.611. This proportion is very close to the golden section, which is 0.618.... Because the golden section is approximated here, the proportion between the durations of part 1 and the whole work is very close to the proportion between the durations of parts 2 and 1. That proportion is 157"/247" = 0.635. Thus, when the golden section is used to determine the the large-scale temporal structure of a work, the proportion between the longer part and the whole is reflected inside-time in the proportion between the parts. This differs from the proportional relationships that were observed earlier with respect to *Metastaseis* and *à r.*, where specific proportions in the temporal structure were reflected in the proportions between categories of materials outside-time. In either situation, proportions in one aspect of the structure are reflected in another aspect. In general, mention of temporal proportions in analysis is restricted to those instances where they are evident in more than one aspect of a work's structure, for only then can it reasonably be assumed that the composer has employed them with the intention of reinforcing the conceptual, and possibly perceptual, unity of the work in question.[84]

Given that part 2 of *Herma* is shorter than part 1, it is not surprising that Xenakis has generally confined the shorter segments to part 2 and has placed the longer segments in part 1. The general difference between the durations of the segments in parts 1 and 2 contributes to the perceptibility of the principal structural division in the work. It may also be interpreted as the projection of the principle of acceleration evident in the introduction onto the large-scale structure of the work. Thus, the proportions of the parts and the durations of the segments contained within them tend to produce a dynamic drive toward the work's conclusion, despite the potentially static nature of its highly abstract pitch structure. In addition, the fact that Xenakis repeats several of the psets in part 2 suggests that he wishes to prepare the listener for the definitive arrival of the final pset F, which is equivalent to $(AB + \bar{A}\bar{B})C + \overline{(AB + \bar{A}\bar{B})}\bar{C}$, for this composite pset is foreshadowed three times through the superimposition of its components, beginning at the end of the sixth system of the flow chart in Figure 2.23. Despite the absence of traditional procedures for achieving continuity and goal-directedness (such as linearity and functional harmony) in this work, it appears that Xenakis wished to imitate at least some of the superficial aspects of these procedures through his use of a dynamic, i.e. asymmetrical, temporal structure, a logical progression of set-theoretic operations, repetition of important psets, and a judicious balance of sound and silence. As his compositional style developed, he would incorporate other devices in order to create the impression of

[84]On the repetition of temporal proportions on different levels of (inside-time) structure in twentieth-century music, see Kramer, *The Time of Music*, particularly chapters 9-11. See also Roy Howat, *Debussy in Proportion* (Cambridge: Cambridge University Press, 1983).

directed motion toward structural goals. Some of these devices will be discussed in the context of the analyses in chapters 3-5.

This introduction to large-scale structure and the articulation of form in Xenakis's music concludes with a brief examination of the way in which maximum, minimum, and intermediate states in the dimensions of segments may be used to create points of special emphasis within the global structure of a composition. The point of departure for this examination is Xenakis's account of the distribution of densities among the segments of *Achorripsis* for orchestra (1956-7), which is found in chapter 1 of *Formalized Music*.[85] A compositional plan for *Achorripsis* in the form of a matrix is shown in Figure 2.57. The matrix contains 196 cells, which is the product of its 7 rows and 28 columns. The rows represent categories of instrumental sounds[86] and the columns represent temporal segments, each of which has a duration of 15 seconds. Because its segments are equivalent in duration, the temporal structure of *Achorripsis* is much simpler than those of the other works that have been discussed so far. What is of special interest in this work, therefore, is not its temporal structure per se but the global distribution of densities and categories of instrumental sounds as the work unfolds in time.

Xenakis chose to arrange the densities and the categories of instrumental sounds within the cells of the matrix in accordance with a discrete probability distribution known as the Poisson distribution. This

[85]*Formalized Music*, pp. 22-32.

[86]The categories of instrumental sounds shown on the right side of the matrix are only partially accurate. The score makes clear that the composition of the categories is: 1) piccolo, Eb clarinet, bass clarinet; 2) oboe, bassoon, contrabassoon; 3) violin, 'cello, double bass (glissando); 4) xylophone, wood-block, bass drum; 5) violin, 'cello, double bass (pizzicato); 6) 2 trumpets, trombone; 7) violin, 'cello, double bass (arco).

distribution was chosen so that the states of these two dimensions could be governed by "the law for the appearances of rare random events."[87] The events referred to here are not individual sonic events but the characteristics of the temporal segments and the categories of instrumental sounds represented by the cells of the matrix. The formula for the Poisson distribution is used to calculate the probability of occurrence of events within the cells. The mean density of a single event is five sounds per measure, where each measure is equivalent to 26 MM, or 2.3". The densities of single and compound events within the cells are indicated by the circled numbers in Figure 2.57. The legend below the matrix shows the shadings that indicate the presence of multiple events within the cells. In cells where double, triple, or quadruple events are represented, the density increases accordingly as a multiple of the mean density of 5 sounds per measure. The process by which the precise densities in the matrix were derived is not explained by Xenakis.

The number of cells containing no events or a single, double, triple, or quadruple event is determined by the formula for the Poisson distribution,

$$P(\omega) = \frac{\lambda^{\omega} e^{-\lambda}}{\omega!}, \omega = 0, 1, 2, 3, \ldots,$$

in which the density parameter λ is arbitrarily given the value of 0.6 events per cell. The probabilities for each number of events ω when $\lambda = 0.6$ are

[87] *Formalized Music*, p. 25.

$$P(0) = 0.5488$$
$$P(1) = 0.3293$$
$$P(2) = 0.0988$$
$$P(3) = 0.0198$$
$$P(4) = 0.0030$$
$$P(5) = 0.0004$$

When multiplied by the number of cells, these probabilities yield the following frequencies:

ω	$196 \times P(\omega)$
0	107
1	65
2	19
3	4
4	1
5	0

Comparison with Figure 2.57 reveals that these are indeed the frequencies of events represented in the matrix. After the frequencies of events within the matrix as a whole were calculated, Xenakis used the Poisson distribution to calculate the frequencies of single, double, triple, and quadruple events individually in both rows and columns. In this way he hoped to achieve maximal variety of texture and timbre throughout the global structure of *Achorripsis*.

One result of these calculations, and the one most relevant to the present topic, is the succession of densities that occurs as each of the temporal segments is presented. The sum of the densities in each column of the matrix is shown graphically in Figure 2.58.[88] A dotted line in the figure connects successive peaks in density as the work progresses. The highest peak occurs

[88]In Figure 2.58 Arabic numerals replace the Greek numerals that appear at the head of each column in the matrix (Figure 2.57).

in segment 25, which is directly preceded by one of the two segments in which the density is zero.[89] The sharp contrast at this point focuses special attention on segment 25, whose maximal density has been prepared by the density peaks in previous segments and retains its special status as the density gradually subsides toward the work's conclusion. The virtual explosion of sounds—*Achorripsis* means "jets of sound"—so close to the end suggests analogies with climactic passages in other styles of music. Examination of other works by Xenakis suggests that, along with his interest in the formalization of compositional processes, he has remained sensitive to listeners' conventional expectations with regard to the overall formal shape of musical compositions. The means by which this shape is articulated differs according to the specifics of the compositional process and medium in which he is working. In the discussions of the solo piano works in chapter 3, several dimensions are noted to contribute to the overall formal shape. In the works for solo strings, however, which are discussed in chapter 4, the range of densities is quite limited, and therefore the activation of specific regions within p-space functions as the principal means for articulating overall formal shape.

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[89]In the finished composition, segments whose density is zero according to the matrix actually contain a very light "fill" by the percussion section. Other modifications in the finished work include slight overlaps between segments, but on the whole the densities and orchestration prescribed by the matrix are followed rather faithfully.

2.4 Conclusion

This chapter has provided an introduction to the structural elements that are basic to Xenakis's music. An attempt has been made to identify the elements that are normative within Xenakis's compositional practice and to give clear explanations of the terms and methods proper to that practice. A proper understanding of the analyses in chapters 3-5 presupposes a familiarity with the concepts presented here. If any of the material in those chapters seems unfamiliar, the reader may wish to refer back to the appropriate sections of this chapter or to the Glossary for clarification.

Chapter 3

Works for Piano Solo

In contrast to other composers of his generation, such as Boulez and Stockhausen, Xenakis did not choose the piano as the medium on which to realize his first distinctive musical ideas.[1] Instead, he turned first to strings (with or without percussion) in *Metastaseis* (1953-4), *Pithoprakta* (1955-6), *Analogique A* (1958), and *Syrmos* (1959), to orchestra (chamber or full) in *Achorripsis* (1956-7) and *Duel* (1959), and to electroacoustic composition in *Diamorphoses* (1957), *Concret PH* (1958), and *Analogique B* (1959), before producing his first work for piano, *Herma* (1960-1). Since then the piano has appeared in twenty or so of Xenakis's nearly one hundred fifty works. It has been used in various combinations: as a member of a small ensemble, as a solo instrument with orchestra, as accompaniment to a vocalist or other instrumentalist, and as a member of a chamber or full orchestra.[2] Only three more works for piano solo, however, were written after *Herma*: *Evryali*

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[1]Early works involving the piano by Boulez include *Notations* (1945), *Sonatine* for flute and piano (1946), *First Piano Sonata* (1946), *Second Piano Sonata* (1948), and *Structures I* for two pianos (1951-2), and by Stockhausen, *Kreuzspiel* (1951) and *Klavierstücke I-IV* (1952).

[2]Representative works in each category include *Morsima-Amorsima* for piano and strings (1956-62), *Eonta* for piano and brass quintet (1963), *Akanthos* for soprano, strings, winds, and piano (1977), *Akea* for piano and string quartet (1986) and *Plekto* for piano, percussion, and strings (1993); *Synaphaï* (1969), *Erikhthon* (1974) and *Keqrops* (1986), all for piano and orchestra; *Dikhthas* for violin and piano (1979), *Pour Maurice* for baritone and piano (1982), and *Paille in the wind* for violoncello and piano (1992); *Lichens* (1983), *Thalleïn* (1984), *Tracées* (1987), *Kyania* (1990), and *Mosaïques* (1993) for orchestra; and *Aïs* for baritone, percussion, and orchestra (1979).

(1973), *Mists* (1980), and *à r.* (1987). These three works are the subject of the analyses in this chapter.

Each of Xenakis's works for piano solo embodies aspects of a distinct phase in his compositional style. *Herma* is an example of what Xenakis calls "symbolic music," i.e. music based upon the principles of symbolic logic as manifested in set theory. Set-theoretic operations are performed upon psets in this work, and the results are realized uniformly in stochastic configurations. *Herma* was discussed in sections 2.2.1.9 and 2.3 of the previous chapter and has been written about elsewhere.[3] No further commentary on it, therefore, will be offered here. In contrast to *Herma*, *Evryali* presents a variety of configuration types, including arborescences, stochastic configurations, and complex time-point sequences. As in the works for piano and orchestra, *Synaphaï* (1969) and *Erikhthon* (1974), many of the arborescences in *Evryali* are complex and extensive. *Evryali* is also related to works for percussion, such as *Psappha* (1975), in which sieve theory has been applied to the formation of time-point sequences.[4] *Mists* contains random walks, arborescences, and stochastic configurations, of which some examples have been given in chapter 2. It also features transpositions of large m-psets constructed according to the principles of sieve theory. *à r.* also features

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[3]See Iannis Xenakis, *Formalized Music*, rev. ed. (Stuyvesant, NY: Pendragon Press, 1992), pp. 170-7; D. Sevrette, "Etude statistique sur 'Herma' de Xenakis," unpublished thesis (Schola Cantorum, Paris, 1973); Yayoi Uno and Roland Hübscher, "Temporal-Gestalt Segmentation: Polyphonic Extensions and Applications to Works by Boulez, Cage, Xenakis, Ligeti, and Babbitt," *Computers in Music Research* 5 (1995): 1-38.

[4]See Ellen Rennie Flint, "An Investigation of Real Time as Evidenced by the Structural and Formal Multiplicities in Iannis Xenakis' *Psappha*," (Ph.D. diss., University of Maryland at College Park, 1989) and E. R. Flint, "Metabolae, Arborescences and the Reconstruction of Time in Iannis Xenakis's *Psappha*," *Contemporary Music Review* 7 (1993): 221-48.

random walks and transposed m-psets along with simultaneities of relatively long duration.

The analysis of all three works follows the same general plan. Each analysis begins with a table of the work's segments, indicating their temporal position, density, configuration type, and intensity. Following the segment table is a graphic transcription of the score, in which each of the segments is labelled for reference. The analysis proper consists of several passes through the work, each focusing on a specific structural feature. First, the general characteristics of the configuration types are described and the evolution of each type is traced over the course of the work. Following the introduction of the configuration types, the formation of larger structural units from patterns in the succession of configuration types is discussed. The temporal relationships among collections of segments, both inside- and outside-time, is then considered, along with the role of pitch in the articulation of the work's large-scale structure. Finally, a summary of the analysis is presented, recalling the ways in which the structural features that have been described separately interact in the articulation of the work's form.

Evryali (1973)

The Greek title of this work signifies both "the open sea" and "Medusa."[5] The title is appropriate, for *Evryali* is certainly expansive in its gestures, and the twisting shapes of its arborescences, particularly in their graphic representation, might easily be associated with the serpentine tangles

[5]Harry Halbreich, trans. Elizabeth Buzzard, "Music for Strings and Piano" in the booklet accompanying *Iannis Xenakis: Musique de chambre 1955-1990* (Disques Montaigne 2 CD 782005): 29-40, p. 39.

of Medusa's hair. A list of the segments in *Evryali* is shown in Table 3.1 and a graphic transcription of the score in Figure 3.1. As Table 3.1 indicates, there are four configuration types in *Evryali.* The first configuration type to appear in the work results from the set-theoretic union of time-point sequences (tpseqs), each of which is assigned to a specific pitch within the configuration. The first configuration in the work, which is an example of this configuration type, has been discussed in section 2.1.7 in the previous chapter, where its time-point structure was examined in terms of sieve theory and set theory. Because of their formation according to the principles of sieve theory, configurations of this type will be referred to here as "time-point sieves" (abbreviated TPS). The second type of configuration introduced in *Evryali* is the stochastic type, abbreviated ST. Arborescences, abbreviated A, constitute the third configuration type. The fourth type is not a kind of configuration at all, but merely a measured silence, or rest, abbreviated R. Rests have been included in the list of configuration types because they constitute a type of musical material (or anti-material) that is used to define the duration of a segment of music. The rests in *Evryali* play an important role in articulating its temporal structure. It is proper, therefore, to consider them as significant structural elements in the work.

The rhythmic structure of *Evryali* is unique among Xenakis's works for solo piano, for it is the only one in which a single unit of duration is adopted throughout. The basic unit is the sixteenth note at ♩ = 60. All of the notated durations in the score are multiples of this unit, with the exception of the thirty-second notes that occur in segments 3-6 (mm. 16-35, 31.75-70") only.[6]

[6]The temporal locations of the segments are given in measures, as they appear in the score, and in seconds, as they appear in the graphic transcription of the score.

These sixteenth- and thirty-second note units are the basic elements in the TPSs that occur in twenty-three of the work's fifty segments. As the work unfolds in time, the nature of the TPS configurations evolves in specific stages. Two structural characteristics are especially important in this evolution. The first is the number of intersections among the tpseqs that make up a given configuration, and the second is the position of the configuration within p-space.

The relative presence or absence of intersections among the tpseqs that make up the TPS configurations can be interpreted in terms of relative degrees of order and disorder, which is a dichotomy to which Xenakis refers frequently with respect to his compositional aesthetics.[7] At the two extremes of this dichotomy are segments 8 (mm. 40-6, 78.125-91.25") and 46 (mm. 198-206, 409.625-426"), portions of which are shown in Figure 3.2. The pitches in segment 8 are members of pset {-7 -5 -3 1 3 6 10}, but no more than four of the seven elements in the set appear on any one time-point.[8] This is an example of apparent disorder created within the completely deterministic (i.e., non-stochastic) system of sieve theory. The impression of disorder is achieved by minimizing the number of intersections among the time-points assigned to the individual pitches within the pset, and thus maximizing the rhythmic variety within the configuration. The structural complexity of this configuration creates a sense of partially directed movement within p-space,

[7]References to this dichotomy are found, explicitly or implicitly, in nearly all of Xenakis's writings. See, in particular the Table (Mosaic) of Coherences in *Formalized Music*, p. viii, and the statement, "The mind of man should be able to travel back and forth constantly, with ease and elegance, through the fantastic wall, of disarray caused by irrationality, that separates determinacy from indeterminacy," (*Formalized Music*, pp. 237-8). This is part of a more extensive statement cited in chapter 1, note 40.

[8]The time-point on which four pitches are sounded occurs in m. 45, which is not shown in Figure 3.2.

which is evident visually in the graphic transcription of the score (Figure 3.1). This sense of motion contrasts with the impression of fixity in p-space given by the remaining TPSs (see, again, Figure 3.1). The qualities of relative disorder and partially directed motion that are characteristic of segment 8 are significant with respect to its location in the work, for it follows the stochastic configuration in segment 7 (mm. 36-40, 70-78.125") and precedes the graphically designed configuration in segment 9 (mm. 46-60, 91.25-119.875"), which is classified as an arborescence for reasons that will be explained later. Segment 8 acts as a link between segments 7 and 9 because it combines aspects of their distinguishing characteristics—relative disorder and directed motion through p-space, respectively—and thereby mediates between these two contrasting configurations.

Segment 46 (mm. 198-206, 409.625-426") could hardly be more different. From the last beat of m. 200 forward, all eight elements in its pset, {-6 -5 -2 1 2 3 5 9}, are sounded simultaneously. This is a maximal time-point intersection that represents a maximum degree of order, in contrast to the disorder represented by segment 8. The INT of the tpseq for this portion of segment 46 is shown in the figure. Like the segment as a whole, the INT for this portion demonstrates a progression from relative disorder to relative order. From the third beat of m. 204 to the fourth beat of m. 206, the tpseq consists entirely of repetitions of the simple iambic pattern <1 2>. In this way, the structural complexity of the tpseq prior to this point is liquidated into a simple pattern.

The reverse of this process occurs twice earlier in the work, in segments 13 (mm. 69-74, 147.125-158") and 15 (mm. 87-92, 183.75-192.5"). The configuration in segment 13 begins with the intersection of all six members of its pset {30 33 37 39 41 46} in a tpseq whose INT is <1 1 5 1 1 4>. This is followed by an intersection on four members of the pset in a tpseq whose INT repeats the pattern <1 1 4> two more times. Afterward the configuration demonstrates a much less regular pattern in the time-point intersections of its pitches. Segment 15 begins with the intersection of all eight members of its pset {0 1 4 5 20 23 26 27} in a tpseq whose INT repeats the pattern <1 1 2> five times before changing to a less regular pattern and eventually abandoning the maximal intersection of its pitch elements entirely. The only other configuration that features maximal time-point intersection for an extended period of time occurs in segment 19 (mm. 100-2, 208.25-212.125"), where maximal intersection is maintained throughout. Here, however, the INT of the tpseq is irregular: <2 1 2 1 1 2 2 2 1 2 2 3 3 2 1 3>.

All of the TPS configurations are characterized by a relatively small interval between their lowest and highest pitches within the total p-space available on the piano keyboard. This restriction in the amount of p-space occupied by them allows their relative positions to be perceived clearly as the work unfolds in time. The bands of p-space that are activated by the successive TPSs appear to have been chosen in order to articulate a specific long-range pattern over the course of the work. The configuration in segment 1 (mm. 1-4, 0-8") occupies a central position within p-space. This is followed by a succession of TPSs in segments 3-6 (mm. 16-35, 31.75-70", intermittently) that articulate a descending sequence of bands in p-space, roughly following the contour of the stochastic stream configuration in segment 2 (mm. 5-35, 8-70") upon which they are superimposed. These four TPSs effectively open up nearly the entire range of the p-space. Segment 8 (mm. 40-6, 78.125-91.25") activates the central position in p-space once again, and is followed by segments 10 (mm. 60-4, 119.875-128") and 13 (mm. 69-74, 147.125-158") which progressively ascend through p-space, segment 13 superseding the upper boundary previously established by segment 3. The configurations in segments 15, 17, 19, 21, and 22 (mm. 87-114, 183.75-237.825", intermittently), are scattered throughout various locations in p-space, but segment 22 once again extends the upper boundary, bringing it to pitch 48, the upper limit of the piano's range.

The succession of TPS configurations in segments 37-46 (mm. 190-206, 392-426") begins at the outer extremes of the p-space and works gradually inward, settling in the central position formerly occupied by segments 1 and 8. This succession of registrally and temporally interlocked configurations effectively reverses the process of gradual expansions of the p-space that has been taking place in the TPSs throughout most of the work and returns to the location in p-space originally associated with the TPS configurations at the beginning. This, in conjunction with the liquidation of rhythmic complexity that takes place in segment 46 (the final TPS in the work), suggests the possibility of structural closure with respect to this configuration type. The topic of closure in *Evryali* will be considered in greater detail below, but for now it is sufficient to regard it as a possibility with respect to the TPS configurations.

There are four stochastic configurations in *Evryali*. The first occurs in segment 2 (mm. 5-35, 8-70") and is followed directly by the second, in segment 7 (mm. 36-40, 70-78.125"). The configuration in segment 2 is a stochastic stream whose contour first ascends from the center of the p-space, beginning with six of the seven pitches in the pset from the TPS in segment 1. From the initial ascent in its contour the stream traces an irregular, but generally descending, path through p-space before ascending rapidly at the very end (m. 34, beat 4 through m. 35, 67.5-70"). The events in this configuration are distinguished in three ways from those in the TPS configurations that are superimposed upon it. First, the density of the stochastic configuration is considerably less than that of the TPSs (see Table 3.1). Second, changes in intensity occur once per event in the stochastic configuration, whereas intensity remains constant within the TPSs. These changes in intensity appear to have been generated stochastically. Although stochastic changes in intensity are mentioned as a possibility in *Formalized Music* and were applied in the works composed with "Free Stochastic Music," their application outside that group of works seems to be relatively rare.[9] Here, however, they serve as one factor that distinguishes two simultaneously occurring configuration types. Finally, the damper pedal is applied when the stochastic configuration appears alone, but is released when the TPSs are active. Thus, for the most part, the stochastic configuration is distinguished timbrally from the TPSs.

The configuration in segment 7 contains some of the characteristics found in segment 2, but in modified form. This configuration is a stochastic field whose density is relatively high and whose range of intensities is limited to *mf-ffff* (see Table 3.1). The pedal is sustained halfway down throughout this configuration, so that its timbre resembles that of the previous stochastic configuration. Due to the increase in density and to the restriction of time-points to sixteenth-note units (i.e., without the grace-notes and arpeggiations found in segment 2), the rhythmic pattern in this configuration is similar to that found in the TPSs. The configuration in segment 7 acts as a fitting transition, therefore, between the more characteristically stochastic rhythm of the configuration in segment 2 and that of the TPS in segment 8.

The other two stochastic configurations also appear as a consecutive pair, in segments 23 (mm. 136-46, 280.125-302") and 24 (mm. 147-79, 302-

[9]See *Formalized Music*, pp. 13 and 141-3.

366.75"). The configuration in segment 17 is a stochastic field whose registral boundary extends gradually from the center of the p-space, starting on pitch 4 (E4), toward the upper and lower limits of the piano's range. The rhythm of this configuration is non-stochastic, consisting entirely of steady sixteenth-notes, and its changes in intensity are gradual, from *p* at the beginning to *ffff* in mm. 142-3 (282-4"), and from there down to *ppp* by the end. This configuration acts as a transition between the extremely complex and lengthy arborescence that precedes it (segment 20, mm. 102-36, 212.125-280.125") and the lengthy stochastic configuration that follows it. The configuration in segment 24 is also a stochastic field, but one whose density is extremely low. Like the configuration in segment 2, it has configurations of another type superimposed upon it, in this case arborescences. The stochastic configuration and arborescences are differentiated primarily in terms of their density, for the pedal is sustained halfway down through most of segment 24. Changes in intensity occur in this configuration, but less frequently than they do in segment 2.

The grouping of the stochastic configurations into consecutive pairs and the strong similarity in terms of duration and in the superimposition of non-stochastic configurations in segments 2 and 24 creates a significant correspondence between the areas of the work occupied by each pair. This correspondence, in fact, articulates a major division within the temporal structure, as will be demonstrated below.

Evryali contains twenty arborescences that vary considerably in their morphological characteristics. Common to them all, however, is a rapid traversal of p-space. Although the majority of them exhibit the branching structures that give this category of configurations its name, some of the configurations that are labeled arborescences in this analysis do not show clear evidence of branching. They are included in this category, however, because they traverse the p-space in a manner that is similar to the branching-type arborescences but unlike any of the configurations found in the TPS or stochastic categories. The first two "arborescences," for example, in segments 9 (mm. 46-60, 91.25-119.875") and 12 (mm. 66-9, 140-147.125"), do not show any branching features. The configuration in segment 9 is notable for its quasi-symmetrical structure, which vaguely resembles a large "W" traced through p-space. At the center of this large W a replica of the larger design is found. This configuration makes several rapid traversals through p-space, the first of which spans the entire range of the piano keyboard. It is unlike anything heard previously in the work for, although the stochastic configuration in segment 7 also covered a large area of p-space in a short period of time, the sense of direction in the stochastic configuration was not as clearly articulated as it is in segment 9. The configuration in segment 12 appears as a slightly modified restatement of the initial descending-ascending arc of the configuration in segment 9.

The directed paths through p-space traced by the configurations in segments 9 and 12 prepare the way for the complex, branching arborescence in segment 14 (mm. 75-87, 158-183.75"). This configuration ends with a rapid movement from the near-extremes of the p-space toward its center. A similar movement toward the center appears at the end of the arborescence in segment 20 (mm. 102-136, 212.125-280.125"), which is the lengthiest and perhaps most spectacular configuration in the entire work. The association of movement toward the center of the p-space with structural closure is appropriate in segment 20, for the end of this configuration coincides with the major division in the temporal structure noted previously in connection with the start of the stochastic configuration in segment 23.

Between segments 14 and 20, two more arborescences are presented, in segments 16 (mm. 92-5, 192.5-198.25") and 18 (mm. 97-100, 202.625-208.25"). These two configurations, which are confined to the upper portion of the p-space, are each followed by relatively brief TPSs. The rapid alternation of these contrasting configuration types, in conjunction with the similar morphology of the two arborescences, results in a passage of music that recalls the function of sequences or stretti in tonal composition, and effectively gathers energy in preparation for the long arborescence that follows. In addition, a degree of synthesis among configuration types is achieved in segments 20-2, for the TPS configurations, which have been alternating with arborescences since segment 12—and even, if the rest in segment 11 is overlooked, since as far back as segment 8—are superimposed upon the arborescence in segment 20.

The process of a large arborescence "growing" out of a succession of smaller arborescences that precede it, evident in segments 16, 18 and 20, is repeated in segments 25-35 (mm. 148-88, 305.625-385.125"). The arborescences in segments 25-34 are superimposed upon the stochastic configuration in segment 24 (mm. 147-79, 302-385.125"). The first seven arborescences, in segments 25-31, are brief and are scattered freely throughout p-space. Their intensity is uniformly *pp*. The next three arborescences, in segments 32-4, are more substantial. They are lengthier, occupy greater portions of the p-space—at both its upper and lower extremes—and have an intensity of *mf* (see Table 3.1). The succession of arborescences culminates in segment 35, in which a large arborescence extends across the entire p-space. This arborescence also summarizes and supersedes the previous ones in terms of intensity, which increases gradually from *pp* to *ffff* as the configuration progresses.

Three more arborescences appear near the conclusion of the work. Following the succession of TPSs in segments 37-46 (mm. 190-206, 409.625-426"), in which interlocking configurations moved progressively from the extremes of the p-space toward its center, the arborescence in segment 47 (m. 207, 426-432.625")[10] moves in a quasi-symmetrical pattern from the center of the p-space out towards its extremes once again. The main branches of this arborescence are connected by pitch proximity to the preceding TPS. The arborescence in segment 48 (mm. 207-12, 432.625-451") is connected in similar fashion to the one in segment 47. Although segment 48 could be regarded as a continuation of segment 47, significant changes in density, intensity, and pedal at this point indicate the start of a new segment. Like the arborescence in segment 47, however, the one in segment 48 is quasi-symmetrical around the center of the p-space. It first wavers back and forth around the center and finally moves out toward both extremes of the p-space. After a ten-second silence in segment 49 (m. 212, 451-461"), a final arborescence is presented in segment 50 (m. 213, 461-471"). The concluding function of this arborescence is reinforced by its presentation at a slower tempo, which is marked "Plus lent" in the score without any change in metronomic indication.[11] The morphology of this arborescence is unusual, since it is uncharacteristically confined to a clearly demarcated band within p-space. In this respect it occupies a position within p-space in a manner similar to the TPS configurations. This, along with the fact that it is located in a position similar

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[10]The lengths of the measures change at this point without there being any explicit change in the meter. See the note to this effect at * on the second page of Table 3.1.

[11]The duration of segment 50 in the graphic transcription, therefore, is only approximate. In order to represent the change in tempo visually, the duration of the sixteenth-note unit has been increased from 0.125" to 0.167".

to that of the final TPS (in segment 46), suggests that the configuration in segment 50 may represent a partial synthesis of characteristics belonging to the TPS and arborescence configuration types. This synthesis is different from the one that occurred in segments 20-2 (see above), in which two TPS configurations were merely superimposed upon an arborescence. A true blending of characteristics occurs here, which is suggestive of a process of synthesis that has reached its conclusion.[12] Further reinforcing the impression of conclusion at this point is the structural redundancy that results from the repeated motions toward and away from the center of the p-space in segments 37-48. Repetition, symmetry, and synthesis are all factors that contrast with the avoidance of repetition, the asymmetry, and the differentiation that characterize the music prior to segment 37. Taken together, and considering their function within the context of the structural characteristics established earlier in the work, all of these factors seem to point toward a contextually defined sense of closure that is effective not only for this section of the work, but for the work as a whole.

Nothing has been mentioned so far about the rhythmic characteristics of the arborescences in *Evryali*, but in this aspect as well a large-scale process is involved that moves from differentiation toward a relative lack of differentiation. The first arborescence, in segment 9 (mm. 46-60, 91.25-119.875"), has a tpseq whose INT divides into smaller tpseqs of period eleven,

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[12]In his "Opening Address" at the International Computer Music Conference at Northwestern University in 1978 (*Proceedings of the 1978 International Computer Music Conference* [copyright 1979, Computer Music Journal], v. 1: 2-4), Xenakis refers to contrasting musical structures in both traditional instrumental music and electroacoustic composition as "entities" and "anti-entities." If entities and anti-entities are interpreted as referring to contrasting configuration types in his instrumental music, the following comment would appear to support the notion of synthesis among contrasting types proposed above: "The identical reproduction of the entity is outlawed in general. On the other hand, the entity modified more or less strongly (until it reaches the anti-entity) becomes indispensable," p. 3.

as shown in Figure 3.3. The duration of eleven sixteenth-notes is divided into slightly different patterns of time-point intervals, but each of these patterns has some features in common. The first INT marked in the figure is <2 1 1 2 2 1 1 1>, which is followed by two INTs <2 1 1 2 1 2 1 1> and then by several that are either <2 1 1 2 3 1 1> or <2 1 1 1 1 3 1 1>, up until m. 57, at which point the pattern changes to <1 1 1 1 1 1 3 2> after a transitional pattern of <2 1 1 2 3 2>, which occurs only once. The final tpseq, <1 1 2 1 1 3>, is incomplete, having a duration of only nine sixteenth-notes. Most of the patterns begin with <2 1 1 2> after which there are variations involving either order permutations of the remaining intervals, the fusion of smaller intervals into larger ones, or the subdivision of interval 2 into <1 1>.

Periodic tpseqs are unusual in this work, as mentioned previously with respect to the TPS configurations. What is even more unusual in the configuration in segment 9 is the close structural resemblance between its tpseq and its pset. The pset in this configuration is

> {-38 -36 -35 -34 -32 -31 -30 -29 -27 -25 -24 -23 -21 -19 -18 -16 -14 -13 -12 -10 -8 -7 -5 -3 -2 -1 1 3 4 5 6 8 9 10 12 13 14 15 17 18 19 20 21 23 25 26 28 30 31 32 34 36 37 39 41 42 43 45 47 48}.

The spacing of this tpseq, subdivided into periods of pitch-interval size 11, is

> <2 1 1 2 1 1 1 2> <2 1 1 2 2 1 2> <2 1 1 2 2 1 2> <2 1 1 2 2 1 1 1> <2 1 1 2 1 1 1 2> <1 1 1 1 2 2 1 2> <2 1 1 2 2 1 2> <2 1 1 2 2 1>.

The spacing pattern that occurs with the greatest frequency is <2 1 1 2 2 1 2>, which bears a close resemblance to patterns in the INTs of the tpseqs in this segment such as <2 1 1 2 2 1 1 1> and <2 1 1 2 3 1 1>. As in the INTs of the tpseqs, the spacing patterns occur in several more-or-less closely related variants, and the final spacing is incomplete, i.e. it does not fill out a complete period of eleven semitones. The periodicity of the subsets that make up the large pset is evident on the surface of the music, where pitch-interval 11 occurs frequently.

As more arborescences are introduced, their rhythmic definition begins to dissolve into a steady pulse of sixteenth-note units. Rhythmic definition may still be found in segment 12 (mm. 66-9, 140-147.125") and in isolated portions of segments 14 (mm. 75-87, 158-183.75") and 20 (mm. 102-36, 212.125-280.125"). In segment 20 the areas of rhythmic definition are reinforced by the introduction of an intensity of *ff* in contrast to the basic intensity of *pp*, and by full depression of the pedal, in contrast to the half-pedal that is indicated elsewhere in the configuration. The rhythm in the remaining arborescences consists almost entirely of a steady sixteenth-note pulse. No further structural correspondences between tpseq and pset structure exist in these arborescences, unless one chooses to associate the successions of time-point interval 1 with pitch-interval 1, for all of the arborescences after segment 12 make use of the undifferentiated chromatic p-space. The pset of the configuration in segment 12, like that of the configuration in segment 9, is periodic. Its component subsets have a period of thirteen semitones, which is manifested on the surface of the music by frequent occurrences of pitch-interval 13. There is no discernable correspondence, however, between tpseq and pset structure in this arborescence.

Three of the segments in *Evryali* are occupied by measured silences of considerable duration, which are labelled "rests" (R) in Table 3.1. The first rest occurs in segment 11 (m. 65, 128-140"). The score indicates that this measure should consist of approximately 12 seconds of silence. In the graphic table of segments and in the graphic transcription (Figure 3.1), this duration is interpreted as 12 seconds exactly. A similar rest occurs in segment 49 (m. 212, 451-461"), but its duration is only 10 seconds (again, approximately). An even shorter rest occurs in segment 36 (m. 189, 386-392"). Approximately 6 seconds of silence are called for here, but the "silence" in this segment is colored by the resonance from the arborescence in segment 35 since the pedal that was depressed during the arborescence is held throughout the rest. The arborescence ends on the third beat of m. 188, after which the remainder of the measure is filled in with conventional rests. In the segmentation of *Evryali*, the remainder of m. 188 has been added to the 6-second rest in m. 189 so that the location of segment 36 is given as mm. 188-9 (385.125-392"). All three rests help to articulate sectional divisions in the work's temporal structure, which will be discussed presently.

Three principal factors will be considered in determining the boundaries of structural units larger than the segment in *Evryali*. The first factor is the formation of patterns in the succession of configuration types. As discussed in section 2.1.5 in the previous chapter, the existence of contrasting configuration types in a work allows Xenakis to generate large structural units based on the perceptible difference between successions of configurations in which contrasting types alternate and those in which similar types succeed one another directly. The second factor considered in determining the boundaries of large structural units is the location of unusual structural features, such as configurations whose morphology is especially striking or segments, such as those containing rests, that have the effect of interrupting a sense of forward motion in the music. The third factor is the coherence of the temporal structure that results from consideration of the first two factors. As indicated in sections 2.1.5 and 2.3, relatively simple systems of proportions tend to be articulated by the durations of large structural units in Xenakis's music. In addition, there may also be direct correspondences between the proportions found in a work's inside-time structure and those found in its outside-time structure. The proportions of a work's temporal structure outside-time are determined by summing the durations of segments containing configurations of a particular type, without regard for the order of their succession or their locations within the temporal structure inside-time.

Based on these general criteria, a major structural division of the work into two parts may be observed between the end of segment 20 (mm. 102-36, 212.125-280.125") and the beginning of segment 23 (mm. 136-46, 280.125-302"). This is the point at which the lengthiest and most complex arborescence draws to a close at the center of the p-space, on pitch 4 (E4). This same pitch is the origin of the stochastic configuration in segment 23. When the work is divided in this way, it can be seen from Table 3.1 that a pair of consecutive stochastic configurations appears at or near the beginning of each part. Part 1 begins with a TPS, but a pair of stochastic configurations follows immediately in segments 2 (mm. 5-35, 31.75-36") and 7 (mm. 36-40, 70-78.125"). (These stochastic configurations are consecutive, despite the numbering system, since the TPSs in segments 3-6 [mm. 16-35, 65.75-70"] are superimposed upon the configuration in segment 2.) Part 2 begins with consecutive stochastic configurations in segments 23 (mm. 136-46, 280.125-302") and 24 (mm. 147-79, 302-366.75"). The stochastic configuration in segment 24, like the one in segment 2, has non-stochastic configurations superimposed upon it. (The configurations superimposed upon segment 24 are arborescences as indicated above.)

With these pairs of stochastic configurations as general indicators of the beginnings of the parts, clear differences may be observed in what occurs subsequently within each part. In part 1, beginning with segment 8 (mm. 40-

6, 78.125-91.25"), and overlooking for the moment the rest in segment 11 (m. 65, 128-140"), an alternating pattern of TPSs and arborescences is established that continues until the arborescence in segment 20 (mm. 102-36, 212.125-280.125"). Two TPSs are superimposed upon this arborescence, which concludes part 1, recalling the similar superimposition that took place near the beginning of the part, in segments 2-6 (mm. 5-35, 8-70"). In part 2, the arborescences and TPSs are grouped together by type, instead of alternating as they did in part 1. The arborescence in segment 35 (mm. 179-88, 366.75-385.125") develops out of the succession of ten arborescences in segments 25-34 (mm. 148-71, 305.625-352"), which are superimposed upon the stochastic configuration in segment 24 (mm. 147-79, 302-366.75"). The rest in segment 36 (mm. 188-9, 385.125-392"), which is filled in with resonance from the arborescence in the previous segment, separates the succession of arborescences from the succession of ten TPSs that follows in segments 37-46 (mm. 190-206, 392-426"). A shorter succession of arborescences, interrupted by a rest, follows in segments 47-8 (mm. 207-12, 432.625-451") and 50 (m. 213, 461-471"). The contrasting TPS and arborescence types are reconciled somewhat in the final configuration, which presents a synthesis of some of their characteristics, as mentioned previously. The change in the presentation of configuration types between the two parts, from the alternation of contrasting types to the grouping together of similar types, is consistent with the general progression from a state of differentiation to one of relative non-differentiation in the rhythmic and pitch structures of the configurations, and from a state of relative irregularity to one of relative regularity in the location of configurations in p-space, both of which occur as the work unfolds in time.

The division into parts partitions the total duration of *Evryali* as shown in the table below. The work's total duration, according to the metrical structure indicated in the score, is 471".[13] Actual performance times are likely to differ from this total but, so long as the performer maintains a steady tempo, the proportions indicated in the table should be represented fairly accurately in performance.

part:	1	2
duration (sec)	280.125	190.875
proportion of total duration:	0.6	0.4

Put simply, part 1 occupies $\frac{3}{5}$ of the total duration and part 2 occupies $\frac{2}{5}$ of it. The relation between the parts may thus be expressed by the simple proportion 3:2.

A similar division of the total duration results when the durations of segments containing configurations of a given type are grouped together outside-time. The total duration of *Evryali* outside-time, determined by summing the durations of all fifty segments in Table 3.1, is 553.825". The table below shows the proportion of the total duration occupied by segments containing configurations of all three types, plus rests.

configuration type:	A	ST	TPS	R
duration (sec):	233.625	156.750	134.575	28.875
proportion of total duration:	0.40	0.30	0.25	0.05

[13]The durations in Table 3.1 are based upon a literal interpretation of the tempo indication of "♩ = 60 MM approx.," which appears at the head of the score, as well as of the lengths of the measured silences (rests). At the indicated tempo, a sixteenth-note is equivalent to 0.125". As mentioned previously, the duration of the sixteenth-note unit is extended to 0.167" in the final segment, whose tempo is marked "Plus lent." The total duration given here, 471", is equivalent to 7'51". Claude Helffer, in his recording of *Evryali* on Montaigne 2CD 782005, has a performance time of 9'58".

Outside-time, approximately $\frac{2}{5}$ of the work is occupied by arborescences, and the remaining $\frac{3}{5}$ is occupied by stochastic configurations, TPSs, and rests combined.

A summary of the temporal structure (inside-time) of *Evryali* is shown in Table 3.2. Part 1 is broken down into supersections and sections, with one of the sections broken down further into subsections. Part 2 is broken down into sections only. Part 1 divides into two supersections between the rest in segment 11 (m. 65, 128-140") and the arborescence in segment 12 (mm. 66-9, 140-147.125"). The durations of these supersections are 140" and 140.125" and their proportions in relation to the work's total duration are 0.3 and 0.3, respectively.[14] These proportions correspond exactly to those of the stochastic configurations and of the TPSs and rests combined in relation to the total outside-time duration, as shown in the table above. Thus, as in the works discussed in sections 2.1.5 and 2.3 of the previous chapter, a close correspondence is established between general features of the concrete, inside-time temporal structure and the abstract, outside-time temporal structure of *Evryali* on the basis of proportions that are common to both.

The temporal structure within part 1 is divided neatly into halves at the level of supersections A and B, which divide in half again into sections 1-4, as shown in Table 3.2. As previously mentioned, the rest in segment 11 (m. 65, 128-140") is the structural feature that articulates the division of part 1 into supersections. The division into sections is less obvious perceptually, but it is notable that both sections 1 and 4 contain lengthy segments (i.e., segments 2 [mm. 5-35, 8-70"] and 20 [mm. 102-36, 212.125-280.125"]) upon which TPSs are superimposed, while sections 2 and 3 contain successions of relatively brief

[14]Each is 0.5 times the 0.6 of the work's total duration occupied by part 1.

segments. Section 3 divides further into subsections a and b, whose boundary is marked by the rapid motion toward the center of the p-space at the end of the arborescence in segment 14 (mm. 75-87, 159-183.75"). This gesture is repeated at the end of the arborescence in segment 20 (mm. 102-36, 212.125-280.125"), which concludes part 1. The proportions of 0.61 and 0.39 that occur within section 3 as a result of its subdivision into subsections a and b replicate in close approximation the proportions of 0.6 and 0.4 by which the work divides into parts 1 and 2. The fact that both subsection a and part 1 close with a similar gesture suggests that motion toward the center of the p-space may be used as a signal to indicate closure on more than one level of structure.

The proportions of 0.6 and 0.4 are also replicated in close approximation within part 2, which divides into sections 5 and 6 according to the proportions 0.59 and 0.41. The boundary between the two sections is articulated by the rest in segment 36 (mm. 188-9, 385.125-392"), which immediately precedes the succession of interlocking TPSs in segments 37-46 (mm. 190-206, 392-426"). Within section 6, these TPSs lead directly into the concluding succession of arborescences in segments 47-8 (mm. 207-12, 426-451") and 50 (m. 213, 461-471"), which is interrupted by the rest in segment 49 (m. 212, 451-461"). Division of the section at the rest results in unequal subsections whose durations are 69" (segments 37-49) and 10" (segment 50), occupying 0.87 and 0.13 of section 6, respectively. This subdivision is not shown in Table 3.2 because it does not reflect the proportions of 3:2 (0.6 and 0.4) and 1:1 (0.5 and 0.5) found elsewhere in the work. Within the context of this section, the rest seems to function less as a point of division in the temporal structure and more as an element in a general dissipation of energy that takes place as the work reaches its conclusion. The sense of dissipation suggested by the rest is confirmed unmistakably by the slackening of the tempo in segment 50. In this respect, as well as in the partial synthesis that takes place between the TPS and arborescence configuration types and the quasi-symmetrical orientation of its component segments' gestures in p-space, section 6 as a whole functions as a fitting conclusion to the work.

The preceding comments have demonstrated that the succession of configuration types and the durations of the segments in *Evryali* have been coordinated in such a way that they articulate a large-scale temporal structure that is organized around a few basic proportional schemes. In addition, the durations of the individual segments seem to have been chosen with a specific global structure in mind. As in *Herma*, both the durations of the segments outside-time and the time-point intervals between the initiations of the segments inside-time in *Evryali* appear to have been calculated according to the exponential distribution.[15] In order to test for the possible use of the exponential distribution in determining the durations of segments outside-time, it is necessary to calculate the mean number of segments per second in the work as a whole. Dividing the total number of segments by their total duration outside-time in *Evryali* gives a mean density of 0.09 segments/second (i.e., 50 segments/553.825"). Using this figure as the density parameter for an exponential distribution, it is possible to calculate the expected probabilities of segments with duration x in the work. The table below shows the expected probability of segments whose durations fall within the specified ranges of values for x and the observed relative frequency of segments in *Evryali* whose durations fall within the same ranges.

[15]The evidence for this in *Herma* was discussed in chapter 2, section 2.3. The exponential distribution was introduced in section 2.2.1.5.

duration (sec)	probability (∂= 0.09)	frequency
$0 \leq x < 10$	0.593	0.680
$10 \leq x < 20$	0.241	0.200
$20 \leq x < 30$	0.098	0.060
$30 \leq x < 40$	0.040	0.000
$40 \leq x < 50$	0.016	0.000
$50 \leq x < 60$	0.007	0.000
$60 \leq x < 70$	0.003	0.060
$70 \leq x < 80$	0.001	0.000
$80 \leq x < 90$	0.000	0.000

Dividing the number of segments by the total duration inside-time gives the mean number of time-point intervals between the starting points of the segments per second. In *Evryali,* this number is 50 segments/471" = 0.11 segments/sec. Taking this figure as the density parameter in an ideal exponential distribution yields the expected probability of time-point intervals whose durations fall within specified ranges of values for x. These probabilities are shown in the table below, along with the observed relative frequency of time-point intervals in the specified ranges.

duration (sec)	probability (∂= 0.11)	frequency
$0 \leq x < 10$	0.667	0.660
$10 \leq x < 20$	0.222	0.220
$20 \leq x < 30$	0.074	0.100
$30 \leq x < 40$	0.025	0.000
$40 \leq x < 50$	0.008	0.020
$50 \leq x < 60$	0.003	0.000
$60 \leq x < 70$	0.001	0.000
$70 \leq x < 80$	0.000	0.000
$80 \leq x < 90$	0.000	0.000

Comparative histograms based on the values in both tables above are shown in Figure 3.4. The histograms show that the observed frequencies are reasonably close to the expected probabilities, both outside- and inside-time.

A comparison of Figure 3.4 with the histograms for *Herma* in Figures 2.55 and 2.56 reveals a close resemblance in the distribution of segment durations and time-point intervals in both works. This resemblance extends even to the value of the density parameters of the distributions in each work. From the examination of the temporal structure of these two works, as well as that of other works yet to be discussed, it appears that one of Xenakis's methods for determining the durations of segments outside-time and their positions inside-time involves the use of the exponential distribution. Although he describes applications of the exponential distribution to the determination of time-point intervals within segments in his stochastic music and, in the case of music composed with "Free Stochastic Music," to the determination of segment durations as well,[16] he has never alluded in his writings to its application to the temporal structure of works that include non-stochastic configurations. In stochastic composition, intervals calculated according to the exponential distribution are placed in random succession but in the temporal structure of works such as *Herma* and *Evryali*, the intervals are ordered at the composer's discretion, similarly to the way that they were in early, pre-computer-assisted stochastic composition. In early stochastic composition, correlation formulas were applied to the successions of intervals in order to test their resemblance to genuine randomness.[17] In applications to non-stochastic temporal structures, however, intervals calculated according to the exponential distribution are apparently arranged so as to fit in with the proportional organization of the large-scale temporal structure, which seems to belong to a prior stage of the compositional process.

[16] See *Formalized Music*, chapters 1 and 5.

[17] See *Formalized Music*, pp. 36-7.

Whether it is applied in stochastic or non-stochastic contexts, Xenakis's application of the exponential distribution to various levels of the temporal structure in his music reveals an interest in statistical homogeneity that is one of the guiding principles of stochastic music. The notion of statistical homogeneity appears to have remained important to Xenakis both as an aesthetic ideal and as a practical aid to composition, despite the changes in style and technique that his music has undergone since stochastic music was first introduced in 1955.

A different perspective on the global structure of *Evryali* may be obtained by examining the succession of maximum, minimum, and intermediate states within its segments. In order that the succession of states may correspond more closely to the listener's experience of the work as it unfolds in time, its fifty segments will be gathered into a smaller number of segment groups. In this way it will be possible to combine the dimensional states of several of the smaller segments with those of the longer segments upon which they are superimposed. In addition, the succession of interlocking TPS configurations in segments 37-46 (mm. 190-206) will be gathered together into a single group. The contents of the twenty-four segment groups are shown in Table 3.3.

As in the examination of densities in the temporal segments of *Achorripsis* in chapter 2, section 2.3, only the temporal succession of states in the segment groups, and not their precise locations within the temporal structure, will be considered here. Four dimensions are included in the examination of states within segment groups in *Evryali*: duration, density, intensity, and the size of the interval between the lowest and highest pitches. These dimensions are treated as independent phenomena for the sake of simplicity in analysis, but it should not be forgotten that they are clearly interdependent in the music as it is experienced by the listener. For example, the actual loudness of a collection of sonic events marked *ff* in the score varies according to several factors, including the instrument it is performed on, the performer's physical strength, the acoustics of the room in which it is performed, the location of the events in p-space, and the density of the configuration in which the collection occurs. For present purposes, however, it will be assumed that a configuration that is marked *ff* throughout will sound slightly louder than one that is marked *f* throughout, more than slightly louder than one that is marked with a crescendo from *pp* to *ff*, and significantly louder than one that is marked *pp* throughout. The measurements that are considered here, therefore, are only intended to indicate the relative strength or weakness of states in different dimensions within and among the segment groups.

Based upon the information in the score to *Evryali*, the maximum and minimum values in each of the four dimensions considered are:[18]

attribute:	duration	density	registral span	intensity
dimension:	d-space	∂-space	r-space	i-space
unit:	second	sounds/second	semitone	level (*ppp*, *pp*, ...)
minimum:	0	0	0	0 (silence)
maximum:	68	43.36	87	8 (*ffff*)

Values are assigned to the states in all four dimensions for each of the segment groups as follows. The value of the state in one dimension, expressed in units appropriate to that dimension, is first divided by the maximum value in that dimension. It is then multiplied by a normalization

[18]The attributes of collections of sonic events and the dimensions with which they are associated are discussed in chapter 2, section 2.1.1.4.

constant. The value of the state of the segment group in each of the other dimensions is obtained in the same way and the values of the states in all four dimensions are summed. The purpose of this simple model is to trace the relative strength of the states in several dimensions at once so that the segment groups may be compared with one another on the basis of a single value. Since four dimensions are considered here, and since all of the dimensions are weighted equally, a normalization constant of 2.5 will allow the value of the multi-dimensional scale (MDS) used in this model to vary within a range from 0 to 10.[19] A rest of brief duration would have an MDS value near 0, whereas a segment group that is simultaneously the longest, most dense, widest (in p-space), and loudest would have an MDS value of 10.

The MDS value of segment group 1 will be calculated as an example. The duration of segment group 1 is 8". This value, divided by the maximum value of 68", is 0.118. The density of segment group 1 is 11.63 sounds/sec. This value, divided by the maximum density of 43.36, is 0.268. The highest pitch in segment group 1 is 9 (A4) and the lowest pitch is 0 (C4), so its registral span is 9. This value, divided by the maximum value of 87, is 0.103. The intensity of segment group 1 is *ffff* throughout, which is equivalent to the maximum value of 8. The maximum divided by itself, of course, is 1. Multiplying each of these values by the normalization constant yields values

[19]The values of the states in different dimensions are weighted individually in segmentation algorithms, such as those proposed in James Tenney and Larry Polansky, "Temporal Gestalt Perception in Music," *Journal of Music Theory* 24/2 (1980): 205-42, and in Uno and Hübscher, "Temporal-Gestalt Segmentation." The weights are assigned empirically, based on intuitions regarding the general criteria for the segmentation of a given work. The weights vary, therefore, from work to work and, especially, from composer to composer. Equal weighting is used in the MDS model because no objective criteria could be determined by which to differentiate the relative perceptual importance of the four dimensions considered. The MDS model is intended to represent the state in several dimensions of each segment group, whereas Tenney and Polansky's model measures differences between states in several dimensions in order to determine points of segmentation.

of 0.295, 0.67, 0.258, and 2.5. The sum of these four values is 3.723, which is the MDS value for segment group 1. In groups in which brief segments are superimposed over a longer segment, the addition of the brief segments will cause the density of the group to be higher than the density of the longer segment alone. The mean density and registral span may increase as well, but the duration will not.

A graph of the MDS values for each of the segment groups is shown in Figure 3.5. The mean value of 4.66 is represented by the line that runs across the graph horizontally. A dashed line connects the peak values that occur in segment groups 2, 16, and 21. Segment group 2, which has an MDS value of 6.447, is a local maximum. It contains segments 2-6, in which four TPS configurations are superimposed over a long arborescence. The conclusion of this segment group coincides with the division between sections 1 and 2 in the temporal structure. Segment group 16, which has a value of 8.413, supersedes the local maximum in segment group 2 and is the maximum for the whole work. This group contains the lengthy and complex arborescence in segment 20 and the TPSs in segments 21-2 that are superimposed upon it. Its conclusion coincides with the division between parts 1 and 2 in the temporal structure. Segment group 21, which has a value of 7.809, has the second highest MDS value in the work. This segment contains the succession of interlocking TPSs, in segments 37-46, which brings that configuration type to its conclusion within the work as a whole. Its beginning coincides with the division between sections 5 and 6 in the temporal structure. The significance of this point in the structure is reinforced by the fact that the proportions between the durations of sections 5 and 6 within part 2 replicate those between parts 1 and 2 within the work as whole. (See Table 3.2.) The positions of these peaks in the MDS model suggest that the distribution of values representing the states in several dimensions within the segment groups has been coordinated with the work's temporal structure. On the basis of this observation, it appears that the positioning of relative maxima in the distribution of these values is one of the factors that helps to articulate the work's temporal structure as it unfolds in time.

The final aspect of *Evryali* that will be examined here is its pitch structure. As mentioned previously, correspondences between adjacent segments may be made on the basis of common pitches. A correspondence of this type occurs between segments 1 and 2 (over the barline between mm. 4 and 5), where six of the seven pitches in the pset of segment 1 appear in the first measure of segment 2. The pset of segment 1 is {0 2 3 4 5 7 9} and the set of pitches that occurs in the first measure of segment 2 is {0 2 3 5 7 9 23 35}. Further analysis of the pitch structure of segment 2 shows that it contains all of the pitches found in segment 1 plus several others and that the spacing of its pset conforms nearly, but not completely, to a regular pattern whose period is 20.

These preliminary observations point out two important characteristics of pitch structure in Xenakis's instrumental music. The first characteristic is that, where large psets are used in a composition, their contents are not always represented in full on the surface of the music. A large pset is often represented partially by its subsets, while explicit presentations of the entire pset are relatively rare. Examination of the spacings of psets allows them to be compared on the basis of their interval structure. A small pset whose spacing is included within that of a larger pset is a literal subset of the larger pset if all of its pitches intersect with pitches found in the larger pset. If only a partial intersection with the larger pset is found, or if there is no intersection at all, the smaller pset is a subset of a transposition of the larger pset.[20] The second important characteristic of pitch structure in Xenakis's instrumental music is the relatively frequent presence of pitches that do not fit within a consistently definable pset or m-pset structure. These "stray" or "rogue" pitches frustrate attempts to define perfect inclusion or transpositional relations among psets, and yet their existence does not totally obscure the general structure into which the remaining pitches fit quite easily.

The reason for the appearance of stray elements in the pitch structure of Xenakis's music is not entirely clear. Possible explanations include: 1) accidental errors in the transcription of pset elements from numerical to musical notation that bypass the editing process (assuming that there is a post-transcription editing process); 2) a desire to create local pitch-structure correspondences independently of the general system because of their contribution to the coherence of the musical surface, or because of their subjective appeal to the composer's ear; or 3) simply a desire to include genuinely random events within the general context of controlled indeterminacy that governs the structure of most of the sonic configurations in Xenakis's music. Regardless of the possible reasons for their existence, these stray, or apparently non-systematic, elements are a regularly occurring feature in the pitch structures of Xenakis's instrumental compositions, and should therefore be recognized in some way in the analysis of his music.

Both of these basic characteristics of pitch structure in Xenakis's music are addressed analytically by means of pset models. Pset models are large psets that are derived from unions of smaller psets which appear on the

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[20]If any of the points made here seem obscure, the reader may wish to refer back to chapter 2, section 2.1.6 on sets and section 2.1.7 on sieve theory.

surface of the music. As such, these models are hypothetical "master" psets to which the psets in the music may be compared on the basis of inclusion and/or transpositional relations. In some cases, the models appear in full on the surface of the music, in which case they are not hypothetical analytical constructions but rather representations of actual phenomena observed in the music. In most cases, however, the models must be regarded as analytical constructions to which the actual phenomena in a composition may be compared for the purposes of classification and for the determination of specific relations among the psets. In cases where the spacings of psets derived from the musical surface exhibit an almost, but not quite, regular structure, the structural irregularities are "corrected" in the model so that its spacing exhibits an absolutely regular structure. The absolute regularity of its spacing allows the model to serve as a point of reference for several psets, each of which may exhibit its own irregularities but still maintain a demonstrable relationship to the common model.

A list of the psets in the segments of *Evryali* is shown in Table 3.4. The pset contents of each segment are labelled by the segment number, followed by the segment's location (in measures and seconds) and its configuration type.[21] "Stray" pitches and the irregularities in the spacings associated with them are shown in italics. The table shows that the segments 1-4 and 6-8 are all related by inclusion to model A except for a few stray elements in some of them. Model A is based mainly on the contents of the psets in segments 2 and 7, which together span a range in p-space from pitch -37 to pitch 47. Any pitches in these two psets that cause deviations in their spacings from a generally regular structure of period 20 have been eliminated in model A.

[21] Abbreviations for the configuration types are the same as those used in Table 3.1.

Segment 5 does not relate as easily by inclusion to model A as the other sets do, for only two of its five elements intersect with model A, but complete intersection occurs between it and a transposed subset of model A. Thus, in terms of their pitch structure, segments 1-8 form a family of psets each of whose members is related to model A.

Segments 8-10 are related to model B, which is made up of several unfoldings of an m-pset whose period is 11. The pset in segment 8, all but one of whose elements intersect with model A, intersects completely with model B. Segment 8 has been discussed previously because of the unusual morphology of its configuration. (See Figure 3.2.) It is the TPS in which the registral boundaries of its pset are least in evidence, due to its time-point structure, which creates the impression of rapid oscillating motions through p-space. These motions seem to anticipate the much wider oscillations found in segment 9, which contains the first arborescence in the work. Thus, on the basis of its morphology, segment 8 functions as a transition from the previous music, which is characterized by TPSs and stochastic configurations, into the remainder of part 1, which is dominated by alternations between arborescences and TPSs. The transitional function of this segment is reflected also in its pset, which forms a link between the psets related to model A and those related to model B.

Segment 9 is the first segment in the work to span the entire range of the piano keyboard (with the exception of the piano's lowest pitch, -39). Model B is therefore based on the contents of this segment, except for pitches -31, 5, 13, and 18, which result in irregularities in the otherwise consistent spacing of period 11. Correspondences between this pset's spacing and the INT of its tpseq were discussed above. (See Figure 3.3.) The pset in segment 10 intersects with a transposed subset of model B. Segment 11 contains a rest, which is followed by segment 12, whose pset also covers the entire range of the piano keyboard. This pset is made up of m-psets of period 13. The slight irregularities in the spacing caused by the inclusion of pitches -37 and 37 have been removed in model C.

The families of psets related to models A and B and the single pset related to model C constitute the basic classes of differentiated pitch material that is used in *Evryali.* The remainder of the psets, from segment 13 to the end (minus the rests), can be related either to the union of models A, B, and C, or to model D, which represents the set of all pitches available on the piano. The large pset $A \cup B \cup C$ contains 83 of the 88 available pitches. This pset is thus similar to model D, which contains all 88 pitches, but the psets related to it have gaps in their spacings that indicate a more restricted use of p-space than the generally unrestricted use found in the psets related to model D. It is significant as well that the segments whose pset contents are related to model C contain either TPS or stochastic configurations, while those related to model D are all arborescences, with the exception of segment 23, which is stochastic. In general, then, the TPS and stochastic configurations show a differentiated use of p-space, while the arborescences do not. Exceptions are the arborescences in segments 9 and 12, which are related to models A and B, respectively, and the stochastic configuration in segment 23, which makes use of the undifferentiated p-space. Each of these exceptions may be regarded in some sense as transitional, for the morphology of segments 9 and 12 does not feature the branching structures found in the subsequent arborescences. In their morphology and in their pitch structure, which is more similar to that of the surrounding TPS configurations than to that of the later arborescences, these two "proto-arborescences" appear to be hybrid structures that do not yet display all of the distinctive features that are most characteristic of their configuration type. Similarly, the stochastic configuration in segment 23 shows some hybrid characteristics, for its steady 16th-note pulse is clearly non-stochastic, but rather imitates the predominant rhythm of the preceding arborescence in segment 20. In its morphology it is neither a stochastic field nor a stochastic stream. Instead, it expands outward from the center of p-space with fairly clearly defined outer margins, effectively reversing the closing gesture at the end of segment 20, thereby opening up the p-space once again in preparation for the unmistakable stochastic field that occurs in segment 24. The pset in segment 24, which is included in $A \cup B \cup C$ with the exception of three of its pitches, may also, with the exception of five of its pitches, be included in model A. Thus, the pset contents of segment 24 demonstrate a close, though imperfect, structural connection between it and the earlier stochastic configurations in segments 2 and 7.

Although the models A, B, C, and D, and the pset consisting of the union of models A, B, and C are hypothetical analytical constructions intended to explain the general outlines of the pitch structure in *Evryali*, the overall progress of the pitch structure described here has some features in common with the pitch structure of *Herma*, in which psets and set-theoretic operations upon them are given explicit treatment in the compositional plan and in the score of the finished work. (See Figure 2.23.) In *Herma*, the set of all available pitches—R, for referential set—is presented first, after which three psets derived from it—A, B, and C—are presented, along with their complements. The remainder of the work consists of a progressive differentiation of its pitch contents through a succession of set-theoretic operations on A, B, and C and their complements. According to the account of its pitch structure presented here, *Evryali* begins with a presentation of psets (or pset models) which have been labelled A, B, and C,[22] and progresses ultimately to a saturation of the available p-space by means of arborescences whose psets are related to model D. Thus, the general thrust of the pitch structure in *Evryali* is from a differentiated use of p-space toward an undifferentiated one, which is the reverse of the general thrust of the pitch structure in *Herma*, which moves from a state of lesser differentiation to a state of greater differentiation over time.

As demonstrated in this analysis of *Evryali*, Xenakis's music, in which the readily identifiable structural units tend to contain a large number of individual events, demands an analytical approach that takes into account the progression of these structural units over time as well as relations between them that are independent of their temporal succession and location within the work. Analysis of this sort differs from the approach usually taken to standard items from the atonal and serial repertoires, in which the structural units are assumed to consist of a small number of individual events and in which the analysis tends to focus primarily on the music's pitch structure, interpreted indirectly and somewhat abstractly in terms of relations between pitch-class sets found on the surface of the music. Xenakis takes a more global, less particular approach to the generation of the musical surface than most of his predecessors and contemporaries. An analytical approach that reveals most clearly the potentially meaningful relationships among the structural units in his compositions, therefore, requires attention first to the music's global characteristics and then to the similarities and differences among the segments, and the regular or irregular patterns of their

[22]The contents of these pset models are different from those of the similarly labelled psets in *Herma*.

succession as the music unfolds in time. From there, more general statements may be made that can be supported by evidence drawn from the music's demonstrable structural features. Once several works have been approached in a similar way, evidence of general structural principles begins to emerge and allows meaningful comparison of different works according to sets of norms derived from close observation of the music. The analytical observations presented in this and subsequent chapters may lead to a clearer understanding of Xenakis's compositional strategies and also to a consideration of how these strategies help to project the music's structure in real time to the attentive and receptive listener.

Mists **(1980)**

Mists is a longer and more diffuse work than *Evryali*. In certain respects its structure is simpler, however, for it contains no superimposed segments and its pitch structure is much more clearly defined than that of the earlier work. A list of segments in *Mists* is shown in Table 3.5 and a graphic transcription of the score is given in Figure 3.6. Four configuration types are represented in in the table. The first type to appear is the random walk, which was introduced in chapter 2, section 2.2.2 as a configuration whose events are presented in a (relatively) continuous manner, either in a single line or in two or more independent lines. Random walks generally feature unpredictable changes in direction and may also demonstrate differences in the speed at which they move through p-space. The random walks that appear at the beginning of *Mists* were discussed in section 2.2.2 in connection with Figures 2.38-41. The second configuration type to appear is the arborescence, examples of which were discussed in section 2.2.3 in connection with Figures 2.42-52. The third type consists of stochastic configurations. The stochastic configurations in *Mists* employ a special type of time-point/pitch notation, which was discussed in section 2.2.1.10 and shown in Figure 2.33. As in *Evryali,* measured silences are included among the configuration types as rests. There are several more rests in *Mists* than in *Evryali,* and their durations vary considerably. Every rest that stands between two sonic configurations has been noted, even if its duration is as brief as a sixteenth-note (0.313"). The point in making note of such brief rests is that it allows for a distinction to be made between sonic configurations whose boundaries are more clearly articulated, i.e. by rests or by other means, and those whose boundaries are blurred. Several examples of the latter are found in the middle portion of the work, which consists almost entirely of stochastic configurations. It is most likely the nebulous impression produced by these configurations that gave rise to the work's title.

As suggested in chapter 2, the first five random walks in *Mists* form a structural unit. (See Figure 2.39.) Their manner of presentation is roughly analogous to canonic imitation, and guarantees a sense of continuity over the entire passage in mm. 1-7 (0-34.375"), designated in Table 3.5 as segment 1. From this point forward, the random walks are presented in pairs, as in segment 2 (mm. 7-9, 34.375-43.125"), or four at a time, as in segments 3 (mm. 9-11, 43.125-52.187"), 7 (mm. 16-8, 76.875-85.375"), 13 (mm. 24-6, 116.875-130"), and 14 (mm. 27-8130-137.187"). In segment 9 (mm. 18-22, 88.75-106.25"), the walks zigzag intermittently in such a way as to suggest compound voices, leading to the impression that as many as six walks are active simultaneously. As with the staggered walks in segment 1, the individual walks in segments 2, 3, 7, 9, 13 and 14 are distinguished by differences in their rhythmic structure so that very few events in different walks occur on the same time-points.

Unlike the walks in segment 1, however, the walks in the later segments are uniformly ascending in their contour. Thus, they exhibit the changes in speed characteristic of genuine random walks, but not the changes in direction featured in the walks in segment 1.

The durations of segments 2, 3, 7, 9, 13 and 14 vary in an irregular pattern. (See Table 3.5.) These variations in duration appear to correspond to the varying durations of the individual walks in segment 1, except that the pattern of variations there is regular, with each successive walk being shorter than the previous one. (See Figure 2.39.) The irregular quality of the variations in duration among these segments is complemented by the irregular variations in duration among the three arborescences that also appear in this portion of the work. These arborescences, which appear in segments 5 (mm. 14-6, 65-76.25"), 11 (mm. 22-4, 106.875-116.25"), and 16 (mm. 28-30, 137.5-150"), are separated from the surrounding random walks by rests, which also vary in duration. (See Table 3.5.) As demonstrated in section 2.2.3, all three of these arborescences are based on the same prototype, which has been extracted form the sketches and reproduced in Figure 2.42. (These arborescences are shown, in graphic transcription and in musical notation, in Figures 2.43-5.) Taken together, the random walks, rests, and arborescences in segments 1-16 comprise the first section of the work, whose boundary is marked by the change in tempo that occurs in m. 31 (150").

At m. 31 the tempo changes from ♩ > 48 to ♩ ≥ 72.[23] The morphology of the random walks also changes from one in which the basic motion is

[23]In calculating the time, in seconds, at which the segments are located (see Table 3.4), the approximate tempi given by Xenakis have been interpreted as if they were exact, e.g ♩ = 48 and ♩ = 72 instead of ♩ > 48 and ♩ ≥ 72. Fermatas have also been disregarded in the calculations, since the degree to which they lengthen the notated sonic events cannot be precisely determined.

ascending to one in which frequent changes in direction occur. The passage in mm. 31-8, in which the new tempo is maintained, features a succession of five segments containing random walks that lead into an elaborate arborescence. Following a rest, a further random walk—actually, a completely non-random ascent through p-space—appears in mm. 39-40, and effects a transition back to the original tempo. This entire passage, which comprises segments 17-24 (mm. 31-40, 150-184.667"), is shown in musical notation in Figure 3.7. A graphic transcription of the passage, in which the random walks and the arborescence are shown as continuous lines, is given in Figure 3.8, since the configurations are rather difficult to distinguish in the pitch/time-point transcription in Figure 3.6.

Although much of the passage consists of steady 32nd-notes, the individual segments it contains may be distinguished clearly on the basis of changes in intensity, timbre (*sourd* versus *sans sourd*), and/or in pset contents, about which more will be said later. Assuming the segmentation to be valid, let us concentrate on the morphological relations among the segments. Segment 17 (mm. 31-2, 150-155.833") contains a number of random walks that are connected at nodal points. In this respect it resembles an arborescence, but does not quite achieve the degree of structural complexity characteristic of the genuine arborescences found elsewhere in the work. Segment 17, therefore, seems to be a hybrid configuration that may be intended to serve as a transition between the arborescence in segment 16 (mm. 28-30, 137.5-150")—see Figure 2.45—and the more typical random walks that follow in segments 18-20. Segment 18 (mm. 32-3, 155.833-159.166") contains a pair of random walks that are symmetrical around the center of the p-space. Variants of this configuration's morphology occur in segments 19 (mm. 33-4, 159.166-162.083") and 20 (mm. 34-5, 162.083-165"), where the axis is of symmetry is tilted so that it ascends first from left to right (segment 19) and then right to left (segment 20). Segment 21 (mm. 35-6, 165-167.917") begins as if it will present a further variant of the symmetrical morphology, but ultimately both of the walks of which it is composed ascend in parallel motion. The lower of the two walks merges with the beginning of the arborescence in segment 22 (mm. 36-8, 167.917-174.583"). The fact that this portion of the passage ends with an arborescence suggests that segment 17 may serve not only as a transition between the arborescence in segment 16 and the random walks in segments 18-21, but also as a precursor to the arborescence in segment 22.

The next, and final, succession of random walks takes place in segments 101-2 (mm. 122-7, 589.667-612.167"). Segment 101 features four voices divided into pairs. The individual walks in each pair are identical in contour but are separated by a slight differences in their time-points. The pairs cross one another three times, creating a complex configuration whose morphology recalls the succession of random walks in segments 17-21 (see Figure 3.8). The two "diamond" shapes in the central portion of segment 101 recall the morphology of segments 18 and 19 in particular. Unlike the random walks at the beginning of the work, however, those in segment 101 conclude in a downward direction. Segment 102 repeats this downward motion in all four of its voices. The random walks in segment 101 thus represent a synthesis of characteristics found in the random walks at the beginning of *Mists*, specifically the time-point differences between the individual walks in segments 1, 2, 3, 7, 9, 13, and 14 (mm. 1-28, 0-137.187") and the changes in direction found among the walks in segments 17-21 (mm. 31-6, 150-167.917"). The downward motion at the end of segment 101 and in 102, on the other hand, complements the generally upward motion found in segments 1, 2, 3, 7, 9, 13, and 14.

All of the stochastic configurations in *Mists* occur within the long middle portion of the work that extends from segment 25 through segment 100 (mm. 41-121, 184.667-579.667"). The segmentation of the stochastic configurations in this long passage has been determined according to changes in one or more of the following characteristics: intensity, density, timbre (pedalling or articulation), and pset contents. Not every segment boundary is easily detectable to the ear or—in the graphic transcription in Figure 3.6—to the eye, but the changes upon which the segmentation is based have the effect of varying the musical surface, thereby keeping it colorful and interesting throughout the passage. Some notable changes in morphology, however, are immediately apparent from the graphic score. Segments 25-7 (mm. 41-3, 184.667-199.667") are stochastic fields. They are followed by a stochastic stream in segment 28 (mm. 44-5, 199.667-208.417") which, in turn, is followed by more stochastic fields in segment 29 (mm. 45-6, 208.417-213.573") and afterward. Segments 25-9 and the beginning of segment 30 are shown in musical notation in Figure 2.33 and segment 28 is shown in graphic transcription in Figure 2.34. In segments 29-31 (mm. 45-9, 208.417-227.792") the registral span remains wide, but there is a discernable movement upward in the lower margin of the configuration in segment 32 (mm. 49-50, 227.792-234.667") which is followed by a rather thin, descending stream in segment 33 (mm. 51-2, 234.667-244.667"). This configuration leads smoothly into the configuration in segment 34 (m. 53, 244.667-249.667"), which is notable for the extremely limited range of p-space that it occupies. The changes in registral span and in relative location in p-space that occur in this group of configurations are indicative of the types of morphological changes that take place throughout the remainder of the middle portion of the work.

The full range of p-space is used once again in segments 35-40 (mm. 54-7, 249.667-269.667), but the upper and lower margins begin to move inward slowly through segments 41-5 (mm. 58-63, 269.667-295.604"). The music settles into a relatively narrow band of p-space, between pitches 8 (G#4) and 40 (E7), in segments 46-57 (mm. 63-76, 295.604-363.417). The long stretch of activity in this area is interrupted temporarily and dramatically by a downward-moving stochastic stream in segment 58 (mm. 76-7, 363.417-368.573"). This configuration presents a morphology that contrasts strongly with that of the surrounding configurations, and anticipates the further contrast that takes place with the arrival of the arborescence in segment 61 (mm. 80-3, 379.667-398.104"). The morphology of segment 58 also seems to prefigure the descending motion found at the end of segment 101 and in segment 102 (mm. 122-7, 589.667-616.542"). This possible association may not be as remote as it first appears, for segment 58 creates a strong aural impression that endures well beyond its local context. After this striking configuration, stochastic fields in the upper region of p-space return in segments 59-60 (mm. 77-9, 368.573-379.667").

After the arborescence in segment 61, the stochastic configurations are scattered irregularly throughout various locations in p-space. In the graphic transcription, they seem to take on the appearance of patchy fog. Near the end of the middle portion of the work, however, are two similar and particularly remarkable configurations. They are contained in segments 96 (mm. 117-8, 564.667-570.917") and 98 (m. 118, 573.104-574.667"). (Segment 97 [m. 118, 570.917-573.104"] is a rest.) The pset in these configurations is limited to two elements in the extreme upper register of the piano, pitches 34 (B-flat6) and 43 (G7). Because of their irregular rhythm and high register, these configurations sound like an imitation of a natural phenomenon, such as the chirping of a bird, in an otherwise abstract context.[24] Like the descending configuration in segment 58 (mm. 76-7, 363.417-368.573"), this other "out of context" configuration appears to function as a signal of an impending, and more significant change. In this case, the change consists of the return of the random walks in segment 101 (mm. 122-6, 589.667-612.167").

The principal contrast among the configuration types in *Mists* occurs between the random walks and the stochastic configurations. The division of the work's temporal structure into parts, which have been referred to informally as "portions" in the previous discussion, coincides with the predominance of one or the other of these configuration types. The arborescences form a secondary source of contrast, interacting with the surrounding material in different ways, depending on whether they are found among random walks or stochastic configurations. The first three arborescences, in segments 5 (mm. 14-6, 65-76.25"), 11 (mm. 22-4, 106.875-116.25"), and 16 (mm. 28-30, 137.5-150"), are closely related in their morphology (see Figures 2.42-5), which differs considerably from that of the surrounding random walks. They share pitch material with the random walks, but present a contrast in terms of density (see Table 3.5) as well as morphology. The arborescence in segment 22 (mm. 36-8, 167.917-174.583") is more closely related to the random walks that precede it. Its morphology seems to "grow" naturally out of the multi-voice texture of the random walks in segments 17-21 (see Figure 3.8).

[24]Perhaps a covert reference is made here to Messiaen's use of birdsong, as in *Catalogue d'oiseaux* for piano (1956-8).

The arborescences that appear among the stochastic configurations stand out in starker relief from the surrounding material than do those that appear among the random walks. The longest and most complex arborescence in the entire work occurs in segment 61 (mm. 80-3, 379.667-398.104"). In graphic transcription it appears as two disconnected, superimposed arborescences, but in performance it sounds like a complex but relatively continuous configuration. Unlike the surrounding stochastic configurations, whose pitch material derives from the transpositionally related m-psets that are also used in the random walks and in the previous arborescences, this arborescence draws its pitch material from the undifferentiated chromatic pset. In its rhythmic notation and in its density, it is comparable to the random walks found at the beginning of the work (see Figure 2.39). Thus, a new element of contrast is introduced with respect to pitch material while the contrast in rhythm that previously differentiated the random walks from the stochastic configurations is preserved.

The group of five small arborescences in segments 72 (mm. 93-4, 445.917-452.479"), 87 (mm. 109-110, 526.542-533.104"), 94 (115-6, 555.292-562.792"), 104 (mm. 129-30, 627.167-634.667"), and 106 (mm. 133-4, 644.667-651.854"), which were discussed previously in section 2.2.3 and shown in Figures 2.47-52, may be regarded as "spawns" of the large arborescence in segment 61, for they are similar to it in their rhythmic notation and pitch material. Three of these arborescences—those in segments 72, 87, and 94—appear surrounded by stochastic configurations, while the last two—in segments 104 and 106—appear after the final random walks and are the last configurations to be heard. Because of their similarity in rhythm and in density to the random walks, the last two arborescences suggest a partial reconciliation between two of the configuration types, and thus form a fitting conclusion to the work. The relationship between the random walks and arborescences here, of course, is different from the one at the beginning of the work, which was based mainly on similarities in pitch material. Thus, the relationship between the configuration types, as well as the morphology of the configurations within the types, both show signs of evolution as the work unfolds in time.

A summary of the temporal structure of *Mists* is shown in Table 3.6. The table shows a division of the work into three parts, which correspond to the areas in which either random walks or stochastic configurations form the principal configuration type. Part 2 is the longest of the three, followed by parts 1 and 3, respectively. Part 2, whose duration is 405", accounts for 0.62 of the work's total duration, which is 654.667". This proportion is very close to the golden section, which is 0.618.... The duration of parts 1 and 3 combined is 249.667", which is 0.38 of the work's total duration. Although this is not properly an inside-time division, since the proportions of 0.62 and 0.38 are not articulated in direct succession, it is clear that the proportions of the large-scale temporal structure of *Mists* are based on an approximation of the golden section. The division of the work into parts thus represents the overall proportioning of the temporal structure according to differences in the predominant configuration type in each part. In this way, aspects of both the outside- and inside-time structures are supported by this proportion.

A more thorough separation of the segments by configuration type further supports the importance of the golden section in the work's large-scale temporal structure. The total durations of segments containing each of the configuration types are shown below, along the with the proportion of the work's total duration occupied by each.

configuration type:	RW	ST	A	R
duration (sec):	149.479	350.938	93.852	60.398
proportion of total duration:	0.23	0.54	0.14	0.09

When the configuration types are paired as in the following table, an approximation of the golden section results.

configuration types:	RW + A	ST + R
duration (sec):	243.331	411.336
proportion of total duration:	0.37	0.63

This pairing suggests that the arborescences are more closely associated with the random walks than with the stochastic configurations. The fact that the arborescences in part 2 form a definite contrast with the surrounding stochastic configurations seems to support this association, in addition to the fact that the rhythmic structure of these arborescences closely resembles that of the random walks in parts 1 and 3. The juxtaposition of the final two random walks with the final two arborescences in part 3 further points out the resemblance between their rhythmic structure. In the inside-time structure of the work, however, the arborescences and the rests are distributed fairly evenly throughout all three parts, which allows the arborescences to associate in different ways with both the random walks and the stochastic configurations.

In addition to the proportions among the parts, coherent systems of proportions may also be found among the smaller units into which the parts divide. Part 1 divides into two sections at the tempo change in m. 31. This makes for an uneven division in which section 1 is approximately four times as long as section 2. The subdivision within section 1, however, which occurs at the entrance of the first arborescence in segment 5, is much more even, with 0.43 of the duration going to subsection 1a and 0.57 going to subsection 1b. Similar proportions appear in the division of part 2 into sections 3 and 4, which occurs at the entrance of the arborescence in segment 61. Here the proportions are 0.48 for section 3 and 0.52 for section 4. Section 4 subdivides according to another uneven proportion, 0.7 for subsection 4a and 0.3 for subsection 4b. This subdivision and the further subdivisions that result in sub-subsections 4aa, 4ab, 4ba, and 4bb all occur at the beginnings of arborescences. Although the subdivision of section 4 is uneven, the internal subdivisions within its subsections exhibit the same proportion, 0.45 and 0.55. This proportion, in turn, resembles the proportion between subsections 1a and 1b within section 1 and sections 3 and 4 within part 2. Observation of the proportions in part 2 reveals that the successive entrances of the arborescences occur with generally increasing frequency as the part unfolds. Part 3 also divides at the entrance of the first of its arborescences. Once again the proportions represent a slightly uneven bisection, 0.58 and 0.42, which resembles the previous proportions that fell within a range between .5/.5 and .4/.6, but this time the larger portion appears first.

The fact that several of the temporal divisions proposed in Table 3.6 coincide with the initiation of arborescences supports the claim made in section 2.2.3 that the placement of these configurations is important to the articulation of the work's temporal structure. Another interesting aspect of the temporal structure is that the duration of part 3, 65", is identical to the duration of subsection 1a. This only adds to the associations that have already been established between these two portions of the work. Further strengthening the association between subsection 1a and part 3 is the fact that random walks related to the type that appear in section 1 do not return until section 5.

Several of the global aspects of structure in *Mists* are organized according to criteria that are readily identifiable. Four such aspects will be considered here. Two of them pertain to the work's outside-time structure and the remaining two to the succession of its segments inside-time. The first aspect to be considered is the statistical distribution of segment durations outside-time. A comparative histogram of the observed frequency of segment durations versus the expected probabilities of the exponential distribution, ∂ = 0.163, is shown in Figure 3.9.[25] Since there are no superimposed segments in *Mists*, the calculation of a separate distribution for the time-point intervals between the initiations of successive segments is not necessary.

The second aspect of the outside-time structure that appears to have been derived from a probability distribution is the determination of segment densities. This is demonstrated in Figure 3.10, which shows a comparative histogram of the distribution of segment densities versus the expected probabilities of the linear distribution. Only the densities of segments containing sonic configurations have been represented in the histogram. (The densities of all segments containing rests are zero and, were they to be included, would significantly increase the frequency of events at the low end of the graph.) Of the works that are examined in detail in this dissertation, only *Mists* shows evidence of the global structuring of densities according to a probability distribution. Possible reasons why this method may have been applied here include the large number of segments in *Mists*, each of which would need to be assigned an appropriate density, and the fact that so many of

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[25]The mean density of ∂ = 0.163 segments/sec is the quotient of the 107 segments in *Mists* and the work's total duration, which is 654.667".

the segments contain stochastic configurations, which require specific values for their mean densities so that their time-point structures may be calculated.

Two aspects of the global structure that appear to have been structured with respect to the succession of segments are the segments' registral boundaries and their MDS values. A graph of the registral boundaries of each segment is shown in Figure 3.11. If a segment consists of a rest, the boundaries from the preceding segment have been carried through it, since changes in boundaries may only occur during sounding segments. The graph does not indicate the durations of the segments, but only the order of their temporal succession. The mean register for each segment is shown with a broken line. The graph shows clearly that a mean register near the center of the keyboard is maintained during part 1 and into the beginning of part 2 (segments 1-27) and from the end of part 2 to the beginning of part 3 (segments 99-105). The central axis is maintained whether the configurations are random walks, in which case they have a wide registral span, or arborescences, in which case the registral span is much narrower. The establishment of a registral axis near the center of the keyboard at the beginning of a work and a return to it at the end is also a feature of *Evryali*, as shown above.

Throughout most of part 2 of *Mists*, the registral boundaries and the mean fluctuate together, as opposed to the outer sections where the boundary fluctuates but the mean does not. This occurs at first gradually (segments 28-57) and then irregularly (segments 58-98), effectively dividing the central portion of part 2 in half on the basis of the manner in which p-space is used. If all of part 2 is included in the calculation, the proportion created by this division—i.e. segments 25-57 versus 58-100—is 0.44 and 0.56, which is consistent with the slightly uneven bisections found elsewhere in the work.

Thus, a secondary point of division in part 2 would occur at segment 58 (mm. 76-77, 363.417-368.573"), which is the segment whose unusual morphology has been discussed above in connection with its resemblance to the contour of the final random walk in segment 102 (mm. 126-7, 612.167-616-542") and its function as an aural signal that prepares for the arrival of the first arborescence in part 2, which occurs in segment 61 (mm. 80-3, 379.667-398.104").

The MDS for *Mists* has been constructed in the same way as the one for *Evryali*. Minimum and maximum values in each of the four dimensions considered are as follows:

attribute:	duration	density	registral span	intensity
dimension:	d-space	∂-space	r-space	i-space
unit:	second	sounds/second	semitone	level (*ppp, pp, ...*)
minimum:	0	0	0	0 (silence)
maximum:	34.375	19.17	87	7 (*fff*)

As in the MDS for *Evryali*, the values have been scaled so that the minimum for each parameter is equal to 0 and the maximum is equal to 2.5. The sums of the scaled parametric values for each segment are shown in graphic form in Figure 3.12. A fine broken line connects the sums that exceed 6.5. This is an arbitrary cutoff point that was chosen in order to call attention to a small group of high peaks within the graph. The segments that have thus been singled out in this way are segments 1, 3, 7, 14, 18, 22, 58, and 101. Each of these segments functions in a special way within the work. Segment 1 (mm. 1-7, 0-34.375"), of course, introduces the random walk configuration type and the work's characteristic m-pset. Segment 3 (mm. 9-11, 43.125-52.187") is the first four-voice random walk. From this point on, the four-voice type of random walk becomes standard. Segment 7 (mm. 16-8, 76.875-85.375") is the next four-voice random walk and introduces a transposed version of the m-pset. Segment 14 (mm. 27-8, 130-137.187") is the final four-voice random walk in part 1. These segments relate clearly to segment 101 (mm. 122-6, 589.667-612.167"), in which the random walk configuration type returns after a prolonged absence. A coarse dotted line in the graph connects the MDS values for segments 1 and 101, which are the highest in the work, in that order.

The three remaining segments with MDS values of 6.5 or above also demonstrate special structural features. Segment 18 (mm. 32-3, 155.833-159.166") is the mirror-symmetrical linear configuration within the collection of random walks in section 2. Segment 22 (mm. 36-8, 167.917-174.583") is the arborescence that immediately follows this collection of random walks. (See Figure 3.8.) The increase in tempo in this section of the work contributes significantly to the density of the configurations it contains. Finally, segment 58 (mm. 76-7, 363.417-368.573") contains the stochastic configuration that results in a secondary division of part 2, due to the fluctuations in registral boundaries that follow from it. (See Figure 3.11.)

The isolation of this small group of segments is not intended to imply that they uniquely represent the most important moments in the work. Consider, for example, the extraordinary stochastic configurations in segments 96 and 98 (mm. 117-8, 564.667-574.667") which, with their minimal pitch contents (B-flat6 and G7), herald the immanent return of random walk configurations at the beginning of part 3. The examination of high MDS values does suggest, however, that Xenakis is sensitive to the way in the which the maximization of values in several dimensions simultaneously can draw attention to particular areas within a work, and he appears to apply this maximization selectively to segments whose contents are basic to the articulation of large-scale structure (e.g., segments 1 and 101, and 58) or ones that contain configurations with unusual morphological features (e.g., segment 18).

The pitch structure of *Mists* is organized more simply than that of *Evryali*. Virtually all of the pitches in *Mists* may be related to a single m-pset by inclusion, transposition, union or intersection of two of its transpositions, or to the pset that contains all of the pitches available on the standard piano keyboard. Thus, only two models are required to describe the pitch structure of *Mists*, as opposed to the four that were used to describe the pitch structure of *Evryali*. These two models, A and B, and the transpositional derivatives of model A, are shown in Table 3.7. Model A is an m-pset whose period is 90, and therefore exceeds the range of the piano keyboard slightly. The contents of this m-pset, from pitch -38 (B-flat0) to 49 (C#8) are given as found in the sketches to *Mists*. Pitch 52 is shown in parentheses because this is the point at which the period of the m-pset completes itself. When the origin is displaced by transposition, the elements of the m-pset wrap around the ends of the keyboard according to the pattern of intervals in its spacing, thus filling up the available p-space.[26]

Transpositional derivatives of model A are listed in Table 3.7 in order of their initial appearance in *Mists*. The origin of each derivative with respect to model A is shown in bold, as is the interval in the spacing with which it is associated. The transposition operator for each derivative gives the number of semitones that the origin has been displaced with respect to the origin of model A. Since there is no absolute up or down in modular p-space, the

[26]For more on m-psets, see chapter 2, section 2.1.6.

positive numbers for the transposition operators refer to displacements of the origin to the right, i.e. "upward," within p-space. Only those elements that fall within the range of -39 to 48 are listed as pitches within the derivative psets of model A. Thus there are slight differences in the number of elements within the derivatives and there are also intervals missing from the spacings of the derivatives as compared to the complete spacing shown below the pitch contents of model A. Model B consists of the set of pitches from -39 to 48 inclusive, and is therefore equivalent to model D in *Evryali.*

All of the random walks, the stochastic configurations, and the first four arborescences are based upon psets derived from model A. These configurations are indicated by their segment numbers in Table 3.8. Below the label for each pset derivative are listed the segments, preceded by the letter "s," that are based on that pset. A minus sign (-) following the segment number indicates that the pset is missing some pitches, i.e. it is represented by one of its subsets. A plus sign (+) following the segment number directly, or following the minus sign to the right of the segment number, indicates that pitches extraneous to the pset are included in that segment. The extraneous pitches are listed in parentheses following the plus sign. As in the other piano works, not all of the pitches in *Mists* are contained within a simple system of psets (or m-psets) and their transformations, but in general the pitch structure of *Mists* follows the plan indicated in Table 3.7 with relatively few deviations. Perhaps because of the complexity of their construction, or because they constitute the most "indeterminate" configuration type, the stochastic configurations contain more deviations than do the random walks or arborescences. If elements of more than one pset appear in a segment, either through the intersection or union of two psets, that segment is listed under the labels of both psets as well as under the label that identifies the set-theoretic operation performed on the pair of psets. The last six arborescences, beginning with segment 61, are listed under the label for model B. These configurations are transcribed directly into the chromatic p-space, as shown in Figures 2.47-51.

The distribution of psets shown in Table 3.8 is summarized in graphic form in Figure 3.13.[27] The graph shows that, except for some doubling back near the beginning and toward the middle of the work, the pset derivatives of model A are generally presented in a succession from A_0 to A_9. A_0 returns before the presentation of A_{10}, with which it alternates near the end of part 2, in preparation for the return of itself along with A_1 in the final two arborescences (segments 101 and 102). In rough outline, then, the work begins with an alternation of A_0 and A_1, after which several other derivatives of model A are presented in succession until the cycle begins again with the return of A_0 and A_1. Presentations of subsets of model B, in the last six arborescences, interrupt the cycling through of the derivatives of model A.

The overall frequency of each of the derivatives of model A and of subsets of model B within segments, outside-time, is represented graphically in Figure 3.14a. The graph shows that A_0 and A_1 appear with the greatest frequency, followed by A_4. A_0 and A_1 are prominent in parts 1 and 3 of the work, and A_4 in part 2. The graph in Figure 3.14b shows the relative duration of each of the derivatives of model A and of subsets of model B within the work as a whole, outside-time.[28] Once again, A_0 and A_1 are especially

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[27]The values 0-10 along the vertical axis of the graph represent the derivatives of model A. The number 11 represents model B.

[28]Because of the representation of unions and intersections of psets in some segments, the total duration used in the calculation of the relative frequencies in the graph is 689.927", as opposed to the work's total duration inside-time, which is 654.667".

prominent on the surface of the music, but the remainder of the psets, including A_4, receive about the same amount of exposure. Because of their greater frequency and duration within the work as a whole, it is possible to regard A_0 and A_1 as relatively stable elements of the pitch structure. The other psets could then be regarded as relatively transitional in character, leading from the initial presentations of A_0 and A_1 at the beginning of the work to their return at the end. Underlying the abstract pitch structure of *Mists*, therefore, appears to be a traditional scheme in which a particular sonority (here represented by a pair of psets) is first established, then is followed by a departure from it before it returns to conclude the work. This general scheme is reinforced in *Mists* by the association of A_0 and A_1 with random walks both at the beginning and near the end.

Turning now from the general features of the abstract pitch structure to some of the details of its concrete manifestation, the first characteristic that should be pointed out is the difference between full and partial representation of the psets on the surface of the music. Psets are represented in full only in the random walks. A_0 is represented in full in segments 1, 2, 3, 9, 14, 19, and 101, all of which contain random walks. A_1 is represented in full in segments 7, 13, 101, and 102, and nearly in full in segments 17 and 18. A_2 is represented in full in segment 20 and A_3 in segment 21. All of these segments are found either in part 1 or part 3 of *Mists* (see Table 3.6). The remainder of the psets are represented only partially and thus their impact on the audible structure of the music is less direct than is that of psets A_0-A_3. This lends further support to the idea that the pitch structure of parts 1 and 3 is more stable, or at least more fully realized, than is that of part 2, which consists mostly of the nebulous, or "misty," stochastic configurations.

The unions and intersections provide links between adjacent or nearly adjacent psets, whether they are are partially or fully represented, and may also be used to demonstrate unusually high or low degrees of intersection among the transpositional derivatives of model A on the musical surface. The number of elements in the intersection between pairs of derivatives is shown in Table 3.9. Each pset, of course, intersects completely with itself. The number of elements in the intersection of any transposition of model A with any of the eighty-nine other possible transpositions, without regard for the limitations placed on the available p-space by the compass of the piano keyboard, varies between 0 and 18, inclusive.[29] This range is represented adequately in the intersections among pairs of the eleven transpositions chosen by Xenakis for inclusion in *Mists*. The maximum intersection between a pair of distinct psets in Table 3.9 is 16, between A_2 and A_3, which is close to the maximum for transpositions of model A, but understandably smaller because of the limitations on the compass of the piano keyboard. This intersection is represented in full in segment 24 (mm. 39-40, 176.667-184.667"; see Tables 3.7 and 3.8). At the opposite extreme is the near-minimal intersection between A_0 and A_{10}, which consists of the pset {-28 14 34 43} (see again Tables 3.7 and 3.8). This intersection appears in segments 96 and 98 (mm. 117-8, 564.667-574.667"). Since the configurations in both of these segments are confined to the upper register, the intersection is represented only partially, by the two pitches 34 (B-flat6) and 43 (G7).

Appearing shortly after this minimal intersection is the union between A_0 and A_1, which occurs in segment 101 (mm. 122-6, 589.667-612.167"). These

[29]The range of elements in the intersections of transpositions of model A was determined with the aid of a T-matrix mod 90, similar to the matrix used to determine $\#(H \cap T_x(H)(\text{mod } 27))$ in chapter 2, section 2.1.6.

two psets share 15 elements, as indicated in Table 3.9, and together form a pset containing 42 elements, which is fully represented on the surface of the music. Segment 101 contains the complex random walk configuration that opens part 3, and the high number of elements in its pset contrasts strongly with the minimal number of elements in the pset in segments 96 and 98. Another large union occurs in segment 58 (mm. 76-7, 363.417-368.573"). This segment contains a stochastic stream that has already been associated with the random walks at the end of *Mists* on the basis of its descending contour. The union of A_3 and A_5 that is contained within this segment has a total of 50 elements, of which 34 are represented on the surface of the music. It is, of course, a common occurrence that the psets upon which stochastic configurations are based are represented only partially on the surface of the music. This, once again, is because of the quasi-random selection of intervals between elements that takes place as part of the process of stochastic composition.

From the comments above it is clear that the compositional placement of maximum and minimum intersections and also of unions of psets is coordinated with the divisions in the temporal structure and with specific morphological associations between configurations. On the basis of this evidence it appears that Xenakis is well aware of the relations among the group of derivatives of model A that he has chosen, even though the criteria for the selection of this particular group out of the ninety possible psets is not entirely clear.

The subsets of model B are handled somewhat differently, perhaps because of the lack of differentiation in their interval structure. Transposition within the undifferentiated p-space amounts to nothing more than shifts in spatial location, with intersections occurring only as a result of overlaps among the regions occupied by the configurations. The arborescence in segment 61 (mm. 80-3, 379.667-398.104") occupies a sizeable portion of the p-space, from -18 (F#2) to 43 (G7). The remaining arborescences that are inscribed in model B are arranged successively in the parabolic pattern that is summarized in Figure 2.52. Spatial location, as opposed to degree of intersection, is also used to articulate structure in undifferentiated p-space in some works for strings, as will be demonstrated in chapter 4.

In summary, *Mists* is a work that, superficially at least, is organized along relatively traditional lines. Although it may not exhibit a classic ABA formal scheme, due to the nature of the temporal proportions among its parts and to the continuous evolution in the presentation of its basic configuration types, its outer parts are strongly associated nonetheless by the prominence of random walks within them and by the concurrent presence of specific transpositions of the work's basic m-pset. The inner part is more diffuse and variable in its structure than are the outer parts. A primary opposition in materials is played out in the contrast between random walks and stochastic configurations, which are confined to different parts of the work, and this contrast is mediated somewhat by the presence of arborescences, which are found in all three parts. The pitch structure of *Mists* is organized more clearly than that of *Evryali*, but the relations between segments based on model A are handled somewhat differently than those between segments based on model B because of differences between the models' respective interval structures.

à r. (hommage à Maurice Ravel) **(1987)**

à r. is an occasional work that was commissioned by the Radio-France International Festival at Montpellier as part of its commemoration of the fiftieth anniversary of Ravel's death. In its dimensions and in its organization it represents a move toward greater simplicity and clarity in Xenakis's writing for piano. *à r.* does not contain any obvious references to the style of Ravel's piano music. With a little imagination, however, one might try to associate the tremolos near its conclusion with similar effects found in *Ondine* and *Scarbo*, but this superficial resemblance is not reinforced by any apparent correspondences in pitch material or in other stylistic features. Instead, the stylistic and structural precedent for much of the material in *à r.* appears to be *Mists*, particularly the two-voice random walks in part 1, section 2 of that work. Moreover, psets A_0, A_1, A_2, and A_3 from *Mists*—the four psets that are featured in section 2 of that work—appear also in *à r.*[30]

An annotated score of *à r.* is shown in Figure 3.15. A list of the characteristics of the segments labelled in the score is given in Table 3.10.[31] Note that there are two configuration types—random walks and simultaneities—and only one rest in the work. As suggested in chapter 2, section 2.1.5, the contrast between these two configuration types is an important factor in the articulation of form in *à r.* The precise nature of the

[30]My thanks go to Scott McCoy for pointing out the aural similarity between portions of *Mists* and *à r.* to me. Subsequent investigation of the pitch structures and configurations in both works confirmed the reappearance of passages from *Mists* in *à r.* These passages are, of course, recontextualized in the new work, as demonstrated in this analysis.

[31]In the table, segment 37 is presented as a unit and is also subdivided, since its pitch structure represents a composite of several psets. The relations among these psets will be discussed at the end of the analysis.

configuration types was not described in that preliminary analysis since the characteristics of the configuration types that commonly occur in Xenakis's music are introduced later on in that chapter. Simultaneities were not introduced as a typical configuration type there since, unlike the other configuration types, they constitute an element of texture that is common to various styles of music, and therefore do not require any special description.

Two graphic transcriptions of the score are shown in Figures 3.16 and 3.17, respectively. The first graphic transcription shows the points of initiation of the sonic events in pitch and in time. It allows the repeated attacks and tremolos found in the latter part of the work to emerge as visually detectable aspects of the work's texture. The second graphic transcription emphasizes the relative continuity between consecutive events in the random walks and also the duration of the simultaneities. The distinction between sustained and tremolo simultaneities is effaced in this transcription, but the morphology of the random walks is more easily visible. The morphology of segments 16, 17, and 19 in particular should be compared with those of segments 17-19 in *Mists*, which they resemble closely (see Figures 3.7 and 3.8).

More specifically, segment 16, in mm. 8-10 of *à r.*, resembles segment 17 in mm. 31-2 of *Mists*, both in terms of its morphology and its pitch contents. Some differences, however, are observable. For example, the group of four 32nd-notes with which segment 16 in *à r.* begins starts on C#5, rather than on the G#4 that occurs at the beginning of segment 17 in *Mists*. In addition, the four events that occur at the beginning of the lower voice of segment 16, i.e. F4, G#4, C#5, and D#5, are not found at the beginning of segment 17 in *Mists*, but they are members of the pset common to both configurations.[32] The rhythmic structure of the upper voice in segment 17 in *Mists*, from its beginning through the group of triplet 16th-notes near its conclusion, is reproduced literally in segment 16 of *à r.*, even though the succession of pitches has been shifted one position to the left. Similarly, starting from the second group of four 32nd-notes in the lower voice of segment 16 in *à r.*, the rhythmic structure of the lower voice in segment 17 of *Mists*, from its beginning to its penultimate event, is reproduced, again with the pitch succession shifted one position to the left. Two pitches in the lower voice of segment 16 in *à r.* have been altered with respect to the comparable passage in *Mists*: the C4 that occurs after the group of nine 16th-notes in the time of seven in *Mists* has been changed to a B-flat3 at the end of the nonuplet in *à r.*; and the B-flat1 that occurs at the end of the lower voice in segment 17 of *Mists* has been changed to B1 in *à r.* Curiously, the F#6 that occurs at the end of the upper voice in segment 17 of *Mists* has been changed to F6 in *à r.*, even though F#6 occurs adjacent to E6 earlier in the segment, as it does also in the pset in segment 17 of *Mists*.

The discussion of rhythmic structure in these configurations so far has ignored the change to doubled articulations that occurs at the beginning of m. 10 in *à r.* This change coincides with a change in tempo, and both changes are important to the work's temporal structure, which will be discussed below. The tempo of ♩ = 36 that is introduced at this point is one half as fast as the ♩ ≥ 72 that is introduced in m. 31 of *Mists*. The original tempo of *à r.* is ♩ ≈ 46, which is close to the principal tempo of *Mists*, i.e. ♩ > 48 (or ♩ ≥ 48, beginning in m. 41 of that work). Another difference between segment 16 in *à r.* and

[32]The pset contents of the segments in *à r.* are shown in Tables 3.13 and 3.14.

segment 17 in *Mists* is that the third, inner voice that appears in the configuration in *Mists* is deleted in *à r.*, thereby preserving a simple two-voice texture throughout.

A slight overlap occurs between segments 16 and 17 in *à r.* This overlap is significant for the work's temporal structure and presents B-flat3 in the lower voice once again in place of the C4 that is doubled between upper and lower voices in *Mists*. Segment 17 reproduces segment 18 in *Mists*, again with some alterations to the relations between pitches and rhythms found in the earlier work. Like segment 18 in *Mists*, the morphology of segment 17 in *à r.* is very nearly symmetrical around a central axis in p-space. The pitch structure of segment 18 from *Mists* is replicated almost exactly in *à r.* although it is cut short at the end. Pitches not found in *Mists* include E2 in m. 10, beat 3 of *à r.*, which is an interpolation from outside the pset (as determined by the transpositions of the m-pset in *Mists* and the context established elsewhere in segments 16 and 17 of *à r.*), and F6 in the upper voice at the beginning of m. 11, which once again replaces F#6. E1 and A#1, which occur in m. 33, beat 2 of *Mists*, are deleted from the lower voice in m. 11 of *à r.*

Segment 19 in *à r.* is related to segment 19 in *Mists*, but the upper and lower voices from *Mists* enter out of phase in *à r.* The lower voice in segment 19 of *Mists* begins on C4. This pitch is preceded in *à r.* by F4 and D#4, both of which are members of the pset upon which both segments are based (see the upper voice in *Mists*). From here the lower voice in *à r.* proceeds similarly to the way that the lower voice in *Mists* does, with the following exceptions: the pitch succession D2, B1, G#/A-flat1 is repeated in m. 11, beat 4; D#2 appears in place of D4 in m. 12, beat 1; and the segment ends on E3 in *à r.*, rather than continuing further as it does in *Mists*. Incidentally, the pset for segment 19 in both works contains B-flat3 and B1, which were introduced into the psets in segments 16 and 17 of *à r.*, apparently as extraneous pitches. Their presence in all three segments creates a contextual common-tone relation that does not exist between the respective transpositions of the m-pset on which both psets are based. The upper voice of segment 19 in *à r.* begins on E6, which occurs in m. 34, beat 1 of *Mists*. This pitch is followed by the apparently erroneous F6 in *à r.*, which was introduced previously in segments 16 and 17, thereby establishing another "unofficial" common-tone relationship. B7 is prolonged through repetition in *à r.* (m. 11, beats 3 and 4). This repetition does not occur in *Mists*. Despite the temporal prolongation in the upper voice caused by the repetition of B7, it is still necessary for a further addition to be made to the upper voice at the end of the segment 19 so that it may end at the same time as the lower voice. This addition, which contains pitches D5, D-flat5, B4, G-flat4, and F4, is drawn from elements that belong to the pset for this segment and for segment 19 in *Mists*. (See m. 33, beat 4 of *Mists*, where these pitches occur in reverse order.)

Segments 7 and 9 from *à r.* also relate to segments from *Mists*, but in these two configurations the pitches from *Mists* are presented in reverse order. Segment 7 begins with the first pitch from segment 21 of *Mists*, G3, and then presents the pitches in segment 20, moving backwards through it and on into the final portion of segment 19. The rhythm is changed slightly—including some triple 32nd-notes in *à r.* where there are none in *Mists*—and some pitches have also been changed. E3 appears in place of E-flat3 near the beginning of segment 7, and G7 and A-flat7, which appear in segment 20 of *Mists*, have been deleted in *à r.*, so that G7 appears only once, flanked on either side by F7. G5 appears at the beginning of m. 5 of *à r.* in place of A5 and, once again, F6 is found in the position occupied by F#6 in *Mists*. In the lower voice in m. 5 of *à r.*, G4 replaces the succession G#4, A#4, C#5 that occurs (in reverse order) in *Mists*. Incidentally, the G#3 that occurs at the end of m. 4 in *à r.*, spelled as A-flat3 in *Mists*, m. 34, beat 4, replaces the A3 proper to the pset in both works (see A3 in the upper voice in *Mists*, m. 35, beat 2).

Segment 9 in *à r.* presents pitches from segment 21 and from the end of segment 20 of *Mists*. In the lower voice of *à r.*, however, C4, which is extraneous to the pset, appears between D4 and G3 in m. 5, beat 3, where no such pitch appears in m. 36 of *Mists*. In the same vicinity A-flat2, also extraneous to the pset, is interpolated between A2 and G-flat2. In m. 5, beat 4, the D5 in the upper voice replaces the D#5 that occurs at the end of m. 35 in *Mists*, and in the lower voice B0 is repeated twice in succession on its way down to A0, but is deleted on the way back up at the beginning of m. 6. The repetition prevents a pitch-class parallelism that would otherwise occur between A0, B0 and A5, B5, but this does not seem to have been a concern in the comparable passage in *Mists*, m. 35, beat 4, where parallel (multiple) octaves are allowed to occur. In m. 6, beat 1, lower voice, C#2 replaces the A#2 of *Mists*, m. 36, beat 1, but this may ultimately be matter a missing ledger line. The end of segment 9 represents a slight recomposition of the end of segment 20 and the beginning of segment 21 of *Mists*. The pitches in the upper voice remain within the pset of segment 21, but the A#2 and C#3 of the lower voice come from the pset of segment 20.

It is apparent from the comparisons of several of the random walks in *Mists* and *à r.* that, once their morphologies have been sketched and their pitch contents decided upon, there is practically nothing random about the walks at all. On the contrary these structures, which have been inspired by the general morphological features of certain categories of actual random walks, show themselves to be susceptible to the compositional processes of repetition and variation typical of small-to-medium-scale structures in other styles. Thus, the cross-referencing that takes place between *Mists* and *à r.* is possible in large part because of the types of material chosen for each work.[33]

The compositional context in which the random walks appear in both works is, however, quite different. In *Mists,* the random walks in segments 17-21 occur as a continuous succession in which the individual configurations are differentiated through changes in intensity, pset contents, and/or pedalling. Together these five configurations constitute the greater part of section 2, which is differentiated from the surrounding music by a change in tempo and by the pairs of independent voices in the walks, a feature that is unique to this section of *Mists.* In *à r.*, however, the five random walks that are related to segments 17-21 in *Mists* participate in the alternations between random walks and simultaneities and in the direct succession of random walks (i.e., segments 16-7) that help to define the work's sections. The tempo is different from section 2 in *Mists,* the intensity of the configurations is not preserved consistently, and there are some differences in morphology and pset contents. The random walks in *à r.*, therefore, are not simply copies of those found in *Mists,* but are integrated fully into the structure of the new work. The temporal structure of *à r.*, for example, requires, in segments 7 and 9, the inclusion of events from more than one of the random walks in *Mists.* The failure to observe the segmental boundaries from the earlier work in the new one results in a disruption of the association of individual random walks with individual psets, and thus

[33]Although it is conceivable that finished passages of stochastic music could also be lifted out of one composition and placed into another, it appears that Xenakis has preferred to generate stochastic configurations fresh for each composition, in keeping with the general aesthetic of indeterminacy out of which stochastic composition in particular was developed.

complicates the pitch structure of *à r.* in ways that are difficult to explain without reference to the origin of these walks in *Mists*.

Now that the configuration types have been introduced and the characteristics of some of the random walks have been described in detail, it is possible to approach the topic of the division of *à r.* into sections. A simplified account of the relationship between patterns in the succession of configuration types and the work's sectional structure was presented in chapter 2, section 2.1.5. Sectional divisions were determined there according to whether the different configuration types alternated or appeared in direct succession. If the letter "a" is taken to symbolize random walk configurations and "b" simultaneities, the relation of the simple table below (adapted from section 2.1.5) to the succession of configurations in *à r.* (see Table 3.10) should be relatively clear.

section	succession	pattern
1	ababababa	X_0
2	bbbbaaa	Y_0
3	ababa	X_1
4	bbbbaaa	Y_1
5	bbbbaaa	Y_2
6	abab	X_2

The succession of configurations given in the table matches that found in *à r.* except for some differences in sections 2 and 4. Segment 14, which is the fifth segment in section 2, consists of a rapid succession of simultaneities. It is unclear, therefore, whether it should be classified as a random walk (in four voices) or placed within the same category as the simultaneities. Furthermore, segment 15 contains a simultaneity rather than a random walk. The actual succession of configurations in section 2, therefore, is bbbb?ba, where "?" indicates the ambiguous status of segment 14. In section 4, a rest is interposed between the first and second random walks. Its succession, therefore, is bbbbaraa.

A revised account of the succession of configuration types is included in Table 3.11. The table shows that the X patterns are associated with a strict alternation of random walks and simultaneities, while the Y patterns are associated, not with strict patterns of succession of similar configuration types as in the earlier model, but merely with the absence of a strict alternation of contrasting types. A strict succession of similar configuration types, as in the earlier model, is found only in Y_2 in the revised model. Y_1 is also close to the earlier model, but for the rest interposed between the first two random walks. Y_0, like all of the Y patterns, begins with a succession of four simultaneities, but becomes more complex thereafter.

The irregular succession of configuration types in the remainder of Y_0 is not merely fortuitous, however, for the direct juxtaposition of the indeterminate type in segment 14 with the simultaneity in segment 15 points out an association between its irregularly descending succession of four-voice simultaneities and the irregularly descending succession of long-held simultaneities in the work as a whole. This association may be observed most easily in Figure 3.18 in which the simultaneities in segment 14 and the long-held simultaneities in the work as a whole are shown in graphic notation. The simultaneities in Figures 3.18a and b are shown in the order of their temporal succession (st-space), independently of their temporal positions in the work. When they are abstracted in this way, it is clear that the long-held simultaneities articulate a pattern that descends gradually and irregularly in p-space as the work unfolds in time. A simplified and compressed version of this pattern occurs in segment 14. The hybrid configuration in segment 14, therefore, is associated with the long-held simultaneities because of the way its component simultaneities are positioned in p-space. Because the characteristics of both configuration types are combined in segment 14, the possibility is raised that further associations between them may be revealed at this point in the work, perhaps involving connections between their respective pitch structures. Exploration of this possibility will be deferred until the end of the analysis, after the work's temporal structure and other aspects of its global structure have been considered.

Some aspects of the temporal structure of *à r.* are revealed in Table 3.11. The lower portion of the table shows that the proportions among the durations of the respective pairs of X and of the Y patterns inside-time all approximate the value 1.3. This proportion is reflected also in the sums of the durations of the X and Y patterns outside-time and in the durations of the random walks, the ambiguous segment 14, and the rest in segment 27 combined versus the sum of the durations of the simultaneities. The proportion of 1.3 is clearly normative for this work, then, in the temporal relations among the sections, both inside- and outside-time, and among the configuration types outside-time.

Further aspects of the work's temporal structure are represented in Table 3.12. The table shows that each pair of sections forms a supersection. Each supersection contains both an X and a Y pattern (see Table 3.11). In supersections A and B, the X precedes the Y, and in supersection C the order of the X and Y is reversed. The reversal of the order of the patterns between supersections B and C causes two Y's to appear in direct succession. Thus, with the order of section types as XYXYYX, the distinction between alternations of contrasting configuration types and successions of similar types within sections is shown to exist in the succession of section types as well. The proportion between the durations of supersections A and B is 1.27, i.e. 48.61" and 38.323", which approximates the proportion of 1.3 which was shown to be normative at the level of the sections inside-time and section types and configuration types outside-time (see again Table 3.11).

The proportion between the durations of supersections B and C, however, is 38.323"/39.996" ≈ 0.97, which is close to 1. The similarity in duration between these supersections tends to associate them more closely together than is the case with supersections A and B. In addition, the tempo changes from ♩ = 46 to ♩ = 36 just before the start of supersection B, at the boundary between segments 16 and 17. Along with this change in tempo comes a change in articulation, from single attacks of pitches to repeated attacks. The repeated attacks appear at the end of segment 16, in segments 17 and 19, and then again in the simultaneities in segments 31, 33, 38, and 40. The similarity in duration, then, plus the similarity in tempo and the introduction of the new, repeated articulation, and also the direct succession of the Y patterns at the level of the individual sections, all contribute to a closer set of associations between supersections B and C than between A and the following supersections. The grouping of the second and third supersections, therefore, suggests a division of the work into parts between supersections A and B.

Note that supersection A, however, is not completely devoid of associations with the characteristics of supersections B and C. The new tempo and articulation introduced at the beginning of m. 10 affect the last portion of the final segment in supersection A. In addition, this segment—segment 16—draws its pitch material from same pset as does the following segment. Segment 17, however, initiates a pattern of alternations between random walks and simultaneities, so that the similarities between the end of segment 16 and segment 17 should be viewed primarily as a means of dovetailing contrasting areas of the work rather than as a weakening of the basic distinctions between supersections A and B described above.

Dovetailing occurs on a local level in this passage as well, for segment 17 apparently begins on the B-flat3 at the end of beat 1 in m. 10, while the E6 and F6 from the end of segment 16 are still sounding. This is the only overlapped segment boundary in the entire work and its inclusion at this particular point has to do with the large-scale temporal structure. If the beginning of segment 17, and with it the beginning of supersection B and of part 2, is taken to occur with the first B-flat3, the work divides into parts whose durations are 48.194" and 78.735", making for a proportion of 0.612. The proportion between the duration of part 2 and that of the entire work is 78.735"/126.929" = 0.620. Both of these proportions approximate the golden section, which is 0.618.... The proportion of the durations between the parts is slightly less than the golden section, by about 0.006, and the proportion of the duration of part 2 versus that of the whole work is slightly more, by only about 0.002. A slightly more accurate golden section occurs if the beginning of segment 17 is set at the first C4, on beat 2 of m. 10. The durations of the parts in this case are 48.61" and 78.319", as shown in Table 3.12, with a proportion of 0.621, which exceeds the golden section by about 0.003. The proportion between the duration of part 2 and that of the whole work in this case is 78.319"/126.929" = 0.617, which is less than the golden section by only about 0.001. A precise golden section, therefore, occurs somewhere between the first B-flat3 and the first C4 in m. 10. The sudden changes in tempo and articulation that take place at the beginning of m. 10, then, seem to function as a preparation for the arrival of this point in the work's temporal structure.

As in the other piano works, the durations of the segments participate in a global outside-time structure that is related to the exponential distribution. The density of this distribution is 40 segments/126.929" = 0.315 segments/second. A comparative histogram of the distribution of segment durations in *à r.* and the exponential distribution, $\partial = 0.315$, is shown in Figure 3.19. As in the earlier works, the values of the durations in the distribution are coordinated with the succession of configuration types in such a way that they help to articulate proportional divisions in the large-scale temporal structure. In addition, the variety of interval sizes contained within the distribution makes possible a varied and locally unpredictable succession of segment durations that nonetheless derives from a conceptually unified global structure.

The MDS values per segment in *à r.* correspond particularly closely to the structural features of the musical surface, for the random walks, on account of their high density, produce generally higher values than the simultaneities. Configurations that feature repeated attacks of the pitches in the simultaneities, however, such as those in segments 31, 33, 38, and 40, have relatively high densities and therefore relatively high MDS values as well. Thus, the introduction of repeated articulations in part 2 permits a partial blending of some of the characteristics of the contrasting random walk and simultaneity configuration types. A graph of MDS values per segment in *à r.* is shown in Figure 3.20. The range of values in the work is as follows:

attribute:	duration	density	registral span	intensity
dimension:	d-space	∂-space	r-space	i-space
unit:	second	sounds/second	semitone	level (*ppp*, *pp*, ...)
minimum:	0	0	0	0 (silence)
maximum:	7.915	38.404	87	6 (*fff*)[34]

As in the previous MDS measurements, the values in each dimension have been normalized to a maximum of 2.5 prior to being summed. A dotted line in Figure 3.20 connects values that supersede those of prior peaks. The segments whose MDS values are selected in this way are segments 1, 7, 16, 17, 19, and 37. The first peak to supersede a previous one occurs in segment 7, in which the first of the random walks adapted from *Mists* occurs. Segments 16 and 17, also adapted from *Mists*, supersede the peak produced by segment 7, partly because the introduction of repeated articulations at the end of segment 16 significantly increases the density of these configurations. The peak in segment 17 is superseded in turn by that of segment 19, which is the last of the random walks adapted from *Mists*. All of the random walks adapted from *Mists*, including the one in segment 9—which does not form an exceptional MDS peak, partly because of its low intensity level (*p*)—are notable for their wide registral span as compared to the majority of the other random walks.

An exception to this general rule is segment 37, whose MDS forms the highest peak in the work. This segment acts as the culmination of the random walks that are not adapted from *Mists*, for it contains portions of several psets that are related by cyclic transposition (T_n(mod 88)) to the pset found in segment 1 and in the majority of the other random walks. Because of the way it is approached contextually, this segment gives the impression of a tremendous release of energy and also of a logical conclusion to a process

[34]The indicators for intensity used in *à r.* are *p*, *mp*, *mf*, *f*, *ff*, *fff*, i.e. 1-6 in numerical values.

begun earlier, for the brief random walks in segments 26, 28, 29, 34, 35, and 36 appear to function as stretti that lead up to the longer and more complex climactic random walk. The progress of these stretti is held up by the rest in segment 27 and also by the simultaneities in segments 30-3, which only contribute to the impression that energy is being accumulated against resistance. Further, the fact that segment 37 contains portions of several transpositions of a pset heard earlier in the work is foreshadowed by the successive transpositions in segments 34-6 of the pset that is heard three times successively in segments 26, 28, and 29. Following the climactic segment 37, the remaining segments, including the registrally expansive random walk in segment 39, produce the effect of a gradual simmering down as the work proceeds to its conclusion.

Because of the relative simplicity of its overall design and the relative stability of its configuration types, compared to the stochastic configurations and arborescences used in earlier works, pitch functions in a particularly significant way in the aural and analytical interpretation of the structure of *à r*. This is not to imply, however, that interpretation of the pitch structure is without its difficulties, especially with regard to the relationship between the simultaneities and the random walks. It is most expedient, given these differences, to begin with an examination of the random walks and then proceed to a consideration of relations between them and the simultaneities. Another reason to begin with the random walks is the fact that they contain many more elements than the simultaneities, and are therefore much easier to associate with particular pset models.

A list of pset models and their derivatives in the random walks in *à r.* is given in Table 3.13.[35] As in the pset table for *Mists* (Table 3.7), the pitch within each derivative of a pset model that corresponds to the model's origin (transposed according to the indicated transposition operator) is shown in bold, as is the interval in the derivative's spacing with which that pitch is associated. And, as in the pset table for *Evryali* (Table 3.4), pitches and intervals within the psets and their spacings that do not correspond to the contents of the derivatives and their spacings are shown in italics. Thus, the interpretation of the pitch structure of the random walks in *à r.* represented in Table 3.13 combines the transpositional derivations (T_n(mod m)) featured in the interpretation of the pitch structure of *Mists* with the detailed observation of the musical surface featured in the interpretation of the pitch structure of *Evryali*. The reduced dimensions of *à r.* and the basically systematic nature of its pitch structure—a quality it shares with *Mists*—allows for a more thorough interpretation of its pitch structure than was possible for either of the works examined previously.

The spacing of model A in Table 3.13, and therefore of its derivatives as well, has been determined by taking the interval successions most commonly observed in the group of transpositionally related psets that are not identical with psets found in *Mists*. Model A, therefore, represents a consensus of the interval structures of psets observed in the context of the music, and not an abstract structure that is imposed from outside the work. The determination that the period of model A is 88 was made by piecing together the spacings of the psets, none of which unfolds a complete period in itself. Each of the

[35]Table 3.13 is limited to the pitch structure of the random walks. The pitch structure of the simultaneities is outlined in Table 3.14 and will be discussed separately.

derivatives of model A, therefore, functions as a potential source of which the psets in the random walks are subsets. Pitches in the psets that do not correspond to the contents of the derivatives usually result either from alternative subdivisions of larger intervals—e.g., interval 4 subdivided into 2 + 2 instead of 3 + 1—or from interpolations between the elements found in the derivatives.

The contents of the derivatives of model B are identical to four of the derivatives of model A in *Mists*. Specifically, B_0 in *à r.* corresponds to A_2 in *Mists*, B_1 to A_3, B_2 to A_1, and B_3 to A_0 (see Table 3.7). As in section 2 of *Mists*, from which the configurations containing these psets were drawn, the derivatives are represented on the surface of the music completely, or nearly so. Elements from the psets that do not correspond to elements in the derivatives result from alternative subdivisions of larger intervals, interpolations, possible copyist's errors,[36] extraneous pitches that seem to have been introduced in order to create local correspondences between segments (cf. the comments on segments 16, 17, and 19 above), or differences in segmentation between *Mists* and *à r.* All of these possible reasons for the presence of "stray" or "wrong" pitches, from the standpoint of the strictly systematic transposition of the m-pset models, have been discussed above, but it is worth repeating that segments 7 and 9 of *à r.* are based on segments 20 and 21 of *Mists*, respectively, with their elements presented in reverse order. Segment 7, however, contains a portion of segment 19 of *Mists*, and segment 9 contains a portion of segment 20, thereby introducing what appear to be extraneous pitches into their respective psets. These extraneous pitches are

[36]The published edition of *à r.* was prepared by Patrick Butin. No credit is given to the copyist in the score to *Mists*.

actually elements from other transpositions of the same m-pset, carried over from the previous work in the process of transferring some of its material into the new work. The differences in segmentation between the two works probably result from differences in their temporal structure, which has been shown in the case of both works to have been determined with considerable precision.

Examination of the succession of psets in the random walks reveals a relatively simple overall design. Segments 1, 3, and 5 contain subsets of derivatives of model A, while segments 7, 9, 16, 17, and 19 contain subsets of derivatives, or complete derivatives, of model B. The remaining random walks, in segments 21, 26, 28, 29, 34, 35, 36, 37, and 39, contain subsets of derivatives of model A. Thus, the pitch structure of the random walks as a whole articulates an ABA design which is audible to the extent that the difference between the pset types is audible.

Aspects of rhythm and contour also reinforce the similarities and differences between the pset types. For example, triple and quadruple subdivision of the 16th-note is especially characteristic of configurations containing A-type psets, while duple subdivision is more characteristic of configurations containing B-type psets. Segments 1 and 3 feature triple subdivisions of the 16th-note, and present derivatives A_0 and A_1, respectively. Segment 5 features both triple and duple subdivisions, and reiterates derivative A_0. Segments 7 and 9, which present B_0 and B_1, respectively, also feature a mixture of triple and duple subdivisions. Thus segment 5, from the rhythmic standpoint, is transitional between the first group of random walks containing A-type psets and the first two random walks containing B-type psets. Segments 16, 17, and 19 feature mainly duple subdivisions of the 16th-note, with or without repeated articulations, against which more complex rhythms appear in segment 16. These segments represent derivatives B_2 (in 16 and 17) and B_3 (in 19).

The return of A-type psets, represented by a subset of A_2 in segment 21, is marked by the return of strict triple subdivisions of the 16th-note. Segments 26, 28, and 29 recall segment 5 in their pset contents, each presenting a subset of A_0. They also resemble segment 5 in their range and in the motion of their voices, which is strictly parallel in segments 26, 28, and 29, and parallel throughout most of segment 5. A new characteristic introduced here, however, is the quadruple subdivision of the 16th-note. This rhythm, along with parallel motion, appears again in segments 34, 35, and 36, which present derivatives A_3, A_4, and A_5, respectively. The increased pace of the pset transpositions, which is due to the brevity of these segments, is carried further in segment 37, whose pitch structure is a composite of subsets of derivatives A_4, A_6, A_7, and A_8. The quadruple subdivisions of the 16th-note in this configuration result in a high density which, combined with its high intensity, wide registral span, and long duration, make this segment the high point on the MDS scale (see Figure 3.20). The extraordinary richness of the pitch structure of this segment is appropriate to its climactic function within the work's form, as indicated in the MDS scale. The final random walk, in segment 39, represents something of a cooling off after the explosive activity of segment 37. Triplet subdivisions of the 16th-note return, as well as derivative A_1, which was last heard in segment 3.

To sum up, the succession of psets in the random walks of *à r.* articulates an ABA structure overall, based on the derivations of the psets from two different models. The first succession of derivatives of model A articulates a miniature ternary structure, for the succession of segments 1, 3, and 5 contains subsets of derivatives A_0, A_1, and A_0, respectively. The succession of derivatives of model B contains four psets found in *Mists*. The second succession of derivatives of model A contains several more distinct psets, but recurrences of A_0—in segments 26, 28, and 29, with morphological features in common with segment 5—and A_1, in segment 39, refer back to the first succession of derivatives of model A and thereby demonstrate a significant instance of return in the pitch structure.

The pset sources of the simultaneities are more difficult to identify. Part of the difficulty stems from the fact that the cardinalities of the simultaneities are quite small in comparison to the cardinalities of the random walks. The establishment of definite inclusion relations within larger psets, therefore, is problematic. Another difficulty comes from the fact that the simultaneities appear to be derived not from either of the models separately, but from the union of various derivatives of models A and B. Because of the special difficulties involved in its interpretation, the pitch structure of the simultaneities is represented separately from that of the random walks, in Table 3.14. This table shows two methods for deriving the pset contents of the simultaneities from unions of the derivatives of models A and B. The first method is based on the union of two additional derivatives that were not represented in the random walks, i.e. A_9 and B_4. These two derivatives show an interesting property in relation to one another. Their intersection is concentrated within their middle-to-upper range, i.e.

$$A_9 \cap B_4 = \{-29\ -26\ -4\ 1\ 9\ 15\ 16\ 20\ 24\ 26\ 30\ 34\ 39\ 43\ 45\ 48\}.$$

This intersection is not, in itself, the source of the pitches in the simultaneities, but the union of these two psets is the source for six out of the eleven distinct psets in the simultaneities, or sixteen out of the twenty-two simultaneities overall. The range of these sixteen simultaneities falls between pitches -4 and 26, or A3 and D6, inclusive. The subset of $A_9 \cup B_4$ containing just these pitches is shown in Table 3.14 following the complete contents of $A_9 \cup B_4$.

This first method of relating the psets in the simultaneities to models A and B is abstract, and is independent of the compositional contexts in which the simultaneities occur. It leaves out of consideration the six simultaneities that contain some pitches that are not found in $A_9 \cup B_4$, although most of their pitches can be found in this union. A second method of relating the pitches in the simultaneities to the models takes into account the positions of the simultaneities within the temporal succession of segments, and relates them in specific ways to the random walks that precede or follow them. Because of the low cardinality of the simultaneities in relation to that of the unions of the derivatives of models A and B, it is often possible to relate a given simultaneity to several different unions. In order to relate each simultaneity to psets within its compositional context in a straightforward way, however, only those unions that involve at least one of the derivatives found in the random walk that precedes or follows the simultaneity are listed in Table 3.14 as likely sources for the pitches in that simultaneity. Out of necessity, the "abstract" derivatives A_9 and B_4, which are not represented in any of the random walks, are included in the candidates for pset unions, for it is impossible to relate the psets in all of the simultaneities to derivatives of models A and B without including them. The contents of the unions are not spelled out in the table, but the reader is free to refer back to the contents of the derivatives in Table 3.13 and in the first part of Table 3.14 in order to verify the inclusion relations between the psets in the simultaneities and the unions with which they are associated.

Note that segment 14 is counted among the simultaneities in the interpretation of the work's pitch structure. Although there were morphological reasons for including this segment among the random walks in calculating the proportions of the temporal structure outside-time, in terms of its pitch structure it clearly belongs among the simultaneities, for it cannot be related directly to any of the derivatives of model A or B. Thus the structural status of segment 14 is truly ambiguous. It is also the first among the simultaneities (interpreted as such according to its pitch structure) that cannot be included in $A_9 \cup B_4$. It therefore signals the beginning of a change in the pitch structure of the simultaneities that has repercussions intermittently throughout the remainder of the work, for segments 24, 30, 33, 38, and 40 all contain at least one pitch that falls outside of $A_9 \cup B_4$, while the remaining simultaneities are all related to this union by inclusion.

In conclusion, *à r.* is a brief work based on two contrasting configuration types. Patterns in the succession of configuration types divide it into six sections. A change in tempo at m. 10 and the simultaneous introduction of repeated articulations suggest a larger division into two parts whose temporal proportions approximate the golden section. Superimposed upon the temporal structure that is articulated by changes in the pattern of succession of configuration types, and by the change in tempo and articulation, is a tripartite pitch structure based on two distinct m-pset models. The psets in every segment of *à r.* may be related, more or less directly, to either or both of these models. The interpretation of the pitch structure presented here is the result of observation of the musical surface and extrapolation of its details into an abstract structure of pset transpositions and unions similar to those found in *Evryali* and *Mists*. Sketch materials to *à r.* have not been made available by the composer for use in this analysis, but the characteristics of the pitch structure indicated in Tables 3.13 and 3.14 are consistent with Xenakis's practice as exemplified in the sketches to *Mists* and also with the composer's theoretical writings.[37] The type of pitch structure evident here, as in the other works for piano, is a loosely constructed system of pset relations that articulates a relatively simple overall design.

Conclusion

This chapter has shown how several of the structural features that were described in relative isolation in chapter 2 may be combined as components of an analytical approach to specific works. One problem encountered in the analyses is that the psets used by Xenakis, because of their generally large cardinalities and their frequently partial representations on the musical surface, defy concise classification. Added to these difficulties is the fact that the transformations of the m-psets that are the apparent source for the psets on the surface of all three works discussed here, whether these transformations consist of unfoldings in p-space (as in *Evryali*) or cyclic transpositions (T_n(mod m)) (as in *Mists* and *à r.*), are not always carried out systematically or, if they are, the results are not always transcribed accurately into the score. A preliminary attempt to deal with these difficulties is the creation of pset models. While this solution is not free of problems, it at least offers an orientation to the pitch structure of the works that takes into account the vast majority of the details on the surface of the music and relates

[37]See, in particular, chapters 6-8, 11 and 12 of *Formalized Music* and the presentation of some of this material in sections 2.1.6 and 2.1.7 of this dissertation.

these details meaningfully to a relatively simple underlying system of m-psets and their transformations. In general, difficulties of this kind will not be encountered in the analyses in the remainder of the dissertation, since two of the three works chosen for examination in chapter 4 do not feature differentiated pitch structures. The remaining work, *Theraps* for double bass (1975-6), does contain some specific psets, but their organization is much simpler than that found in the works for piano. Pitch levels may only be determined in a relative sense in the electroacoustic work *Mycenae-Alpha* (1978), which is examined in chapter 5. Psets, therefore, play no part in the analysis of that work.

Aspects of these analyses that reappear in subsequent chapters, however, include examinations of temporal structure and configuration types, both inside- and outside-time. Because string instruments do not allow for as wide a range of densities as does the piano, the interpretation of form according to an MDS scale is less revealing in these works than in the works for piano. (An exception is *Theraps*, however, in which a reduced version of the MDS—one in which density is not taken into consideration—points out some interesting structural features.) Xenakis compensates for this limitation in the range of densities by focusing special attention on the relative location of events in p-space in the music for strings. As a consequence, the articulation of regions of p-space with respect to time plays an important role in the definition of form in these works that is comparable to the distribution of states in several dimensions in the piano works. Thus, although random walks are used both in music for strings and for piano, their structural significance is defined in each case according to the specific characteristics of the instrument for which the music has been composed. Because of the change in structural priorities in the music for strings, the amount of quantitative information presented in chapter 4 is reduced in comparison with what has been presented here, although the compositional principles underlying both groups of works are clearly related.

Chapter 4

Works for Solo Strings

String instruments have figured prominently in Xenakis's compositions since the beginning of his career. His first important works, *Metastaseis* (1953-4) and *Pithoprakta* (1955-6), both for orchestra, feature an instrumentation that is balanced heavily in favor of the strings. He has also written several works for string ensemble, including *Analogique A* for 9 strings (1958), *Syrmos* for 18 or 36 strings (1959), *Aroura* for 12 strings (1971), *Shaar* for string orchestra (1983) and, most recently, *Voile* for 20 strings (1995). His output also includes four string quartets: *ST/4* (1956-62), *Tetras* (1983), *Tetora* (1990), and *Ergma* (1994). He seems to have been attracted to string instruments as vehicles for his musical ideas because of the great variety of timbres and articulations that they offer, including the possibility of continuous modulations in pitch with respect to time, i.e. glissandi, which have been a staple feature of his compositional style.

Xenakis's music for solo strings represents the characteristics of his writing for these instruments in microcosm. His first work for a solo string instrument was *Nomos Alpha* for violoncello (1966). As explained in chapter 2, this work features a variety of configuration types whose characteristics are sequenced according to operations based on group theory. *Nomos Alpha* has been discussed at length by Xenakis and others, and will not be discussed further here.[1] Xenakis's next work for a solo string instrument was *Mikka* for violin (1971), which features an intensive exploration of continuous random walks in a single voice. *Mikka "S"* for violin (1976), a companion piece to *Mikka*, consists principally of continuous random walks in two voices. *Theraps* for double bass (1975-6) combines non-glissando random walks in a single voice with two-voice, glissando random walks and sustained harmonics in two voices. These three works—*Mikka, Mikka "S"*, and *Theraps*—form a representative sample of the music for solo strings that encompasses both extremes of the pitch register available to these instruments. More importantly, this selection presents a gradual increase in textural variety from work to work, a characteristic that has specific structural consequences for each individual work. Thus these three works provide a coherent introduction to Xenakis's music for solo string instruments. Two additional works for solo strings are *Kottos* for violoncello (1977) and *Embellie* for viola (1981). These works, in addition to containing continuous random walks in one or two voices, include rhythmic patterns that take place within small psets, similar to those found in *Evryali* for piano, as well as other textures involving sounds of both definite and indefinite pitch.[2] Because of the difficulties involved in the graphic representation of sounds of indefinite pitch, and because the three works selected provide an adequate

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[1]See Iannis Xenakis, *Formalized Music*, rev. ed. (Stuyvesant, NY: Pendragon Press, 1992), chapter 8; Fernand Vandenbogaerde, "Analyse de Nomos Alpha de Iannis Xenakis," *Mathématiques et Sciences Humaines* 24 (1968): 35-50; Thomas DeLio, "Iannis Xenakis' Nomos Alpha: The Dialectics of Structure and Materials," *Journal of Music Theory* 24/1 (1980): 63-95; Jan Vriend, "'Nomos Alpha' for Violoncello Solo (Xenakis 1966): Analysis and Comments," *Interface* 10 (1981): 15-82; and my comments in chapter 2, section 2.3.

[2]All of the works for solo strings have been recorded by members of the Arditti Quartet on Disques Montaigne 2 CD 782005, with the exception of *Theraps*, which has been recorded by Robert Black on Neuma CD 450-71.

sample of the variety in Xenakis's writing for solo string instruments, these two works will not be considered in detail here.

Mikka for violin (1971)

Mikka means *small* in the ancient Ionian and Dorian languages, as well as in modern Romanian. The title is also a play on the name of Xenakis's Romanian-born publisher, Mica Salabert, to whom the work is dedicated. *Mikka* is a brief work, as its title suggests: its duration is just under 4 minutes. It is also extremely restricted with regard to texture, consisting entirely of a single, continuous glissando line from beginning to end. As indicated in chapter 2, section 2.2.2, the model for the configuration type used throughout the work is the continuous random walk, a model in which the position of a particle changes as a function of its velocity. In musical terms this means that the pitches at the endpoints of the glissando's arcs are determined by its speed and direction—up or down—at specific points in time. The flexibility of the line that results from these fluctuations in speed and direction is clearly visible in the annotated score, shown in Figure 4.1. The scoring of the work is unusual in that no barlines are indicated, thereby emphasizing all the more the continuity of the glissando line. The unit of temporal measurement in this work is the sixteenth-note at ♩ > 60 MM. In order to facilitate analysis the indicated tempo has been simplified to ♩ = 60 MM, at which the sixteenth-note is equivalent to 0.125". In lieu of measure numbers, seconds have been indicated to the left of each system of the score.

Despite the undeniable continuity on the surface of the work there are some factors that serve to articulate various degrees of structural differentiation. First, there are differences of contour within the line. These differences are more readily apparent in the graphic transcription, shown in Figure 4.2, than in the score itself.[3] Some areas of the work show a relatively smooth contour, while other areas reveal a markedly jagged contour. This general difference in contour type has important structural ramifications, which will be treated in more detail as the discussion progresses. For now, however, it is sufficient to point out that a smooth contour results from the traversal of relatively small pitch intervals with respect to time, while a jagged contour results from the traversal of large intervals with respect to time.

Other factors that contribute to structural differentiation include differences in the intensity, articulation, and duration of sonic events. Intensities vary within a range from *ppp* to *ffff*. There are three types of articulation: *arco normale, sul ponticello*, and *sul ponticello tremolo*. Durations vary from a sixteenth-note to four whole-notes tied together (i.e., from 0.125" to 8"). (See Figure 4.1, system 1, which contains both extremes in the range of durations.) Four factors—change in pitch interval size with respect to the size of the surrounding intervals, change in intensity, change in articulation, and change in duration—have been considered in determining the segmentation that is represented in Table 4.1. The segment numbers found in the table have been indicated in the score above the appropriate staves and along the upper margin of the graphic transcription. This segmentation, which is based upon an intuitive interpretation of how the four factors contribute to structural differentiation, compares favorably with

[3]The p-space in this work, and in Xenakis's works for strings in general, is calibrated in quarter-tones. The origin (0) is set at middle C (C4). An octave is equivalent to 24 quarter-tones.

the results of an application of the temporal gestalt method for segmentation proposed by Tenney and Polansky.[4]

The articulation of form in this work differs from its articulation in the piano works. This is due in part to the fact that the range of densities in *Mikka* is quite narrow in comparison to the ranges of densities in the piano works. (Compare Table 4.1 with Tables 3.1, 3.5, and 3.10). Changes in density, therefore, do not serve as a significant means for achieving structural differentiation in *Mikka*. Also missing from *Mikka* is the variety of configuration types found in the piano works. Further, there are no discernable pset transformations in *Mikka*. Differences in pset contents from segment to segment are merely the result of changes in speed and direction as the line moves through p-space. The formal design of *Mikka*, therefore, is articulated principally through differences in the range and position of segments within p-space.

A graph of maximum, minimum, and mean pitch levels per segment is shown in Figure 4.3.[5] It is immediately apparent from the graph that

[4]James Tenney with Larry Polansky, "Temporal Gestalt Perception in Music," *Journal of Music Theory* 24 (1980), 205-41. I applied Tenney and Polansky's method to the grouping of sonic events into segments in *Mikka*. No applications to higher levels of structure were attempted, since I did not have access to a computer program which could perform the necessary calculations. According to their method, the various structural factors considered in the segmentation of a given work are weighted numerically. These numerical weights are determined empirically and have a definite effect on the resulting segmentation. Once appropriate weights were determined, segmentation according to their method helped me to fine-tune my initial attempts at an intuitive segmentation of *Mikka*. The segmentation finally settled upon is the one given in Table 4.1. Tenney and Polansky's method puts into practice some of the ideas found in Tenney's *Meta/Hodos and Meta Meta/Hodos* (Hanover, NH: Frog Peak Music, 1992). Their method is limited to single-line textures, and therefore is ideally suited to a work like *Mikka*. For an extension of Tenney and Polansky's method to more complex textures, see Yayoi Uno and Roland Hübscher, "Temporal-Gestalt Segmentation: Polyphonic Extensions and Applications to Works by Boulez, Cage, Xenakis, Ligeti, and Babbitt," *Computers in Music Research* 5 (1995): 1-38. The Xenakis work analyzed by Uno and Hübscher is *Herma* for piano.

[5]The horizontal axis of the graph in Figure 4.3 is calibrated in seconds so that the relative durations of the segments may be appreciated. The locations of the segments are not labelled individually in the figure due to space limitations. The locations of several segments are given

changes in pitch range take place more frequently between (approximately) 50-100" than they do between 0-50". The reason for this difference is that an alternation between smooth and jagged contours is established in segment 4 (47.25") with the introduction of the first jagged contour. (This is also clearly visible in the graphic transcription, Figure 4.2.) This alternation effectively ceases with the resumption of smooth contour alone in segments 28 (98.625") and 30 (102"), interrupted only briefly by a slightly jagged contour in segment 29 (101.5"). Another striking feature of the graphs in Figures 4.2 and 4.3 is the nearly uninterrupted descent in the line from segment 40 (142") to the end. The slow descent helps to connect the extremely high register used in the alternation of smooth and jagged contours, found in segments 9-27 (62.125-98.625"), to the low register used in the beginning of the work and in segments 31-39 (121.125-142").

Segments may be grouped into sections according to changes in the pattern with which smooth and jagged contours are presented. When the segments are grouped in this way, the sections are as follows:

section	segments	seconds	contour
1	1-3	0.000-47.250	smooth
2	4-27	47.250-98.625	alternating
3	28-30	98.625-121.125	smooth
4	31-39	121.125-142.000	alternating[6]
5	40-48	142.000-193.25	smooth

in seconds in the text, however, and a full list of locations in seconds is given in Table 4.1. The segments are labelled individually in Figures 4.1 and 4.2, which may be compared with the more concise representation in Figure 4.3.

[6]In segments 36 (131.5") and 37 (135") smooth contours succeed each other. These two segments are differentiated from one another, however, by their intensity and degree of rhythmic activity. Segment 36 consists of two events of long duration, *pp*, and segment 37 consists of steady 16th-notes, *fff*. The impression of alternating contrasts, therefore, is maintained, although not strictly on the basis of contour in this case. The apparent exception to the rule at this point is analogous to the status of segment 29 (101.5") in section 3, whose slightly jagged contour does not disrupt significantly the prevailingly smooth contour of the surrounding segments because it is similar to them in intensity and articulation.

This division of the work into sections suggests an a ABABA formal plan that is based on patterns in the presentation of contour types.

This segmentation, however, fails to account for differences in the degree to which the contrasting patterns are separated on the surface of the music. In particular, the arrival of the first episode of alternating contour types is prepared by an increase in the complexity of the preceding smooth contour. The first jagged contour, which initiates the first episode of alternating contour types, appears in segment 4 (47.25"). A transition between the very smooth contour of segment 1 and the jagged contour of segment 4 is effected by the increasingly active, but not yet jagged, contours of segments 2 (34.875") and 3 (41.25"). The arrival of segment 4 is further prepared by the increase in intensity from *mf* to *fff* at the end of segment 3. Segments 4-7 (47.25-54.625") articulate a pattern of contours that alternates between smooth and jagged, but at segment 8 (54.625") the pattern is disrupted as the smooth contour of this segment follows the smooth contour of the preceding segment. At this point a pattern is initiated in which extremely smooth contours alternate with extremely jagged contours. The jagged contours in this pattern, which extends through segment 27 (98"), are extremely brief and contain very wide intervals, with the result that they appear like spikes in their graphic representation in Figure 4.2. The disjunction between segments 7 and 8 is further emphasized by the sudden change in intensity (*fff* to *ppp*) and articulation (*arco normale* to *sul ponticello*) that occurs at the beginning of segment 8 (54.625"). These considerations suggest a need for a revision of the preliminary segmentation given above. A new segmentation is given below, in which the boundary between sections 1 and 2 is pushed ahead so that it occurs between segments 7 and 8 instead of between segments 3 and 4.

section	segments	seconds	contour
1	1-7	0.000-54.625	mixed (smooth, then alternating)
2	8-27	54.625-98.625	alternating
3	28-30	98.625-121.125	smooth
4	31-39	121.125-142.000	alternating
5	40-48	142.000-193.25	smooth

In this interpretation of the work's form, section 1 appears to be structurally more complex than the other sections. If this section is viewed as a sort of exposition, in which both contours are given in direct succession, then the difference in its degree of complexity may be seen in a fairly traditional structural context, with the following sections playing out the implications of the material that is introduced at the beginning of the work. Despite the change in contour pattern within section 1, there are more factors that favor structural cohesion within it than there are within the originally proposed section 2 (segments 4-27). These factors include the gradual change in contour in segments 2 and 3 and the general consistency in intensity and articulation within segments 1-7, as opposed to the striking changes in intensity and articulation between segments 7 and 8. There is also the strictly consistent pattern of contrasts in contour in segments 8-27 that holds the new section 2 together in a particularly coherent fashion. Nonetheless, some ambiguity over the exact location of the boundary between sections 1 and 2 remains. An additional possibility is that segments 4-7 form a separate section. The ambiguity of the precise boundaries of the opening sections will be explored further as the discussion continues, for upon consideration from several perspectives this feature seems to function significantly within the work's overall structure.

The boundaries between the remaining sections are more straightforward. The boundary between sections 2 and 3 (at segment 28, 98.625") is marked by the introduction of a new articulation. In this case the new articulation is *sul ponticello tremolo*. The boundary between sections 3 and 4 occurs in the vicinity of 120", where three changes in different dimensions occur successively. There is a crescendo from *ppp* to *fff*, a change in timbre from *sul ponticello tremolo* to *arco normale*, and a change in contour from smooth to jagged. The latter change is the most noticeable of the three, and therefore serves as the definitive criterion for determining the boundary between these sections, which occurs between segments 30 and 31 (121.125"). The boundary between sections 4 and 5 is very clear, for it is articulated by the sudden jump upward in p-space at segment 40 (142"). The long descent that occupies the whole of section 5 proceeds directly from this point.

In light of the ambiguity of the boundary between sections 1 and 2, neither of the segmentations suggested above should be adopted or dismissed without further reflection. So far, the second segmentation, in which the boundary between sections 1 and 2 occurs between segments 7 and 8, appears to be the stronger of the two. As such, it will be referred to in the remainder of this discussion as the *primary segmentation*. The first segmentation, in which the boundary between sections 1 and 2 occurs between segments 3 and 4, will be referred to as the *secondary segmentation*. Each segmentation is supported structurally by slightly different factors, and each has particular implications for the interpretation of the work's temporal structure. The distinction between them, therefore, should be kept in mind as the discussion proceeds.

The primary segmentation results in an inside-time durational structure that is remarkable for its elegance and simplicity. The duration of each of the sections in this segmentation is given below:

section	duration (sec)
1	54.625
2	44.000
3	22.500
4	20.875
5	51.250

The list shows that the durations of sections 1 and 5 are similar. Further, a little bit of addition shows that the duration of sections 3 and 4 combined, 43.375", is very similar to the duration of section 2.

The implications of these similarities for the large-scale temporal structure of *Mikka* are explored further in Table 4.2. In the table a two-part structure is proposed, in which the boundary between parts 1 and 2 coincides with the boundary between sections 2 and 3. The reader will recall that at this point, 98.625", the articulation *sul ponticello tremolo* appears for the first time. The boundary between sections 2 and 3 is stronger than any of the boundaries between segments in section 2 because of the multiple contrasts in duration (dotted whole note D♭6 at 95-8" surrounded by sixteenth-notes), contour (alternation between smooth and jagged), intensity (*fff* to *ppp*) and articulation (*arco normale* to *sul ponticello tremolo*) that occur near the end of section 2 and the beginning of section 3. The change in articulation to *sul ponticello tremolo* is particularly significant here, for it differentiates the timbral quality of the material in section 3 (and therefore also of the beginning of part 2) from that of the previous material, even though it is related in its contour, intensity, and register to the smoothly contoured portions of section 2.

The work is nearly bisected at the boundary between parts 1 and 2, where its total duration is divided into proportions of 0.51 and 0.49. This proportion is approximated in the inner divisions within the parts: the durations of sections 1 and 2 represent .55 and .45, respectively, of the total duration of part 1, and the durations of sections 3 and 4 combined (represented as supersection A in Table 4.2) and of section 5 (supersection B) represent .46 and .54, respectively, of the total duration of part 2. This proportion is also reflected in the relation between the durations of sections 3 and 4 within supersection A, which divide its duration into proportions of 0.52 and 0.48.

A summary of the temporal structure of *Mikka*, outside-time, is shown in Table 4.3. The table shows divisions in the outside-time temporal structure with respect to contour and articulation. Sections 1, 3, and 5 in the secondary segmentation are made up entirely of segments with smooth contours, with the sole exception of the jagged contour in the very brief segment 29. The combined duration of this group of sections is 121". Sections 2 and 4, also in the secondary segmentation, consist of segments with alternating smooth and jagged contours, and have a combined duration of 72.250". This grouping of sections according to patterns in the presentation of contour types divides the work's duration outside-time into proportions of 0.626 and 0.374. The outside-time temporal structure with respect to articulation shows remarkably similar proportions, with 119.375" of *arco normale* and 73.875" of *sul ponticello* and *sul ponticello tremolo* combined, producing proportions of 0.618 and 0.382 respectively. This partition of the total duration corresponds precisely to the golden section and its complement. Note that the partition of the total duration according to articulation is independent of the division of the work into sections. Nevertheless, the partition according to patterns in the presentation of contour types, which recognizes the work's sectional divisions (in the secondary segmentation), results in proportions that closely resemble those of the partition according to articulation.[7]

The outside-time partition of the total duration of *Mikka* according to articulation and pattern in the presentation of contour types is matched by an inside-time partition at the boundary between sections 3 and 4. This occurs at 121.125", which divides the work into proportions of 0.627 and 0.373. The reader may recall that the precise location of this boundary was somewhat unclear, due to the fact that changes in intensity, articulation, and contour occur out-of-phase at this point. In the interpretation given above, contour was selected as the decisive criterion, and therefore the boundary between the sections was set at 121.125" (sixteenth-note C5 at the end of the crescendo mark, preceded by eighth-note C#5 and followed by sixteenth-note G#4) rather than at the beginning of the crescendo (C/5, 119.75") or at the change in articulation back to *arco normale* (B4, 120.375"). These alternative segmentations would partition the work into proportions of 0.62 and 0.38, or 0.623 and 0.377, respectively. Thus, various approximations of the golden section, which correspond closely to the outside-time temporal structure of the work, are reflected on the surface of the music inside-time. These

[7]If the duration of segment 29, however, is added to the duration of the sections with alternating contours and is subtracted from the duration of section 3, the durations for each category would be 120.5" for smooth and 72.75" for mixed jagged and smooth contours, yielding proportions of 0.624 and 0.376, respectively. These proportions are slightly closer to 0.618 and 0.382 than the 0.626 and 0.374 given above. The slight gain in accuracy of the proportions is offset by the confusing segmentation required to obtain it, however, so this minor revision in segmentation cannot be recommended without reservations.

multiple approximations occur as a result of the staggering of changes in several dimensions as the actual point of division between sections 3 and 4 is approached.

The outside-time temporal structure presents an additional approximation of the golden section that corresponds to an aspect of the inside-time temporal structure. The combined duration of *sul ponticello* and *sul ponticello tremolo* articulations, 73.875", is divided into 48.25" for *sul ponticello* alone and 25.625" for *sul ponticello tremolo.* This produces proportions of 0.65 and 0.35, respectively, which is a fairly rough approximation of the golden section nested within the precise golden section that partitions the work into *arco normale* and *sul ponticello* (with and without *tremolo*) articulations. (See Table 4.3.) The outside-time duration of the *sul ponticello* articulation without *tremolo*, 48.25", corresponds very closely to the combined inside-time duration of segments 1-3, 47.25", which constitute section 1 in the secondary segmentation.

In summary, the primary segmentation shows a bisection of the work into parts of approximately equal duration, with similar proportions reflected within the parts. The unusual elements within this segmentation are sections 3 and 4, which are notably shorter than the other sections. The boundary between these two sections, however, approximately bisects supersection A within part 2. (See Table 4.2.) A comparably elegant outside-time segmentation of the work was also proposed which, like the inside-time segmentation, is based on patterns in the presentation of the contour types. This outside-time segmentation, however, relies upon a different grouping of segments into sections. Despite its differences from the primary segmentation, however, this segmentation corresponds closely to the outside-time segmentation according to articulation type. (See Table 4.3.) In addition, the outside-time segmentations by pattern in the presentation of contour types and by articulation types correspond to a possible segmentation of the work into two parts between sections 3 and 4, instead of between sections 2 and 3. (See again Tables 4.2 and 4.3.) This segmentation, like the outside-time segmentations, is based on the golden section. Thus, ambiguities in the large-scale structure, involving the location of the boundaries between sections 1 and 2 and parts 1 and 2, can be integrated into an interpretation of the whole that recognizes two distinct proportional systems—one based on bisection and the other based on the golden section—operating simultaneously in the work's temporal structure.

One further aspect of the temporal structure of *Mikka* deserves mention. As in the piano works analyzed in the previous chapter, the durations of the segments appear to have been chosen in accordance with the exponential probability distribution. The density of the expected distribution in this case is 48 segments/193.25 seconds, which is equivalent to 0.248 segments/second. A comparative histogram of the expected and observed frequencies of segment durations is shown in Figure 4.4. Once again, the durations indicated by the exponential distribution have been assembled by the composer into a succession that articulates the proportions of the large-scale temporal structure.

In conclusion, although *Mikka* at first appears to be a single, uninterrupted random walk through p- and tp-space, it is revealed on closer examination to consist of a succession of random-walk fragments that have been assembled in such a way that they articulate both a well-defined large-scale temporal structure and a relatively simple large-scale structure in p-space. These large-scale temporal and p-space structures are shown together in Figure 4.5, which presents a graph of the minimum, maximum, and mean pitch values within sections. The section durations illustrated in the graph follow the primary segmentation, which emphasizes the approximate bisection of the work into parts and of the parts into sections. The alternative large-scale segmentation, in which the work divides approximately according to the golden section at the boundary between sections 3 and 4, is also given concrete expression in the graph, since the registral span and position of section 1 return in section 4.

While the basic characteristics of the inside- and outside-time temporal structures in this work are generally consistent with the temporal structures found in the piano works, the manner in which form is projected in *Mikka* is particularly appropriate to the "melodic" nature of the violin. The articulation of form in *Mikka*, therefore, differs considerably from that of the piano works, which relies heavily on the articulation of strong contrasts in configuration type and in density, features that are not available within the more limited medium of music for solo violin. Thus, while certain general principles, specifically those having to do with the organization of the temporal aspect of the music, are fairly consistent among these works, other features of the compositional organization differ considerably depending on the medium in which he writes and on the basic materials chosen for each work.

***Mikka "S"* for violin (1976)**

Mikka "S", like its predecessor *Mikka*, is dedicated to Mica Salabert. A companion piece to the earlier work, it may be performed immediately following *Mikka* or on its own, according to the composer's instructions in the score. (An annotated score of *Mikka "S"* is shown in Figure 4.6.) *Mikka*

"*S*" is not a companion piece to *Mikka* in name only. Both works are based on continuous random walks, realized as glissandi, and both involve issues of continuity and discontinuity as their structures unfold in time. There are, however, significant differences in the ways these basic characteristics are manifested in each work. The glissandi in *Mikka "S"* are of two basic types, as opposed to the single type found in *Mikka*. The first type consists of long, slow glissandi whose entrances overlap, resulting in a two-voice texture. The second type consists of short, disconnected gestures that are cut off abruptly by upward strokes toward the frog of the bow.[8] In *Mikka* continuity was expressed by means of the contact between bow and strings that was maintained throughout. Relative discontinuity, on the other hand, was expressed by the changes in intensity, contour, duration, and timbre that formed the basis for segmentation. In *Mikka "S"* the play between continuity and discontinuity is much different. Contact between the bow and strings is maintained for the first 60 measures of this 76-measure work through the overlapping of the slow glissandi. The final 16 measures, however, consist almost entirely of brief, disconnected glissandi, interspersed occasionally with sustained pitches that are presented either singly or in double stops. Thus, in *Mikka "S"* there is an unmistakable separation between continuous and discontinuous textures. This separation differs markedly from the integration of glissandi with smooth and jagged contours in *Mikka*, which was accomplished through the establishment of patterns of alternation between the contour types. The change from continuous to discontinuous textures in *Mikka "S"* is marked by a halving of the tempo, from ♩ ≥ 54 M.M.

[8]The performance instructions on the last page of the score (see Figure 4.6) refer to the frog, or heel (French *talon*) of the bow as the hell [sic].

at the beginning to ♩ ≥ 54 M.M. at m. 61, thus suggesting a primary structural division at that point.[9] Despite the differences in their basic materials, there are some similarities of a more general nature between the two works. Both works have a duration of between three and three-and-a-half minutes[10] and each divides more or less clearly into five sections. Each work also exhibits a similar pattern in the use of p-space as it unfolds in time. These general similarities will be explored further as the discussion proceeds.

Segmentation in *Mikka "S"* is less clear-cut than it is in *Mikka*, mainly because of the overlapping glissandi in its first part. Its segmentation, therefore, will be dealt with in two stages, the first based on the durations of the segments outside-time and the second on the duration between the initiations of the segments as they occur inside-time. In the outside-time segmentation, represented in Table 4.4, the durations correspond to the beginning and end points of the glissandi, whether they overlap or not. The table shows that overlaps occur only among segments 1-13. The boundaries of the segments consisting of short glissandi are determined by the points at which motion toward the frog of the bow is completed. This motion is indicated in the score by lines that terminate in grace notes. There are several instances where the lines that indicate glissandi between the notated events are missing from the score. In the segmentation used here it has been assumed that no break is intended unless motion toward the frog is indicated

[9]As in the works analyzed previously, the metronomic indications have been interpreted as precise values in the preparation of the graphic transcription (Figure 4.7) and the list of segments (Tables 4.4 and 4.5).

[10]The timings for Irvine Arditti's recording of these works, on Disques Montaigne 2 CD 782005, are each about half a minute longer than the durations indicated in their scores: 3'58" for *Mikka* and 4'10" for *Mikka "S"*, versus the 3'13" and 3'24" indicated, respectively, by the scores.

specifically by means of a grace note, or a single or double-stopped event of long duration intervenes between the glissandi. Events of the latter type are introduced in m. 66. Segments 14-129 all occur within the space of mm. 61-76. Due to considerations of space and legibility, these segments have not been indicated individually in the annotated score (Figure 4.6) or in the graphic transcription (Figure 4.7).

An inside-time segmentation is represented in Table 4.5. The durations given in the inside-time segmentation correspond to the time spans from the initiation of each numbered segment to the initiation of the following segment. The duration of the time span from the beginning of segment 129 to the end of work is included so that both the inside- and outside-time segmentations contain the same number of segments. The inside-time segmentation differs from the outside-time segmentation only with respect to segments 1-13, because of the overlaps that occur among these segments. The durations of the remainder of the segments in Table 4.5 are the same as in Table 4.4 because no overlaps occur among segments 14-129.

The total duration of the segments outside-time, as presented in Table 4.4, is 312.778." Examination of the durations in the table reveals a wide range of values, from 0.278" (e.g., segments 15, 16, 19) to 86.111" (segment 5). The difference between the minimum and maximum values may seem extreme, but it may be accounted for easily if the distribution of durations is considered as a whole. For a sample size of 129 segments with a total duration of 312.778", the mean number of segments per second is 0.412. The actual distribution of durations in *Mikka "S"* compares favorably with an expected exponential distribution, $\partial = 0.412$, as shown in the comparative histogram in Figure 4.8. As in other works featuring overlapping segments, such as *Herma* (see chapter 2, section 2.3 and Figures 2.55 and 2.56), the distribution of durations inside-time in *Mikka "S"* also corresponds to an exponential distribution. In this case, the mean density of the distribution is 0.631 segments/second. This value corresponds to the total of 129 segments divided by the total inside-time duration, which is 204.444". A comparative histogram showing the distribution of segment durations inside-time versus the expected probabilities of the exponential distribution, ∂ = 0.631, is shown in Figure 4.9. As in Figure 4.8, the curves for both the observed and expected distributions closely resemble one another.

The boundaries of the segments are significant not only for the global temporal structure they articulate cumulatively, but also for the way they partition the p-space as the work unfolds in time. Figure 4.10 shows maximum, minimum, and mean pitch levels for segments 1-13 and for groups of segments from segment 14 to the end. The boundaries of the segment groups are determined by the appearances of the isolated events, whose durations are longer than the surrounding glissando gestures. Each group ends with an isolated event, with the exception of the final group, whose end coincides with the conclusion of the work. In this way segments 14-129 are gathered into ten groups, as indicated below:

group	segments	measures	seconds
1	14-57	61-66	133.333-158.889
2	58-65	66-67	158.889-163.611
3	66-74	67-68	163.611-168.333
4	75-78	68-69	168.333-170.833
5	79-84	69	170.833-173.333
6	85-94	70	173.333-172.222
7	95-112	70-73	172.222-186.944
8	113-122	73-75	186.944-200.000
9	123-127	76	200.000-203.889
10	128-129	76	203.889-204.444

The figure shows that segments 1-6 (0.000-59.444") occupy a low position in p-space, while a gradual ascent is discernable in segments 7-13 (59.444-133.333"). The segments in group 1 (133.333-158.889") once again occupy the low register, while those in group 2 (158.889-163.611") regain the extreme high register formerly occupied by segments 12 and 13 (118.056-133.333"). From the end of group 2 until the conclusion of the work there is a steep but irregular descent into the low register.

In its general outline the path traced through p-space in *Mikka "S"* resembles that found in *Mikka*. The general outlines of both paths include a gradual (and irregular) ascent from the low register followed by a return to that register. This is followed, in turn, by a succession of descending segments (or segment groups) that fill in the entire range of the p-space used in the work, effectively summarizing it as the work reaches its conclusion. (Compare Figure 4.10 with Figure 4.3.) Use of p-space, therefore, is one more aspect of structure that relates *Mikka "S"* to *Mikka*.

The use of p-space in *Mikka "S"* also serves as a basis for delimiting sections. On this basis the work may be divided into five sections, as follows:

section	segments	measures	seconds	duration
1	1-6	1-27	0.000-59.444	59.444"
2	7	27-47	59.444-103.611	44.167"
3	8-11	47-54	103.611-118.056	14.445"
4	12-13	54-60	118.056-133.333	15.277"
5	14-129	61-76	133.333-204.444	71.111"

The boundaries of the segments listed above are those used in the inside-time segmentation, which is given in full in Table 4.5. Minimum, maximum and mean pitch levels for each section are shown in Figure 4.11. The result is a combined registral profile and temporal structure that compares interestingly with the graph of pitch levels per section in *Mikka* (Figure 4.5). In the earlier work, however, the highest mean pitch level is attained in section 2, whereas it is approached gradually by means of a steady ascent in *Mikka "S"*, not arriving until section 4. After an ascent within narrow bands of p-space in sections 1 and 2 of *Mikka "S"*, the low register returns in section 3, at about halfway through the work. In *Mikka*, on the other hand, the return of the low register is delayed until section 4, coinciding approximately with the golden section in that work's large-scale temporal structure.

The criteria for grouping segments into sections in *Mikka "S"* are perhaps less obvious than they are in some of the works analyzed previously, principally because they are not based on any single factor such as patterns in the presentation of configuration types. Instead, each section is defined within the context of the work according to certain characteristics that are proper to it. Section 1 consists of a succession of overlapping segments that are confined within the range from G#3 to G4 (-8 to 14 in the graph), i.e. within the instrument's lowest octave. Only one new segment, segment 7, is introduced in section 2. The line in this segment creates a two-voice texture with the line at the end of segment 6 for most of the section. Here A4 and pitches near it, such as A/4 and A#4, act together as a focal point in p-space around which the line in segment 6 moves in close proximity. This results in some interesting acoustical effects, such as the appearance and disappearance of beats as the focal pitch is approached and left by the second line. Special articulations, such as the quarter-tone trill in m. 32 and and the eighth-tone and quarter-tone oscillations around focal pitches in mm. 40-45, are characteristic of section 2. These special articulations provide a degree of sonic variety to the section despite its extremely restricted pitch range, which extends only from E4 to C5 (8 to 24 in the graph). An eighth-tone oscillation also occurs in section 1 (m. 10), but this is an isolated incident that does not detract strongly from the association of such effects with section 2. Such effects are notably absent from the remainder of the work.

In section 3 a succession of overlapping segments occurs, calling to mind perhaps the similar succession in section 1, but this time the range of the p-space is extended upward significantly, first to B5 (m. 47) at the beginning of the section and finally to C-three-quarter-sharp6 (m. 54) at the end. The lowest pitch in section 3, G/3 (m. 50), is one quarter-tone lower than G#3 (m. 2), which had been the lowest pitch heard in the work so far. Section 3, therefore, includes and supersedes the p-space used in sections 1 and 2, and with its brief duration—at just over 14 seconds, it is the shortest section in the work—represents a marked acceleration in the coverage of p-space in comparison to the earlier sections. Section 3 provides a link between these earlier sections and section 4, which is confined to a high register. Like section 2, section 4 features two overlapping segments containing lines that operate at a close proximity to one another in p-space, only this time the p-space covered is within the range from E7 (m. 54) down to C#5 (m. 60), its low end directly above the high end of the p-space in section 2, i.e. C5 (m. 47).

Section 5 begins with a change in tempo (m. 61) and features a combination of short, abrupt glissando gestures and sustained pitches, both described previously. The use of p-space in section 5, taken as a whole, summarizes and supersedes the use of p-space in the previous sections. The lowest note in the section, G3 (m. 61, beat 2), is a quarter-tone lower than the G/3 in section 3 (m. 50), and the highest note, E/7 (m. 67, beat 1) is a quarter-tone higher than the E7 in section 4 (m. 54). Thus, section 5 functions in relation to the pitch structure of the entire work similarly to the way that section 3 functions in relation to sections 1 and 2.

The durations of the sections form a temporal structure whose proportions are reflected in several ways within the work, as shown in Table 4.6. The table demonstrates that the most prominent structural division occurs at the boundary between sections 4 and 5, which also coincides with the boundary between parts 1 and 2, at the tempo change in m. 61. At this point the total duration of the work divides into proportions of 0.65 and 0.35, which is slightly larger than the golden section. The importance of this proportion within the temporal structure of the work is reinforced by its approximate reappearance at the strongest internal division within section 5. This division occurs between segments 57 and 58, after the sustained E4 in m. 66. This is the point of greatest discontinuity with respect to p-space in the section and, in fact, within the work as a whole. There is also a maximal discontinuity with respect to intensity at this point, from *pp* to *ffff*. This division coincides with that between segment groups 1 and 2 within part 2, referred to above and shown in Figure 4.10, where its proportional aspect is evident with respect to the horizontal time line. The proportions of this division are 0.36 and 0.64, which are very close to the 0.65 and 0.35 of parts 1 and 2, but the order of the respective longer and shorter portions is reversed in section 5. The proportions of 0.36 and 0.64 are also reflected outside-time: the combined duration of the inner sections, 2 through 4, in relation to the duration of the whole work (73.889"/204.444") is 0.36 and and the combined duration of the outer sections, 1 and 5, in relation to the whole (130.555"/204.444") is 0.64.

In addition to the proportions described above, there are two instances of durational equivalence (or near-equivalence) that play a potentially significant role in the temporal structure of *Mikka "S"*. The first, and more important, of these relates sections 2-4 to section 5 inside-time. (The relation is inside-time since section 5 follows directly from sections 2-4.) The duration of sections 2-4 is 73.889" and the duration of section 5 is 71.111". In relation to the total duration of the work this makes for the proportions of .36 and .35, respectively, that were pointed out above. The durational near-equivalence of these two portions of the work may, subliminally at least, help section 5 to relate more strongly to the rest of the work in spite of the differences in texture, articulation, and segment duration that separate its music so markedly from the music in part 1. The second instance of equivalence relates section 1 to sections 2 and 4 outside-time. The duration of section 1 is 59.444" which is the precise duration of sections 2 and 4 combined. The actual perceptibility of this equivalence may be somewhat remote since section 3, which does not take part in the relation, intervenes between sections 2 and 4, but the fact that a precise outside-time durational equivalence exists between these two sections and section 1 suggests that Xenakis did in fact wish to associate them temporally in addition to the fact that all three sections feature pairs of overlapping lines that move in close proximity to one another in p-space.

In summary, the principal proportions in the inside-time temporal structure of *Mikka "S"* are 0.65 and 0.35, which occur at the boundary between parts 1 and 2. These proportions are closely approximated by the similar proportions of 0.64 and 0.36 that occur elsewhere in the work, both inside- and outside-time. There are some durational equivalences (or near-equivalences), at least one instance of which appears to be structurally important in relating part 2 to a significant portion of part 1. The principal proportions in this work are different from those that were identified in *Mikka*, where the temporal structure was seen to be complicated by competing proportional schemes that appeared to be almost equal in their structural weight. The first of these schemes relied upon a bisection of the work into nearly equal parts and the second relied upon division according to the golden section. These proportions do not figure as strongly in *Mikka "S"*, but it is worth pointing out that particularly conspicuous details appear in this work near the midpoint and at the golden section. The first of these details is the appearance of B5 in m. 47, which represents the most sudden change in pitch in the work up to that point. This occurs at 103.611", 0.51 of the way through the work, and marks the beginning of section 3. The golden section is marked by the arrival of the two lines in section 4 on G#5 and A#5, at the end of m. 57. This location serves as a focal point in p-space for the remainder of the section, and thus represents a temporary point of arrival. Neither of these details possesses primary structural importance, but their conspicuousness in relation to their surrounding material and their location within the temporal structure of the work suggest the possibility of a veiled reference to the temporal structure of *Mikka* within the structure of its companion piece, *Mikka "S"*. Despite the differences in the proportions of their temporal structures, both works consist of five sections in which the first and fifth are the longest and the third and fourth are significantly briefer than the others. In addition, the use of p-space in both works is coordinated with the division into sections so that the low and high regions in p-space used in the previous sections are brought together in the final section. (Compare Figures 4.5 and 4.11.)

Theraps for double bass (1975-6)

Theraps is Xenakis's only work for double bass. It is discussed here as an example of Xenakis's handling of the timbral resources available to string instruments other than the violin. The range of pitches in *Theraps* is unusually wide, from the lowest pitch available on the standard double bass, -64 (E1), to 55 (D-three-quarter-sharp6), a span of just under six octaves. In his introductory notes to the published score of the work, bassist Barry Guy explains that in order to realize the pitches in the extreme upper register, the performer must use "an Italian technique of fingering (now almost defunct) where the string is pulled to the side rather than pressed onto the fingerboard thereby creating the possibility of a constant glissando up into the highest registers beyond the fingerboard."[11] Harmonics, which are notated at the pitch levels at which they are to sound, also contribute to the articulation of frequencies in the upper register. According to the composer's instructions, also found in the introductory notes to the score, passages marked *sul ponticello* should be played in such a way that the notated fundamental frequencies are masked almost completely by their upper harmonics. Thus, a variety of timbres is demanded from the instrument along with the unusually wide expanse of its p-space.

Like *Mikka, Theraps* contains long random walks in which *arco normale* and *sul ponticello* articulations alternate (see Figure 4.12, beginning at m. 4). Unlike the random walks in *Mikka,* however, those in *Theraps* are not articulated glissando, but are to be articulated legato instead. More specifically, according to the composer's instructions in the introductory notes to the score:

> Starting from measure 4, the chromatic lines (scales) should be played as much as possible with just one finger, which slides abruptly from a note to the following one without leaving the string, while respecting as much as possible the duration of the

[11]Iannis Xenakis, *Theraps* (Paris: Editions Salabert, 1976), introductory notes.

> note during each momentaneous [sic] stop of the finger. The bow should not articulate each note of the scale. Instead a legato execution should be used changing where necessary at suitable points in the music, taking care not to break the line.

Thus relatively continuous random walks are produced in which some differentiation in the pitch intervals should be perceptible, since the individual events are members of a specific pset whose elements are calibrated in equal-tempered quarter-tone units. (The structure of this pset will be discussed in detail below.) Furthermore, the rhythmic notation indicates that there should be a perceptible differentiation in the speed of the walks as they unfold in time, similar to the differentiation found at the opening of *Mists* (see Figure 2.39).[12] Because these random walks are perceived as relatively continuous, and some actually contain brief episodes of glissando articulation, they are represented by continuous lines in the graphic transcription of the work in Figure 4.18.

Like *Mikka "S"*, *Theraps* also contains some random walks in two voices. These are performed glissando, with only the endpoints of the glissandi indicated in the score (see Figure 4.13). These random walks are performed at an unvarying intensity of *fff* whenever they appear. They are also performed at a faster tempo than the rest of the configurations, i.e. at ♩ = 80 M.M. rather than the predominant tempo of ♩ = 58 M.M.[13] Unlike the

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[12]Since the meter, which is predominantly $\frac{4}{4}$, is specified in the score, several errors in the rhythmic notation are detectable: the pair of 32nd-notes at the end of m. 15 should be 16th-notes; the 16th-note quintuplet in m. 31 should consist of 32nd-notes; the eighth-notes on F#3 and G3 in m. 33 should be 16th-notes; the 64th-note nonuplet in m. 43 is missing a note; the stems of the first ten notes in m. 59 should each carry an additional flag; the 32nd-note nonuplet in m. 159 should consist of 64th-notes; the 32nd-note septuplet in m. 160 should consist of 64th-notes; and the 16th-note quintuplet in m. 190 should consist of 32nd-notes. The copyist for *Theraps* was J. L. Sulmon.

[13]In the calculation of segment durations, changes in tempo have been taken to coincide with changes in configuration type, even if the new configuration begins before or after the bar

primarily gentle two-voice random walks in *Mikka "S"*, therefore, the effect of the two-voice walks in *Theraps* is raucous and disruptive. In the table of segments (Table 4.7), these configurations are listed as TVG (two-voice glissandi) in order to distinguish them from the single-voice random walks (RW) whose elements are articulated legato but generally not glissando.

Theraps also contains short, clipped glissandi, similar to those found in part 2 (or section 5) of *Mikka "S"*. A succession of these glissandi occurs over a single interval at the beginning of the work and again at its conclusion, where all four strings are played in parallel fourths (see Figure 4.14, mm. 1-3 and m. 192). These sections have the effect of framing the remaining portions of the work. Both occur at the low end of the p-space, thereby producing a gruff sound whose precise pitches are difficult to discern. This type of configuration is labeled SG (short glissando) in Table 4.7. As in the segmentation of *Mikka "S"*, the termination of each SG is regarded as the end of a segment, despite the extreme brevity and repetitiveness of these gestures in *Theraps*. Analysis of the global temporal structure of *Theraps* demonstrates, however, that the frequency of brief SG segments forms part of an exponential distribution of segment durations. (This will be discussed in more detail below.) Incidentally, the repeated, staccato pitch 42 (D#2) in m. 2 is an anomaly from the standpoint of the categorization of sonic configurations. Thus it is marked "?" in Table 4.7. Because of its similarities

marked with the tempo change. For example, a single-voice random walk extends one beat into m. 94 but the tempo change is marked at the beginning of the bar. Conversely, the single-voice random walk that begins on the fourth beat of m. 186 is assumed to be at the tempo indicated at the start of m. 187. The tempo changes that should occur at the start of the two-voice random walk in m. 152 and at the beginning of the single-voice random walks in m. 154 do not appear in the score.

in register and duration to the SGs, however, it clearly belongs among them and should probably be regarded as a momentary variant in articulation.

The fourth and final configuration type found in *Theraps* consists of sustained harmonics (marked H in Table 4.7). These configurations are notable for their pure timbre and for their relatively static rhythm, which is in marked contrast to the hyperactivity of the surrounding material. The harmonics are notated at the pitch level at which they are to sound (see Figure 4.15). The roman numeral above each notehead gives the string on which the sound is to be produced and the arabic numeral with the circle above it gives the number of the harmonic on that string. Like the TVGs, the harmonics produce a structure of two independent voices. The harmonics are rhythmically independent, however, in a way that the TVGs are not, for within the TVG segments activity is maintained throughout by both voices and the delineation of individual contours is difficult to discern due to the nearly constant glissando articulation. The duration of the TVG segments, therefore, is calculated by taking both voices as a single unit, while the beginning and end points of each harmonic are calculated individually. The harmonics thus constitute the only configuration type in *Theraps* in which the boundaries of the segments overlap. This produces a slight discrepancy between the calculation of the inside-time and outside-time temporal structures of the work, as was the case also in *Mikka "S"*, *Evryali*, and *Herma*.

The segment durations indicated in Table 4.7 reveal a global organization, outside-time, according to the exponential distribution with ∂ = 0.109 segments/sec (98 segments/897.181"), as shown in Figure 4.16. The distribution of intervals between points of initiation of the segments inside-time also appears to be organized according to the exponential distribution, with ∂ = 0.137 segments/sec (98 segments/713.56"), as shown in Figure 4.17.

The correspondence between the expected and observed relative frequencies is not as close inside-time (Figure 4.17) as it is outside-time (Figure 4.16), but the exponential distribution nonetheless provides a reasonable model for both observed distributions. In this respect the global temporal structure of *Theraps* resembles those of the works mentioned at the end of the previous paragraph. (Compare Figures 4.16 and 4.17 with Figures 2.55, 2.56, 3.04, 4.08, and 4.09.)

Table 4.7 and the graphic transcription of the score in Figure 4.18 demonstrate distinct patterns in the alternation of configuration types. After the opening succession of SGs (segments 1-46, mm. 1-3, 0-12.414"), random walks alternate with successions of harmonics until segment 61 (mm. 66-80, 263-308.25"), where the first TVG is introduced. From this point until the closing group of SGs, the succession of configuration types follows the pattern TVG, RW, TVG, H for two and one-half cycles. The introduction of the first TVG in segment 61, then, constitutes an important event in the work's large-scale structure. Division of the work at this point results in two parts whose durations are 263.276" and 450.284", respectively, constituting proportions of 0.369 and 0.631 of the total inside-time duration, which is 713.56". This proportion, which falls somewhere between the golden section and 2:3, is similar to the large-scale proportional divisions found in other works by Xenakis.

The inside-time temporal structure of *Theraps* is summarized in Table 4.8. The table shows that the work divides into seventeen subsections. These subsections, which are labeled with consecutive lower-case letters, represent single segments or groups of segments that present events belonging to distinct configuration types. The subsections have been labeled consecutively in the table since changes in configuration type constitute the most immediately perceptible feature of the musical surface, regardless of the number of segments within a subsection and regardless of the number of subsections within a section. Strict hierarchical criteria, therefore, are not employed in the grouping of segments into subsections, of subsections into sections, or of sections into supersections. Instead, the grouping of structural units in the table follows, at the level of the subsections, the perceptual criterion of change in configuration type, while grouping at higher levels follows a combination of perceptual and proportional criteria. Perceptual criteria include the separation of the short glissandi from the main body of the work in supersection A, which is coextensive with section 1 and subsection a, and supersection D, which is coextensive with section 6 and subsection q. Proportional criteria guide the division of supersections B and C. Supersection B divides into sections 2 and 3, with proportions of 0.629 and 0.371, which resemble the proportions of 0.369 and 0.631 of parts 1 and 2, but in reverse order. Section 3, which is coextensive with subsection f and segment 60, is different from the preceding subsections in section 2 because of its long duration and wide registral span, which includes the work's registral high point, 55 (D-three-quarter-sharp6). Likewise, supersection C divides into sections 4 and 5, with proportions of 0.65 and 0.35, at subsection k, which is where the second cycle of TVG, RW, TVG, and H configuration types begins. The second and third (incomplete) cycles differ from the first in that the TVGs become much shorter. Thus, there is a perceptual difference within these sections that supports the division at subsection k. Incidentally, subsection i, which contains the longest segment in the work, a TVG, begins at 363.077", 0.509 of the way through the work. Thus, a simple proportional division into halves overlays the more complex system of nested (approximate) 2:3 proportions at this point in the work.

The subsections also provide a basis for examining the distribution of configuration types in the work overall. Table 4.9 shows a classification of the subsections by configuration type along with the durations of the subsections within each category. Note that the durations of the subsections represent the successions of segments inside-time. The only category for which the distinction between inside-time and outside-time durations is significant is the harmonics. In the process of classification the decision was made that the perceptual difference between the succession of the configuration types is greater than that between individual, overlapping segments. The classification, therefore, is outside-time with respect to the subsections but inside-time with respect to the segments within subsections.

Below each category is the sum of the durations of the subsections contained within it. To the right of this sum is the proportion of the work's total inside-time duration represented by the sum. At the bottom of Table 4.9 is a summary which shows that the random walks occupy 0.405 of the work's total duration and that the remainder of the configuration types combined occupy 0.595 of it. The random walks occupy a greater proportion of the work's duration than any other single configuration type, and it is arguable that the sound and contour of the segments containing this configuration type are so striking as to establish the relatively continuous random walk in one voice as the most characteristic, or principal, configuration type in the work. The harmonics clearly function as a contrasting configuration type to the random walks, and they are the only type that alternates with the random walks until the first TVG arrives in subsection g, marking the beginning of part 2. The TVGs combine the two-voice texture of the harmonics with the glissando articulation found in the SGs and the long, arching contours of the random walks. They thus represent a partial synthesis of characteristics found in the previously introduced configuration types. For a number of reasons, then, the separation of the random walks from the other configuration types for the purposes of general classification makes sense, both on the basis of perceptual criteria and with respect to proportional balance. As the summary shows, the outside-time proportion between the random walks and the other configuration types combined approximates 2:3, which is the proportion articulated by the division of the work into parts and of the supersections into sections, inside-time (see Table 4.8).

The subsections are also notable for the succession of registral spans they articulate inside-time. Figure 4.19 shows the minimum, maximum, and mean pitch levels per subsection. The durations and temporal locations of the subsections are represented along the horizontal axis of the graph. The figure shows that the second and fourth subsections, b and d, which contain random walks, are separated registrally from the third and fifth subsections, c and e, which contain harmonics. (The subsections are not labeled on the graph, but are discernable by means of the order of their succession.) The random walks occur in the lower portion of the p-space, while the harmonics are situated above them. This situation changes in the sixth subsection, f, where the registral span is wider and extends to the extreme upper boundary of the p-space. In effect, subsection f connects the distinct registers found in subsections b-e and extends the p-space further upward. The distinctiveness of its appearance on the graph emphasizes its difference from the preceding subsections and thereby reinforces the structural division here between sections 2 and 3 that is proposed in Table 4.8.

The seventh through tenth subsections, g through j, demonstrate a directly ascending pattern in their mean pitch levels. This pattern contrasts with the more complex, but generally descending pattern among the mean pitch levels in the eleventh through sixteenth subsections, k through p. This distinction in the contour of the mean pitch levels among these groups of subsections reinforces the structural division between sections 4 and 5 that is proposed in Table 4.8. The short glissandi in sections 1 and 6 (also supersections A and D or subsections a and q) are confined to the low end of the p-space and therefore frame the main body of the work with distinctive gestures.

The proportions among the larger sections in *Theraps* are particularly evident in Figure 4.20, which shows the minimum, maximum, and mean pitch levels per section. The figure shows that sections 2 and 5 are nearly equal in duration. This arises from the fact that the system of proportions involved in the division of the work into parts and of the supersections into sections roughly approximates the golden section. The larger section within the smaller part, therefore, is similar in duration to the smaller section within the larger part. The figure also demonstrates that the mean pitch level ascends overall in part 1 and descends in part 2. It is also the case that the registral span is widest in the last major section, section 5. The overall contour of the p-space within subsections and within sections in *Theraps* resembles the contours within segments (or segment groups) and sections in both *Mikka* and *Mikka "S"*. (Compare Figures 4.19 and 4.20 with Figures 4.3 and 4.5 and Figures 4.10 and 4.11, respectively.)

Because of the extreme contrasts in registral span, intensity and duration within and among the different configuration types in *Theraps*, the construction of a multi-dimensional scale (MDS) with respect to the subsegments can serve to highlight some general perceptual differences among them. The SGs, as a class, constitute the briefest of the subsections, but two of the TVGs, in subsections k and m, are also quite brief. The TVGs and

SGs, on the other hand, are almost uniformly loud, with an intensity of *fff*, except for the decrescendo at the very end of subsection a. The random walks are medium to long in duration and vary in intensity, while the successions of harmonics are also medium to long in duration, but are uniformly soft, with an intensity of *p* throughout. The longest, and one of the widest, configurations is the TVG in subsection i, whose beginning, as mentioned previously, marks the approximate midway point in the work's duration. MDS values for the subsections are shown in Figure 4.21.[14] The figure shows that all of the local peaks in part 1 (subsections a-f) are formed by random walks in alternation with either SGs (subsection a) or harmonics (subsections c and e). Subsection f forms the highest peak within part 1. All of the peaks in part 2 (subsections g-q) belong to TVGs, including the highest peak in the work, in subsection i. The pattern of peaks and valleys is somewhat irregular in section 4 (subsections g-j) but the local peaks ascend steadily in section 5 (subsections k-p) even as the mean pitch level descends overall (see Figure 4.18). This distinction in the pattern of peaks and valleys within part 2 further reinforces the perceptual basis of the division between sections 4 and 5. The ascending pattern of peaks in section 5, which is coordinated with a generally descending mean pitch level, seems to contribute to the forward momentum in this portion of the work and also seems to prepare for the arrival of the abrupt SGs in the final section which, given the careful

[14]Specific high and low values in the dimensions of duration, intensity, and registral span are not given here. The procedure for the construction of the MDS is similar to that used in chapter 3, except that the values are scaled by a factor of 3.333... instead of 2.5, since only three dimensions are considered. Density, which is easily measured in textures that consist of discrete attacks, cannot be quantified so easily in textures that consist of continuous articulations (glissandi). Density, therefore, did not enter into consideration in the construction of the MDS for *Theraps*.

organization of the music that leads up to them, assume a relative degree of structural inevitability.

Though *Theraps* is the longest of the works analyzed in this dissertation, the relative simplicity of its design, from the standpoint of its temporal structure and of the patterned presentation of the distinct configuration types it contains, makes it particularly amenable to the analytical approach adopted here. The relative simplicity of its design is reflected as well in its pitch structure which, unlike the pitch structures of *Mikka* and *Mikka "S"*, shows some evidence of the kind of intervallic differentiation employed in the pitch structures of the piano works. Two pset models are given in Table 4.10, since only the random walks and the successions of harmonics show clear evidence of structural differentiation within the work's equal-tempered quarter-tone p-space. No period is proposed for either model since neither one of them shows any repeated patterns in its spacing, nor does either of them span the entire available p-space. Further, neither model undergoes cyclic transposition, which would help to reveal the endpoints of their spacings. The sole criterion for the inclusion of elements within the models, then, is the repeated traversal of specific pitches through several passes across particular locations in p-space. Pitches that occur only in one random walk or succession of harmonics within areas of p-space that are traversed by other random walks or successions of harmonics are not included in the models but are shown in italics in the psets in which they are found. Because of their rarity in the context of configurations of the same type, these pitches are taken to have a status similar to the extraneous pitches with respect to the psets in the piano works.

Model A consists of a collection of relatively closely packed pitches. This collection is represented indirectly in the random walks means of its subsets. The random walks in subsections f and l cover the same area of the p-space but the pset in subsection f contains pitches 7 and 12, which are not found in subsection l. Only pitch 7 is shown in italics, since pitch 12 also appears in the pset in subsection h. Model B contains a variety of intervals in its spacing. The spacing of its subset in subsection c is symmetrical but this characteristic is not present in any of the other subsets. One perceptually important aspect of the variety of intervals in the spacings of these psets is the presence of several intervals of size one, some of which are realized in the ordered successions of pitches on the surface of the music. When pitches a quarter-tone apart are sounded together, audible beats result. (The calibration of pitches among the harmonics is approximate, of course, since all of the harmonics are natural, but Xenakis's notation does not take this into account.) The existence of beats, in addition to the special timbre of the natural harmonics, produces an almost unearthly quality to the sound in the subsections that contain harmonics whose pitches are in close proximity. Harmonics a quarter-tone apart occur in all of the H subsections except for subsection e.

The pitch structure of *Theraps* contributes little to the articulation of form in the work, other than to reinforce the differentiation of subsections and sections by configuration type and registral span that has been described above. Local pitch connections between distinct configuration types, however, do contribute to the coherence of the musical surface, particularly toward the end of the work. The D4 at the end of the TVG in subsection k is one quarter-tone above the C-three-quarter-sharp4 at the start of the RW in subsection l (m. 154, 558.258"). Similarly, the C5 at the end of the TVG in subsection m is close to the B4 and B/4 harmonics at the beginning of subsection n (m. 163, 593.224"). In addition, the D2 at the end of the TVG in subsection o connects with the C-three-quarter-sharp2 at the beginning of the RW in subsection p (m. 186, 683.819"). Finally, the C/3 at the end of the RW in subsection p is close to the C#3 at the beginning of the SGs in subsection q (m. 192, 708.129").

Specific pitch connections also exist among configurations of the same type, separated in time by other configurations. The first few RWs are connected in this way, from the D#2 at the end of subsection b (m. 16, 65") to the D/2 at the beginning of subsection d (m. 26, 106.552") and from the D3 at the end of subsection d (m. 35, 141.5") to the C-three-quarter-sharp3 at the beginning of subsection f (m. 42, 170.172"). These connections suggest the possibility that subsections b, d, and f derive from a single, long RW that was then segmented in order to conform to the work's temporal structure, which may have been conceived separately from it. Similarly, the F-three-quarter-sharp4 and A/4 at the end of the succession of harmonics in subsection c (m. 26, 106.552") reappear at the beginning of subsection e (mm. 35 and 36, 141.724" and 144.828").

Given the prevalence of glissando articulations in the works for strings and the special difficulties in the production and perception of the equal-tempered quarter-tone units frequently used in these works, extensive structuring of the pitch material, such as that found in the works for piano, does not appear to struck Xenakis as a particularly rewarding enterprise. Whatever the ultimate reasons may be, after *Nomos Alpha* complex pitch structures do not seem to have played a predominant role in the composition of his works for strings.[15]

Conclusion

The analyses in this chapter have shown how the textures characteristic of Xenakis's music for strings can be shaped into coherent compositional structures. Particularly problematic in this regard are glissandi which, rather than functioning as portamenti between structural pitches, are to be interpreted as structural gestures in their own right. Thus, they must be regarded as primary material despite the volatile nature of their pitch structure, which changes continuously in ways that can only be approximated by traditional musical notation. Continuous random walks in a single voice provide a model for the glissandi in *Mikka* and also for the legato "scales" based on quarter-tone increments in *Theraps*. Glissandi in two voices, such as those found in *Mikka "S"* and *Theraps*, also seem to be modelled after continuous random walks. A greater variety of textures is evident in *Theraps* than in the other works. A similar variety of materials is evident in the other works for solo strings as well as in the works for string ensemble and string orchestra.

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[15]See *Formalized Music*, chapter 8 and Vriend, "'Nomos Alpha'," cited in note 1, for more on the pitch structure of *Nomos Alpha*. A possible exception is *Tetora* for string quartet (1990), in which pitch is calibrated in equal-tempered semitones and glissandi are notably absent. These two conditions promote the invention and transformation of complex m-psets. This work is the subject of the unpublished paper, "Residue-Class Sets in the Music of Iannis Xenakis: An Analytical Algorithm and a General Intervallic Expression," by Evan Jones of the Eastman School of Music.

Chapter 5

Electroacoustic Music

In the brief overview of Xenakis's work in the electroacoustic medium given at the end of chapter 1, it was pointed out that his early electroacoustic works were composed according to the principles of *musique concrète* and that in more recent works he has incorporated electronic sound synthesis, including an original synthesis technique known as "dynamic stochastic synthesis."[1] Of all Xenakis's projects in the composition of electroacoustic music, however, perhaps none has made quite as direct an appeal to the imagination as the UPIC system, which features a graphic interface both for the synthesis of sound and for the organization of sonic events into complete compositions. Since the generation of a graphic score is an integral part of composition on the UPIC system, works produced on this system are ideally suited to the type of score-based analysis that is featured in this dissertation. The complete graphic score of only one UPIC work has been freely disseminated by Xenakis. This work, *Mycenae-Alpha* (1978), will be discussed in detail in the analytical portion of this chapter.

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[1]See Iannis Xenakis, *Formalized Music*, rev. ed. (Stuyvesant, NY: Pendragon Press, 1992), chapters 9, 13, and 14; Marie-Hélène Serra, "Stochastic Composition and Stochastic Timbre: *Gendy3* by Iannis Xenakis," *Perspectives of New Music* 31/1 (1993): 236-57; and Peter Hoffmann, "Implementing the Dynamic Stochastic Synthesis," *Les cahiers du GREYC* 4 (1996): 341-7.

The UPIC System

The UPIC (Unité Polyagogique Informatique du CEMAMu) was first developed in the late 1970s by Xenakis and a team of engineers and computer scientists at CEMAMu (Centre d'Etudes Mathématiques et Automatiques Musicales), Xenakis's research facility near Paris.[2] Since the construction of the original UPIC in 1977, several newer versions have been produced.[3] An organization called Atéliers UPIC, directed by Gerard Pape, is in charge of research, software development, and the manufacture of UPIC systems as well as instruction in composition on the UPIC.[4] Since its invention, efforts have been made to reduce production costs and to make the system more compatible with standard hardware. So far these efforts have met with limited success, but a new version of the system is due to be released in 1997.[5] Because of difficulties related to the cost and portability of the UPIC, relatively few people have had an opportunity to work with the system on anything but a short-term basis. Xenakis himself has produced only three complete works on the UPIC so far: *Mycenae-Alpha* (1978), *Taurhiphanie* (1987), and *Voyage absolu des Unari vers Andromède* (1989). He has also used the UPIC in the composition of portions of *La Légende d'Eer* for electroacoustic sounds (1977) and *Pour la Paix* for chorus, narrator, and electroacoustic sounds (1981).

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[2]CEMAMu is currently located at CNET E 655, 3, avenue de la République, 92131 Issy-les-Moulineaux, France.

[3]See Henning Lohner, "The UPIC System: A User's Report," *Computer Music Journal* 10/4 (1986): 42-9; *Formalized Music*, Appendix III; and Gérard Marino, Marie-Hélène Serra, and Jean-Michel Raczinski, "The UPIC System: Origins and Innovations," *Perspectives of New Music* 31/1 (1993): 258-69.

[4]Its current location is Atéliers UPIC, 18, rue Marcelin Berthelot, 94140 Alfortville, France.

[5]My thanks to Brigitte Robindoré of Atéliers UPIC, and to composers Hans Mittendorf and Robert Scott Thompson for information on the technical aspects of the UPIC system.

A schematic of a relatively early version of the UPIC is shown in Figure 5.1. The version of the system represented in the figure is similar to the original version, on which *Mycenae-Alpha* was composed. The graphics tablet, pictured near the top of the figure, contains a drawing board and a set of controls, which are located to the right. These controls are used to select the function of the drawing board. If the voice editor function is chosen, the electromagnetic pen can be used to design waveforms and envelopes for the generation of sounds via a synthesis unit. If the score editor function is chosen, the pen can be used to generate lines or curves representing pitch versus time, which are called "arcs." Each arc represents a sonic event. Horizontal arcs produce steady pitches and curved arcs produce discrete or continuous changes in pitch with respect to time. The sonic result of a curved arc, depending on whether a discrete or continuous setting has been chosen, resembles either a scale or a glissando. Collections of arcs are known as "pages." Each page represents a segment of music, and a succession of pages constitutes a complete composition. In the early days of UPIC composition, pages were often designed on tracing paper which would then be taped to the drawing board as a guide for the movements of the electromagnetic pen. The designs inscribed by the pen could then be viewed on a graphic display and the final results could be printed, thereby producing a hard copy of the graphic score for a page of music. In more recent versions of the UPIC, the graphics tablet has been replaced by a computer monitor and the functions of the electromagnetic pen are performed by a mouse.

In terms of compositional theory, the operations of the voice editor are performed at the level of microcomposition and those of the score editor constitute the macrocompositional level. The distinction between these levels is crucial for an understanding of the operation of the UPIC system, for while the graphic score of a page of music corresponds symbolically to its sonic structure, at least with respect to the dimensions of pitch and time, such is not the case with the waveforms and envelopes used to produce the sounds. The timbral quality of a synthesized sound is not easy to predict from the shape of its waveform and envelope. Further, the UPIC allows the user to extract waveforms and envelopes from sampled sounds (see the microphone for sampling at the bottom of Figure 5.1) and to modify them if desired. If the timbres are designed graphically, therefore, considerable experimentation may be required to obtain the desired sounds. The arcs, on the other hand, generally represent the fundamental frequencies of the pitches as indicated along the vertical axis of the score, calibrated in units and oriented around a tuning pitch, both of which may be chosen by the user. The audible frequencies of the sounds generally contain the fundamentals represented by the arcs, but the upper harmonics may mask the fundamentals partially or completely, depending upon the timbral characteristics chosen for the sounds. An effect of this sort is analogous to the articulation of a notated pitch *sul ponticello* on a string instrument, as indicated in the scores of *Mikka* and *Theraps*, which were discussed in chapter 4. Arcs that are located in close proximity in p-space may produce beats, or sum or difference tones, thereby generating acoustical phenomena that do not seem to relate directly to the sonic structures represented in the graphic score.[6] Since Xenakis tended to prefer dense clusters of arcs in the composition of *Mycenae-Alpha*, such effects tend to occur quite frequently in that work. In general, it would appear that Xenakis takes a certain enjoyment in the somewhat unpredictable sound

[6]The reader who desires a general introduction to the physics of sound may wish to consult Thomas Rossing, *The Science of Sound* (Reading, MA: Addison-Wesley, 1990).

world of the UPIC system. Nonetheless, the graphic score of a UPIC work represents the composer's general sense of its sonic structure and, all things considered, serves as a useful guide for a preliminary study of that structure. More detailed investigation of the sonic properties of specific areas in a UPIC work would require special techniques, such as spectral analysis of the recorded performance of the work. No special techniques, other than observation of the score accompanied by close listening, have been applied in the following analysis of *Mycenae-Alpha*.

Mycenae-Alpha (1978)

The first performance of *Mycenae-Alpha* was given in the context of a visual and musical spectacle that took place at the Mycenae Acropolis in Greece, from which the work derives its name. Subsequent, non-site-specific performances have sometimes included the projection of the graphic score via photographic slides along with playback of the recording that was generated during the work's creation on the original UPIC system.[7] Clearly, then, the work *Mycenae-Alpha*, as conceived by the composer, comprises both its sonic structure and the graphic score used to organize that structure. It is significant in this regard that both releases of the recording of *Mycenae-Alpha* for private audition are accompanied by reproductions of its graphic score.[8] By including the display of its graphic score in both public and private

[7]Cf. the instructions for public performance contained in the catalog of Xenakis's works by Editions Salabert (Paris: Editions Salabert, 1993).

[8]Recordings of *Mycenae-Alpha* have been released on the cassette tape accompanying *Perspectives of New Music* 25 (1987) and on compact disc Neuma CD 450-74. The score of *Mycenae-Alpha* (Paris: Editions Salabert, 1978) has been reproduced in *Perspectives of New Music* 25/1 (1987): 12-5 and in the booklet accompanying the Neuma CD.

performances of this work, Xenakis has made the connection between graphic composition and sonic structure explicit to a greater degree than he has for any other of his works.[9] The consideration of the relationship between the sonic and graphic aspects of the structure of *Mycenae-Alpha* in the following discussion is therefore particularly appropriate, and supports by implication the similar relationship that been assumed to obtain in the analyses of instrumental works in the previous two chapters.

An annotated graphic score of *Mycenae-Alpha* is shown in Figure 5.2. The arcs in the score represent the fundamental frequencies of the sounds and their duration. The p-space spans five octaves. Each octave is marked in the score as C_n, where n = 1, 2, ..., 6, and C_3 represents middle C.[10] A 440 is also indicated, as A_3. The starting and ending time for each of the score's pages is indicated below each system.[11] Numbers have been added above the systems at the start of each page. These numbers correspond to the segments listed in Table 5.1, in which each segment corresponds to a page of the score. The segmentation indicated in the figure and in the table follows the page divisions in the original score. It is notable that sounds are produced continuously in each of the segments, except for numbers 7 and 13. These two segments contain multiple configurations separated by silences. The configurations they contain, and the silences that separate them, would likely

[9]Portions of the graphic sketches for some works, however, have been published. Most of these appear in *Formalized Music.*

[10]According to the system for pitch identification used in this dissertation, the p-space extends from C2 to C7, with C4 equivalent to middle C.

[11]The timing indications have been enlarged in the annotated score. An error in the original has also been corrected: 17", a timing derived from the CD recording of the work, replaces the 55" that originally appeared at the beginning of the second page. 55" has been restored to its proper place, at the beginning of the third page, which was left blank in the original score.

be counted as individual segments in an analytical reading if this were an instrumental work whose score indicated the precise beginning and end of each configuration or rest. Given the difficulties involved in obtaining precise measurements of the durations of the configurations (and rests) and the fact that the analysis works quite well without the subdivision of segments 7 and 13, the segmentation will follow Xenakis's division of the work into pages.[12]

Table 5.1 shows that the arcs in the segments are generally of a single orientation. The arcs in segment 1, for example are mainly horizontal except for a few sloped arcs in the second cluster from the bottom, between C_3 and C_4 (see Figure 5.2). The slight unevenness of some of the horizontal arcs in this and other segments is likely due to the fact that each arc was drawn into the score editor by hand from an original design on tracing paper. All of the arcs in segment 2, on the other hand, are curved. Horizontal arcs result in steady pitches, while curved arcs result in continuous modulations of pitch with respect to time, generating sounds that resemble glissandi on string instruments or the sounds produced by sirens.

An exception to the general rule regarding similarity of arc orientation within segments is segment 11, which contains clusters of horizontal arcs connected by curved arcs. The result is a hybrid type of configuration that is similar in concept to the hybrid configuration types found in some other works, such as *Evryali*. It is perhaps significant that this hybrid configuration appears near the end of the work, thereby implying that the opposed types—

[12]Segmentation of this particular UPIC work, incidentally, is simplified by the fact that, at the time it was composed, it was not possible to superimpose pages or portions of pages. In more recent versions of the UPIC, a "mixing" function allows for such superimpositions to take place. It is also now possible for the UPIC to read pages backwards, to change the speed at which the pages are read, and to draw sounds from various parts of the page out of sequence. See *Formalized Music*, Appendix III.

based on the general orientation of the arcs contained in the segments—have achieved a partial synthesis as the music moves toward its conclusion. Another slight exception to general rule regarding arc orientation is the succession of sloped and curved arcs that connects segments 9 and 10. This short passage at the end of segment 9 seems intended to create a link in p-space between it and segment 10, which is notable for the way in which it starts near the extreme upper limit of the p-space. Both of these exceptions to the general rule play interesting roles in the work's temporal structure, outside-time. Before examining the outside-time temporal structure, however, it will be useful to focus on similarities and differences among configurations in their inside-time context.

Segments 1 and 4 are related by arc orientation and by the fact that their arcs appear in clusters within specific bands of p-space. The sound of segment 1 in particular changes noticeably over time on account of the sculpted morphology of its clusters, which activate different portions of the p-space bands as the segment unfolds. Both configurations are notable for a noisy timbre in which individual pitches, and even individual bands of p-space, are difficult to discern.

Segments 2 and 5 both feature configurations made up mainly of curved arcs. Both begin within a narrow band of p-space and then expand outward. Segment 5 returns to a narrow band of p-space at the end, but segment 2 does not. A difference that is perhaps more significant is the fact that the arcs are generally farther apart in segment 5 than in segment 2. As a result, the p-space in segment 5 is much more expansive. The curved arcs in both segments, however, generate collections of sounds that are clearly related to another and that are distinct from the sounds produced by the surrounding configurations. These sounds suggest associations with war and devastation, resembling war sirens or the cries and wails of collective mourning. From a more technical perspective, segment 5 in particular resembles the massed glissandi heard at the beginning and end of *Metastaseis* for orchestra (1953-4). In that work the difference between narrow and wide bands in p-space is likewise explored, and the effect there is similarly hair-raising.

Segments 3 and 6 begin almost identically, with horizontal arcs confined to the extreme low end of the p-space, but segment 6 ends with a gradually ascending cluster that is not found in segment 3. The timing of the initiation of this cluster is significant within the work's global temporal structure, inside-time, which will be discussed separately below. In addition to the fundamental frequencies represented in the graphic score, both configurations contain prominent upper harmonics and also some extremely low frequencies. These effects possibly result from the close proximity of the arcs in p-space in conjunction with the waveform and envelope chosen by the composer for the production of these sounds.

As suggested by the comments above, the first six segments may be regarded as a group made up of three distinct types of configurations. Representatives of each type are presented one at a time in a pattern that unfolds twice in direct succession. The complete group represents a large structural unit within the work as a whole, as does the remaining group of seven segments.

Segments 7 through 13 form a group that is less obviously unified than the one formed by segments 1 through 6. Nonetheless, specific correspondences among the segments contained within it generate a coherent network of associations. Segments 7 and 13, which begin and end the group, are morphologically identical. This pair of segments presents the only exact morphological correspondence in the entire work. Clearly, this single instance of literal morphological repetition relates these two segments quite strongly and, by implication, helps to gather the intervening segments into a larger structural unit. The durations of segments 7 and 13, however, are quite different. The duration of segment 7 is 24", while that of segment 13 is 61", which is just over two-and-a-half times as long as segment 7. The relationship of segment 13 to segment 7, then, is roughly analogous to that of a literal motivic repetition with rhythmic augmentation. The change in duration also affects the perception of pitch and timbre between the two segments. The pitches of the individual arcs in segment 7 are more difficult to discern, so that some of the configurations sound like noises rather than pitch clusters. The pitches are easier to discern in the longer segment 13, so that the pitch-cluster aspect of the configurations is more clearly perceptible. The details in the drawing of the arcs are also more audibly perceptible in segment 13, at times lending a sense of hesitancy to the sonic structure as the arcs begin and end at slightly different times. These imperfections are masked in segment 7 because of its faster "tempo," which helps the configurations to be perceived more as gestalts and less as collections of individual arcs.

A morphological correspondence also exists between segments 11 and 12. Both begin with clusters of arcs that ascend unevenly through the p-space and both end in narrow bands near the center of the p-space. Both also contain numerous "holes," i.e. areas in which arcs in particular locations in p-space drop out temporarily and then resume afterward. The holes are a morphological feature that these segments share with segment 9. Because of the variety in the attacks and releases of the arcs caused by the holes, they add a sense of mobility to the sonic structure of these configurations that is less evident in some of the other configurations, where the horizontal arcs are presented in more regularly sustained clusters. Among the differences between segments 11 and 12 are the curved arcs in segment 11 that connect the clusters of horizontal arcs, and the activity in the upper and lower portions of the p-space at its end, both of which are absent from segment 12. In addition, the vertical density of the horizontal arcs is greater in segment 11, as is the segment's total duration. Overall, however, segments 11 and 12 sound as if they are related, even if the basis of that relation is difficult to define precisely in the act of aural perception. The fact that both segments are juxtaposed directly reinforces the perception of the similarities between them.

The remaining segments are more individual in the morphology of their configurations, but each one shares at least some characteristics with at least one other segment. As previously mentioned, segments 9 and 11 contain prominent holes. They are also similar in the vertical density of their arcs. Segment 10 is similar to segments 7, 11 and 13 in the vertical density of its arcs. The morphology of its clusters also resembles those of segments 7 and 13, but unlike the clusters in those segments, which are located mainly near the bottom of the p-space, several of those in segment 10 occupy positions in the upper portion of the p-space, though two of them occupy wide bands in p-space that extend all the way to the bottom. The clusters in segment 10 are all connected by thin bands, which function analogously to common tones in instrumental music. Thus there are no intervening silences between the clusters as there are in segments 7 and 13. In fact, the connections between the clusters in segment 10 appear to represent a "development" of the connection that exists between the last two clusters in segment 7, a connection that also exists in the repetition of the same succession of configurations in segment 13.

Segment 8 presents the most visually arresting morphology of any of the segments in the work. It is composed entirely of curved arcs and displays several branching configurations that are clearly related to the arborescences found in some of the instrumental works (see especially the graphic transcription of *Evryali* in Figure 3.1). Sonically as well as visually it is a distinct entity within within the group that comprises segment 7-13. Of all the segments in this group, it provides the strongest link with segments in the first group, specifically segments 2 and 5, which are also composed entirely of curved arcs. Its relation to other configurations in the second group rests mainly on the basis of its wide registral span and long duration, both of which qualities it shares with configurations 9, 10, and 11.

This brief survey of the visual and sonic relations among the segments in *Mycenae-Alpha* clearly suggests a primary structural division of the work into two parts. This division is illustrated in Table 5.2, which shows a summary of the temporal structure of *Mycenae-Alpha*. The duration of the whole work is the same inside- and outside-time, since there are no overlaps among the segments. The whole duration divides into parts according to the proportions 0.405 and 0.595, which form a close approximation to the simple proportion 2:3. The table also shows that the duration, outside-time, of all of the segments that contain curved arcs versus the duration of those that contain only horizontal arcs likewise approximates the simple proportion 2:3. This proportion between segments with curved and horizontal arcs is maintained within the parts as well. The proportion is exact in part 1 but more approximate in part 2. If the 6-second passage of curved arcs that joins segments 9 and 10 is included in the total duration of curved arc material in part 2, the proportion comes a little closer to 2:3.

Because of the small number of segments and the relatively small range of differences in their durations, the distribution of segment durations in *Mycenae-Alpha* does not conform to the exponential distribution, nor is there a complex system of nested temporal proportions in this work comparable to those observed in the instrumental works in chapters 3 and 4. There is, however, a secondary system of proportions operating in the inside-time structure. This is in the form of a golden section, the smaller part of which consists of the music from the beginning of the work up to the initiation of the ascending cluster in segment 6. This overlaid golden section, which does not coincide with any of the divisions along the time line articulated by the boundaries of the segments, is comparable to the secondary systems of proportions that operate in *Mikka* and *Mikka "S"*.

Conclusion

Although only one of Xenakis's electroacoustic works has been discussed here, it is evident that the structural principles of at least this one work are clearly related to those of the instrumental works examined in the previous chapters. Particular features in common include the graphic design of sonic configurations, the organization of the inside-time temporal structure according to a specific system (or specific systems) of proportions, and a correspondence between aspects of the inside- and outside-time temporal structures on the basis of proportions (or approximations thereof) common to both structures. Missing from the analysis of *Mycenae-Alpha*, however, is a precise account of its pitch structure, or even of the boundaries of the regions of p-space occupied by its segments. This is due to limitations in the symbolic representation of pitch information in the graphic score and lack of information regarding the calibration of units in p-space. It is conceivable, however, that a grid could be superimposed upon the score, following the calibration into octaves shown at the beginning of the first system (see Figure 5.1). This was not done here because it was felt that this might obscure the visibility of the arcs.

Study of the graphic scores of Xenakis's other UPIC works, were these to be made available to interested scholars, would likely reveal additional correspondences between their structural principles and those of the instrumental works. In addition, analysis of Xenakis's electroacoustic works that were not composed on the UPIC system—analysis based on aural discrimination among the types of textures used and the recording of the times at which changes in texture occur—may confirm the general findings presented here or yield further insights into his work in electroacoustic media.

Chapter 6

Conclusion

This study of Xenakis's music began with a brief discussion of the role of indeterminacy in his compositional process.[1] This topic was explored further in chapter 2, where techniques for the composition of stochastic configurations and random walks were discussed and demonstrated (in sections 2.2.1 and 2.2.2, respectively). The analyses in chapters 3 through 5, on the other hand, have emphasized the role of non-indeterminate procedures in generating large-scale structure in Xenakis's music. These procedures include the formation of successions of segments into sections through the establishment of patterns in the presentation of configuration types, either by direct repetition of similar types or by the alternation of contrasting types, and the organization of large-scale temporal structures according to systems of nested proportions. The analyses point out, therefore, an apparent disjunction between the principles that are used to organize the details of the musical surface and those that are used to organize large-scale structure.

This difference in the approach to the organization of details versus the organization of sections and parts—a difference that is most strikingly apparent in works that consist entirely, or almost entirely, of stochastic configurations—may seem to be evidence of a logical inconsistency within

[1]This was the topic discussed at the beginning of the Introduction.

the compositional process. At issue with respect to the music, however, is not logical consistency for its own sake but rather compositional expediency and the aesthetic attributes of the results of the compositional process. It is true that, early on in his career Xenakis proposed, and then attempted several times to realize, the composition of a logically consistent, "pure" stochastic music. These attempts achieved their most thorough realization in the instrumental works composed with the program "Free Stochastic Music" (1956-62) and in *Gendy3* (1991), which was composed with the dynamic stochastic synthesis program. The large-scale structure of these works in their finished form, however, is not a direct expression of the stochastic principles used in their composition, but rather results from a patchwork of segments assembled at the composer's discretion into complete works.[2] The ideal of a logically consistent and aesthetically satisfactory stochastic music, then, has never been fully realized.

The type of thinking that led to the pursuit of such an ideal is, of course, characteristic of certain strains in musical thought and compositional practice after the Second World War, the most famous examples of which are the attempts to achieve the ideal of totally serialized compositional process, as in Boulez's *Structures Ia* (1952) and Stockhausen's *Gruppen* (1955-7).[3] Further developments in the music of both of these composers may or may not have come about as a reaction to the difficulties involved in the

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[2]Peter Hoffmann, who has worked at CEMAMu on the development of an improved version of the dynamic stochastic synthesis program, revealed to me in conversation that Xenakis selected material for *Gendy3* by deleting portions of several of the superimposed stochastic "voices" generated by the program, thereby creating greater contrasts in density and timbre than were found in the original program output.

[3]See György Ligeti, "Pierre Boulez: Decision and Automatism in *Structure Ia*," *Die Reihe* 4 (1960): 36-62, and Karlheinz Stockhausen, "... how time passes ...," *Die Reihe* 3 (1959): 10-40.

composition of music in which several dimensions are serialized simultaneously. Notably absent from their later projects, however, is the inflated rhetoric that typically accompanied the production of their important early works. The situation is similar with Xenakis who, after proposing and attempting to realize several highly formalized approaches to composition, both stochastic and non-stochastic, settled into a highly individual but less thoroughly formalized approach in the works composed since 1970. The fact that his last thorough explanation of a compositional process and of the resultant musical structure, "Towards a Philosophy of Music," dates from the late 1960s is probably connected with a change in his compositional aesthetics at around this time, as well as with a lessening of his confidence in written testimony to support his compositional endeavors.[4] The analyses in this dissertation represent a preliminary attempt to come to terms with the nature of this change in his compositional aesthetics and practice.

On the one hand, Xenakis's recent music recaptures some of the exploration of varied textures that characterized his works prior to the almost exclusive dominance of stochastic configurations that began with *Achorripsis* (1956-7) and lasted roughly until the composition of *Nomos Alpha* (1966). On the other hand, his music written since 1970 retains stochastic configurations as a particular type of texture but generally combines them with other textures, including random walks, arborescences, and simultaneities, or abandons them completely, as in the works for strings and in *Mycenae-Alpha* (1978). In addition, the exponential distribution, which was used for the generation of small- and large-scale temporal structures in the works

[4]"Towards a Philosophy of Music" has been reprinted as chapter 8 of Iannis Xenakis, *Formalized Music*, rev. ed. (Stuyvesant, NY: 1992).

composed with "Free Stochastic Music," is apparently retained in the recent music as a generator of segment durations outside-time and also of the points of initiation of segments inside-time (in works in which there is a distinct difference between the two). The sequence of the durations, however, is no longer randomized, but is rather "composed" according to a desire for local similarities and contrasts in segment duration and in conformity to the proportions that govern the large-scale temporal structure of a given work. In place of the conceptual unity afforded by a thoroughly stochastic compositional process, an abstract unity between the inside- and outside-time temporal structures, observable in early works such as *Metastaseis* (1953-4) and *Pithoprakta* (1955-6), reappears in subtler form in the recent music. This unity is generally given realization as a correspondence between the proportions used in the inside-time temporal structure of a work and those found among the total outside-time durations of groups of segments that contain particular configuration types. As suggested in chapter 2, section 2.1.5, it is possible that this "unity" may not be merely conceptual, but may actually contribute to the sense of structural completeness of a work as it unfolds in time.[5]

The remarks above indicate that practical, rather than idealized or formalized, criteria have come to the fore as Xenakis's compositional approach has developed over the last 40 years or so. Without doubt, the formalized adventures from the beginning of his compositional career have influenced the nature of the intuitive choices he has made all along, including the choices involving which formalized techniques to use and how

[5]See chapter 2 and Jonathan Kramer, *The Time of Music* (New York: Schirmer, 1988), chapters 2 and 10.

to employ them compositionally. Whenever the use of precise calculations has proved too burdensome, or simply inadequate to realize the structures that have arisen in his imagination, Xenakis has turned to his abilities in graphic design in order to seek solutions to specific compositional problems. His reliance upon graphic methods for the organization of musical structures has been demonstrated indirectly in the graphic transcriptions of the published scores of his instrumental works and directly in the reproduction of the graphic score to *Mycenae-Alpha*. Whether calculations or graphic methods are used in the design of the sonic configurations in his music, much of the frankness and vitality of the music seems to come from the almost direct realization of abstract, non-acoustic concepts into sonic structures. The results often seem rough and are not uniformly satisfying, but the energy and apparent spontaneity of the music seems to betray the working of an impatient imagination that strives for perfection while at the same time taking an almost perverse delight in the art of approximation. These characteristics of Xenakis's aesthetics and working methods are hinted at in the following remarks from the composer:

> The hand, itself, stands between randomness and calculation. It is both an instrument of the mind—so close to the head—and an imperfect tool. ... Industrialization is a forced purification. But you can always recognize what has been made industrially and what has been made by hand. Industrial means are clean, functional, poor. The hand adds inner richness and charm.[6]

Another feature of Xenakis's music that has emerged in the analyses is the relative, or even optional, structural role of pitch relations. The diminished status of pitch relations—more specifically of small- and large-

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[6]Iannis Xenakis, "Xenakis on Xenakis," *Perspectives of New Music* 25/1 (1987): 23.

scale structures based on pitch or pitch-class relations—in Xenakis's music is certainly striking in comparison to the principal role of pitch and pitch-class relations in the music of most other composers, as well as in most theoretical and analytical discourse about musical structure. Xenakis's attitude toward the structural role of pitch relations is expressed in a number of ways. First, there is his preference for large psets (often m-psets, or sieves) whose spacings deliberately avoid periodicity at the interval of the octave, and thus inhibit the formation of meaningful small-scale structures based on pitch-class intervals or interval-classes.[7] Second, the stochastic or graphically designed configurations favored by Xenakis represent movements through p-space that occur essentially independently of the intervallic characteristics of that space. As shown in Figures 2.43-5, a single arborescence may be realized in more than one pset or, as in Figures 2.47-51, a slightly different arborescence may just as easily be realized in an undifferentiated semitonal p-space. Third, there is the issue of Xenakis's seeming unwillingness to generate large-scale pitch structures in which the transformations on the elements of the pset are carried out with absolute consistency. An apparently deliberate vagueness is involved here that interferes with any attempt to formulate a definite analytical account of the operations used in the composition of the works. Finally, the presence of continuous modulations in pitch with respect to time (glissandi) and of non-pitch-specific sounds—through the use of certain percussion instruments, unusual methods of articulating sounds on pitched acoustic instruments or with the human voice, or by particular approaches to

[7]That is, of intervals comprising from 0 to 11 semitones, or their interval-class equivalents from 0 to 6. The distinction between pitch-class intervals and interval-classes is made clear in Robert Morris, *Composition with Pitch-Classes* (New Haven: Yale University Press, 1987), chapter 3.

electronic sound synthesis or to the processing of sampled sounds—works against the perception, and thus the structural role, of discrete pitches. Structure in the pitch dimension in such cases is limited to the articulation of sounds within specific or general locations within p-space, depending on the method of sound production and musical notation (if any) that are used in a given situation.

Pitch relations, then, constitute one potential factor in the structural articulation of Xenakis's music, but not a necessary structural criterion. Because of the optional structural status of pitch relations, it is necessary for an analytical approach to this music to take into account those factors that account for the music's structure with or without the presence of definite pitch relations. In the analyses presented here, similarities and differences in texture and the temporal structures that they help to articulate were shown to be basic structural features of the music regardless of the medium in which a particular work was composed. Where events are articulated discretely, whether or not they are pitch-specific, changes in density with respect to time may also constitute a basic element in the articulation of structure. It is possible that these basic structural criteria could serve as a starting point for the analysis of works by Xenakis in media that have not been considered in detail here. For example, though the method of graphic transcription used in chapters 3 and 4, which is quite helpful in determining the boundaries of segments, may prove too cumbersome for use in the study of orchestral scores, segmentation based on changes in timbre and in the use of p-space may form the basis for an analytically feasible investigation into the structure of works for chamber and full orchestra. Analysis of percussion works in addition to *Psappha,* which has already been studied in considerable detail,[8] may similarly be guided by the same general criteria, although more attention will have to be paid to the details of the rhythmic structure of this repertoire than was done in the analyses presented here. Xenakis's choral and vocal works may also respond well to the analytical approach proposed here, but attention should also be paid to the use of phonetics in these works, which frequently proceeds independently of the semantic contents (if any) of the phonemes employed in them. Finally, further analysis of Xenakis's UPIC works along the lines suggested here, and of his electroacoustic music in general, may benefit from spectral analysis, such as that demonstrated by Robert Cogan in *New Images of Musical Sound,* in part because of the difficulties involved in the symbolic representation of frequency and timbre with respect to time in electroacoustic music.[9]

Finally, the applicability of the analytical approach developed here to the analysis of works by other composers whose music shares some structural characteristics with Xenakis's should be considered. Much of Messiaen's music, for example, particularly the music composed from the 1950s onward, is made up of successive segments whose texture, timbre, and other qualities contrast markedly as the musical structure unfolds in time.[10] The structuring

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[8]See Ellen Rennie Flint, "Metabolae, Arborescences and the Reconstruction of Time in Iannis Xenakis' Psappha," *Contemporary Music Review* 7 (1993): 221-48, and E. R. Flint, "An Investigation of Real Time as Evidenced by the Structural and Formal Multiplicities in Iannis Xenakis's Psappha," (Ph.D. diss., University of Maryland at College Park, 1989).

[9]Robert Cogan, *New Images of Musical Sound* (Cambridge, MA: Harvard University Press, 1984). Cogan's book includes spectrographs of excerpts from electroacoustic works by Babbitt, Risset, and himself.

[10]The graphs of musical form in Robert Sherlaw Johnson, *Messiaen* (Berkeley: University of California Press, 1989), particularly those in chapter 12, on *Catalogue d'oiseaux,* make this clear.

of pitch and time in Messiaen's music differs significantly from the structuring of these dimensions in Xenakis's music, however, and these differences need to be taken into account in analysis. Jonathan Bernard has demonstrated a general lack of meaningful octave-equivalence relations in the structuring of pitch in Varèse's music, a feature that invites comparison with Xenakis's general disregard of octave equivalence.[11] But Bernard has also demonstrated transformations based on relations between the sizes of pitch intervals and an approach to the "spatialization" of pitch relations that reveals a greater reliance on specific details than the approach found in much of Xenakis's music. Ligeti is known to have designed sonic structures in pitch and time that, when transcribed from musical notation into graphic notation, reveal striking images that are often asymmetrical in their morphology.[12] These designs are associated, however, with pitch, pitch-class, and rhythmic structures that are generally quite foreign to Xenakis's aesthetic preference for extreme irregularity of detail on the musical surface. Thus, although a disciplined approach to segmentation and to the role of texture and temporal factors in the projection of form in the works of these and other composers may help to focus investigations into the structural principles of their music, any attempt to uncover general structural characteristics in the music of

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[11]See Jonathan Bernard, "Pitch/Register in the Music of Edgard Varèse," *Music Theory Spectrum* 3 (1981): 1-25.

[12]See Jonathan Bernard, "Inaudible Structures, Audible Music: Ligeti's Problem, and His Solution," *Music Analysis* 6 (1987): 207-36; J. Bernard, "Voice Leading as a Spatial Function in the Music of Ligeti," *Music Analysis* 13 (1994): 227-53; Jane Clendinning, "The Pattern-Meccanico Compositions of György Ligeti," *Perspectives of New Music* 31/1 (1993): 192-234; Pozzi Escot, "'Charm'd Magic Casements': György Ligeti's *Harmonies*," in *Contiguous Lines: Issues and Ideas in the Music of the 60's and 70's*, ed. Thomas DeLio (Lanham, MD: University Press of America, 1985), 31-56; Michael Hicks, "Interval and Form in Ligeti's *Continuum* and *Coulée*," *Perspectives of New Music* 31/1 (1993): 172-90; and Miguel A. Roig-Francolí, "Harmonic and Formal Processes in Ligeti's Net-Structure Compositions," *Music Theory Spectrum* 17 (1995): 242-67. Further references to the literature on Ligeti may be found in the Roig-Francolí article.

avant-garde composers from before and after the Second World War should not lose sight of the important distinctions in aesthetics and compositional practices among this group of strikingly individual musicians.

Appendix I

Sieve Program Listings

Program listings for the synthesis and analysis of sieves are included in *Formalized Music.*[1] The listings are in the C programming language, in a version that may be run on the IBM PC. The programs may also be run on the Macintosh, after a few adjustments to the code. Unfortunately, the program listings given in *Formalized Music* contain quite a few typographical errors. A list of errata is provided below. Code changes necessary for operation on the Macintosh have also been included in the errata. These changes are preceded by the indication "for Mac only:" so that they will not be confused with genuine errata. Virtually error-free versions of the program listings may be also be found in *Perspectives of New Music* 28 (1990): 69-78, as part of an alternative translation (by John Rahn) of Xenakis' introduction to sieve theory. The second of the two programs in that version, however, generates output in French. Readers who wish to compare the two versions may do so, but those who incorporate the corrections given below into the *Formalized Music* versions should be able to run the programs without any difficulty. Following the list of errata, complete listings for both programs are given below. These versions have been run successfully on the Macintosh with Symantec THINK C 5.0 software. They include a few cosmetic refinements, such as the capacity for printing data to files, that are not found

[1]Iannis Xenakis, *Formalized Music,* rev. ed. (Stuyvesant, NY: Pendragon Press, 1992), chapter 12.

in either of the two other versions. Macintosh users with THINK C may wish to transcribe the listings as they are given here, rather than make the necessary adjustments to the versions found in *Formalized Music.*

* * * * *

Errata for program listings in *Formalized Music,* 279-88:

Program A: Generation of points on a straight line from the logical formula of the sieve

page	line#	should read
279	1	#include <stdio.h>
	3	#include <conio.h>; *for Mac only:* change "<conio.h>" to "<console.h>"
280	73	for (u = 0; u < unb; u++)
	84	for (i = 0; i < fCrib[u].clnb; i++)
	96	for (u = 0; u < unb; u++)
	100	for (i = 0; i < fCrib[u].clnb; i++)
281	114	for (u = 0; u < unb; u++)
	117	for (i = 0; i < fCrib[u].clnb; i++)
	126	*for Mac only:* between lines 126 and 127 insert "getchar();"
	127	*for Mac only:* change "getche()" to "getchar()"
	139	for (u = 0; u < unb; u++)
282	157	for (u = 0; u < unb; u++)
	181	if (ptnb >= n0)
	185	*for Mac only:* change "getch();" to "getchar();"
	204	for (n = 1; n < fCrib[u].clnb; n++)
283	208	if (cl1.mod < cl2.mod)
284	279	comment should read "/* a1 >= a2 > 0*/"
	279	*for Mac only:* should read "short Euclide (short a1, short a2)"; then delete lines 280 and 281
	293	comment should read "/* c1 >= c2 > 0 */"
	293	*for Mac only:* should read "short Meziriac (short c1, short c2)"; then delete lines 294 and 295

Program B: Generation of the logical formula of the sieve from a series of points on a straight line

page	line#	should read
285	4	#include <conio.h>; *for Mac only:* change "<conio.h>" to "<console.h"
286	58	for (p = 0; p < ptTotNb; p++)
	60	*for Mac only:* lines 60 and 61 may be deleted (this code does not produce the text layout shown in *Formalized Music,* 285)

	65	ptnb < p && ptval > ptCrib[ptnb];
	68	if (ptnb < p)
	70	if (ptval < ptCrib[ptnb])
287	85	for (p = 0; p < ptTotNb; p++)
	95	for (p = 0; p < ptTotNb; p++)
	107	ptnb < ptTotNb && ptval >= ptCrib[ptnb];
	117	while (ptnb < ptTotNb);
	120	ptnb < ptTotNb;
288	138	for (p = 1; p < perTotNb; p++)
	140	if ((long) perCrib[p].mod >= percrib)
	150	for (p = 0; p < perTotNb; p++)
	156	printf (" + ");
	163	comment should read "/*a1 >= a2 > 0*/"

* * * * *

Complete program listings:

A. Generation of points on a straight line from the logical formula of the sieve

```
/* sieveA.c - Generation of points on a straight line from the */
/* logical formula of the sieve, cf. Formalized Music, 279-84  */

#include <stdio.h>
#include <stdlib.h>
#include <console.h>

/* ----------------- types definitions ---------------------------------------- */
typedef struct                  /* period (congruence class)                   */
{
        short       mod;        /* modulus of the period                       */
        short       ini;        /* starting point                              */
}       periode;
typedef struct                  /* intersection of several periods             */
{
        short       clnb;       /* number of terms in the intersection         */
        periode     *cl;        /* terms in the intersection                   */
        periode     clr;        /* resulting period                            */
        unsigned    long ptval; /* current point value                         */
}       inter;

/* ----------------- function prototypes -------------------------------------- */
periode     ReducInter(short u);     /* computation of the intersections       */
short       Euclide(short m1,short m2);/* computation of the LCD               */
short       Meziriac(short c1,short c2);/* computation of "dzeta"              */
void        Decompos(periode pr);  /* decomposition into prime factors         */
```

```
/* ---------------- variables ------------------------------------------ */
FILE    *fp;

inter           *fCrib;              /* sieve formula                     */
short           unb = 0;             /* number of unions in the formula   */

short           u0, u1, u = 0;       /* current union index               */
short           i = 0;               /* current intersection index        */
unsigned long   lastval,n0,ptnb = 0;
periode         CL_EMPTY = { 0, 0 }; /* empty period                      */

#define         NONEMPTY   1
short           flag = 0;
short           decomp = 0;

/* ====================================================================== */
void main(void)
{
        fp = fopen("SieveA Data", "a"); /* append type file */

        printf("SIEVES: user's guide\n\n"
                "A. GENERATION OF POINTS ON A STRAIGHT LINE FROM\n"
                "  THE LOGICAL FORMULA OF THE SIEVE\n\n"
                "Example:\n"
                  "--------------------------\n"
                "DEFINITION OF A SIEVE:\n"
                "       L =  [() * () * ... * ()]\n"
                "            + [() * () * ... * ()]\n"
                "            + ...\n"
                "            + [() * () * ... * ()]\n\n"
                "In each parenthesis are given in order:  modulus, starting point\n"
                "       (taken from the set of integers)\n"
                "[] + [] is a union\n"
                "() * () is an intersection\n\n");
        printf("--------------------------------\n"
                "Given the formula of a sieve made out of unions and\n"
                "intersections of moduli, the program reduces the number of\n"
                "intersections to one and keeps only the given unions.\n"
                "Then, the abcissa of the final points of the sieve are\n"
                "computed from these unions and displayed.\n\n");
        /* ----------- get the formula of the sieve ---------------------- */
        while (unb == 0)
                {
                printf("NUMBER OF UNIONS ? = ");
                scanf("%d",&unb);
                }
        fCrib = (inter *)(malloc (sizeof(inter) * unb));
        if (fCrib == NULL)
                {
                printf("not enough memory\n");
                exit(1);
                }
        printf("-----------------------\n");
        for (u = 0; u < unb; u++)
```

```
        {
        printf("union %d: number of moduli ? = ", u + 1);
        scanf("%d",&fCrib[u].clnb);

        fCrib[u].cl = (periode *)(malloc (sizeof(periode) * fCrib[u].clnb));
        if (fCrib[u].cl == NULL)
                {
                printf("not enough memory\n");
                exit(1);
                }
        for (i = 0; i < fCrib[u].clnb; i++)
                {
                printf("\n        modulus %d ? = ", i + 1);
                scanf("%d",&fCrib[u].cl[i].mod);
                printf("          start ?     = ");
                scanf("%d",&fCrib[u].cl[i].ini);
                }
        printf("------------------------\n");
        }
/* -------------- reduction of the formula -------------------------- */
printf("FORMULA OF THE SIEVE:\n\n"
        "L =  [");
fprintf(fp, "\n\n\nFORMULA OF THE SIEVE:\n\n"
        "L =  [");
for (u = 0; u < unb; u++)
        {
        if (u != 0)
                {
                printf("    + [");
                fprintf(fp, "    + [");
                }
        for (i = 0; i < fCrib[u].clnb; i++)
                {
                if (i != 0)
                        {
                        if (i % 4 == 0)
                                {
                                printf("\n");
                                fprintf(fp, "\n");
                                }
                        printf(" * ");
                        fprintf(fp, " * ");
                        }
                printf("(%d,%d)", fCrib[u].cl[i].mod, fCrib[u].cl[i].ini);
                fprintf(fp, "(%d,%d)", fCrib[u].cl[i].mod, fCrib[u].cl[i].ini);
                }
        printf("]\n");
        fprintf(fp, "]\n");
        }
printf("---------------------------\n");
fprintf(fp, "---------------------------\n");

printf("REDUCTION OF THE INTERSECTIONS:\n\n");
for (u = 0; u < unb; u++)
```

```
		{
		printf("union %d\n      [ ",u + 1);
		for (i = 0; i < fCrib[u].clnb; i++)
			{
			printf("(%d,%d) ", fCrib[u].cl[i].mod, fCrib[u].cl[i].ini);
			if (i != fCrib[u].clnb - 1)
				{
				printf("* ");
				}
			}
		fCrib[u].clr = ReducInter(u);/* reduction of an intersection       */
		printf("] = (%d,%d)\n\n", fCrib[u].clr.mod, fCrib[u].clr.ini);
		printf("          decomposition into prime modules ?\n"
		       "          (press 'y' for yes, 'n' for no): ");
getchar();
		if (getchar() == 'y')
			{
			printf("\n\n  (%d,%d)", fCrib[u].clr.mod, fCrib[u].clr.ini);
			Decompos(fCrib[u].clr);
			}
		else
			{
			printf("\n\n");
			}
		}
	printf("---------------------------------\n");

	/* -------------- display the simplified formula ------------------ */
	printf("SIMPLIFIED FORMULA OF THE SIEVE:\n\n");
	fprintf(fp, "SIMPLIFIED FORMULA OF THE SIEVE:\n\n");
	printf("L =   ");
	fprintf(fp, "L =   ");
	for (u = 0; u < unb; u++)
		{
		if (u != 0)
			{
			if (u % 5 == 0)
				{
				printf("\n    ");
				fprintf(fp, "\n    ");
				}
			printf("+ ");
			fprintf(fp, "+ ");
			}
		printf("(%d,%d) ", fCrib[u].clr.mod, fCrib[u].clr.ini);
		fprintf(fp, "(%d,%d) ", fCrib[u].clr.mod, fCrib[u].clr.ini);
		}
	printf("\n--------------------------\n");
	fprintf(fp,  "\n--------------------------\n");

	/* ------------- points of the sieve --------------------------------- */
	printf("POINTS OF THE SIEVE CALCULATED WITH THIS FORMULA:\n");
	fprintf(fp, "POINTS OF THE SIEVE CALCULATED WITH THIS FORMULA:\n");
	printf("rank of first displayed point ? = ");
```

```
        scanf("%lu",&n0);
        n0 = n0 - n0 % 10;
        printf("Press <return> or <enter> to get a series of 10 points.\n");
        printf("Select 'Quit' from File menu to end program.\n");
        printf("\n\nRank          | \n");
        fprintf(fp, "\nRank      | ");
        for (u = 0; u < unb; u++)
                {
                if (fCrib[u].clr.mod != 0 | | fCrib[u].clr.ini != 0)
                        {
                        fCrib[u].ptval = fCrib[u].clr.ini;
                        flag = NONEMPTY;
                        }
                else
                        fCrib[u].ptval = 0xFFFFFFFF;
                }
        if (flag != NONEMPTY)
                return;
        u0 = u1 = 0;
        lastval = 0xFFFFFFFF;
        while (1)
                {
                for (u = (u0 + 1) % unb; u != u0; u = (u + 1) % unb)
                        {
                        if (fCrib[u].ptval <= fCrib[u1].ptval)
                                u1 = u;
                        }
                if (fCrib[u1].ptval != lastval)    /* new point */
                        {
                   lastval = fCrib[u1].ptval;
                        if (ptnb >= n0)
                                {
                                if (ptnb % 10 == 0)
                                        {
                                        getchar(); /* get a character from the keyboard */
                                        printf("%7lu  | ", ptnb);
                                        fprintf(fp, "\n%7lu  | ", ptnb);
                                        }
                                printf("%5lu ", fCrib[u1].ptval);
                                fprintf(fp, "%5lu ", fCrib[u1].ptval);
                                }
                        ptnb++;
                        }
                fCrib[u1].ptval += fCrib[u1].clr.mod;
                u0 = u1;
                }

        fclose(fp);
}
/* ==================== reduction of an intersection =================*/
periode ReducInter(short u)
{
        periode         cl,cl1,cl2,cl3;
        short           pgcd,T,n;
```

```
	long		c1,c2;

	cl3 = fCrib[u].cl[0];
	for (n = 1; n < fCrib[u].clnb; n++)
		{
		cl1 = cl3;
		cl2 = fCrib[u].cl[n];
		if (cl1.mod < cl2.mod)
			{
			cl = cl1;
			cl1 = cl2;
			cl2 = cl;
			}
		if (cl1.mod != 0 && cl2.mod != 0)
			{
			cl1.ini %= cl1.mod;
			cl2.ini %= cl2.mod;
			}
		else
			return CL_EMPTY;
		/* module resulting from the intersection of 2 modules */
		pgcd = Euclide(cl1.mod, cl2.mod);
		c1 = cl1.mod / pgcd;
		c2 = cl2.mod / pgcd;
		if (pgcd != 1
			&& ( (cl1.ini - cl2.ini) % pgcd != 0 ))
			return CL_EMPTY;
		if (pgcd != 1
			&& ((cl1.ini - cl2.ini) % pgcd == 0)
			&& (cl1.ini != cl2.ini) && (c1 == c2) )
			{
			cl3.mod = pgcd;
			cl3.ini = cl1.ini;
			continue;
			}
		T = Meziriac((short) c1, (short) c2);
		cl3.mod = (short) (c1 * c2 * pgcd);
		cl3.ini = (short) ((cl1.ini
				+ T * (cl2.ini - cl1.ini) * c1) % cl3.mod);
		while (cl3.ini < cl1.ini || cl3.ini < cl2.ini)
			cl3.ini += cl3.mod;
		}
	return cl3;
}
/* =================== decomposition into an intersection ===========*/
/*========================of prime modules==================*/
void Decompos (periode pr)
{
	periode		pf;
	short		fct;

	if (pr.mod == 0)
		{
		printf(" = (%d,%d)\n", pr.mod, pr.ini);
```

```
                return;
                }
        printf(" =");
        for ( i = 0, fct = 2; pr.mod != 1; fct++)
                {
                pf.mod = 1;
                while (pr.mod % fct == 0 && pr.mod != 1)
                        {
                        pf.mod          *= fct;
                        pr.mod          /= fct;
                        }
                if (pf.mod != 1)
                        {
                        pf.ini = pr.ini % pf.mod;
                        pr.ini %= pr.mod;
                        if (i != 0)
                                printf(" *");
                        printf(" (%d,%d)", pf.mod, pf.ini);
                        i++;
                        }
                }
        printf("\n\n");
}
/* ==================== Euclide's algorithm =====================*/
short Euclide (short a1, short a2)          /* a1 >= a2 > 0 */

{
        short tmp;

        while ((tmp = a1 % a2) != 0)
                {
                a1 = a2;
                a2 = tmp;
                }
        return a2;
}
/* ==================== De Meziriac's theorem ===================*/
short Meziriac (short c1, short c2)                 /* c1 >= c2 > 0 */

{
        short T = 0;

        if (c2 == 1)
                T = 1;
        else
                while (((++T * c1) % c2) != 1)
                        ;
        return T;
}
```

Program B: Generation of the logical formula of the sieve from a series of points on a straight line

```
/* sieveB.c - Generation of the logical formula of the sieve from a */
/* series of points on a straight line; cf. Formalized Music, 285-8 */

#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <console.h>

/* -------------------- type definitions ------------------------------ */
typedef struct                          /* period (congruence class)       */
{
        short   mod;                    /* modulus of the period           */
        short   ini;                    /* starting point                  */
        short   couv;                   /* number of points covered        */
}       periode;

/* ---------------- function prototypes -------------------------------- */
unsigned long Euclide(unsigned long m1,
        unsigned long m2);              /* computation of the LCD          */

/* -------------- variables and constants ------------------------------ */
FILE            *fp;

periode         *perCrib;       /* periods of the sieve                    */
short           perTotNb = 0;   /* number of periods in the formula        */
long            *ptCrib;        /* points of the sieve                     */
long            *ptReste;       /* points outside the periods              */
short           ptTotNb = 0;    /* number of points in the sieve           */
short           p,ptnb;
long            ptval;
unsigned long   percrib;

periode         per;

#define         NON_REDUNDANT  0
#define         REDUNDANT      1
#define         COVERED        -1L
short           flag;

/* ================== principal function ====================== */
void main(void)
{
        fp = fopen ("SieveB Data", "a");

        printf("B. GENERATION OF THE LOGICAL FORMULA OF THE SIEVE\n"
               "  FROM A SERIES OF POINTS ON A STRAIGHT LINE\n\n"
               "Example: \n"
               "-----------------------------\n"
               "Given a series of points, find the starting\n"
               "points with their moduli (periods).\n\n");
```

```
/* ---- entry of the points of the sieve and their sorting ------ */
while (ptTotNb == 0)
        {
        printf("NUMBER OF POINTS ? = ");
        scanf("%d",&ptTotNb);
        }
ptCrib  = (long *)(malloc (ptTotNb * sizeof(long)));
ptReste         = (long *)(malloc (ptTotNb * sizeof(long)));
perCrib         = (periode *)(malloc (ptTotNb * sizeof(periode)));
if (ptCrib == NULL || ptReste == NULL || perCrib == NULL)
        {
        printf("not enough memory\n");
        exit(1);
        }
printf("----------------------------\n"
          "abscissa of the points:\n\n");
for (p = 0; p < ptTotNb; p++)
        {
        printf("          point %2d = ", p + 1);
        scanf("%ld", &ptval);
        for (ptnb = 0;
                ptnb < p && ptval > ptCrib[ptnb];
                ptnb++)
                ;
        if (ptnb < p)
                {
                if (ptval < ptCrib[ptnb]) /* new point */
                        memmove(&ptCrib[ptnb + 1], &ptCrib[ptnb],
                                        sizeof(long) * (p - ptnb));
                else /* point already exists */
                        {
                        p--;
                        ptTotNb--;
                        }
                }
        ptCrib[ptnb] = ptval;
        }
printf("\n----------------------------------\n");
/* ------------ points of the sieve ------------- */
printf("POINTS OF THE SIEVE (by order of increasing abscissa):\n\n"
          "Rank         |");
fprintf(fp, "\n\nPOINTS OF THE SIEVE (by order of increasing abscissa):"
          "\n\nRank    |");
for (p = 0; p < ptTotNb; p++)
        {
        if (p % 10 == 0)
                {
                printf("\n%7d |", p);
                fprintf(fp, "\n%7d |", p);
                }
        printf("%6ld ", ptCrib[p]);
        fprintf(fp, "%6ld ", ptCrib[p]);
        }
printf("\n\n----------------------------------\n");
```

```
fprintf(fp, "\n\n-------------------------------\n");
/* ----------- compute the periods of the sieve ----------- */
memcpy(ptReste, ptCrib, ptTotNb * sizeof(long));

for (p = 0; p < ptTotNb; p++)
        {
        if ( ptReste[p] == COVERED )
                continue;
        /* ---- compute a period starting at current point ---- */
        per.mod = 0;
        do
                {
                per.mod++;
                per.ini = (short) (ptCrib[p] % (long)per.mod);
                per.couv = 0;
                for (ptnb = 0, ptval = per.ini;
                         ptnb < ptTotNb && ptval >= ptCrib[ptnb];
                         ptnb++)
                        {
                        if (ptval == ptCrib[ptnb])
                                {
                                per.couv++;
                                ptval += per.mod;
                                }
                        }
                }
        while (ptnb < ptTotNb);
        /* -------- check the redundancy of the period ----------- */
        for (ptnb = 0, ptval = per.ini, flag = REDUNDANT;
                 ptnb < ptTotNb;
                 ptnb++)
                {
                if (ptval == ptCrib[ptnb])
                        {
                        if (ptval == ptReste[ptnb])
                                {
                                ptReste[ptnb] = COVERED;
                                flag = NON_REDUNDANT;
                                }
                        ptval += per.mod;
                        }
                }
        if (flag == NON_REDUNDANT)
                perCrib[perTotNb++] = per;
        }
/* -------- compute the period of the sieve ---------------- */
percrib = perCrib[0].mod;
for (p = 1; p < perTotNb; p++)
        {
        if ((long) perCrib[p].mod >= percrib)
                percrib *= (long) perCrib[p].mod /
                Euclide((long)perCrib[p].mod, percrib);
        else
                percrib *= (long) perCrib[p].mod /
```

```
                Euclide(percrib, (long)perCrib[p].mod);
            }
    /* ---------- display the formula of the sieve ------------- */
    printf("FORMULA OF THE SIEVE:\n"
            "In each parenthesis are given in order:\n"
            "(modulus, starting point, number of points covered)\n\n");
    fprintf(fp, "FORMULA OF THE SIEVE:\n"
            "In each parenthesis are given in order:\n"
            "(modulus, starting point, number of points covered)\n\n");
    printf("L =     ");
    fprintf(fp, "L =     ");
    for (p = 0; p < perTotNb; p++)
        {
        if (p != 0)
            {
            if (p % 3 == 0)
                {
                printf("\n      ");
                fprintf(fp, "\n      ");
                }
            printf("+ ");
            fprintf(fp, "+ ");
            }
        printf("(%5d,%5d,%5d) ",perCrib[p].mod,perCrib[p].ini,
            perCrib[p].couv);
        fprintf(fp, "(%5d,%5d,%5d) ",perCrib[p].mod,perCrib[p].ini,
            perCrib[p].couv);
        }
    printf("\n\n    period of the sieve: P = %lu\n", percrib);
    fprintf(fp, "\n\n    period of the sieve: P = %lu\n", percrib);

    fclose(fp);
}
/* ======================= Euclid's algorithm ====================== */
unsigned long Euclide (a1, a2)  /* a1 >= a2 > 0 */
unsigned long a1;
unsigned long a2;
{
    unsigned long tmp;

    while ((tmp = a1 % a2) != 0)
        {
        a1 = a2;
        a2 = tmp;
        }
    return a2;
}
```

Appendix II

Simple Method for the Calculation of Sieve Formulas by Hand

Each element within a single period of a sieve may be associated with a unique intersection of ordered pairs (m_i, r_j) in which m_i indicates the moduli and r_j indicates the residue classes associated with that that element, and where $i = 0, 1, 2, \ldots, n$ and $j = 0, 1, 2, \ldots, m_i - 1$. (See chapter 2, section 2.1.7.) The number of elements in one period of a sieve is equivalent to the lowest common multiple (LCM) of its moduli. When the intersection that produces each element in the sieve is known, it is possible to write out the formula for the sieve. Matrices provide a simple way of identifying the intersections associated with the elements found in a single period of a sieve.

If the sieve is generated with two moduli, matrices may be constructed in which the columns represent the residue classes of one modulus and the rows represent the residue classes of the other modulus. Each element of the sieve appears at the intersection of the row and column associated with the ordered pairs whose intersection produces that element. As an example, let us consider the intersections of the residue classes of moduli 8 and 11 that can be used to represent the pitches on the standard piano keyboard. The residue classes of modulus 8 are represented in the columns and those of modulus 11 are represented in the rows.

11\8	0	1	2	3	4	5	6	7
0	0	33	66	11	44	77	22	55
1	56	1	34	67	12	45	78	23
2	24	57	2	35	68	13	46	79
3	80	25	58	3	36	69	14	47
4	48	81	26	59	4	37	70	15
5	16	49	82	27	60	5	38	71
6	72	17	50	83	28	61	6	39
7	40	73	18	51	84	29	62	7
8	8	41	74	19	52	85	30	63
9	64	9	42	75	20	53	86	31
10	32	65	10	43	76	21	54	87

The entries in the matrix follow a definite pattern. Beginning from the intersection of (8, 0) and (11, 0), the numbers from 1 to 7 appear along the diagonal from the upper left to the mid-lower right. When all the columns have been passed through, the sequence of numbers continues in the next row by wrapping around to the first column. Thus, 8 is the intersection of (8, 0) and (11, 8). Once again the numbers continue along the diagonal from left to right and top to bottom. When the last row is reached with number 10, the sequence continues into the next column by wrapping around to the first row, where 11 is shown to be the intersection of (8, 3) and (11, 0). The remainder of the numbers are entered in similar fashion, wrapping around columns and rows as needed. Note that the first column contains only multiples of 8 and the first row only multiples of 11. This is consistent with the designation of the first column as (8, 0) and the first row as (11, 0).

If the sieve is generated from more than one modulus, it is necessary to construct submatrices. For example, for a sieve with moduli 2, 5, and 9, and period 90, one matrix can be constructed on the basis of the residue classes of moduli 2 and 5 and another on the residue classes of moduli 10 and 90. Two such matrices are shown below.

5\2	0	1
0	0	5
1	6	1
2	2	7
3	8	3
4	4	9

10\9	0	1	2	3	4	5	6	7	8
0	0	10	20	30	40	50	60	70	80
1	81	1	11	21	31	41	51	61	71
2	72	82	2	12	22	32	42	52	62
3	63	73	83	3	13	23	33	43	53
4	54	64	74	84	4	14	24	34	44
5	45	55	65	75	85	5	15	25	35
6	36	46	56	66	76	86	6	16	26
7	27	37	47	57	67	77	87	7	17
8	18	28	38	48	58	68	78	88	8
9	9	19	29	39	49	59	69	79	89

As an example, in order to find the intersection of (2, r_0), (5, r_1), and (9, r_2) that produces the element 37 using these matrices, we first use the larger matrix to determine that 37 is the intersection of (9, 1) and (10, 7). Using the smaller matrix, we determine that (10, 7), i.e. 7, is equivalent to the intersection of (2, 1) and (5, 2). Thus, 37 is produced by the intersection of (2, 1), (5, 2), and (9, 1).

Appendix III

Program Listings for Calculating and Generating Values for the Exponential and Linear Distributions

A. Calculation of the Exponential Distribution

```
/* expCalc.c - Calculates probabilities according to an expected */
/* exponential distribution (see Formalized Music, Appendix I, 323-5) */

#include <stdio.h>
#include <math.h>

/*================variables=================*/

double  p = 0.; /* probability of x */
double  d = 0.; /* density */
double  w = 0.; /* increment */
double  z = 0.; /* value for dx */
double  s = 0.; /* number of intervals in sample */
double  r = 0.; /* number of intervals in range */
double  sum = 0.;
double  tmp1 = 0.; /* temporary variables */
double  tmp2 = 0.;
double  tmp3 = 0.;
int     n = 0; /* number of values desired */
int     i = 0; /* "for" loop increment */

/*==============main function===============*/

main()
{

        printf("Enter the values requested in order to obtain probabilities"
                "\nfor an expected exponential distribution.");

        printf("\n\nEnter density: ");
        scanf("%lf", &d);
        printf("Enter increment value: ");
        scanf("%lf", &w);
        printf("Enter number of probability values desired: ");
        scanf("%d", &n);
        printf("Enter number of intervals in sample: ");
        scanf("%lf", &s);
```

```
        z = (1 - (exp(-d*w))) / d;

        printf("\n\nProbabilities for x                          Number of intervals\n\n");

        for (i = 0; i < n; i++)
                {
                tmp1 = i; /* convert value of i to float variable */

                p = (exp(-d*tmp1*w)) * (d*z);

                tmp2 = tmp1*w;

                tmp3 = (tmp1 + 1)*w;

                r = p*s;

                printf("%.3lf ≤ x ≤ %.3lf = %.3lf                 %.0lf\n", tmp2, tmp3, p, r);

                sum = sum + p;
                }

        printf("\n\nThe sum of the probabilities is %.3lf.", sum);

}
```

B. Generation of Values According to the Exponential Distribution

```
/* expon.c (generates values according to the exponential distribution) */[1]

#include <stdio.h>
#include <math.h>

main()
{
        FILE *fp;
        int j, total = 100, seed = 2987; /* seed may be any integer */
        double x[100];
        double parm = 1.0; /* density parameter */
        double Expon ();

        fp = fopen( "Expon Data", "w" );
        srand(seed);
        for (j = 0; j < total; j++)
                {
                x[j] = Expon(parm);
                printf("%.1f\n",x[j]); /* returns values to 1 decimal place to screen */
                fprintf(fp,"%.1f\n",x[j]); /* returns values to a file */
                }
```

[1]This program listing is based on "Expon()" in Phil Winsor and Gene DeLisa, *Computer Music in C* (Blue Ridge Summit, PA: Windcrest, 1991), 186-7.

```
        fclose( fp );
} /* end of main */
/*======================= Expon function ==================*/
/* Expon() function */
double Expon(spread)
double spread;  /* horizontal scaling factor */
{
double u; /* random number > 0 & < 1.0 */
double result; /* preliminary Expon value */

double final; /* final Expon value */

u = rand() / 32767.;
result = -log(u) / spread;
final = result * 1.; /* final allows result to be scaled if desired */
return final;
} /* end of Expon() function */
/*========================================================*/
/* end of expon.c */
```

C. Calculation of the Linear Distribution

```
/* linCalc.c - Calculates probabilities according to expected */
/* linear distribution (See Formalized Music, Appendix I, 326-7) */

#include <stdio.h>

/*=====================variables=====================*/

double  p = 0.; /* probability of x */
double  minVal = 0.; /* lower limit of range */
double  maxVal = 0.; /* upper limit of range */
double  a = 0.; /* max range interval */
double  c = 0.; /* variable for dx */
double  w = 0.; /* output increment */
double  s = 0.; /* number of intervals in sample */
double  r = 0.; /* number of intervals in range */
double  sum = 0.;
double  tmp1 = 0.;
double  tmp2 = 0.;
double  tmp3 = 0.;
double  tmp4 = 0.;
int     n = 0; /* number of probability values */
int     i; /* "for" loop increment */


/*=============main function=============*/

main()
{

        printf("Enter the values requested in order to obtain probabilities"
```

```
                "\nfor an expected linear distribution.");

        printf("\n\nEnter minimum range value: ");
        scanf("%lf", &minVal);
        printf("Enter maximum range value: ");
        scanf("%lf", &maxVal);
        printf("Enter number of probability values desired: ");
        scanf("%d", &n);
        printf("Enter number of intervals in sample: ");
        scanf("%lf", &s);

        a = maxVal - minVal;
        tmp1 = n;
        w = a / tmp1;
        c = a / (tmp1 + 1);

        printf("\n\nProbabilities for x                    Number of intervals\n\n");

        for (i = 0; i < n; i++)
                {
                tmp2 = i; /* convert value of i to float variable */

                p = (2 / a) * (1 - ((tmp2 * w) / a)) * c;

                tmp3 = tmp2 * w;

                tmp4 = (tmp2 + 1) * w;

                r = p * s;

                printf("%.3lf ≤ x ≤ %.3lf = %.3lf                 %.0lf\n", tmp3, tmp4, p, r);

                sum = sum + p;
                }

        printf("\n\nThe sum of the probabilities is %.3lf.", sum);

}
```

D. Generation of Values According to the Linear Distribution

```
/* linear.c (generates values according to the linear distribution) */[2]

#include <math.h>
#include <stdio.h>

main()
{
        FILE *fp;
        int j, total = 100, seed = 67676; /* seed may be any integer */
```

[2]This program listing is based on "Linear()" in Winsor and DeLisa, *Computer Music in C*, 194-5.

```
        float x[100], Linear();

        fp = fopen ("Linear Data", "w");
        srand(seed);
        for (j = 0;j < total;j++)
                {
                        x[j] = Linear();
                        printf("%.0f\n",x[j]); /* prints to screen */
                        fprintf(fp, "%.0f\n",x[j]); /* prints to a file */
                }
        fclose (fp);
} /* end of main */
/*========================= Linear() =======================*/
/* Linear() function */
float Linear()
{
        double result; /* final Linear value */
        double u;   /* random number > 0 & < 1.0 */
        double a = 0; /* maximum interval value */

        a = 20.;

        u = rand() / 32767.;
        result = (1 - (sqrt(u))) * a;
        return result;
} /* end of Linear() function */
/*=========================================================*/
/* end of linear.c */
```

Appendix IV

Program Listings for Stochastic Composition Programs

A. STMus1

```
/* STMus1.c - produces stochastic time-point and pitch-position values within limits
        specified by the user */

#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <time.h>

main()
{
        FILE    *fp;

        int     seed; /* initialized by clock function below */
        float   parm = 0.; /* density parameter for expon function */
        float   exp; /* exponential distribution variable */
        float   t; /* time variable */
        float   tMin = 0.;
        float   tMax = 0.;
        float   tFactor = 0.; /* time scaling factor */
        float   fT; /* factored time */
        int     pInt; /* pitch-position interval variable */
        int     p; /* pitch-position variable (initialized randomly below) */
        int     pMin = 0;
        int     pMax = 0;
        float   u1,u2,u3; /* random variables */
        time_t  now; /* time and date variables */
        struct  tm *date;
        char    s[80];

        fp = fopen ("STMus1 DATA", "a");

        seed = clock(); /* initializes seed value */
        srand(seed); /* initializes random number generator */

        time (&now);
        date = localtime (&now);
        strftime (s, 80, "%c", date);
        fprintf (fp, "\n\n%s", s);
```

```
printf ("This program, STMus1, is based on two functions that are found in Iannis\n"
        "Xenakis's program 'Free Stochastic Music' (1956-62).  The first function\n"
        "determines a time point (moment of occurrence) for each sound within a\n"
        "passage of music.  The second function determines a pitch-position value\n"
        "for each time point.  Pitch-position values are numerical labels for\n"
        "elements in a pitch set.  (The  contents of the pitch set must be selected\n"
        "in advance by the user.)  The simplest way to assign pitch-position values\n"
        "is to list the elements of the set in increasing order of pitch irrespective\n"
        "of any differences in interval size between successive elements.  For\n"
        "example, possible pitch-position values for the elements in set\n"
        "{C#4, D#4, F#4, A#4, C#5} are 0, 1, 2, 3, and 4, respectively.\n\n"
        "Both functions model probability distributions.  The first function\n"
        "calculates time-point intervals according to the exponential\n"
        "probability distribution and the second calculates pitch-position\n"
        "intervals according to the linear probability distribution.\n"
        "(Cf. Xenakis, Formalized Music, chaps. 1 & 5 and Appendix 1).\n\n"
        "(Press <RETURN> to continue.)\n");

getchar();

printf ("The program asks the user to input the following information:  initial and\n"
        "terminal time-point values, minimum and maximum pitch-position values,\n"
        "and a density parameter for the time-point function.  This value\n"
        "determines the average number of sounds per second that are expected to\n"
        "occur in the passage of music.  Time-points and pitch positions are printed\n"
        "both to the console and to a file, 'STMusic1 Data.'  Time-points are given\n"
        "in seconds unless the user prefers to subdivide the standard one-second\n"
        "units into smaller units.  The default maximum pitch-position interval\n"
        "is the same as the interval between the minimum and maximum pitch-\n"
        "position values.  The default interval size results in a stochastic field\n"
        "configuration.  A stochastic stream configuration may be produced by\n"
        "choosing a smaller maximum interval size.  This option is presented\n"
        "to the user during each run of the program.\n\n");

printf ("Enter start time (in seconds): ");
scanf ("%f", &tMin);
printf ("Enter end time: ");
scanf ("%f", &tMax);
printf ("Do you wish to subdivide the (1-second) beat?\n"
        "(Enter 'y' for yes, 'n' for no.): ");
getchar();
if (getchar() == 'y')
        {
        printf ("Subdivide into how many parts?: ");
        scanf ("%f", &tFactor);
        }
else
        tFactor = 1.;
printf ("Enter the density (in sounds per second): ");
scanf ("%f", &parm);
printf ("What is the minimum pitch-position value?: ");
scanf ("%d", &pMin);
printf ("What is the maximum pitch-position value?: ");
scanf ("%d", &pMax);
```

```
	printf ("Do you want to set a maximum pitch-position interval size that\n"
		"is smaller than the size of the interval between the minimum and\n"
		"maximum pitch-position values?\n"
		"(Enter 'y' for yes, 'n' for no.): ");
	getchar();
	if (getchar() == 'y')
		{
		printf ("What is the size of the new maximum pitch-position interval?: ");
		scanf ("%d", &pInt);
		}
	else
		pInt = pMax - pMin; /* default max pitch-position interval size */

	fprintf (fp, "\nStart time (in seconds) is %.2f.", tMin);
	fprintf (fp, "\nEnd time is %.2f.", tMax);
	fprintf (fp, "\nNumber of subdivisions per beat is %.2f.", tFactor);
	fprintf (fp, "\nNumber of sounds per second is %.4f.", parm);
	fprintf (fp, "\nMinimum pitch position is %d.", pMin);
	fprintf (fp, "\nMaximum pitch position is %d.", pMax);
	fprintf (fp, "\nMaximum pitch-position interval is %d.", pInt);
	fprintf (fp, "\n\nTIME            PITCH\n");

	t = tMin; /* initializes time value */
	p = pMin + ((rand() / 32767.) * (pMax - pMin)); /* initializes pitch-position  value */

	while (t < tMax) /* within specified time limits */
		{

		u1 = rand() / 32767.; /* pick a random number between 0 and 1 */
		u2 = rand() / 32767.; /* pick a second random number */
		if (u1 >= 0.5)
			p = p - (pInt * (1.0 - sqrt(u2))); /* linear distribution */
		else
			p = p + (pInt * (1.0 - sqrt(u2)));

		if (p >= pMin && p <= pMax) /* if pitch position within range */
			{
			u1 = rand() / 32767.; /* exponential distribution */
			exp = -log(u1) / parm;
			t = t + exp;
			fT = t * tFactor;

			if (t < tMax) /* if time point within range */
				{
				printf ("\n%.2f ", fT); /* prints values */
				fprintf (fp, "\n%.2f ", fT);
				printf ("        %d", p);
				fprintf (fp, "             %d", p);
				}
			}
		}

	fclose (fp);
}
```

B. RWV

```
/* RWV.c - random walk based on velocity */


#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <time.h>

main()
{
FILE    *fp;
int     seed; /* initialized by clock function below */
time_t  now; /* time and date variables */
struct  tm *date;
char    s[80];
float   x = 0.; /* position variable */
float   v = 0.;  /* velocity variable */
float   a = 0.; /* acceleration variable */
float   u; /* 0 < random variable < 1 */
float   epsilon = 0.;     /* "collision" variable */
float   dt = 0; /* temporal increment */
float   alpha = 0; /* friction coefficient */
int     total = 0; /* number of steps */
int     i;

fp = fopen("RWV Data", "a");

seed = clock(); /* initializes seed value */
srand(seed); /* initializes random number generator */

time (&now);
date = localtime (&now);
strftime (s, 80, "%c", date);
fprintf (fp, "\n\n%s", s);

printf("This program calculates a random walk based on changes in velocity.\n"
        "It is based on the Langevin equation which expresses Newton's law of\n"
        "motion for an object of unit mass under the influence of friction and\n"
        "a random external force epsilon. The program output represents changes\n"
        "in the position of a particle with respect to time, which is measured\n"
        "in uniform increments dt. In musical applications, changes in position\n"
        "are usually associated with changes in pitch. The values in the program\n"
        "specific instruments\n\n");
printf("Please enter a starting pitch value: ");
scanf("%d", &x);
printf("Please enter a time increment value: ");
scanf("%f", &dt);
printf("Please enter a value for the friction constant: ");
scanf("%f", &alpha);
printf("Please enter the number of steps you would like the walk to have: ");
scanf("%d", &total);
```

```
printf("\nRandom walk based on changes in velocity:\n\n");
fprintf(fp, "\n\nRandom walk based on changes in velocity:\n\n");
fprintf(fp,       "dt = %.3f\n", dt);
fprintf(fp, "alpha = %.3f\n\n", alpha);

for (i = 0; i < total; i++)
        {

        printf("%.3f\n", x);
        fprintf(fp, "%.3f\n", x);

        u = ( rand() / 32767.);
        epsilon = ( u - 0.5 );
        a = ( ( (-alpha)*v ) + epsilon );
        v = ( v + (a*dt) );
        x = ( x + (v*dt) );
        }

fclose(fp);

}
```

Glossary

arborescence a branching structure, realized musically as a quasi-polyphonic configuration in which individual voices branch off into pairs from nodal points held in common between them. (2.2)[1]

basis a system of axes used as a reference for orientation in a vector space. (2.1.3)

basis vector a unit length proceeding from the origin of a basis along one of its axes, also called a unit vector. (2.1.3)

Brownian motion the irregular motion of small particles suspended in a liquid or gaseous medium. Mathematical functions used to model Brownian motion, such as the Langevin equation, have been used by Xenakis in the design of some of the random walks found in his music. (2.2.2)

c-space contour space. (2.2.2)

complement in set theory, the set of elements that are not contained in a given set (but are contained within a defined reference or universal set, R or U). (2.1.6)

component a variable representing the value of a vector in one of its dimensions. Components are enclosed in angle brackets, e.g. $\langle x, y, z \rangle$ (2.1.3)

congruence an equivalence relation among elements in a modular set. Elements that differ by a multiple of some value m are said to be congruent mod m. (2.1.6)

continuous distribution in probability theory, a numerical or graphic expression of the expected or observed probabilities for events in the outcome set of a continuous probability space. (2.2.1.2)

continuous probability space a probability space in which events are represented by segments of the real number line. (2.2.1.3)

[1]The numbers in parentheses following each definition indicate the section of chapter 2 in which the term is first introduced.

contour space a space in which the elements are arranged from low to high without regard for the size of the intervals between them, abbreviated c-space. (2.2.2)

coordinate a variable representing the value of a state in one of its dimensions. Coordinates are enclosed in parentheses, e.g. (x, y, z). (2.1.1)

coordinate space a space for representing the states of sonic events in one or more dimensions. (2.1.1)

cumulative distribution function in probability theory, a function that measures the sum (in a discrete probability space) or integral (in a continuous probability space) of the probabilities from a lower limit (usually $-\infty$ or 0) to a given upper limit, abbreviated CDF (2.2.1.2)

cyclic transposition Xenakis's term for the transposition of a sieve, defined more precisely as the operation T_n(mod m) on an m-set. (2.1.6)

d-space the dimension of musical space that pertains to duration. (2.1.1)

∂-space the dimension of musical space that pertains to density. (2.1.4)

density a quantitative measure of textural thickness, expressed in sounds per second (s/s). (2.1.4)

dimension a quantitative representation of a particular attribute of a sound, such as pitch, intensity, duration, time-point, or timbre. (2.1.1)

discrete distribution in probability theory, a numerical or graphic expression of the expected or observed probabilities for events in the outcome set of a discrete probability space. (2.2.1.2)

discrete probability space a probability space in which events are represented by discrete real numbers. (2.2.1.2)

duration a measurement of the time from the initiation of a sound (its time-point) to its release or termination. Also, a measurement of the time spanned from the time-point of the first event in a collection of sonic events to the termination of the last event in the collection, or to the initiation of the first event in the next collection of sonic events. (2.1.1, 2.1.4)

event in probability theory, a subset of the elements in an outcome set. An event consists of one or more outcomes (see below). Also, an informal term for a sonic event (see below). (2.2.1.1) *histogram* a graphic representation of the expected or observed probabilities of events in an outcome set. (2.2.1.2)

i-space the dimension of musical space that pertains to intensity. (2.1.1)

inside-time structure a term used to describe the condition of sonic events in a fully realized compositional setting. (2.1.4)

intensity the relative loudness of a sonic event. (2.1.1)

intersection in set theory, the set of elements held in common by two or more sets. (2.1.6)

interval succession an ordered listing of intervals between adjacent elements in a ordered sequence, abbreviated INT. (2.1.6)

inversion in set theory, an operation in which the direction of the intervals between adjacent elements is reversed. Inversion is rarely used in Xenakis's music, but has wide application and a variety of formal and informal definitions in other repertoires. (2.1.6)

m-seq a modular seq. (2.1.6)

m-set a modular set. (2.1.6)

metabola Xenakis's term for a transformation, such as transposition or cyclic transposition, on a sieve. (2.1.7)

modular in set theory, having an interval structure (spacing or interval succession) that repeats periodically. (2.1.6)

module in sieve theory, an ordered pair of integers (m, r) that gives the modulus and residue class of a set of elements. (2.1.7)

modulus in set and in sieve theory, the size of the period at which the intervallic structure of a set repeats itself. (2.1.6)

morphology a general term for the shape of a configuration in p- and tp-space. (2.2)

musical space an abstract, quantitative representation of the attributes of a sound or collection of sounds. (2.1.1)

notelist the numerical output of a computer program, representing the states of sonic events in one or more dimensions. (2.1.3) *outcome* in probability theory, one of the elements in the set of things that may occur under the conditions specified for a probability space. In discrete probability spaces, the probability of each outcome is $1/\Omega$, where Ω is the set of all possible outcomes. In continuous probability spaces, the probability of a single outcome is 0 since each outcome is defined as a point along the real number line. An outcome is usually represented by the variable ω. (2.2.1.1)

outcome set in probability theory, the set of all possible outcomes under the conditions specified for a probability space, usually represented by the variable Ω. (2.2.1.1)

outside-time a general category for the attributes of a sonic event, including duration, that are independent of the temporal succession or location of the event. (2.1.4)

p-space the dimension of musical space that pertains to pitch. (2.1.1)

part a unit of temporal structure that contains one or more sections. (2.3)

period the size of the interval at which a spacing or interval succession begins to repeat itself; modulus. (2.1.6)

pitch the perceptible frequency of a sonic event. (2.1.1)

pitch position the location of a pitch within a pitch set whose elements are arranged from low to high. (2.2.1.7)

position vector a vector whose initial point is located at the origin of a coordinate space. (2.1.3)

probability density function in a continuous probability space, a function whose curve, when bounded by the x axis and by x_0 and x_1, produces an area that indicates the probability that a particular value of x will exist between x_0 and x_1. (2.2.1.3)

probability mass function in a discrete probability space, a function that determines the probabilities of events within the outcome space. (2.2.1.2)

probability theory a branch of mathematics that is used to predict the likelihood of particular outcomes in situations that cannot be determined in advance with certainty. (2.2.1.1)

pset a set of pitches. (2.1.6)

pseudo-random number generator a computer function that produces successions of numbers whose statistical properties resemble those of genuine randomness. (2.2.1.3)

r-space the dimensional of musical space that pertains to registral span. (2.1.4)

random walk a mathematical model of the changes in position of an object or entity with respect to time. Also, in Xenakis's compositional practice, a sonic configuration or waveform based on a mathematical model of a random walk or on a graphic design inspired by the morphology of a random walk. (2.2.2)

registral span the interval from the lowest to highest elements in a pset. (2.1.4)

residue class an integer between 0 and m - 1, representing a class of elements that are equivalent (or congruent) modulo m in a modular set (or sieve). (2.1.7)

sampling with replacement a method of extracting elements from a statistical sample (such as the values produced by a pseudo-random number generator) in which each element may be drawn more than once. (2.2.1.9)

sampling without replacement a method of extracting elements from a statistical sample (such as a probability table) in which each element is drawn only once. (2.2.1.9)

section a unit of temporal structure that contains one or more segments. (2.3)

segment a unit of temporal structure that contains one or more sonic events. (2.3)

sequence an ordered collection of elements, abbreviated seq. The elements of a seq are enclosed in angle brackets (<>). (2.1.6)

sequential time the temporal order in which sonic events occur in a musical setting. (2.1.1)

set an unordered collection of elements. The elements of a set are enclosed in curly brackets ({}). (2.1.6)

sieve in Xenakis's sieve theory, a modular set (or seq) produced by the union and/or intersection of one or more modules. (2.1.7) *sieve theory* a theory and a method for the generation of musical structures, such as pitch sets or time-point sequences, from the union and/or intersection of simple modular sets, called modules. (2.1.7)

sonic configuration a collection of sonic events inside-time. (2.2)

sonic event an individual sound, represented quantitatively in one or more dimensions. (2.1.1)

spacing an ordered listing of intervals between adjacent elements in a set, abbreviated SP. (2.1.6)

st-space the dimension of musical space that pertains to sequential time. (2.1.1)

state the attribute(s) of a sonic event, represented quantitatively in a coordinate space. (2.1.1)

stochastic configuration a sonic configuration in which the intervals between the states of the sonic events are modelled after one or more probability distributions. (2.2)

stochastic field a stochastic configuration in which sonic events are dispersed fairly evenly within specific pitch and temporal boundaries. (2.2.1.7)

stochastic stream a stochastic configuration in which the maximum pitch (or pitch-position) interval is considerably smaller than the interval between the lowest and highest elements in the pitch set in which the configuration is inscribed. This difference in interval size produces a configuration that appears to meander through p- and tp-space. (2.2.1.10)

subset a set, all of whose elements are also elements of another set. (2.1.6)

superset a set that contains all of the elements of another set, plus some additional elements of its own. (2.1.6)

T-matrix a matrix that demonstrates the number of elements in the intersection of two sets that are related by transposition. (2.1.6)

temporal structure an aspect of musical structure consisting of the intervals between the time-points of sonic events. Temporal structure outside-time is based on time-point intervals regardless of the order of their succession in a work, and temporal structure inside-time takes their order of succession into account. (2.1.4) *time-point* a quantitative representation of the instant at which a sonic event is initiated, measured in seconds or beats. Time-points are usually measured from the initiation of the first sonic even in a work or in a passage of music. (2.1.1)

tp-space the dimension of musical space that pertains to time-points. (2.1.1)

tpseq a sequence of time-points. (2.1.6)

transposition an operation in which a pitch is displaced n units in p-space or in modular p-space, i.e. pitch-class space. (2.1.6)

union in set theory, the set of all elements contained by two or more sets. (2.1.6)

unit vector alternate term for a basis vector (see above). (2.1.3)

vector a quantity that represents both magnitude and direction in one or more dimensions. (2.1.3)

vector space a space for representing distances between the states of sonic events in one or more dimensions. (2.1.3)

velocity a measure of distance and direction with respect to time. (2.2.2)

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An Analytical Approach to the Music of Iannis Xenakis: Studies of Recent Works

A Dissertation
Presented to the Faculty of the Graduate School
of
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in Candidacy for the Degree of
Doctor of Philosophy

by
Ronald James Squibbs

Volume 2: Figures and Tables

Dissertation Director: Professor Robert P. Morgan

November 1996

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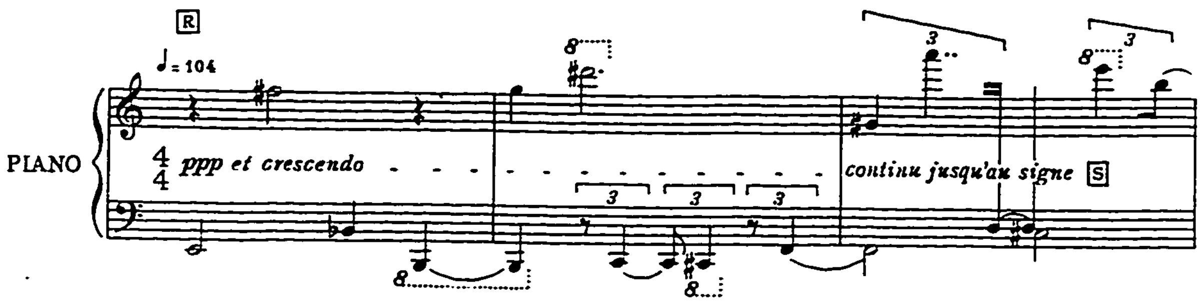
Dediée à Yuji Takahashi

HERMA

IANNIS XENAKIS



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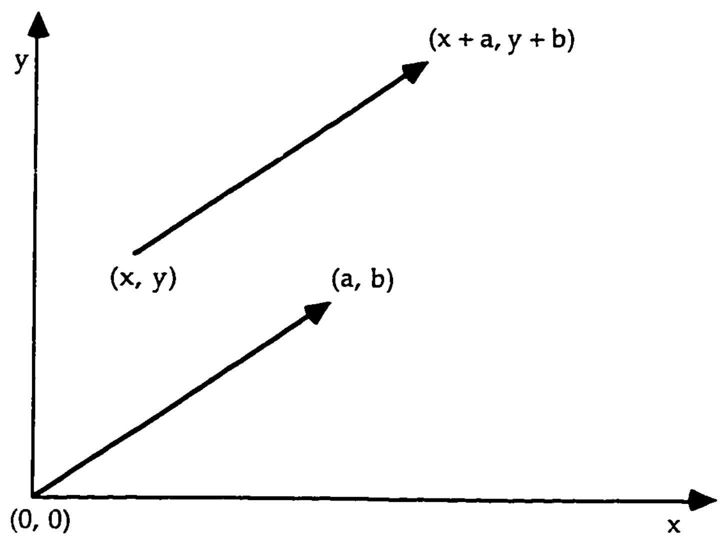

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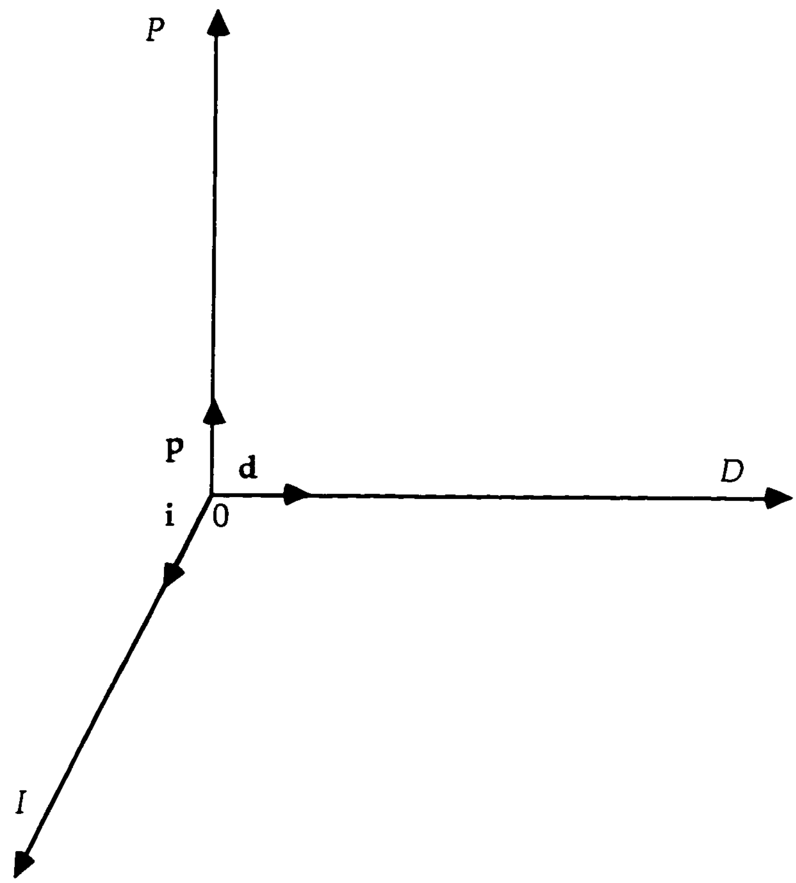

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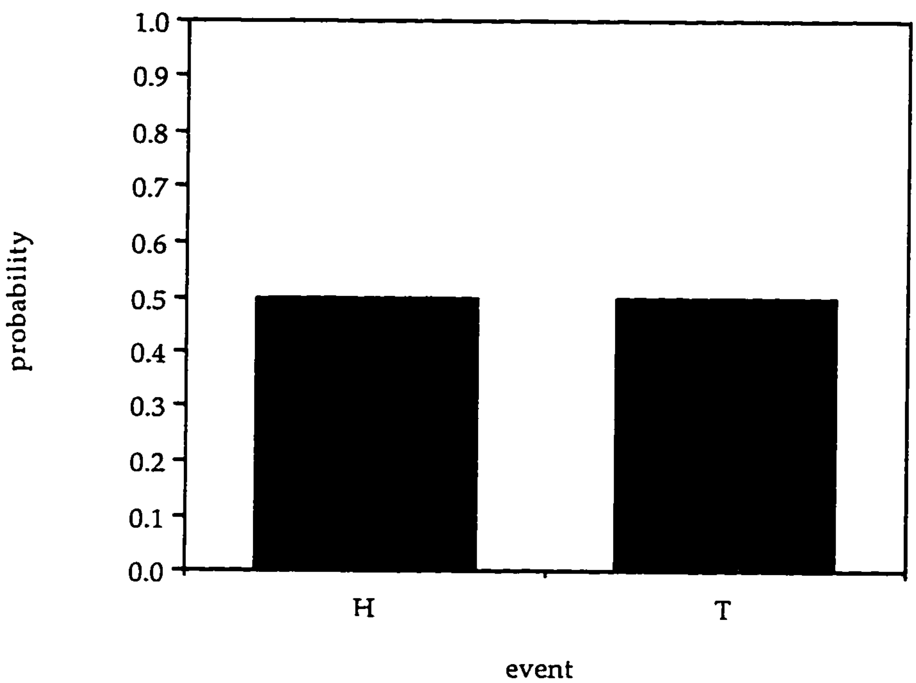

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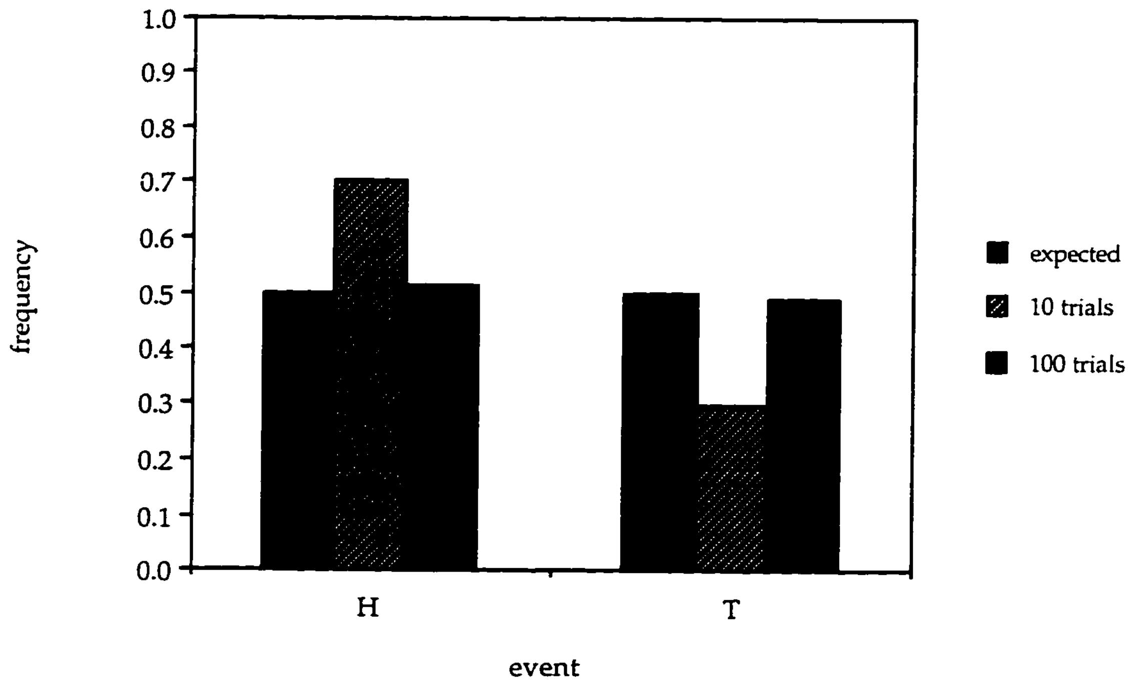

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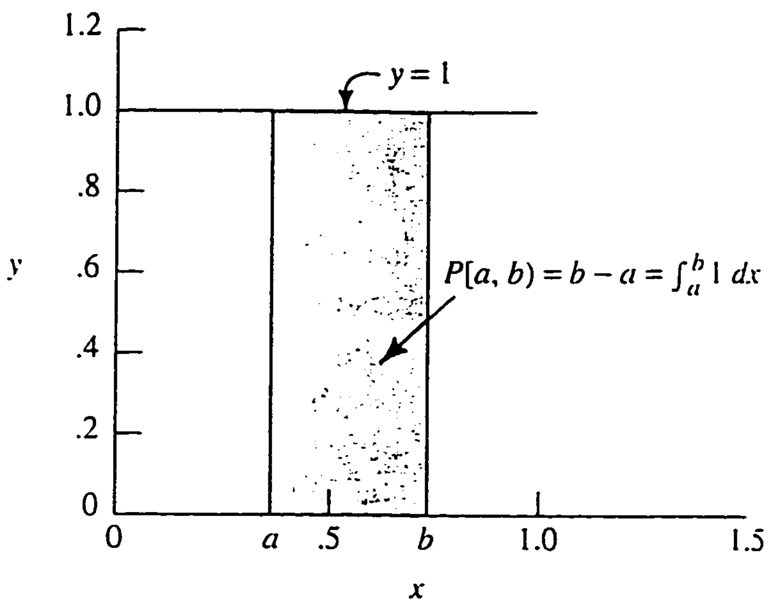

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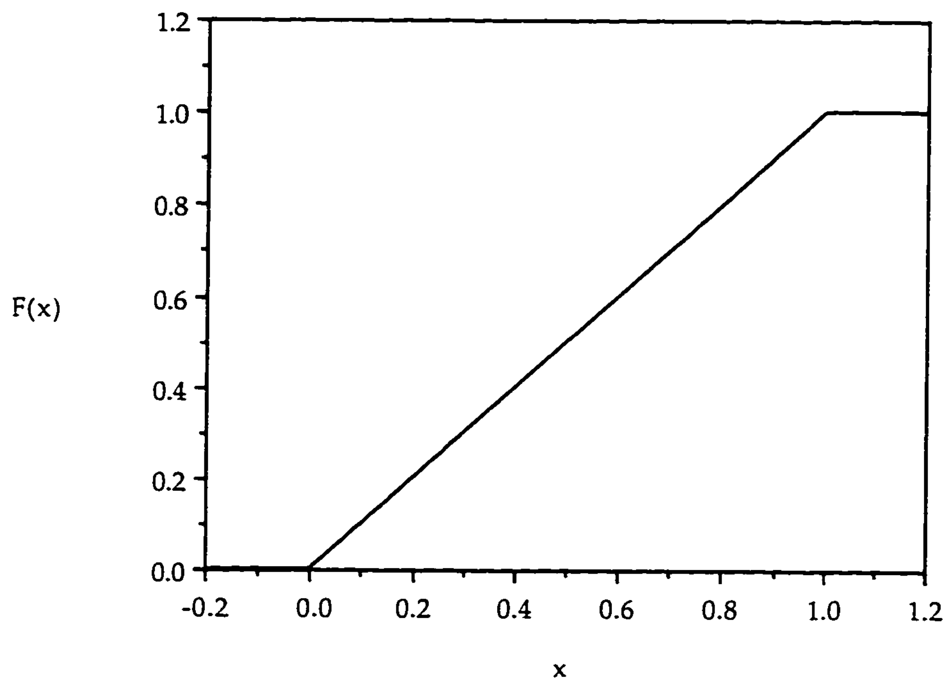

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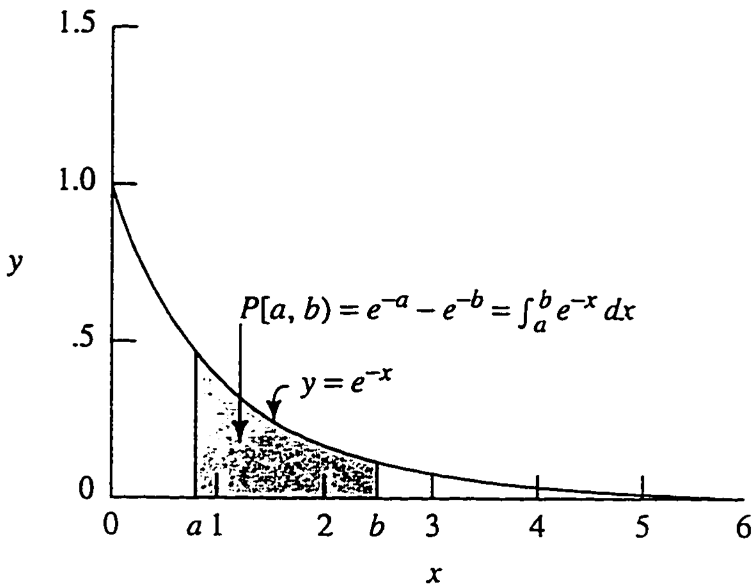

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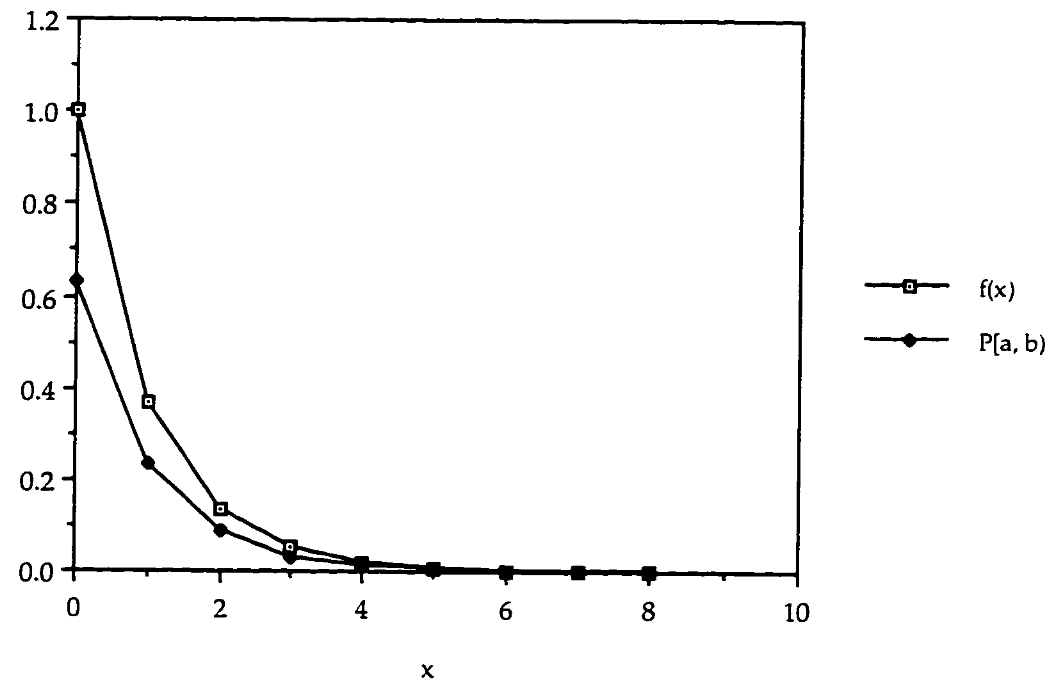

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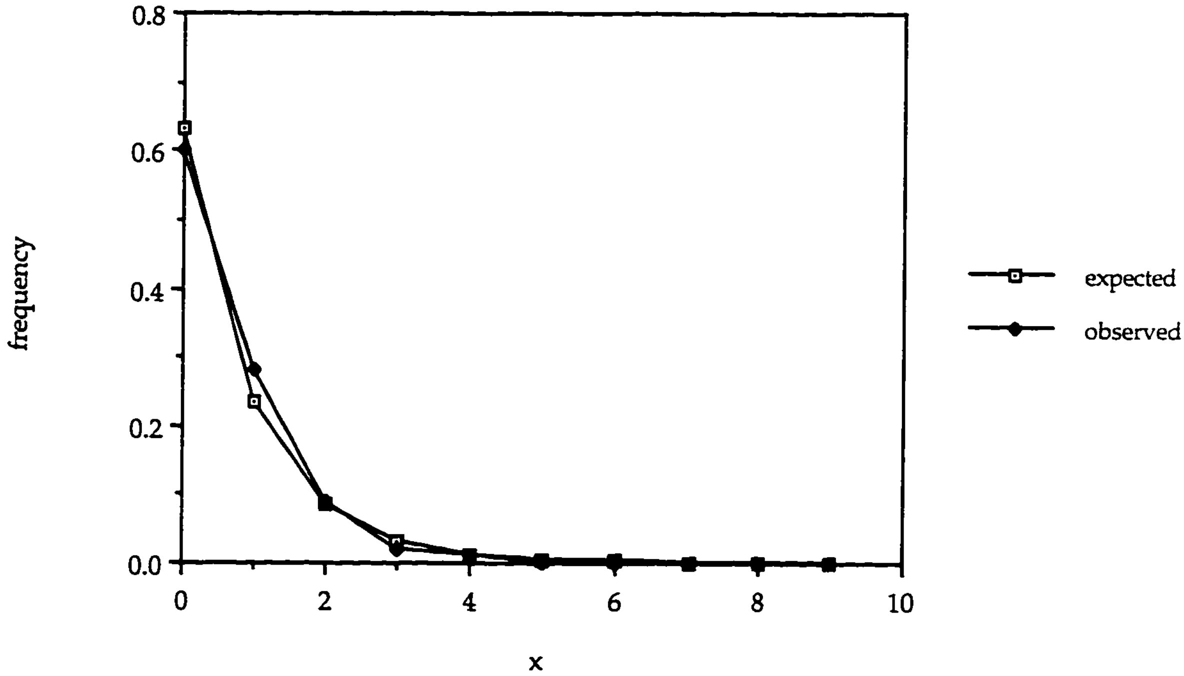

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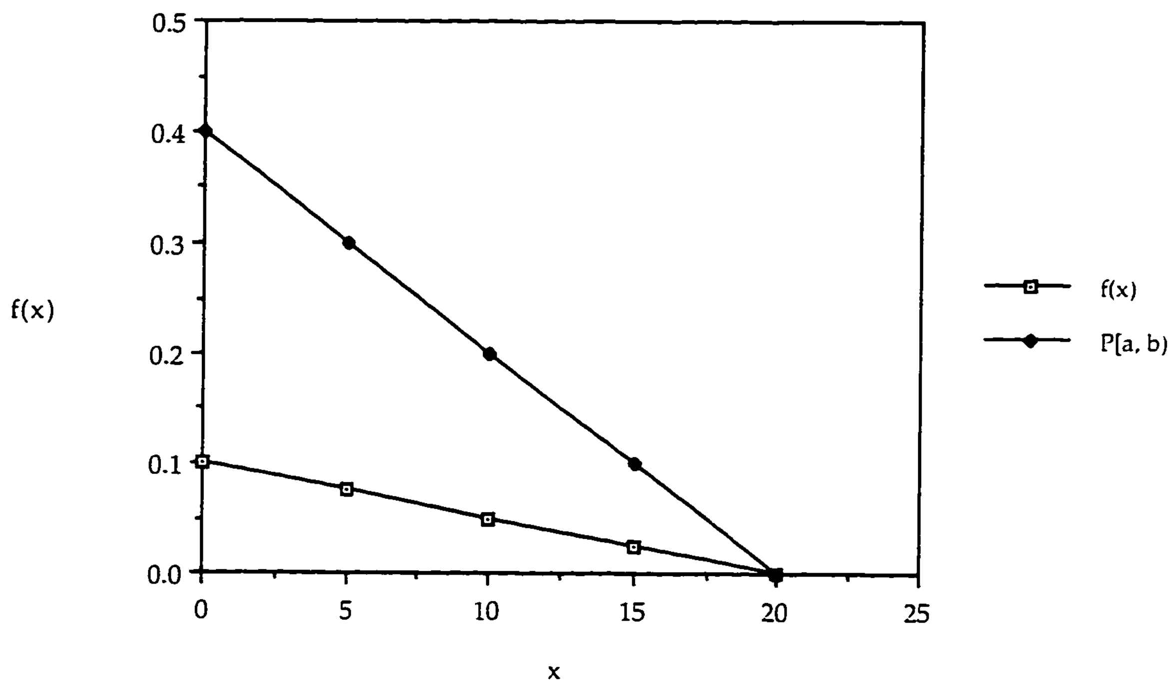

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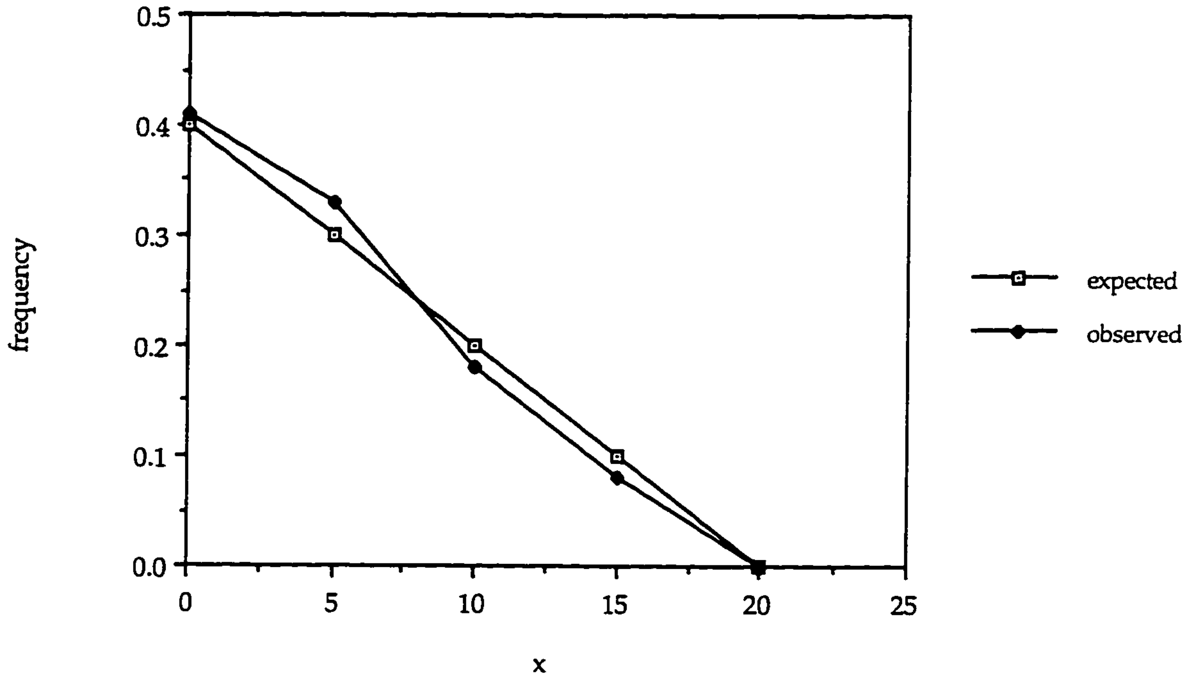

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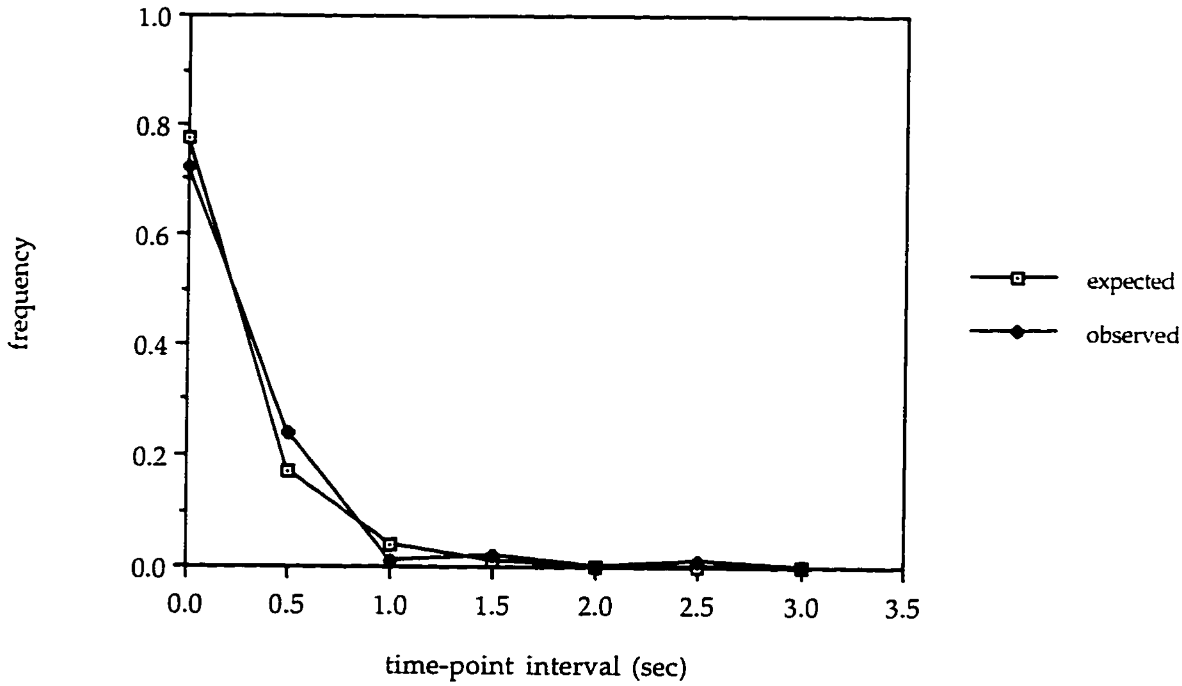

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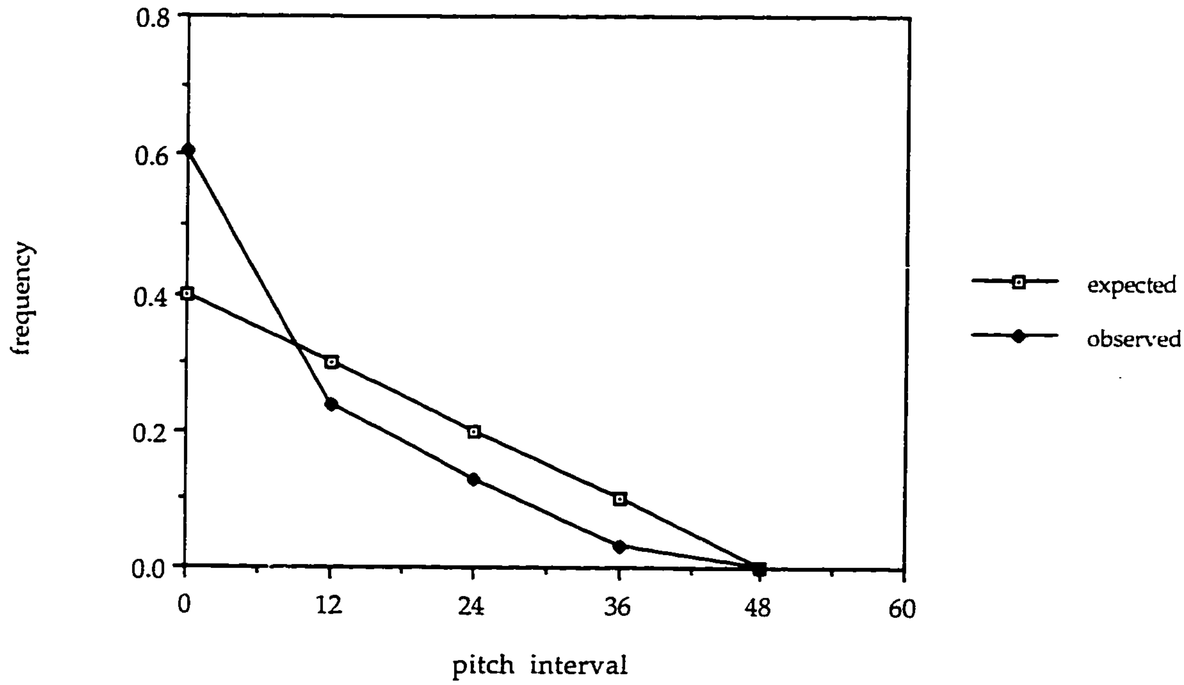

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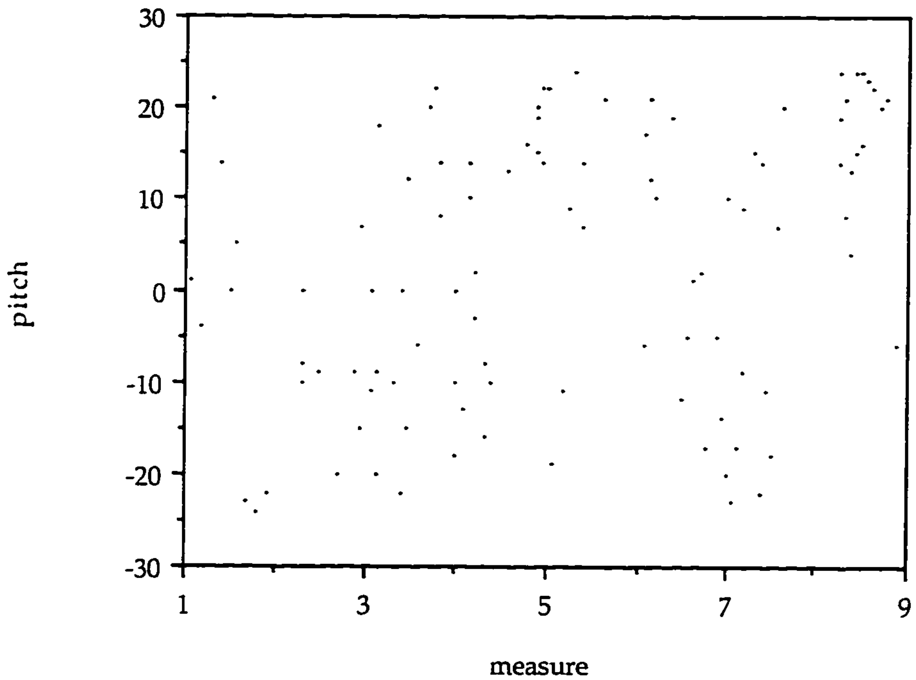

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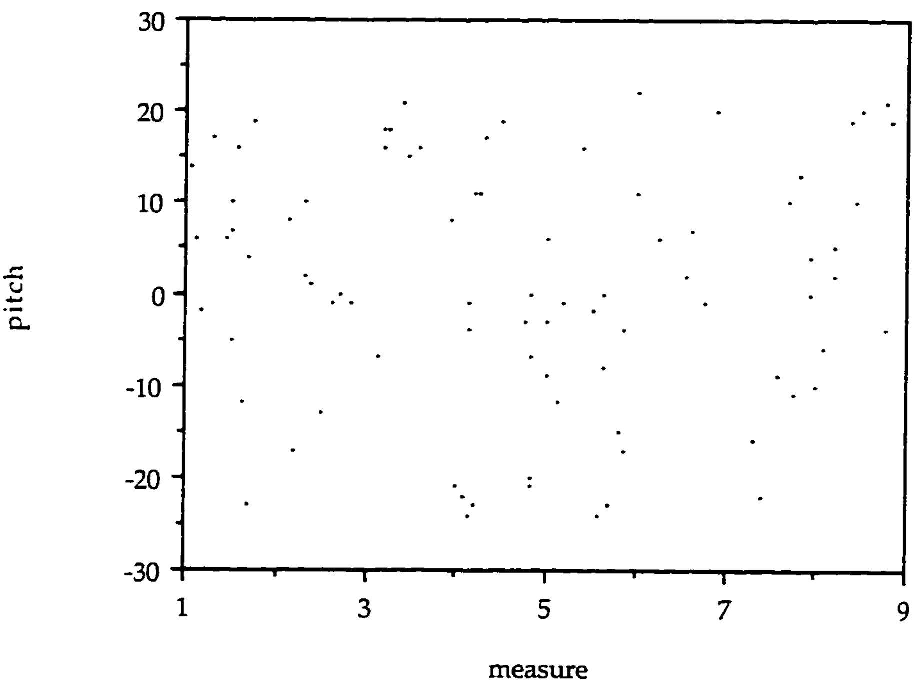

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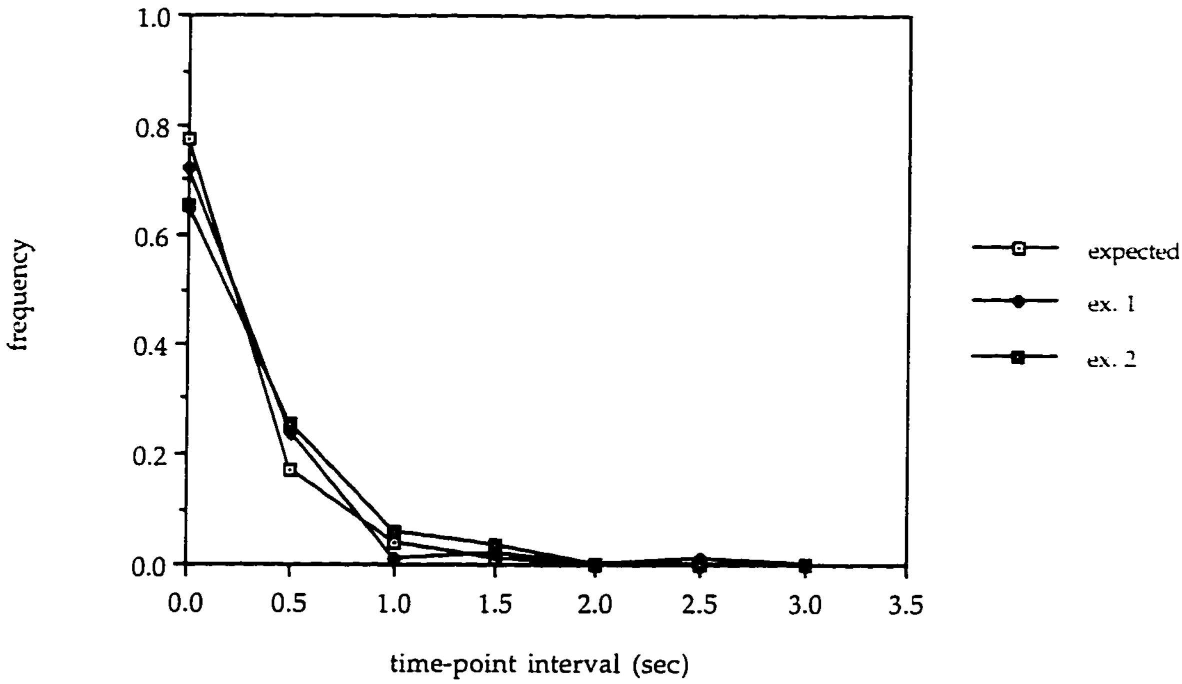

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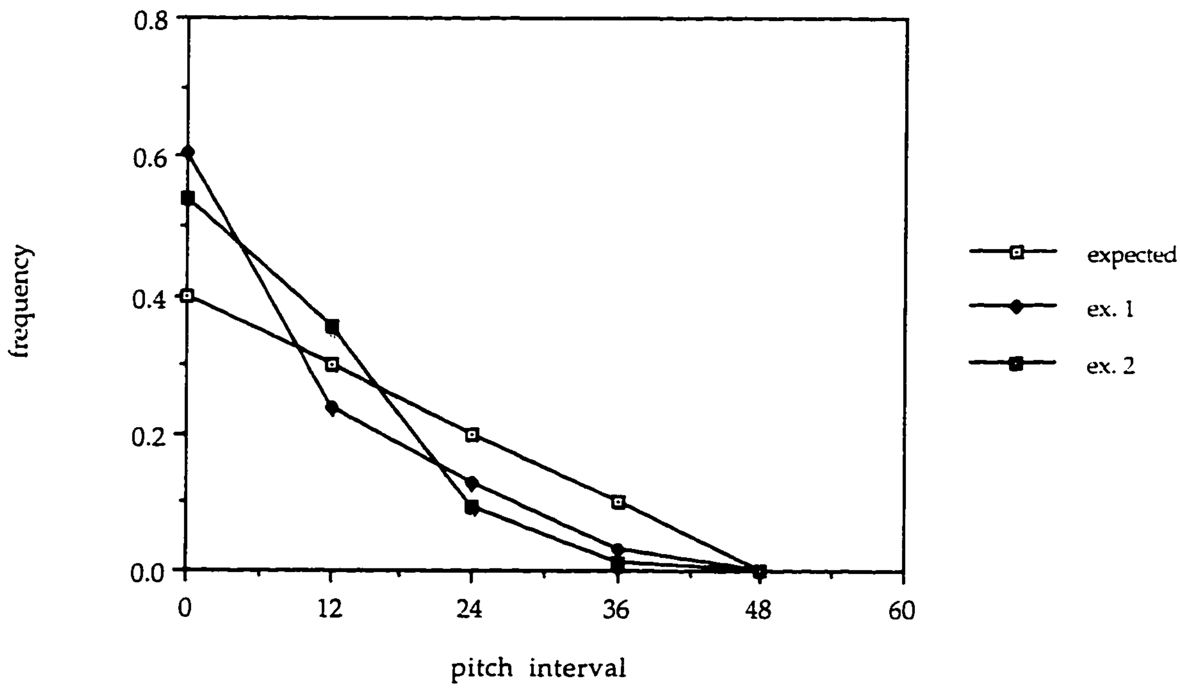

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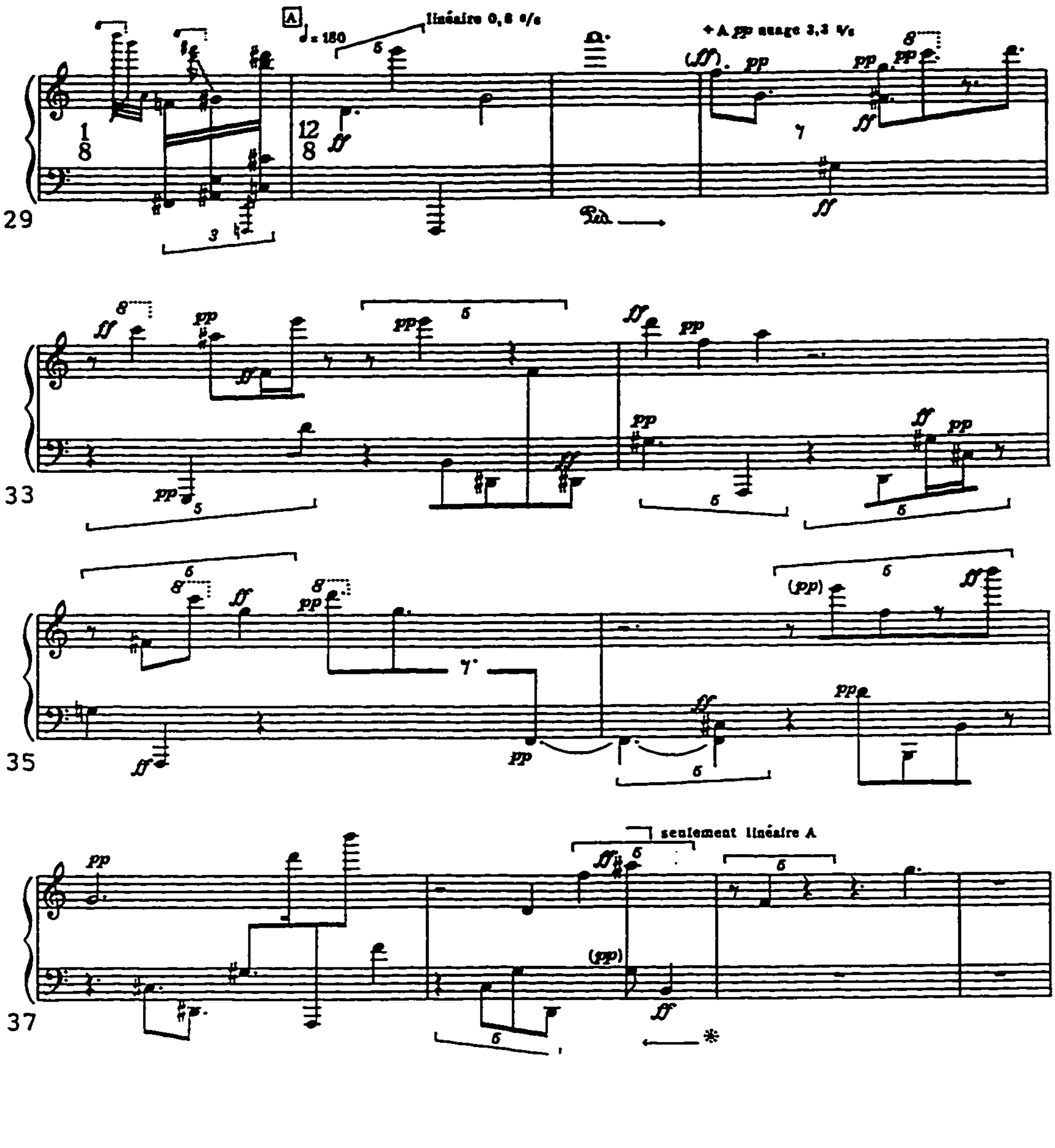

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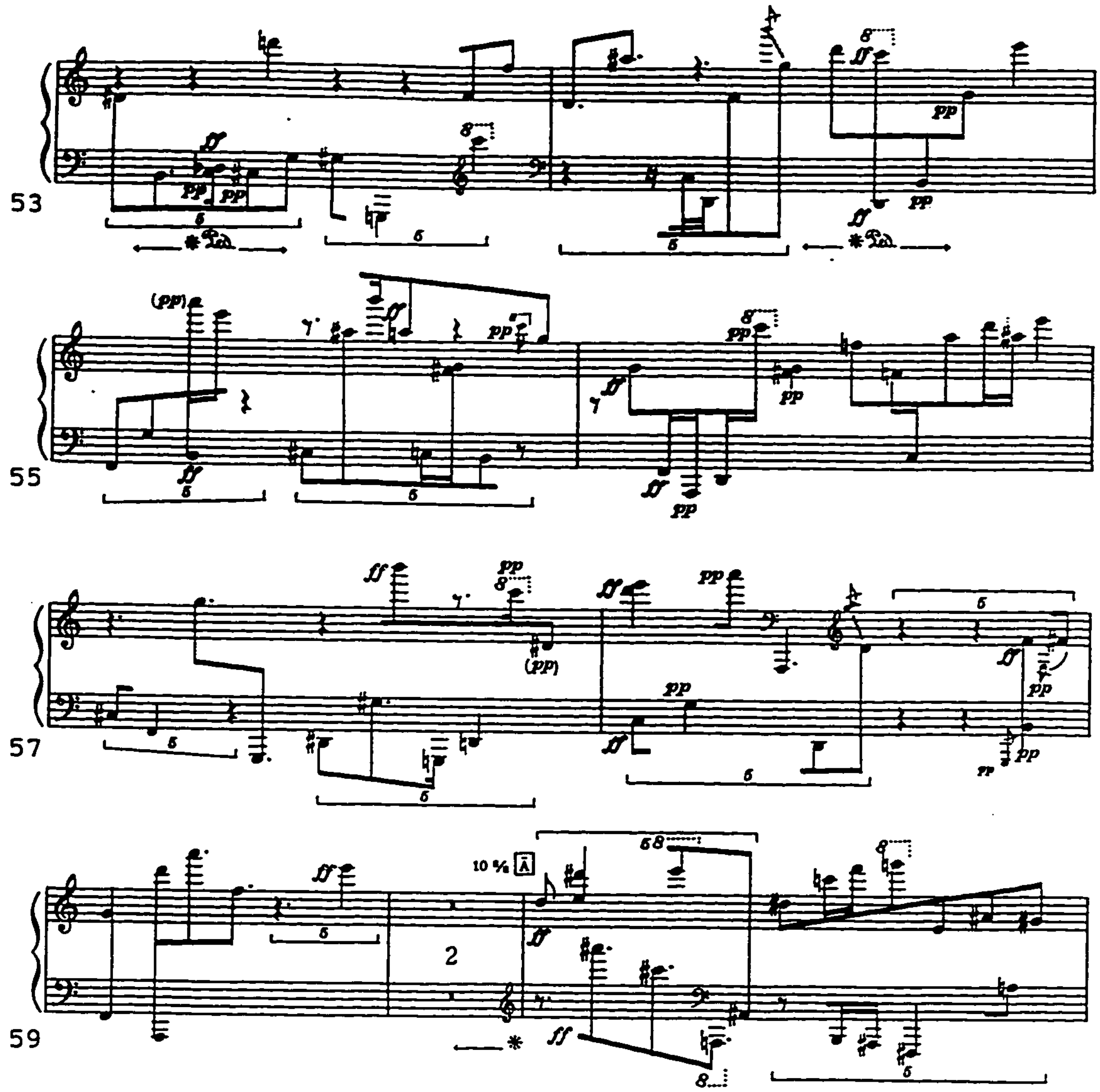

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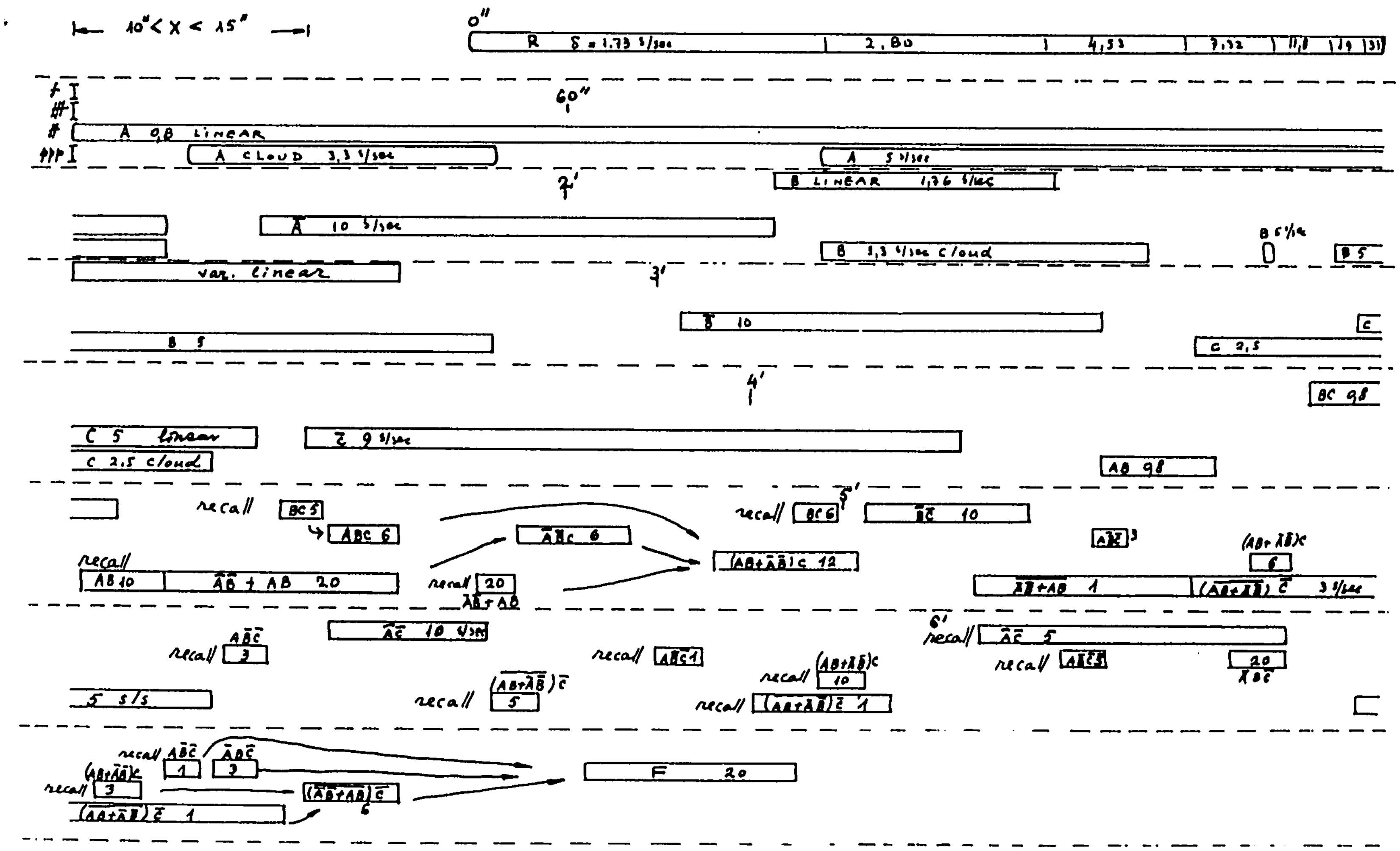

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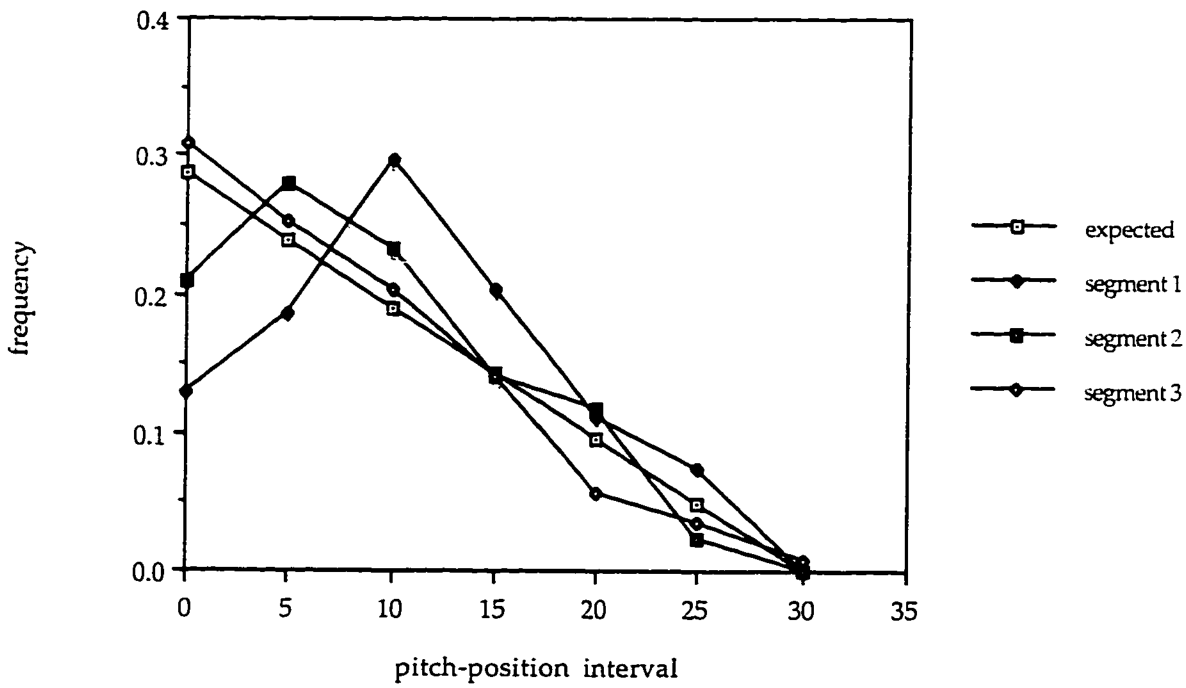

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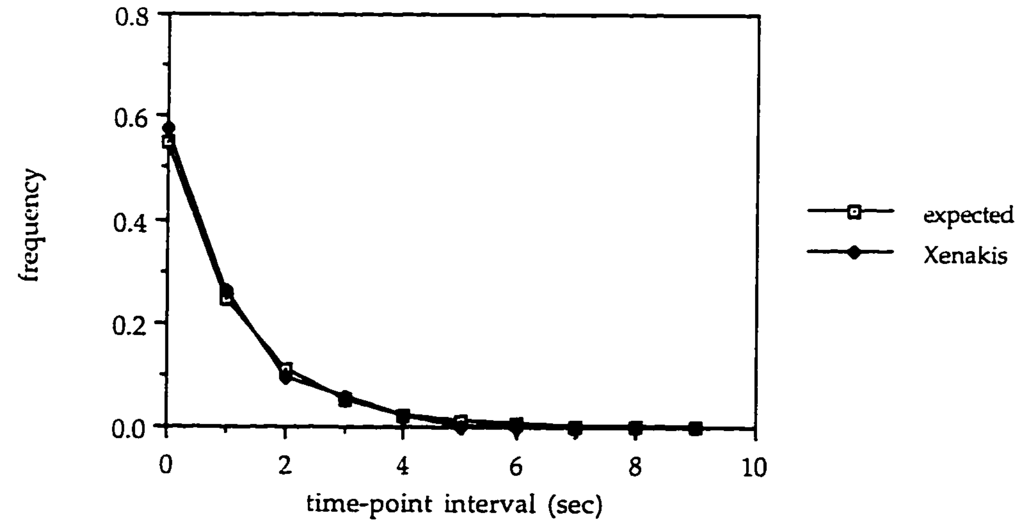
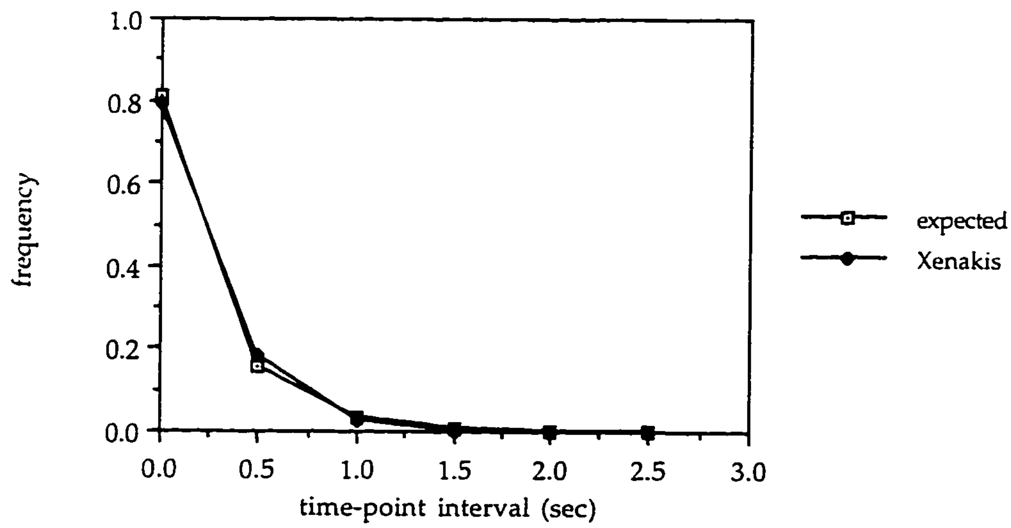


a. $\partial = 0.8$



b. $\partial = 3.3$

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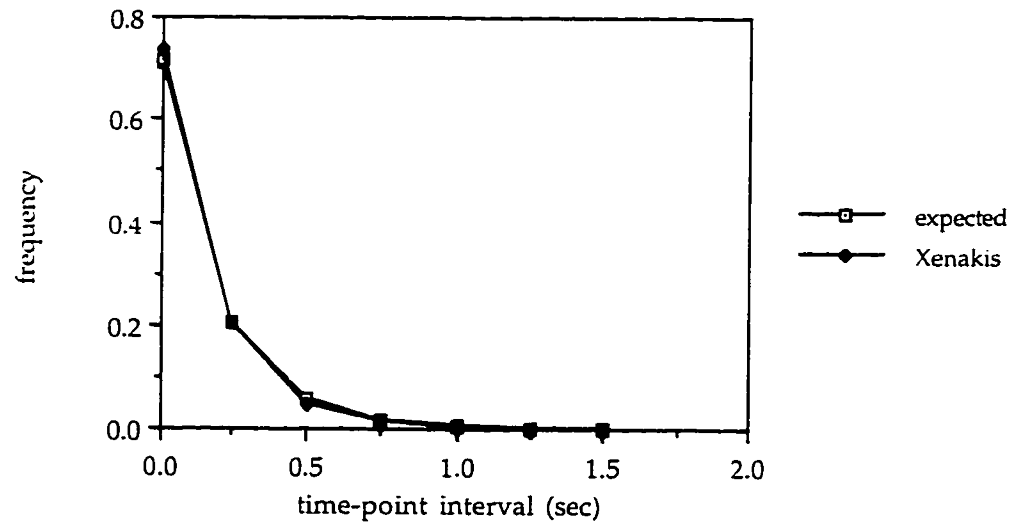

c. ∂ = 5.0

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Figure 2.26: Passage of stochastic music based on the general structural features of *Herma*, mm. 30-59, composed with STMus1, cont.

Figure 2.26: Passage of stochastic music based on the general structural features of *Herma*, mm. 30-59, composed with STMus1, cont.

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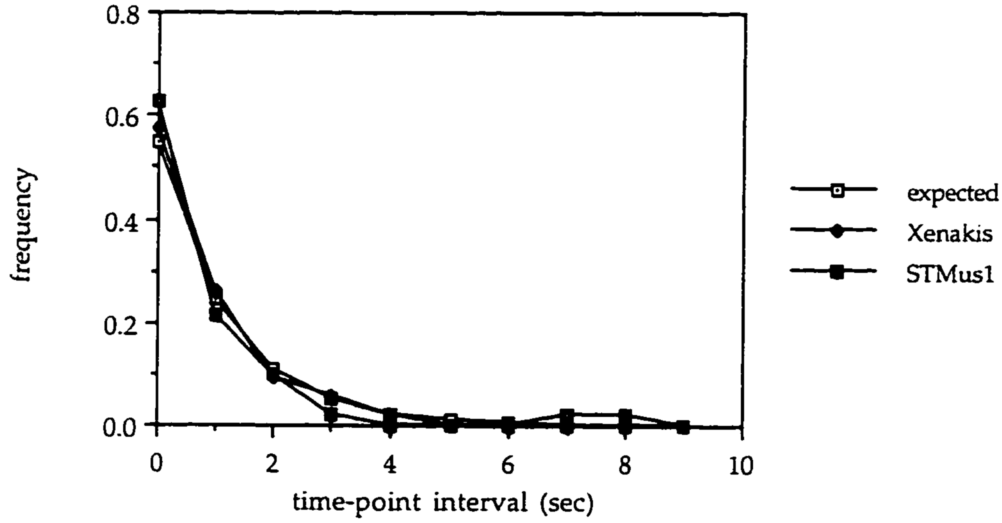
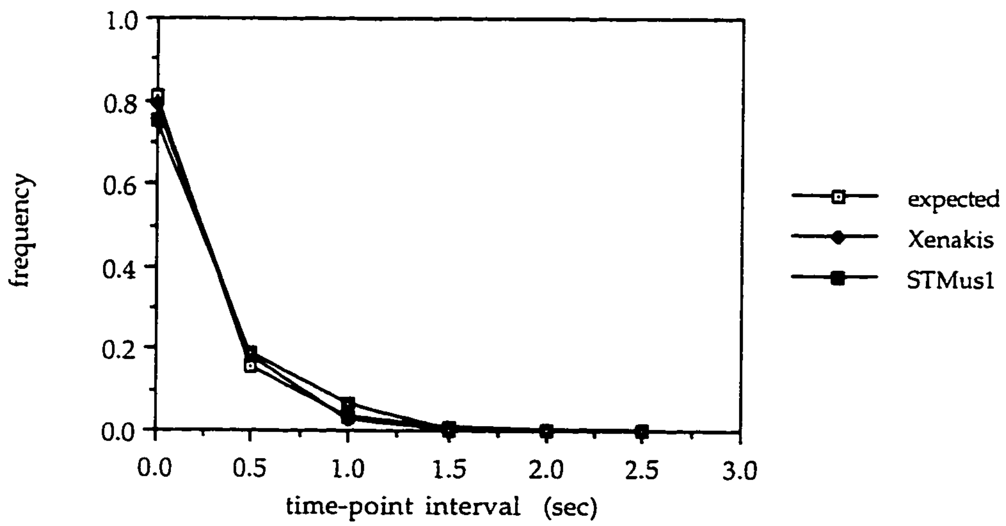


a. ∂ = 0.8



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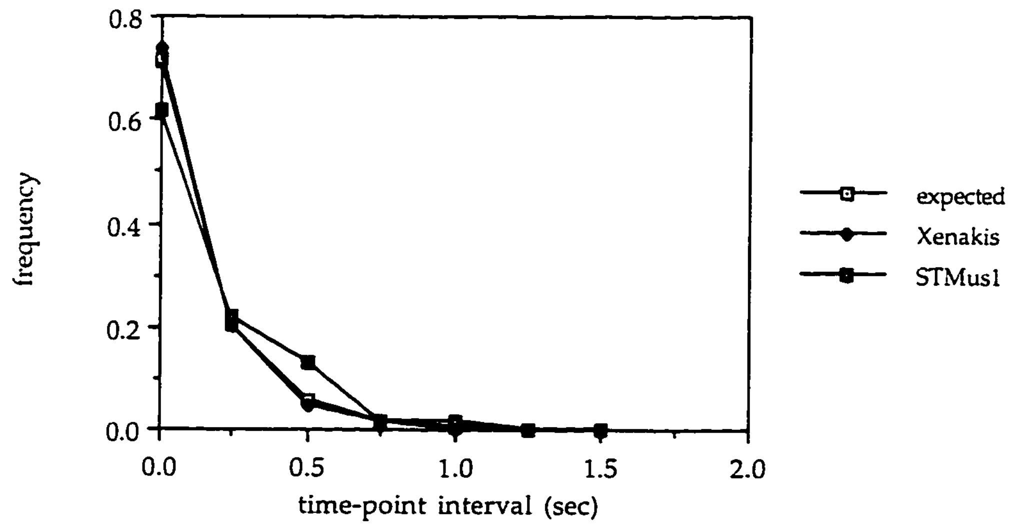

c. $\partial = 5.0$

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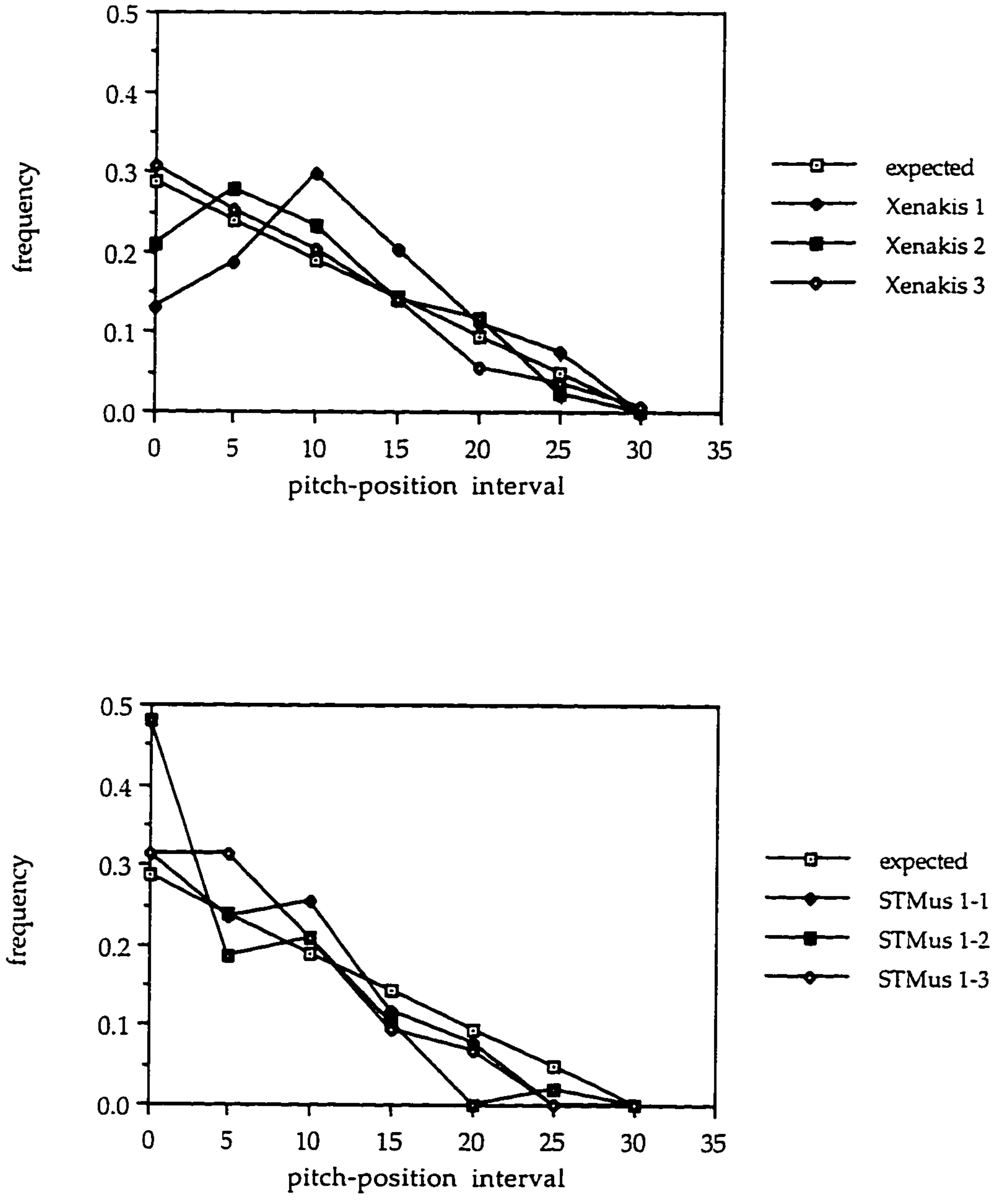

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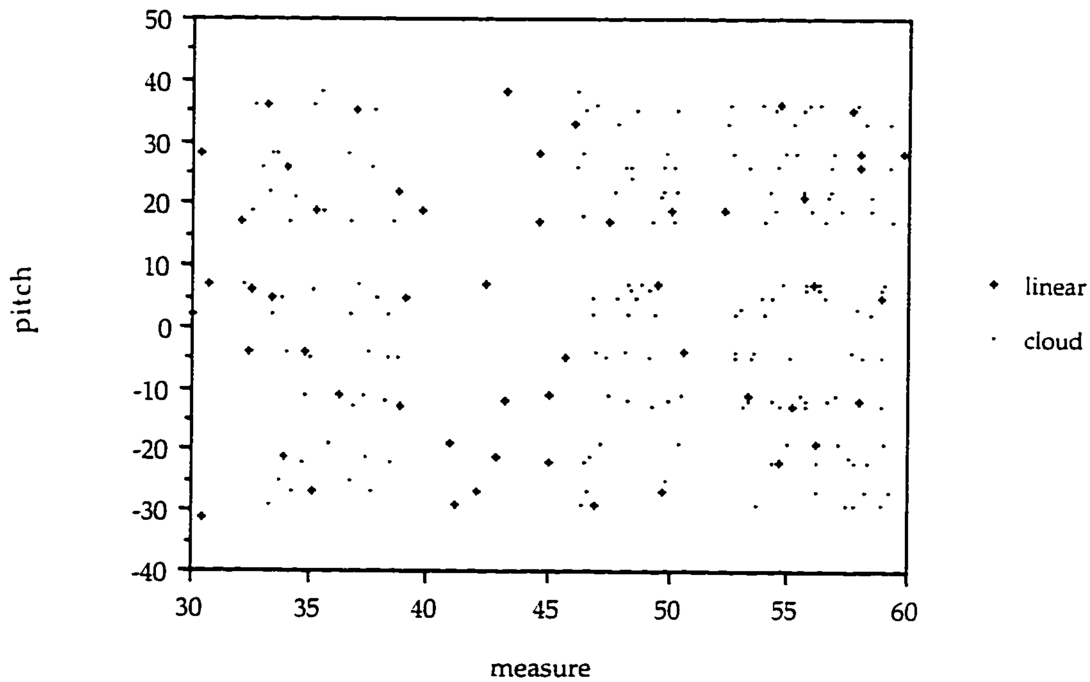

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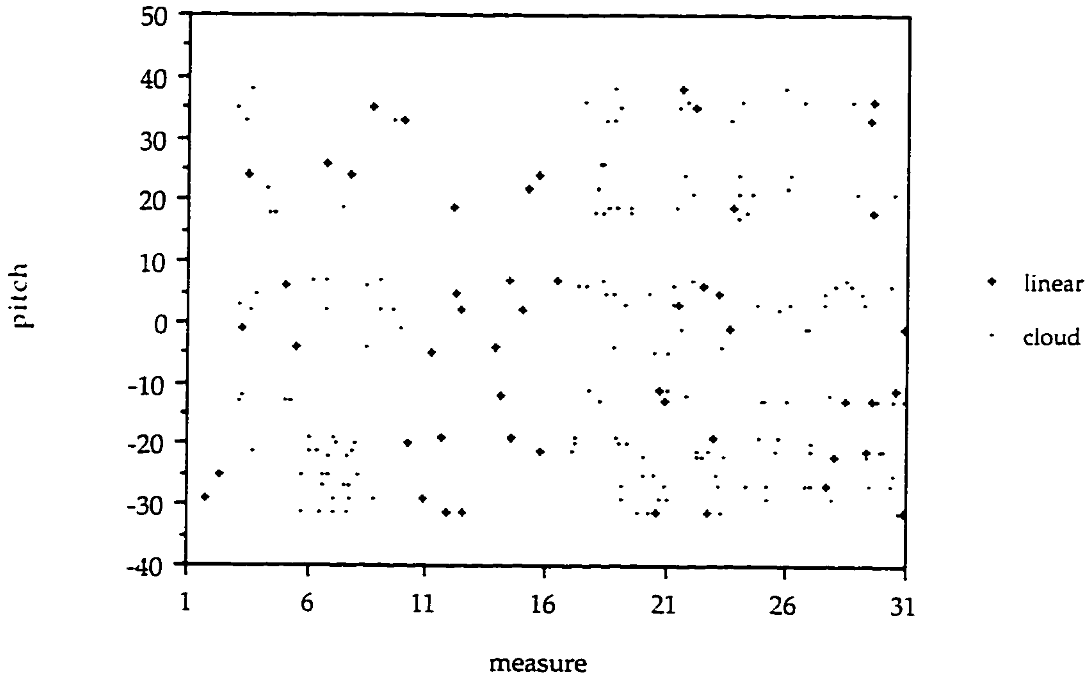

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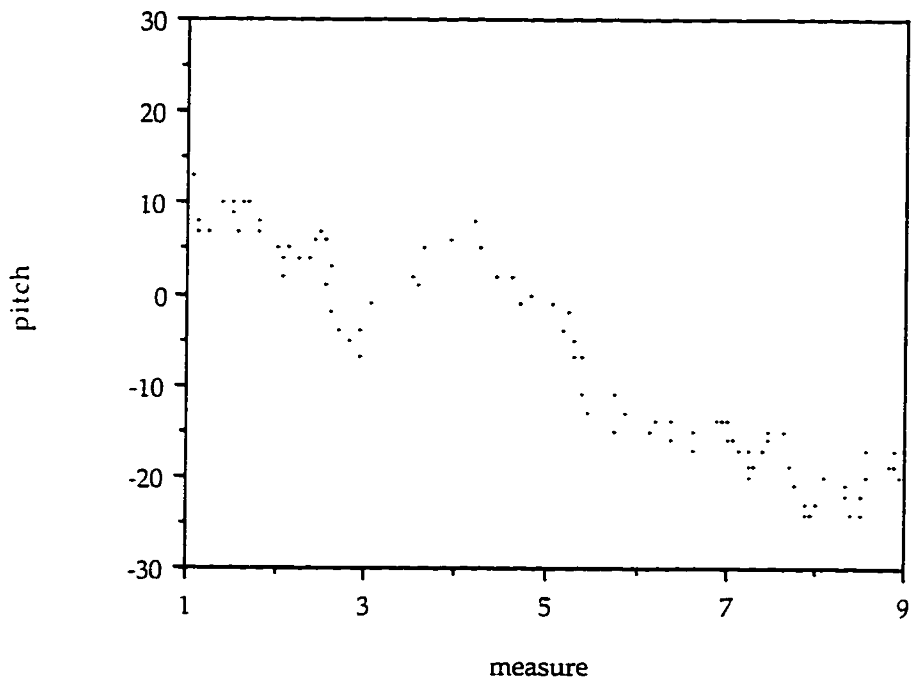

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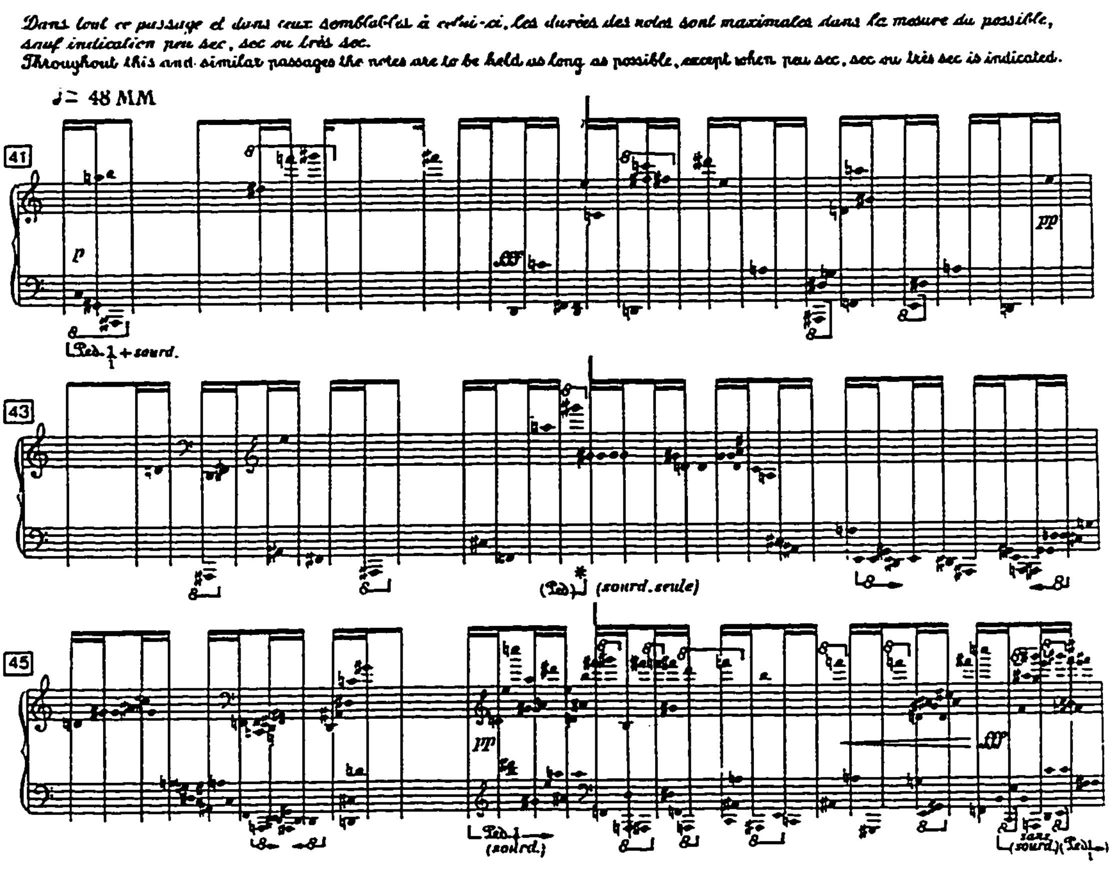

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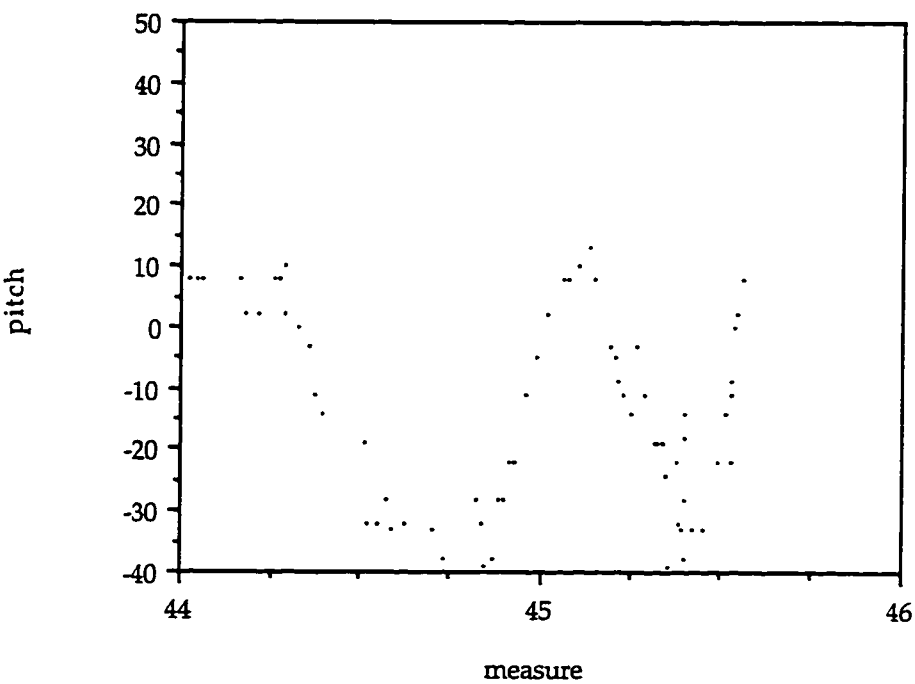

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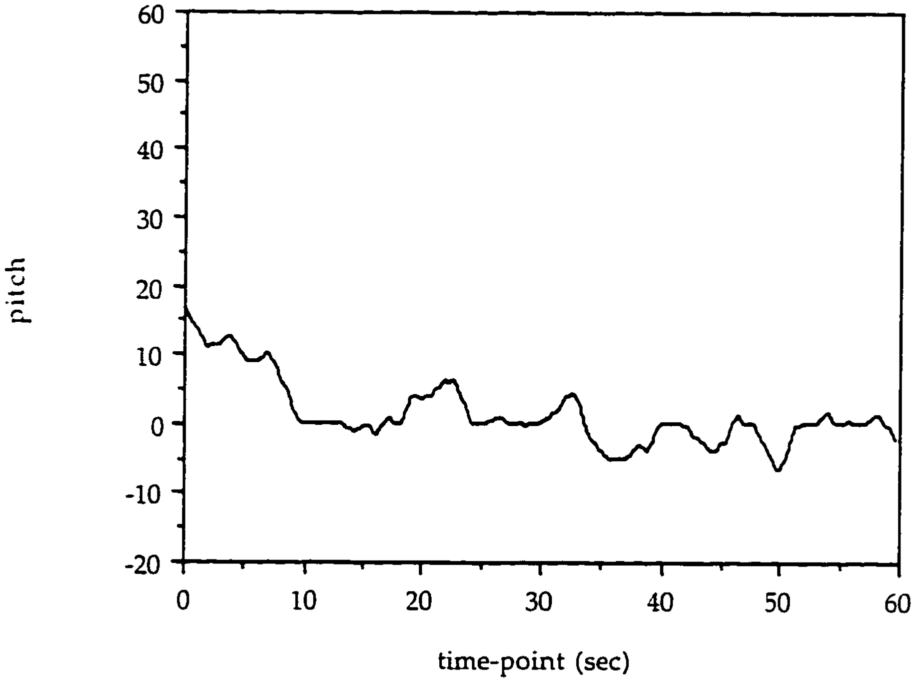

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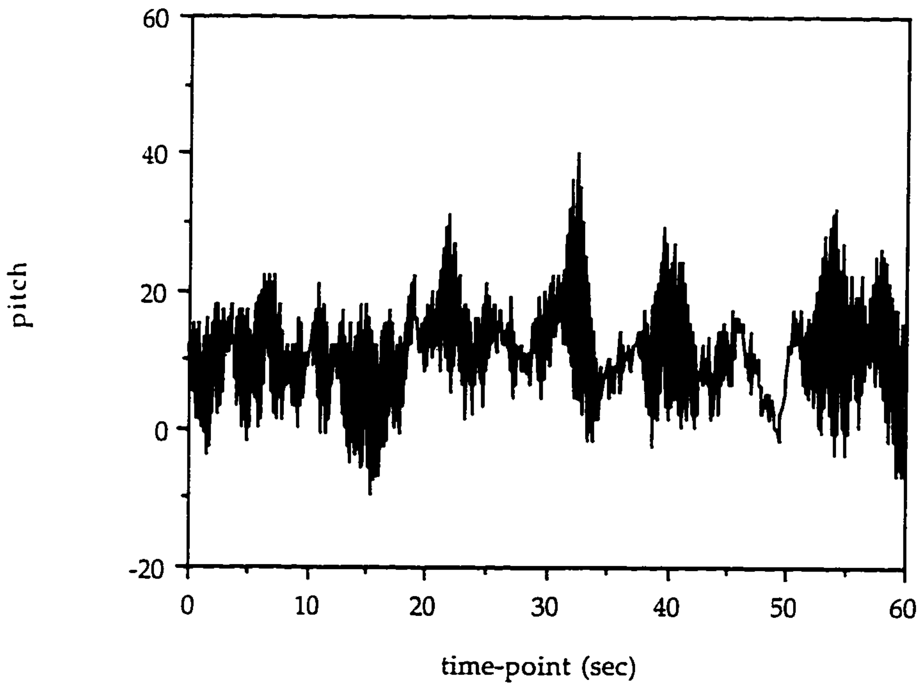

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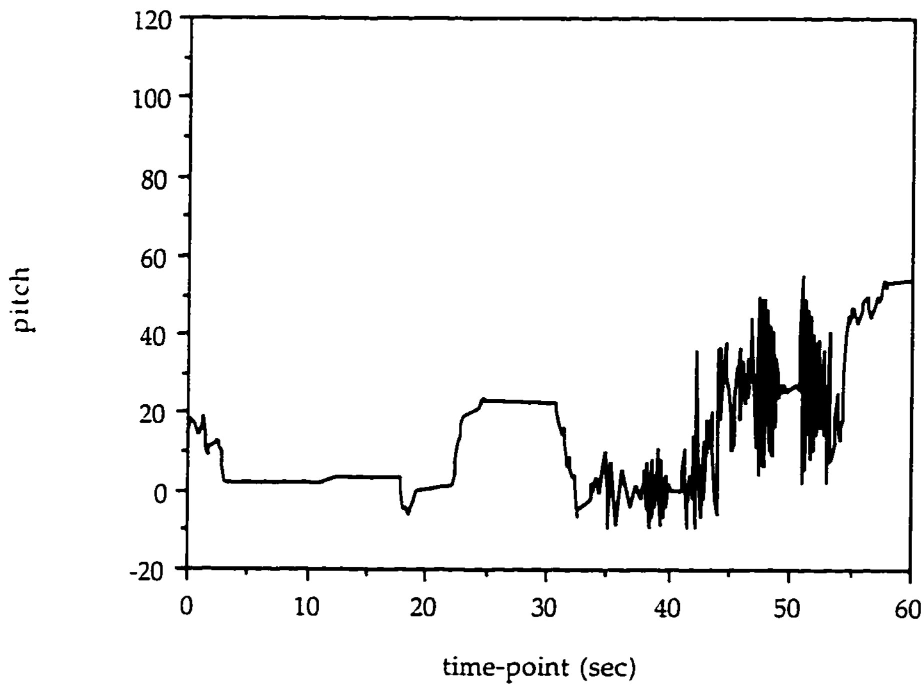

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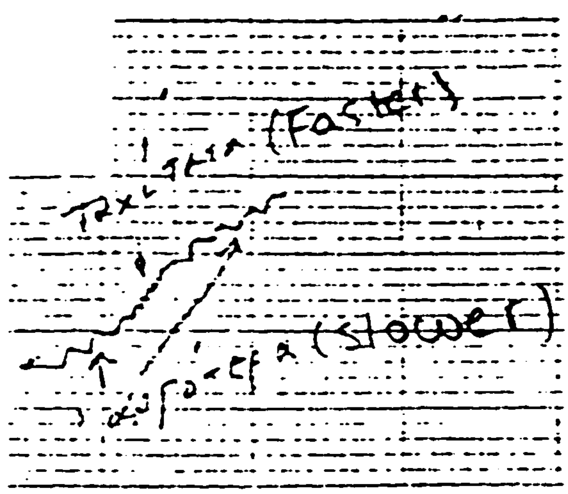

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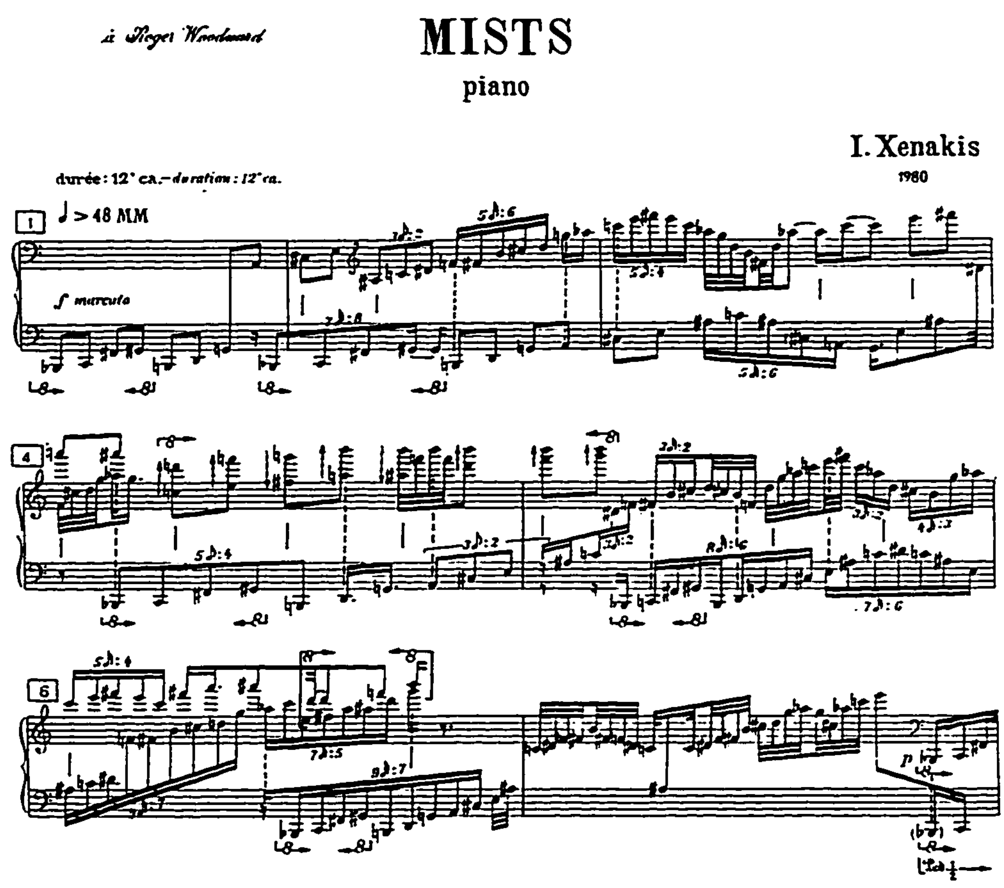

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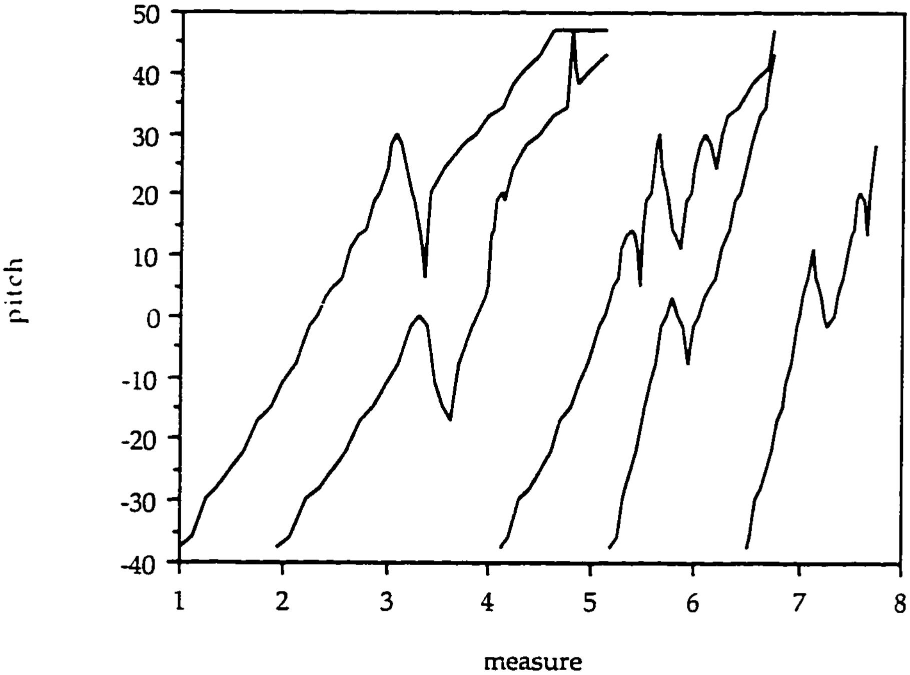
Iannis Xenakis, *Mists* (Paris: Editions Salabert, 1981). Used with permission.



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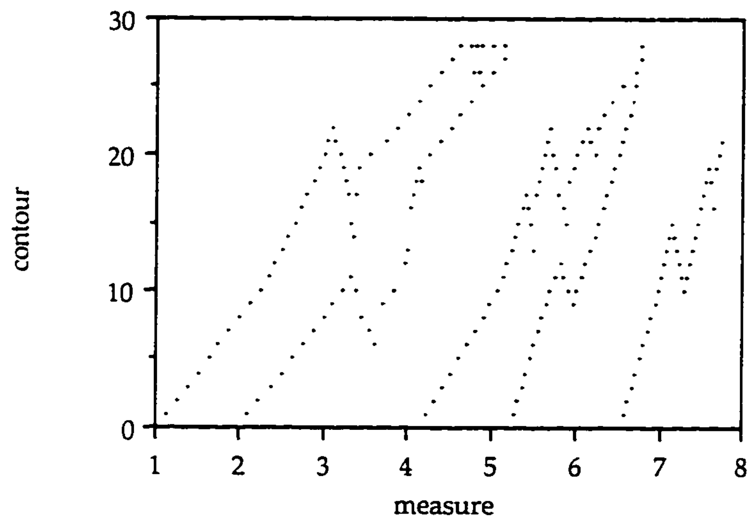

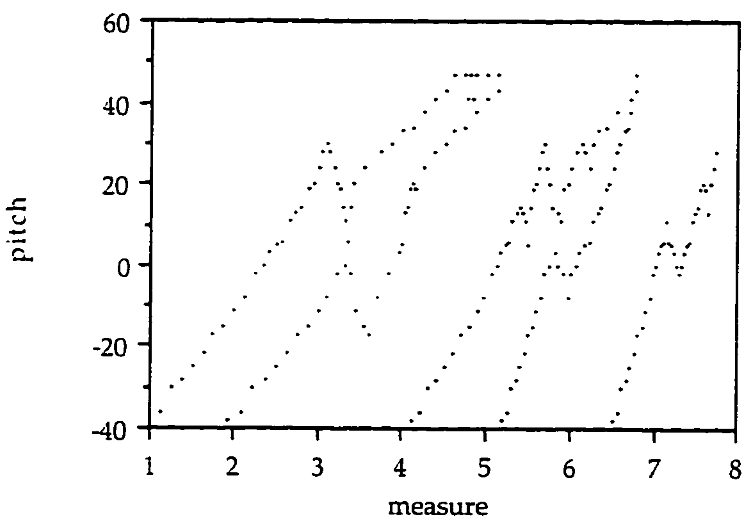

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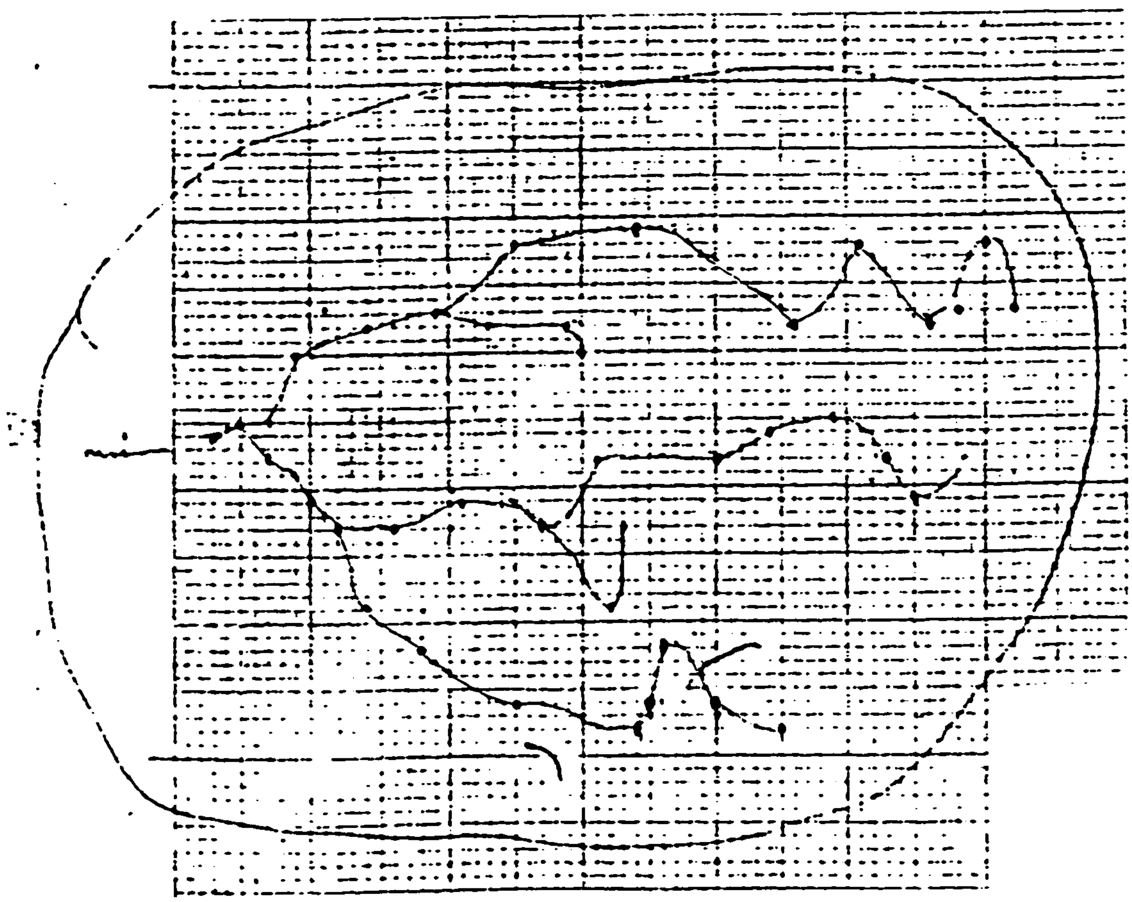

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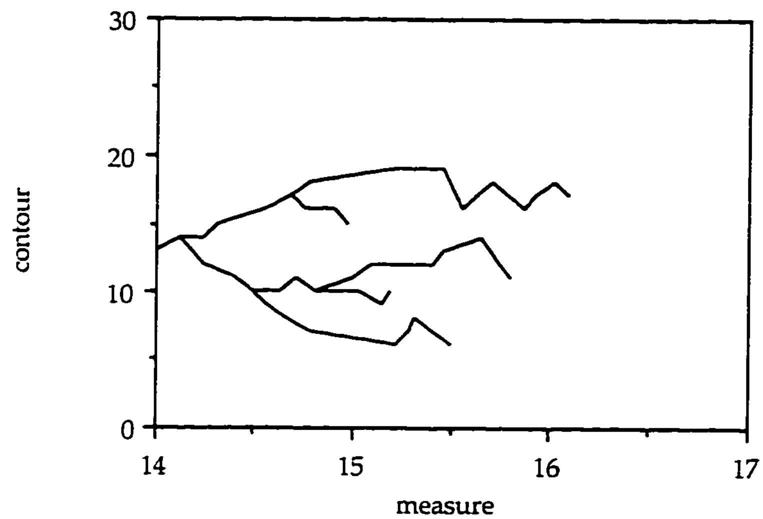

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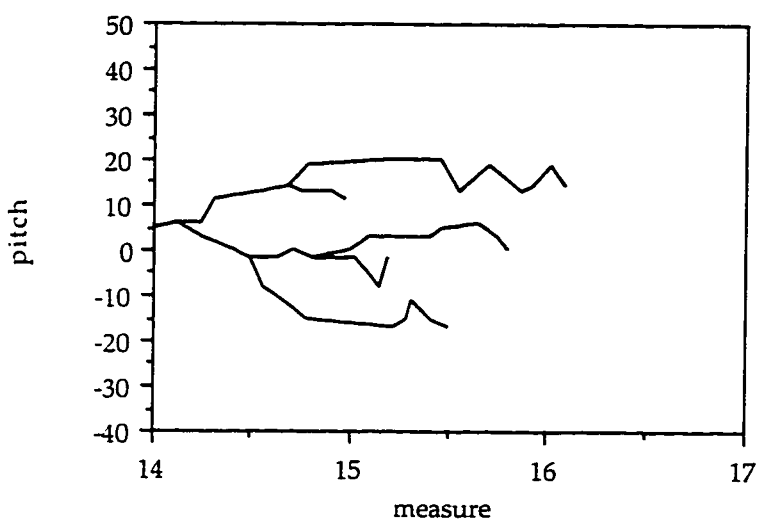

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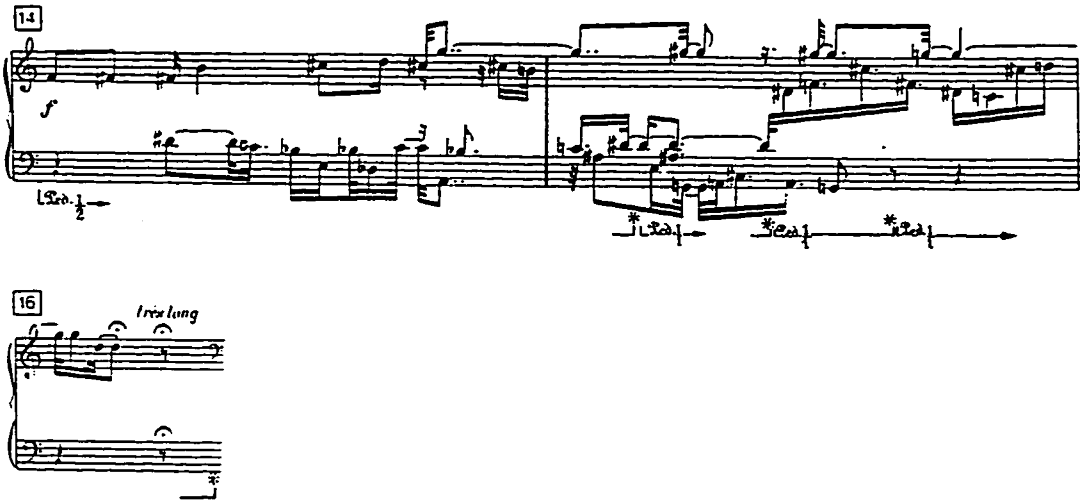

c. score

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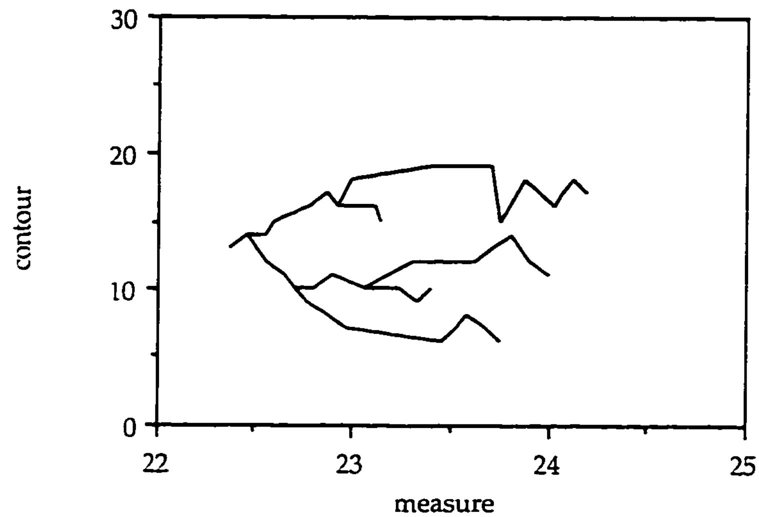
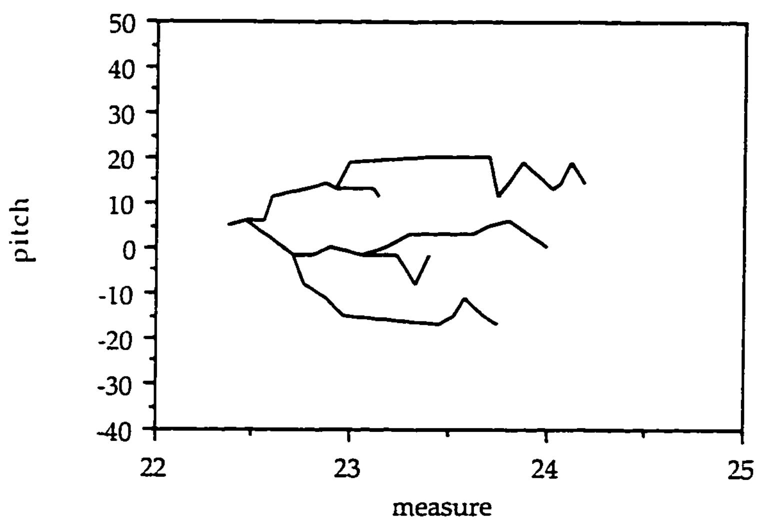
Iannis Xenakis. *Mists* (Paris: Editions Salabert, 1981). Used with permission.



a. c-/tp-space



b. p-/tp-space

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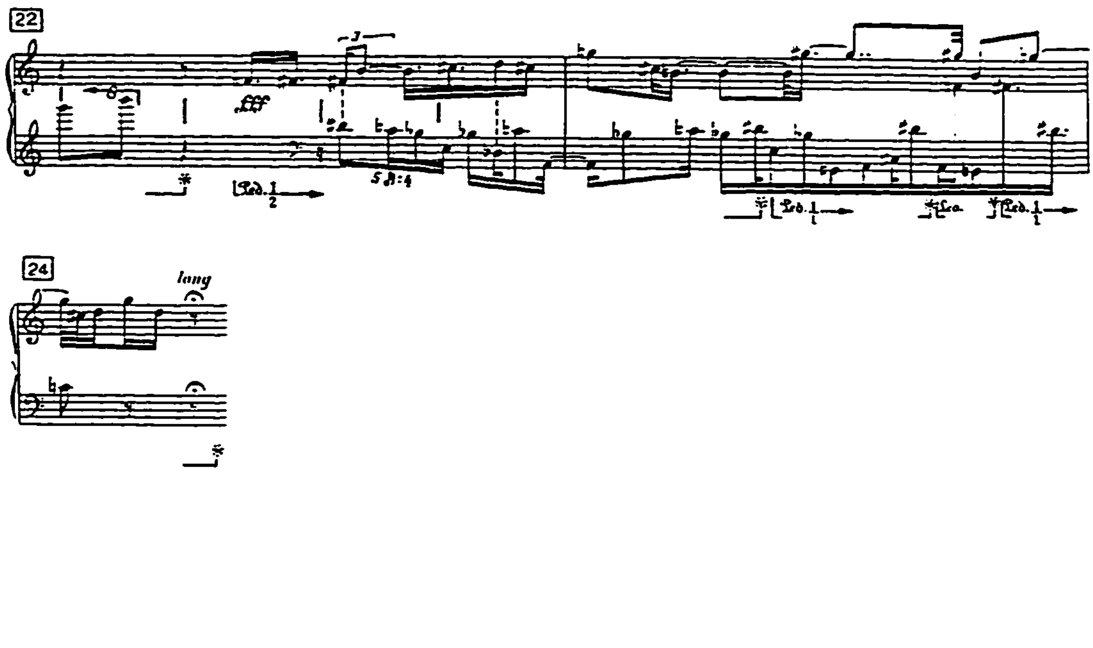
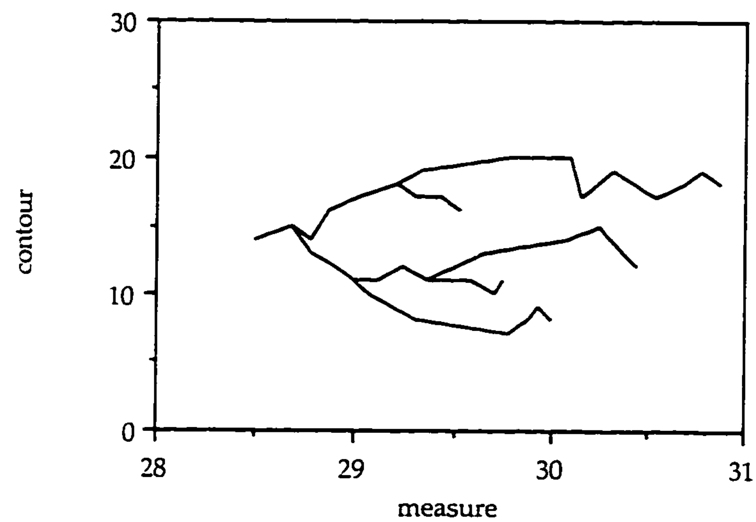
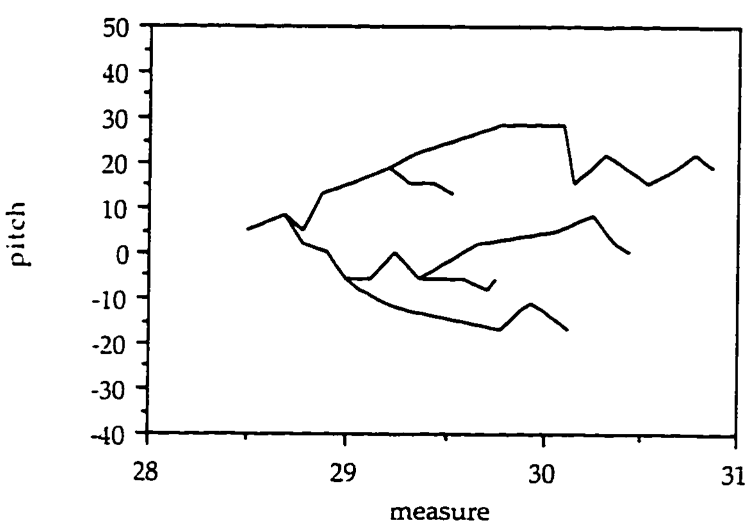


c. score

Figure 2.44: Graphic representations and musical notation of arborescence in *Mists*, mm. 22-4, cont.
Iannis Xenakis, *Mists* (Paris: Editions Salabert, 1981). Used with permission.



a. c-/tp-space



b. p-/tp-space

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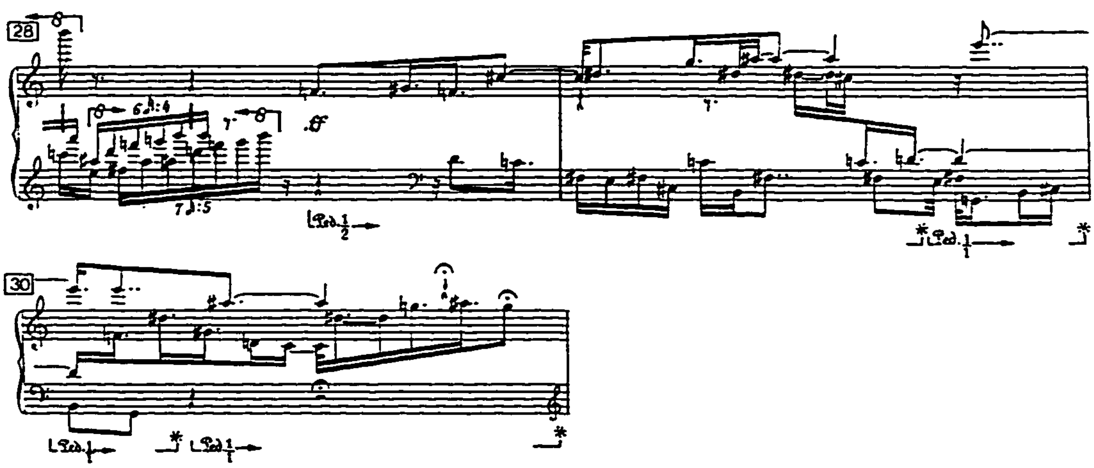

c. score

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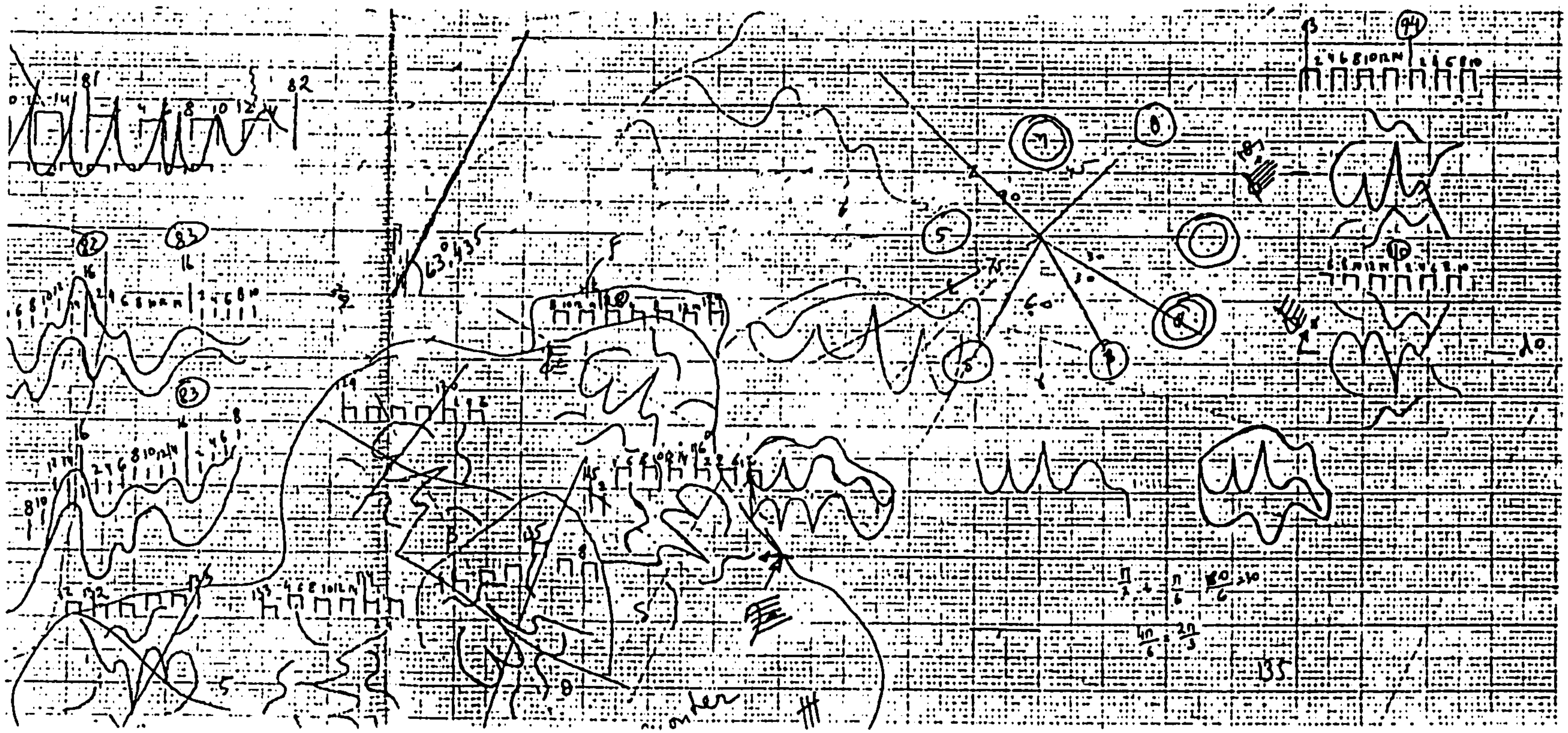
Iannis Xenakis, *Mists* (Paris: Editions Salabert, 1981). Used with permission.



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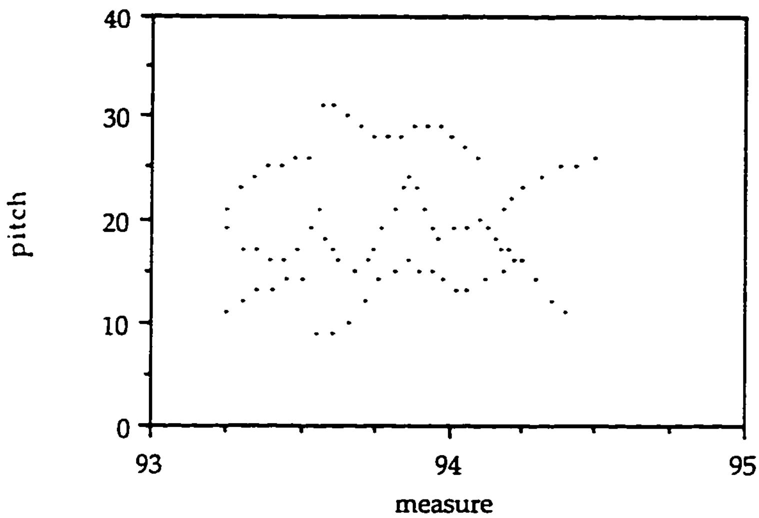
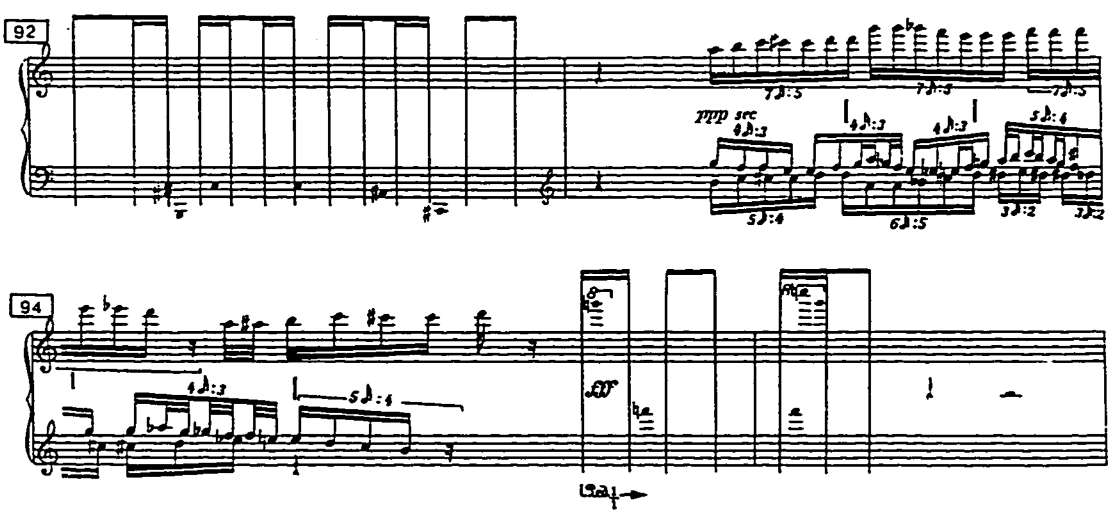


a. p-/tp-space



b. score

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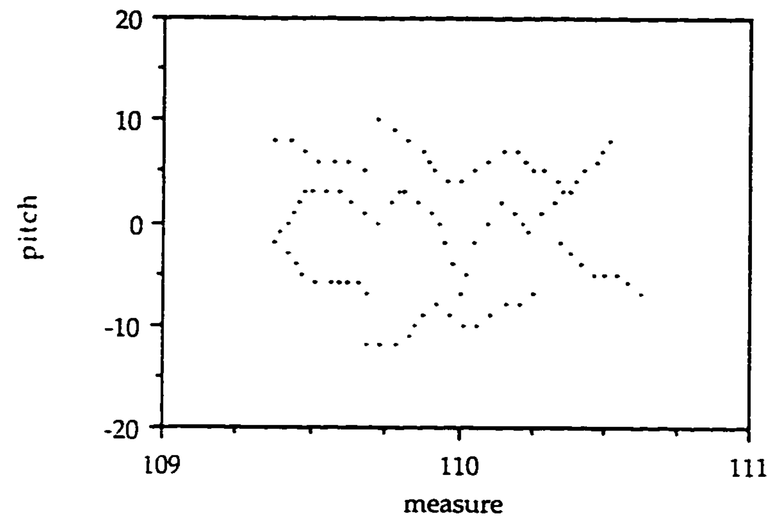
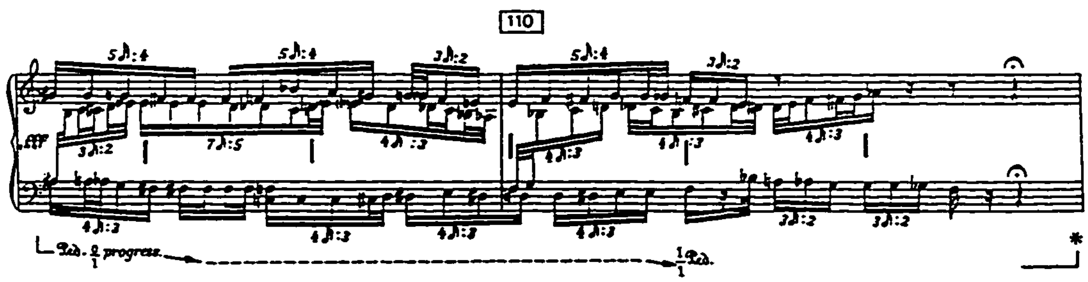
Iannis Xenakis, *Mists* (Paris: Editions Salabert, 1981). Used with permission.

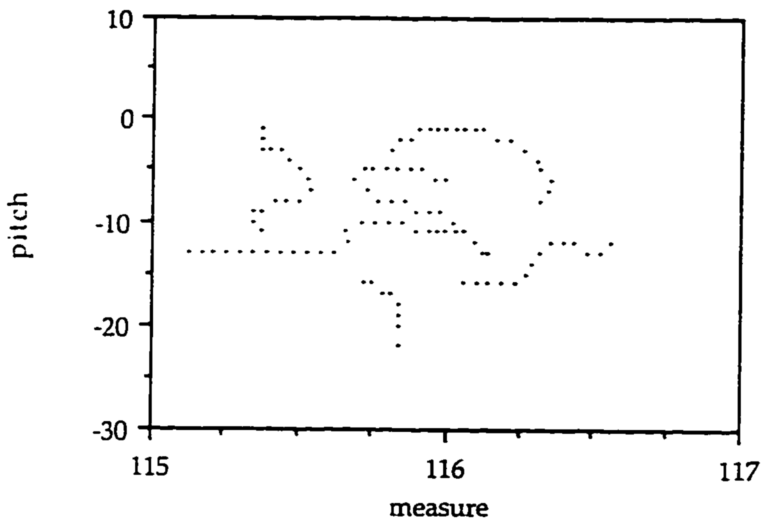


a. p-/tp-space

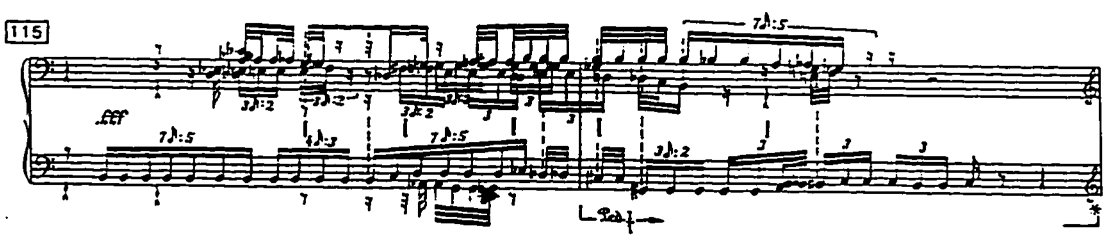


b. score

Figure 2.48: Graphic representation and musical notation of arborescence in *Mists*, mm. 109-10
Iannis Xenakis, *Mists* (Paris: Editions Salabert, 1981). Used with permission.

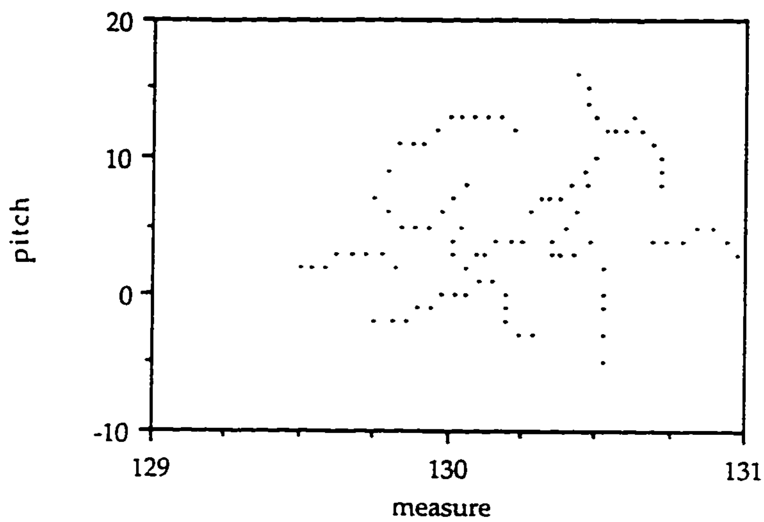


a. p-/tp-space

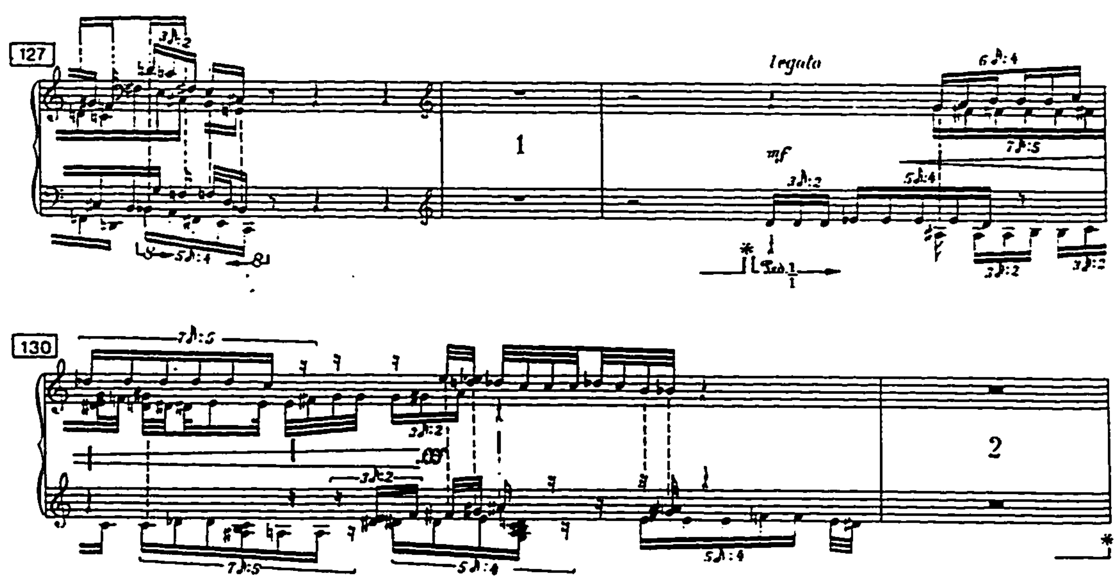


b. score

Figure 2.49: Graphic representation and musical notation of arborescence in *Mists*, mm. 115-6
Iannis Xenakis, *Mists* (Paris: Editions Salabert, 1981). Used with permission.

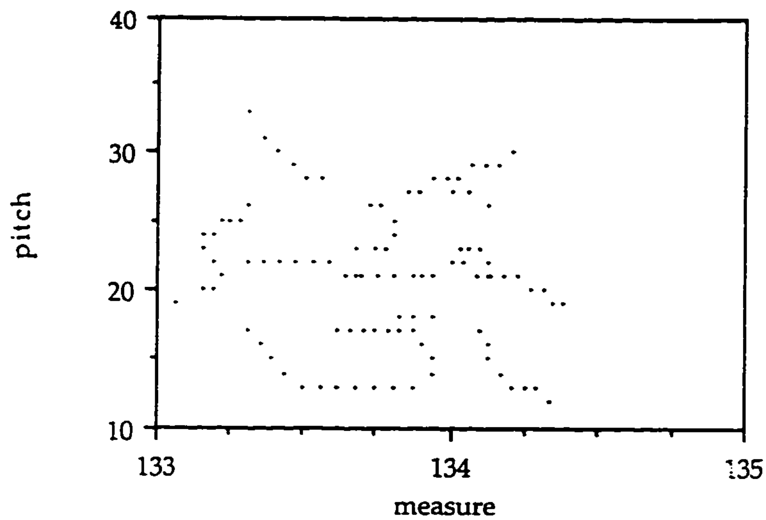


a. p-/tp-space

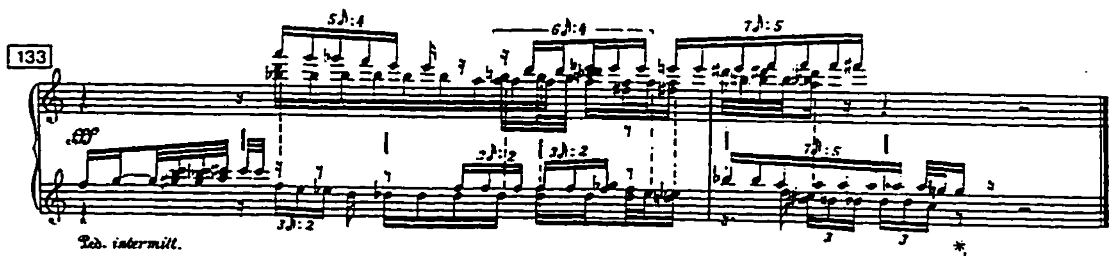


b. score

Figure 2.50: Graphic representation and musical notation of arborescence in *Mists*, mm. 129-30



a. p-/tp-space



b. score

Figure 2.51: Graphic representation and musical notation of arborescence in *Mists*, mm. 133-4
Iannis Xenakis, *Mists* (Paris: Editions Salabert, 1981). Used with permission.

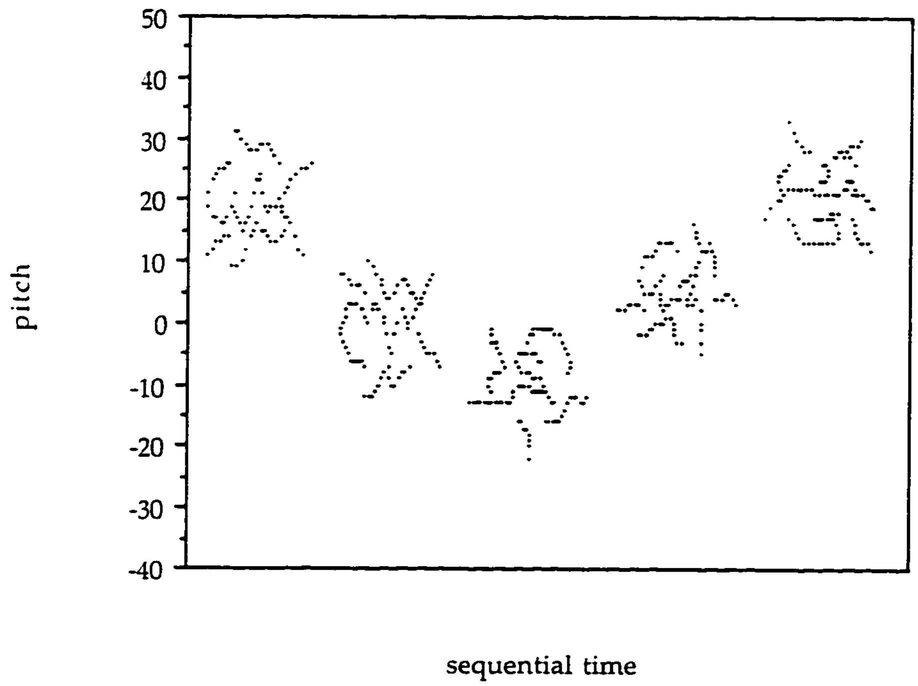


Figure 2.52: Graphic representation of sequence of five arborescences in *Mists*

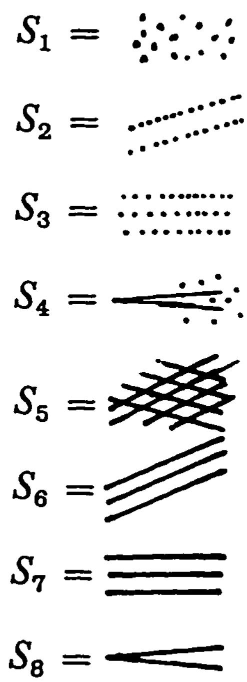


Figure 2.53: Configuration types in *Nomos Alpha*
Reprinted from Iannis Xenakis, *Formalized Music* (Stuyvesant, NY: Pendragon Press, 1992), pp. 232-3. Used with permission.

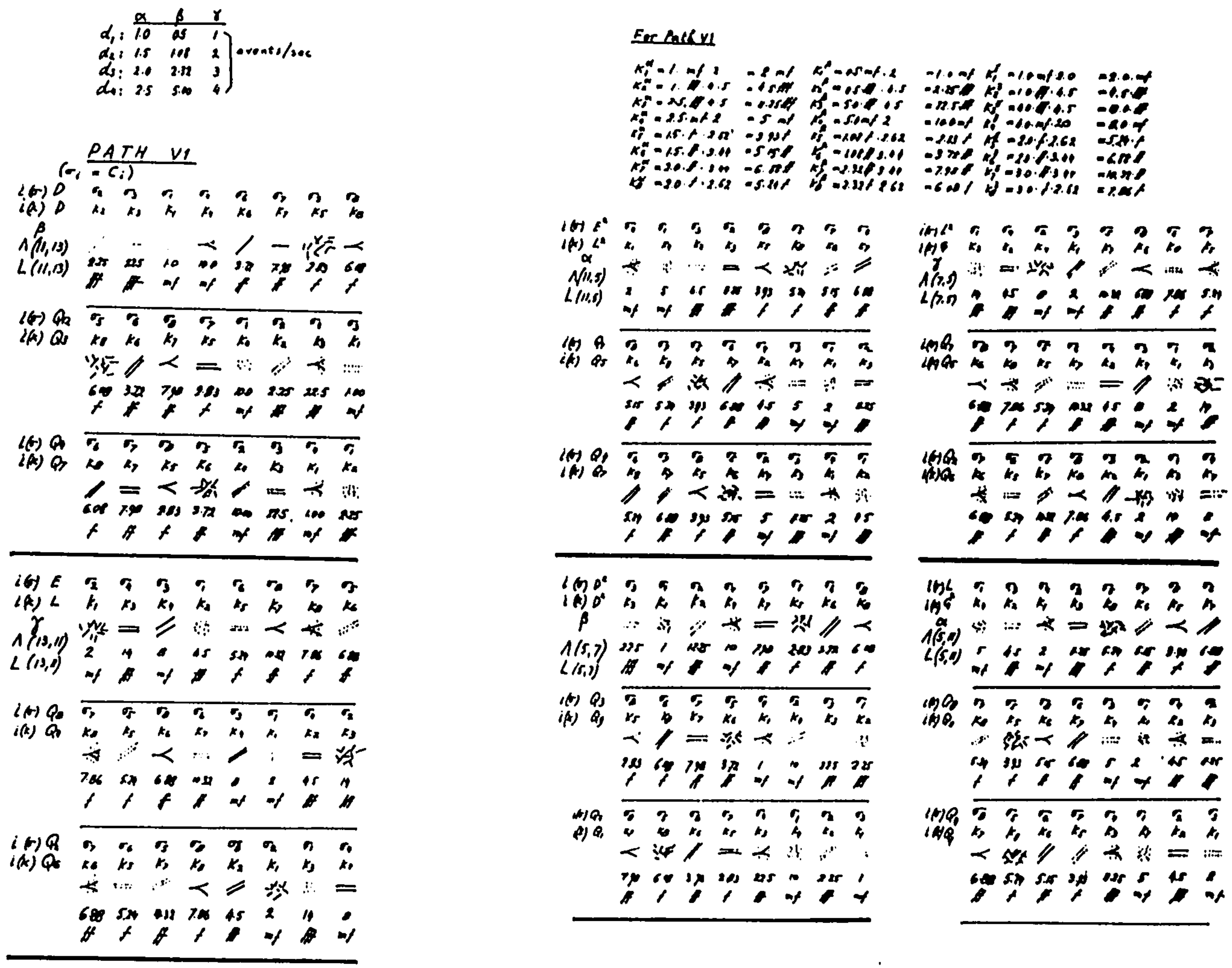


Figure 2.54: Compositional plan for *Nomos Alpha*
Reprinted from Iannis Xenakis, *Formalized Music* (Stuyvesant, NY: Pendragon Press, 1992), pp. 226-7. Used with permission.

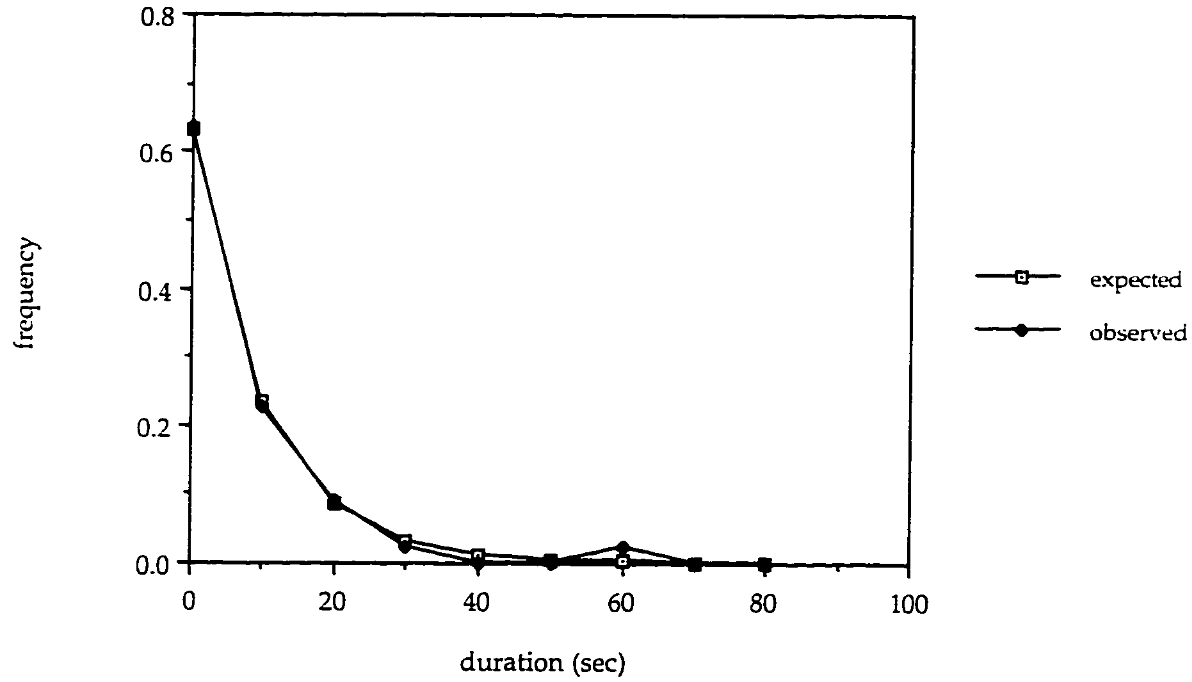


Figure 2.55: Comparative histogram of expected probabilities for the exponential distribution, ∂ = 0.1, and segment durations in *Herma*

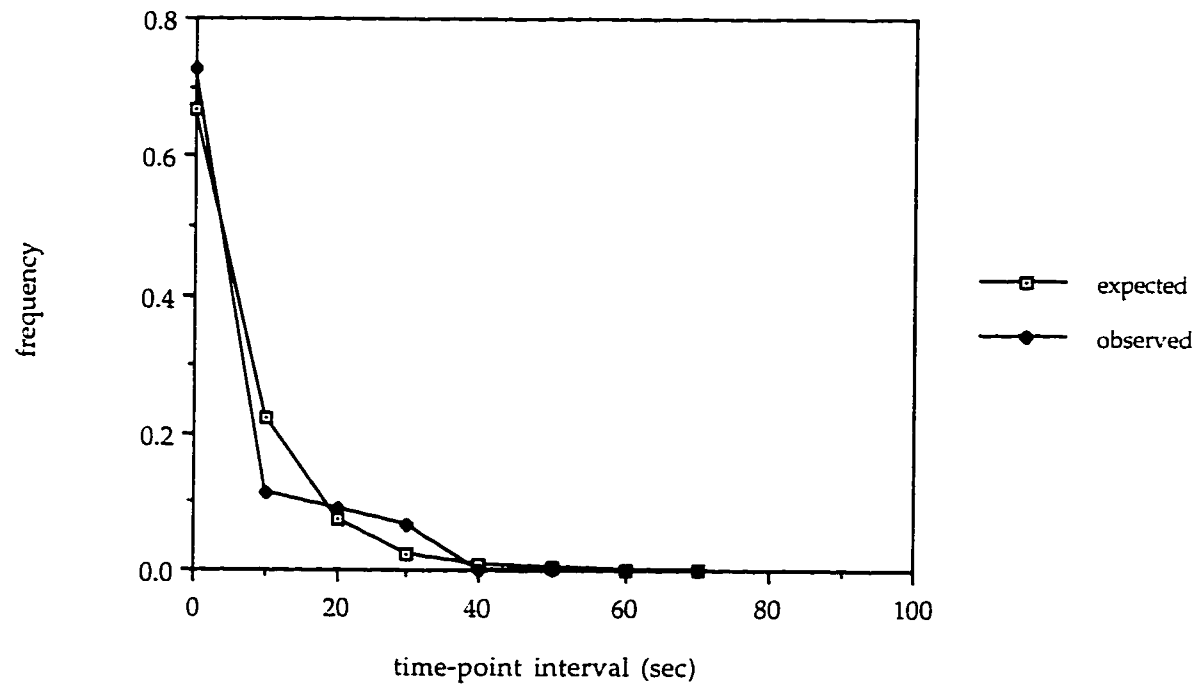


Figure 2.56: Comparative histogram of expected probabilities for the exponential distribution, $\partial = 0.11$, and time-point intervals between initiations of segments in *Herma*

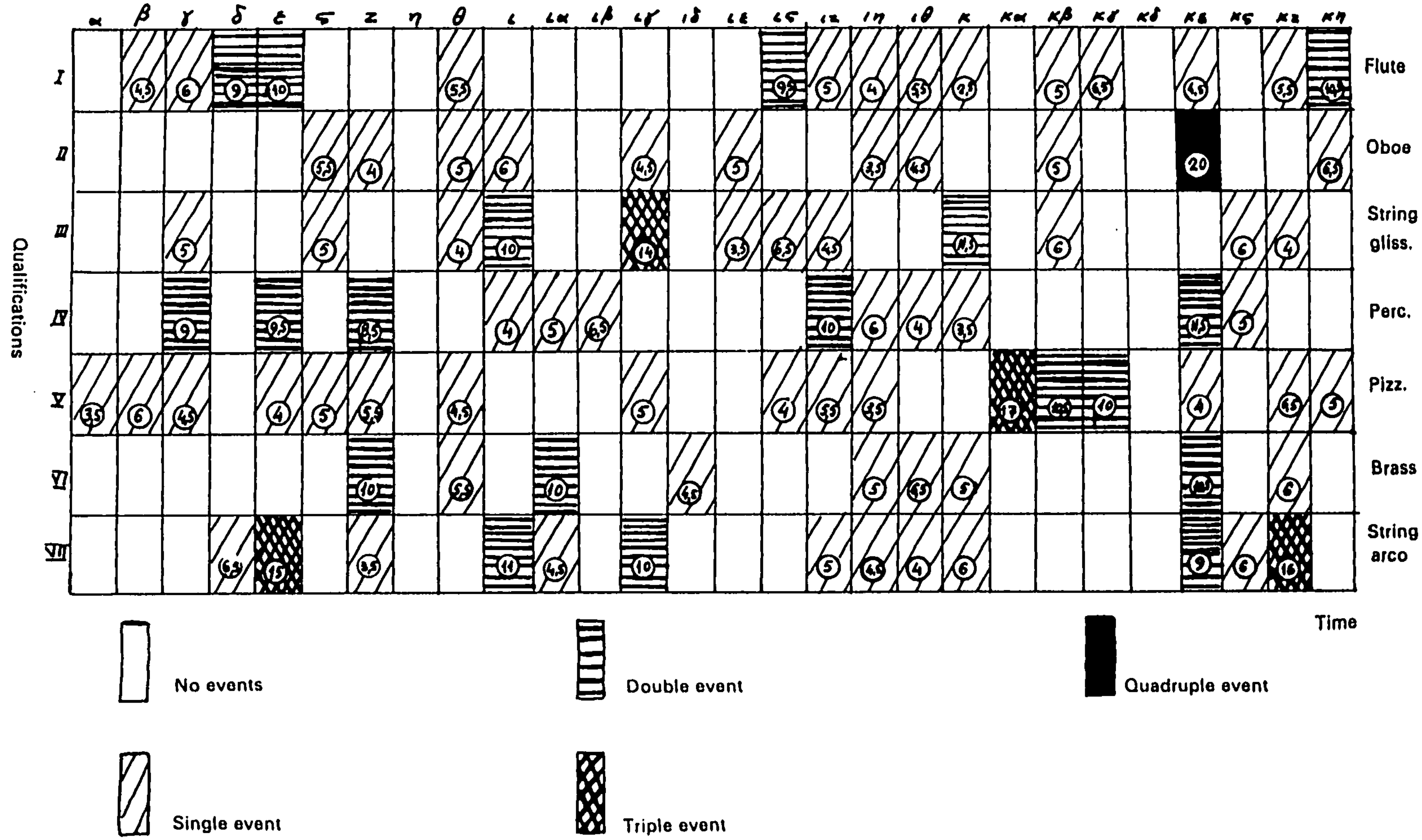


Figure 2.57: Compositional plan for *Achorripsis*
Reprinted from Iannis Xenakis, *Formalized Music* (Stuyvesant, NY: Pendragon Press, 1992), p. 28. Used with permission.

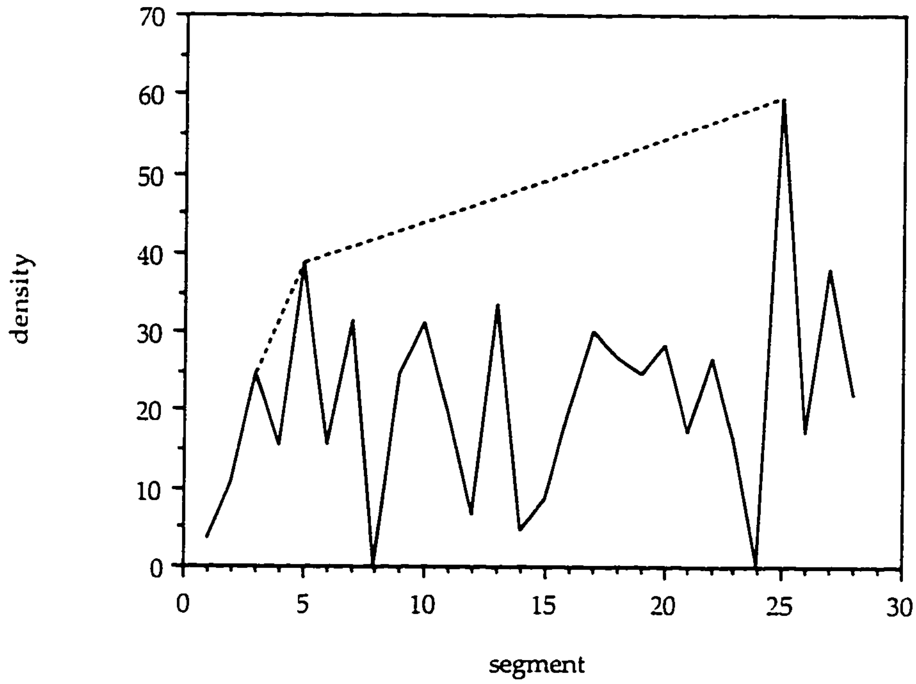


Figure 2.58: Total density per temporal segment in *Achorripsis*

Table 3.1: Segments in *Evryali*

segment	measures	seconds*	duration (sec)	configuration type	density	intensity
1	1-4	0.000-8.000	8.000	TPS	11.63	*ffff*
2	5-35	8.000-70.000	62.000	ST	2.80	*ppp-ffff* (d)†
3	16-18	31.750-36.000	4.250	TPS	10.82	*ffff*
4	25-28	49.750-54.500	4.750	TPS	21.26	*ffff*
5	28-31	55.250-62.000	6.750	TPS	11.11	*ffff*
6	33-35	65.750-70.000	4.250	TPS	10.59	*ffff*
7	36-40	70.000-78.125	8.125	ST	7.75	*mf-ffff* (d)
8	40-46	78.125-91.250	13.125	TPS	9.75	*ffff*
9	46-60	91.250-119.875	28.625	A	11.84	*pp-fff* (c)
10	60-64	119.875-128.000	8.125	TPS	31.38	*ffff*
11	65	128.000-140.000	12.000	R	0.00	-
12	66-69	140.000-147.125	7.125	A	13.05	*pp*
13	69-74	147.125-158.000	10.875	TPS	19.40	*pp*
14	75-87	158.000-183.750	25.750	A	18.10	*pp-fff* (c)
15	87-92	183.750-192.500	8.750	TPS	40.91	*fff*
16	92-95	192.500-198.250	5.750	A	32.35	*pp*
17	95-97	198.250-202.625	4.375	TPS	38.86	*mf*
18	97-100	202.625-208.250	5.625	A	28.44	*pp*
19	100-102	208.250-212.125	3.875	TPS	34.84	*ff*
20	102-136	212.125-280.125	68.000	A	30.21	*pp, ff*
21	107-109	222.000-228.000	6.000	TPS	17.17	*mf*
22	113-114	235.750-237.825	2.125	TPS	32.29	*fff*
23	136-146	280.125-302.000	21.875	ST	14.40	*ppp-ffff* (c)
24	147-179	302.000-366.750	64.750	ST	0.93	*pp-ff* (d)
25	148-149	305.625-308.000	2.375	A	10.53	*pp*
26	150-151	309.625-311.000	1.375	A	10.91	*pp*
27	152	313.125-314.000	0.875	A	11.43	*pp*
28	154	316.625-318.000	1.375	A	8.73	*pp*
29	155	318.250-319.625	1.375	A	9.45	*pp*
30	158-160	325.250-329.000	3.750	A	8.80	*pp*
31	159-160	327.000-328.375	1.375	A	10.91	*pp*
32	161-166	330.125-340.375	10.250	A	16.49	*mf*
33	165-170	338.500-348.625	10.125	A	17.28	*mf*
34	168-171	345.500-352.000	6.500	A	17.69	*mf*
35	179-188	366.750-385.125	18.375	A	29.12	*pp-ffff* (c)
36	188-189	385.125-392.000	6.875	R	0.00	-
37	190-192	392.000-396.250	4.250	TPS	20.71	*pp-ffff* (c)
38	191-194	394.625-400.250	5.625	TPS	20.44	*pp-ffff* (c)
39	192-194	396.250-401.875	5.625	TPS	20.44	*pp-ffff* (c)
40	194-196	400.125-404.875	4.750	TPS	20.21	*pp-ffff* (c)
41	194-196	401.875-405.375	3.500	TPS	19.43	*pp-ffff* (c)
42	196-197	404.875-407.125	2.250	TPS	21.78	*pp-ffff* (c)

* refers to the location of the segments in the graphic transcription (Figure 3.3)
† d = discrete changes in intensity; c = continuous (gradual) changes in intensity
configuration types: TPS = time-point sieve; A = arborescence; ST = stochastic; R = rest

Table 3.1: Segments in *Evryali*, cont.

segment	measures	seconds	duration (sec)	configuration type	density	intensity
43	196-197	405.375-407.500	2.125	TPS	19.29	*pp-ffff* (c)
44	197-198	407.125-409.750	2.625	TPS	20.19	*pp-ffff* (c)
45	197-198	407.375-409.625	2.250	TPS	21.33	*pp-ffff* (c)
46	198-206	409.625-426.000	16.375	TPS	43.36	*pp-ffff* (c), *ffff*
47	207*	426.000-432.625	6.625	A	51.62	*pp-ffff* (c)
48	207-212	432.625-451.000	18.375	A	21.17	*pp-ffff* (c)
49	212	451.000-461.000	10.000	R	0.00	-
50	213†	461.000-471.000	10.000	A	32.40	*pp-ffff* (c, d)

* The lengths of the measures change at this point from 4 quarter-note beats to different lengths. The reading given here follows the divisions created by solid barlines in the score.

† The tempo changes in this measure to "Plus lent." In order to represent this change, the temporal unit has been changed here from 0.125 sec to 0.167.

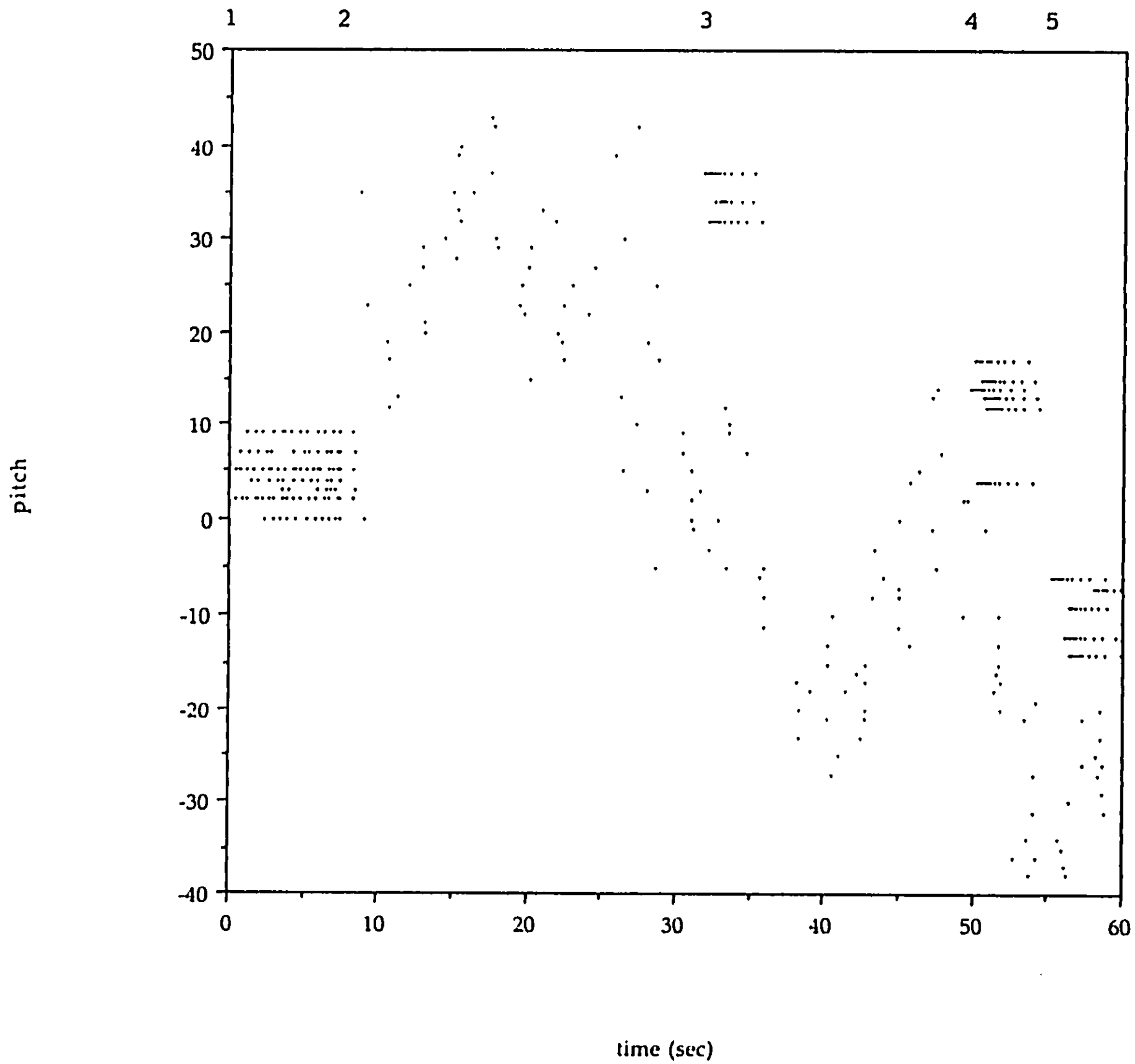


Figure 3.1: Graphic transcription of *Evryali*

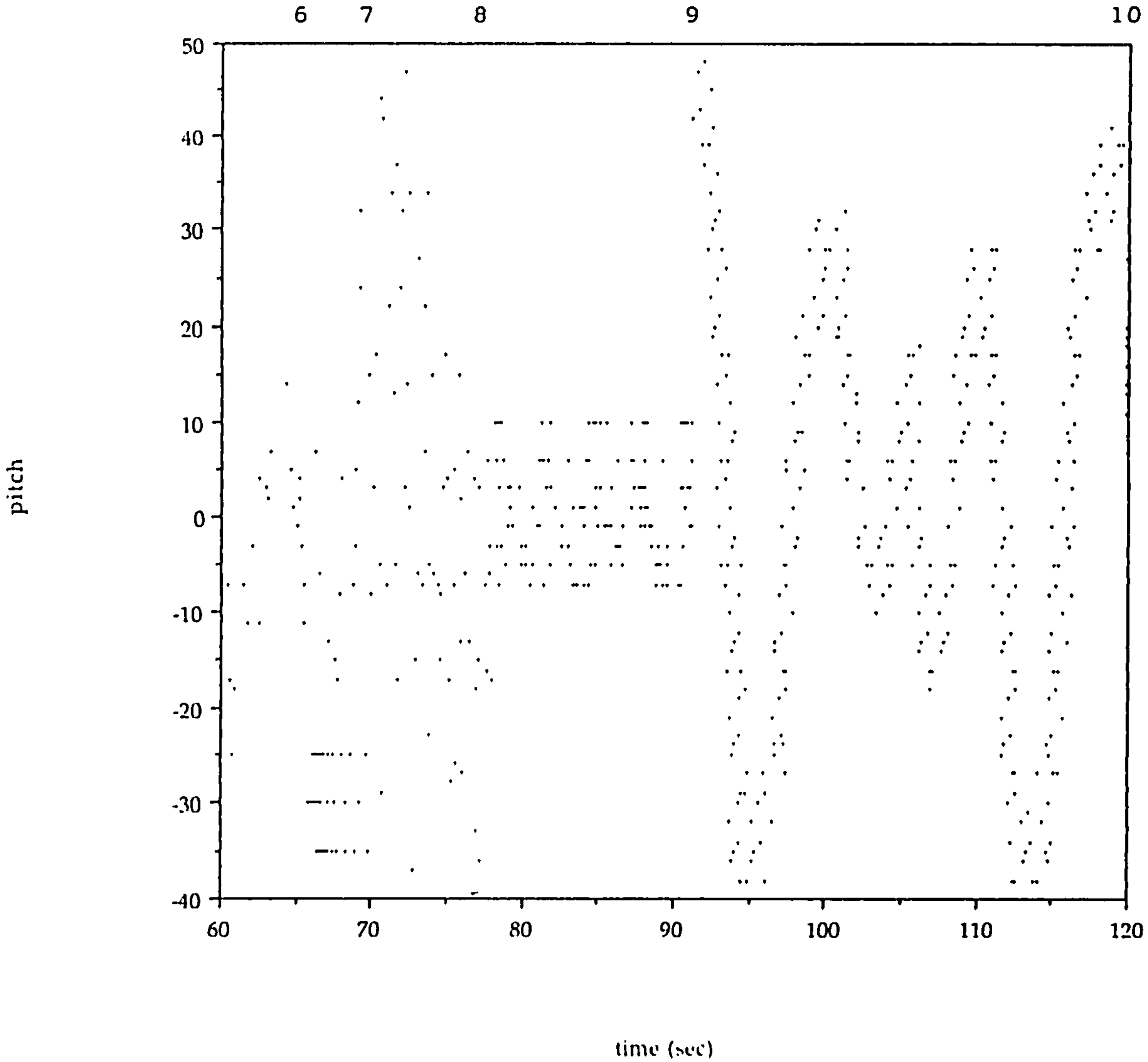


Figure 3.1: Graphic transcription of *Evryali*, cont.

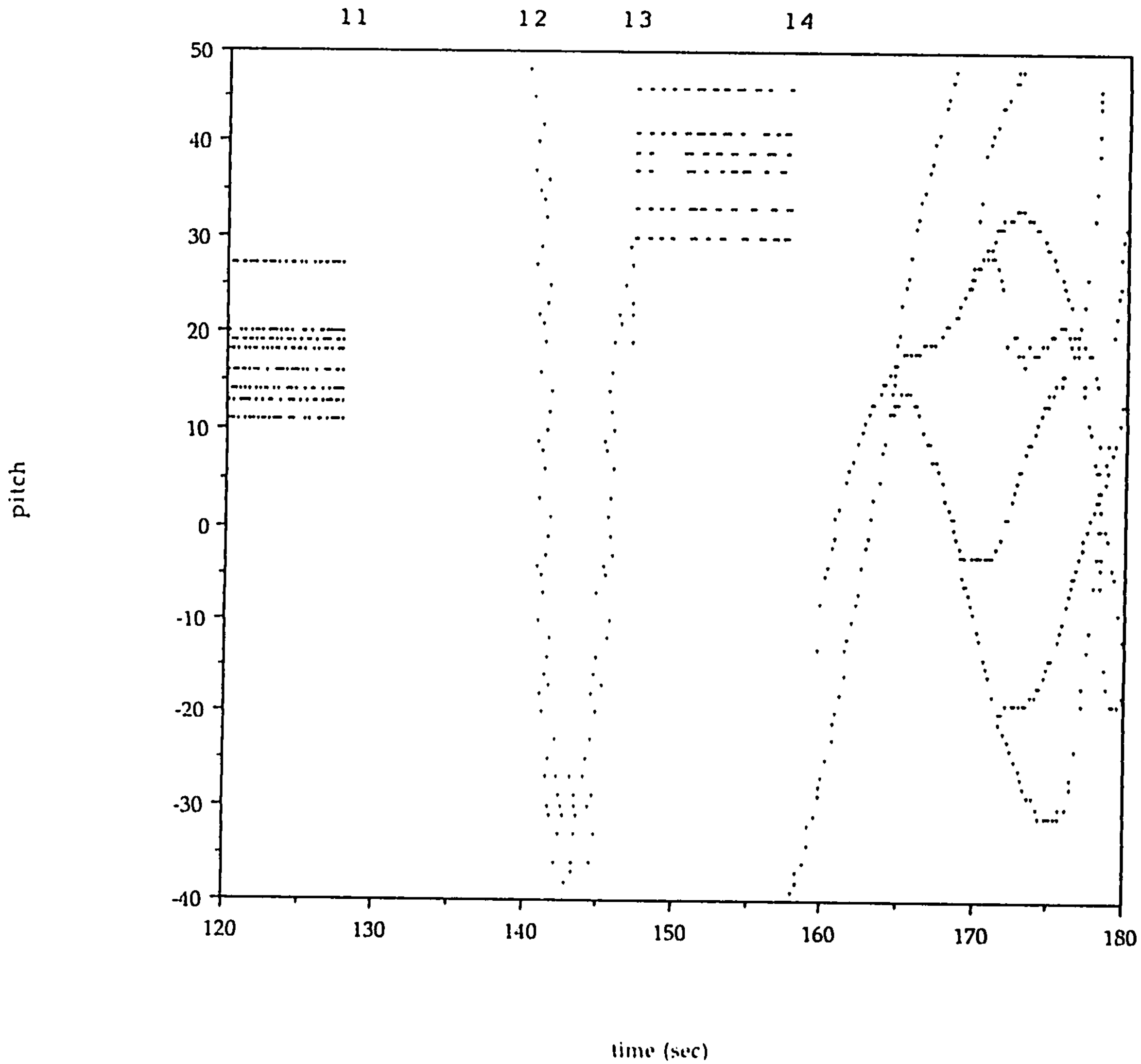


Figure 3.1: Graphic transcription of *Evryali*, cont.

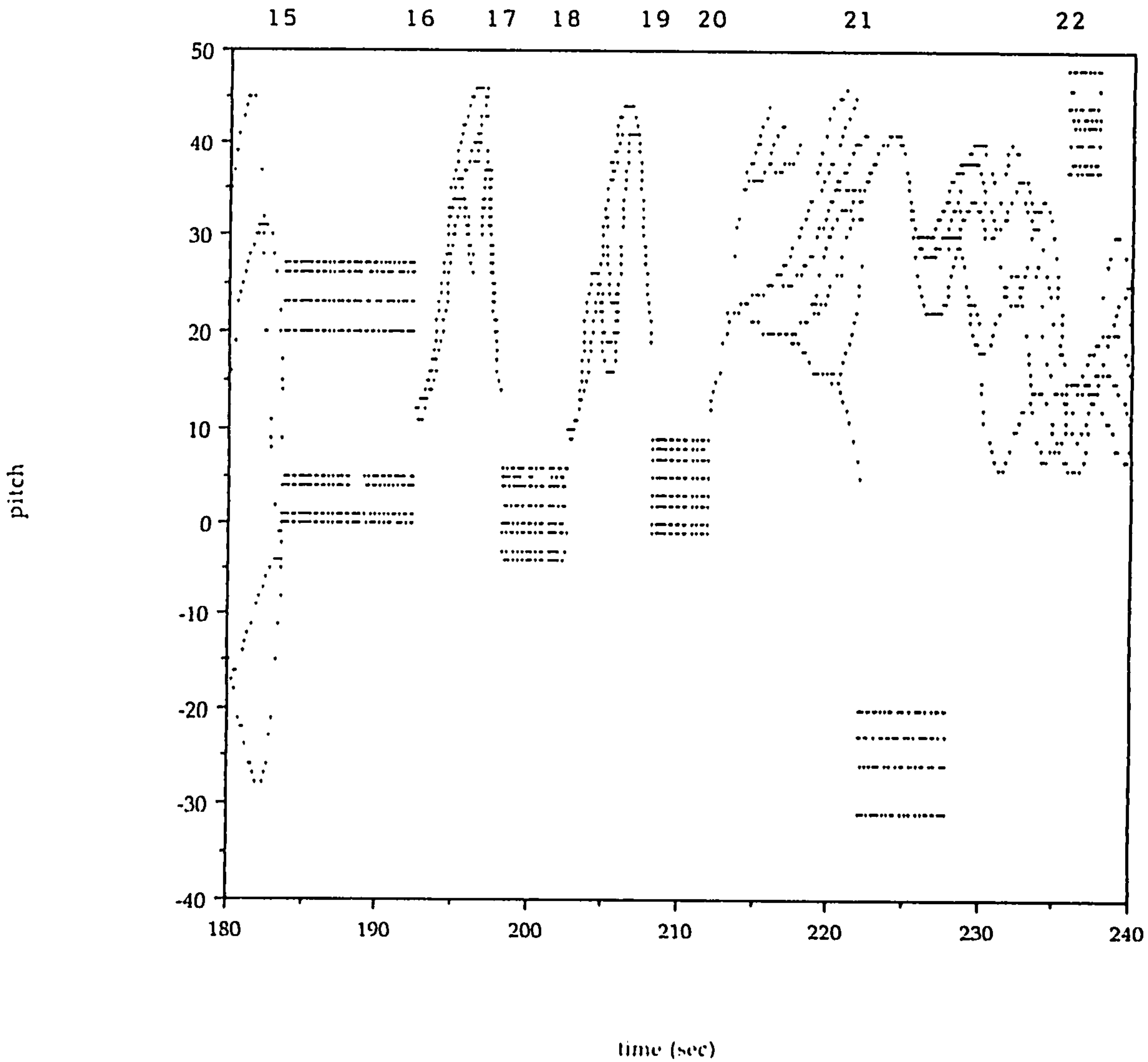


Figure 3.1: Graphic transcription of *Evryali*, cont.

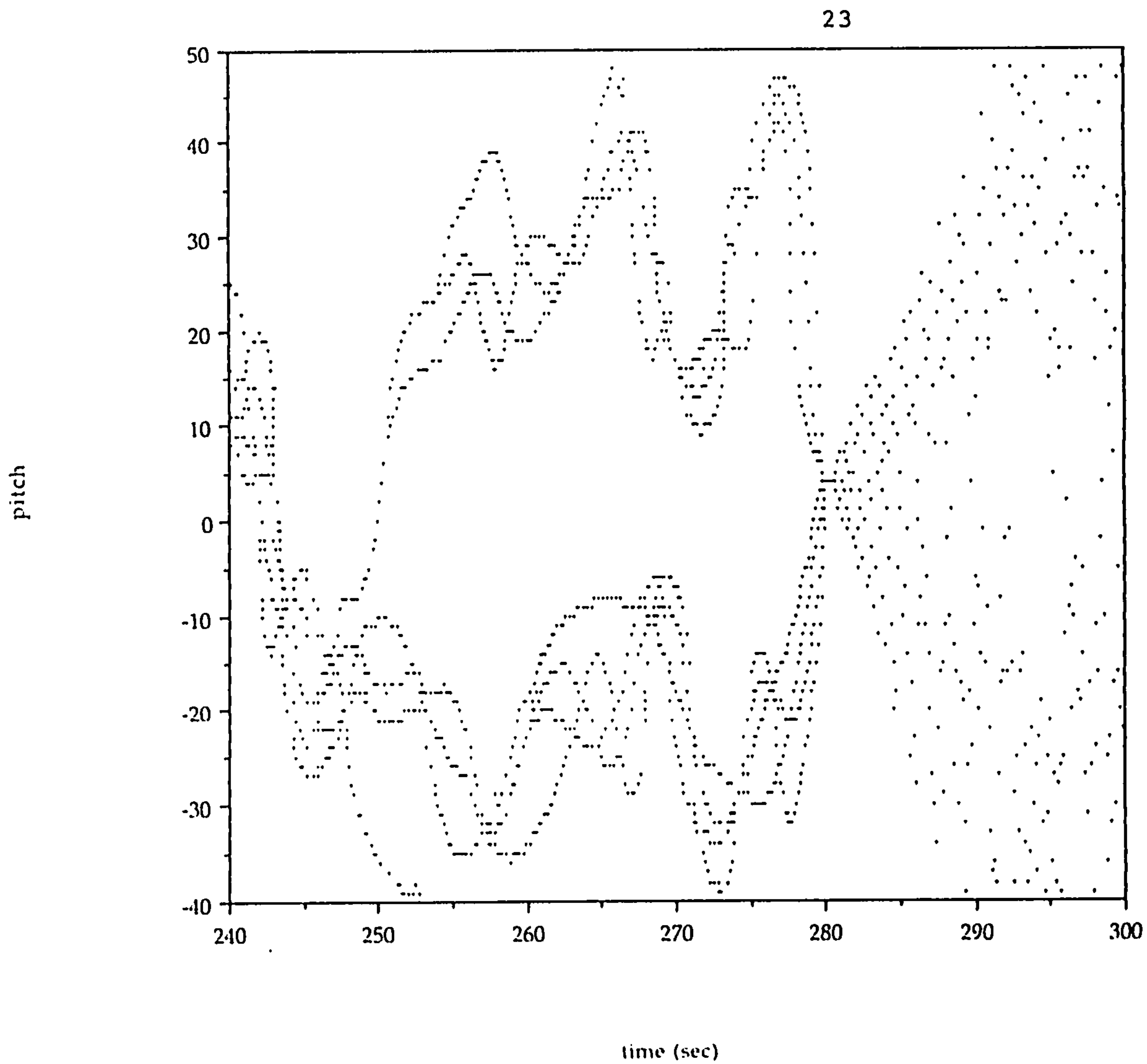


Figure 3.1: Graphic transcription of *Evryali*, cont.

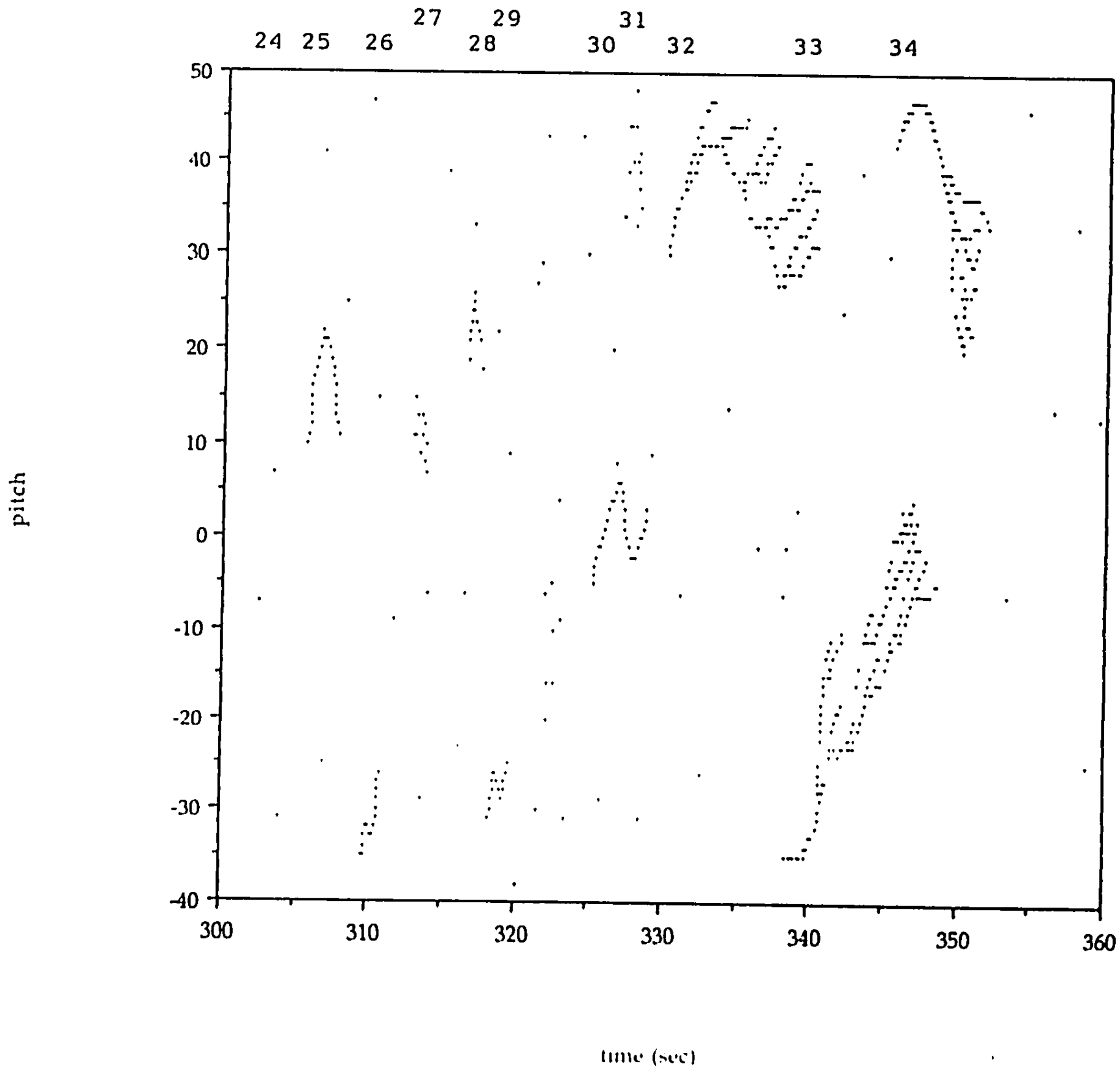


Figure 3.1: Graphic transcription of *Evryali*, cont.

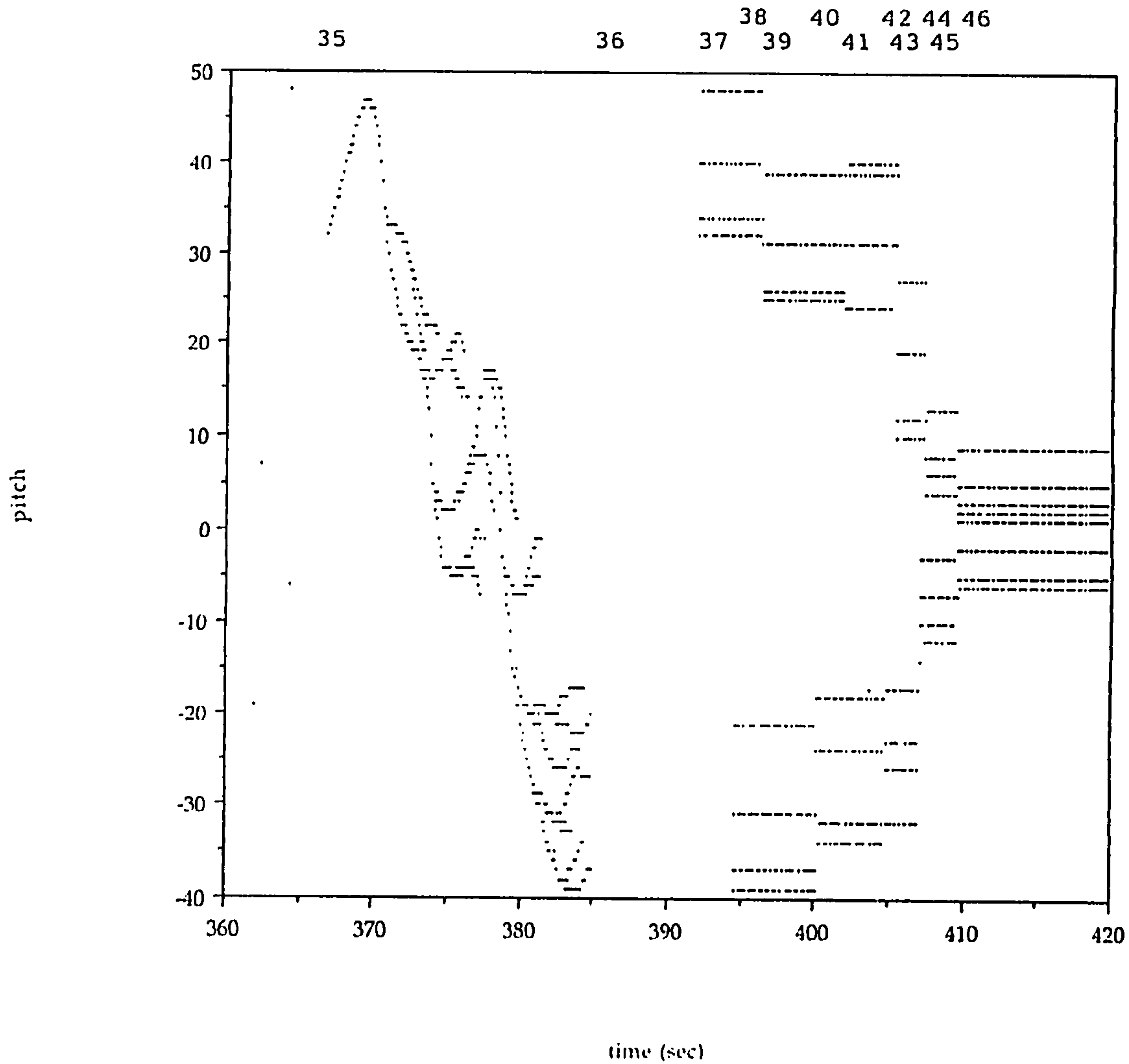


Figure 3.1: Graphic transcription of *Evryali*, cont.

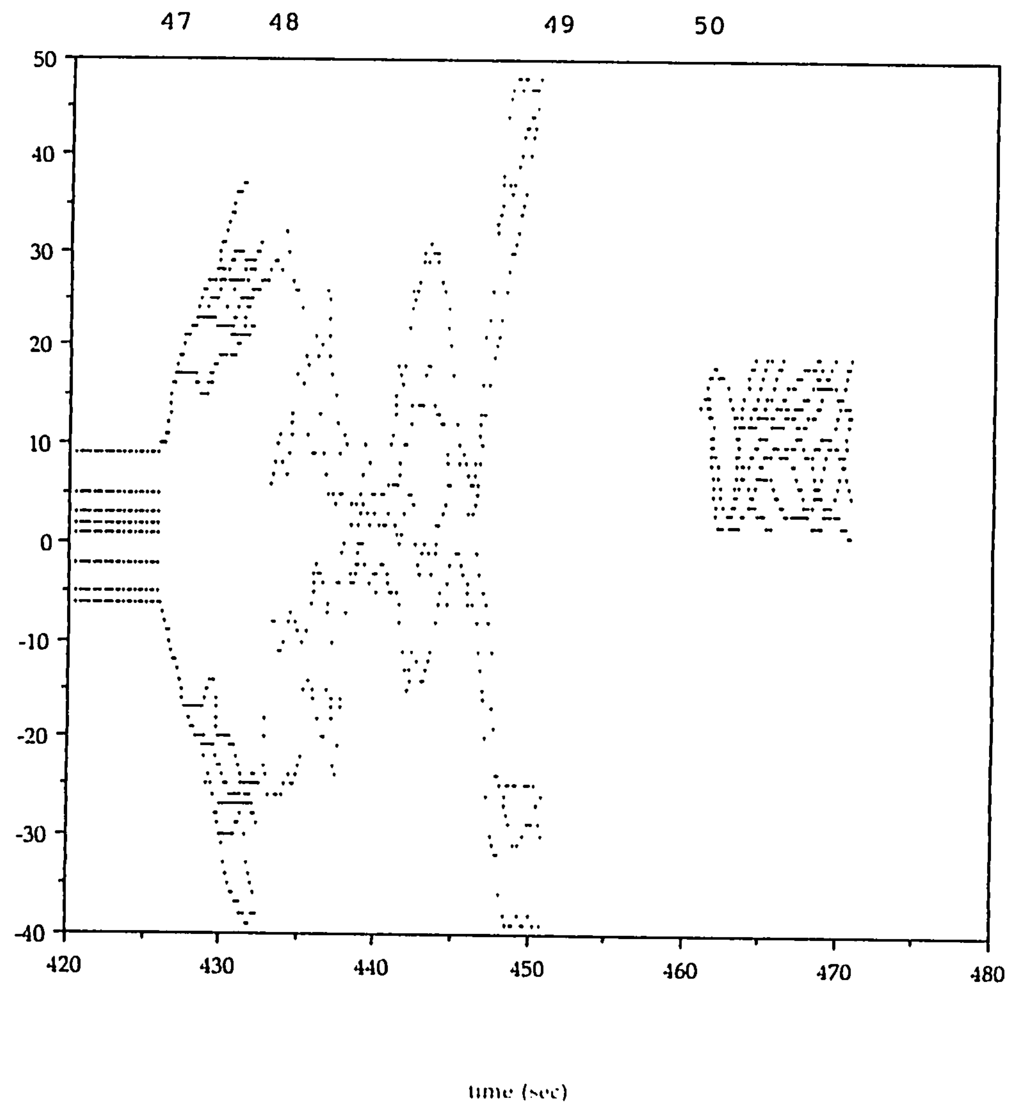
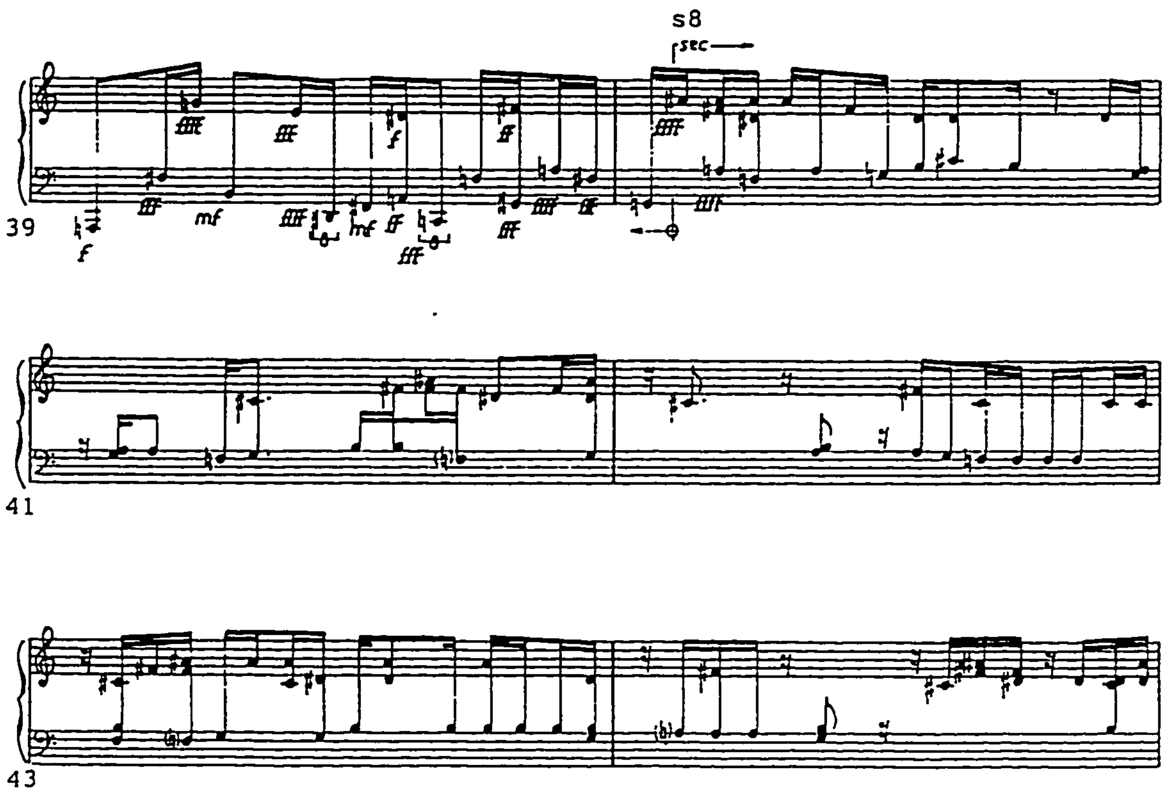
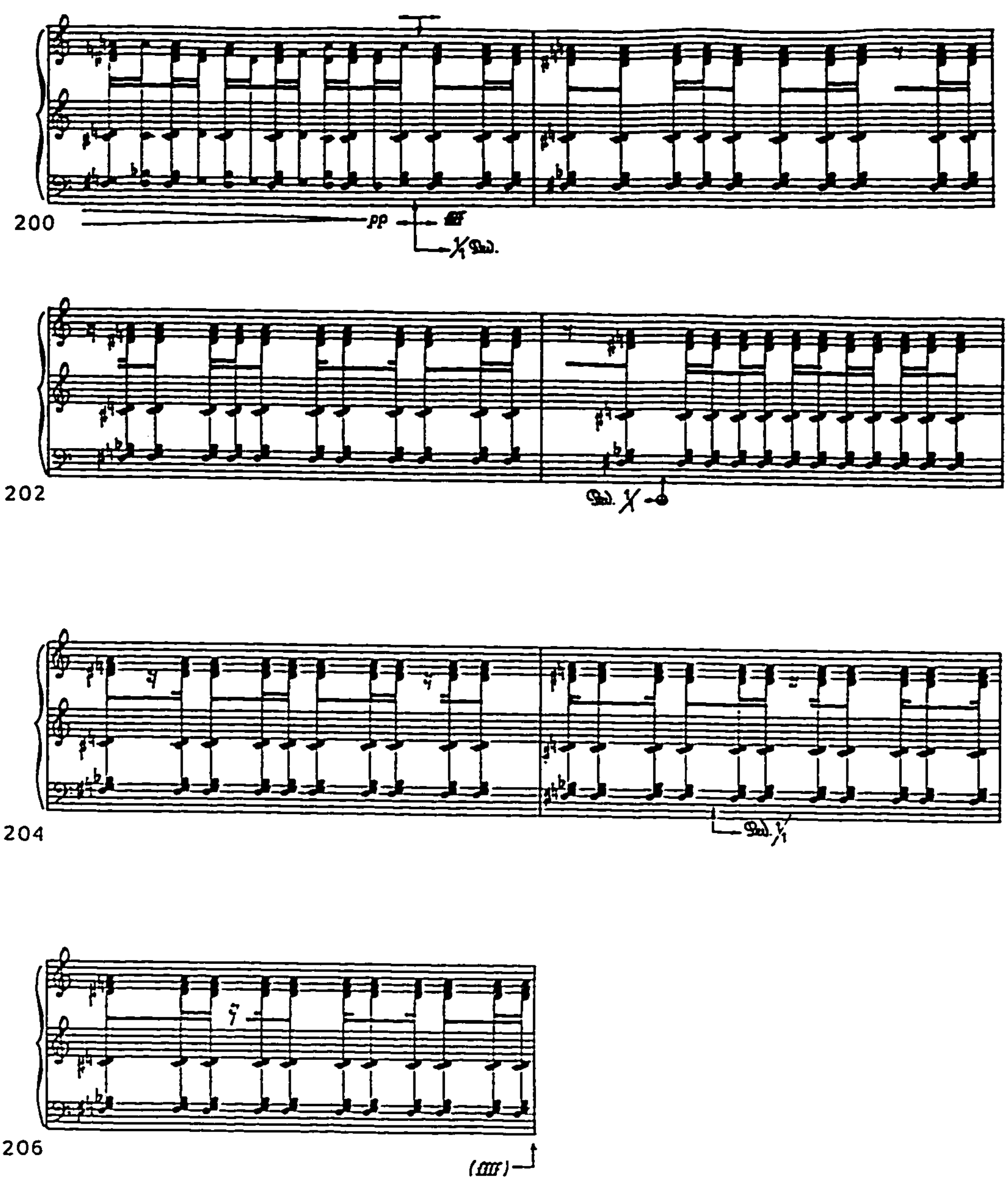


Figure 3.1: Graphic transcription of *Evryali*, cont.



a. mm. 39-44 (from segment 8)

Figure 3.2: TPS configurations in *Evryali*
Iannis Xenakis, *Evryali* (Paris: Editions Salabert, 1974). Used with permission.



b. mm. 200-6 (from segment 46)

Figure 3.2: TPS configurations in *Evryali,* cont.
Iannis Xenakis, *Evryali* (Paris: Editions Salabert, 1974). Used with permission.

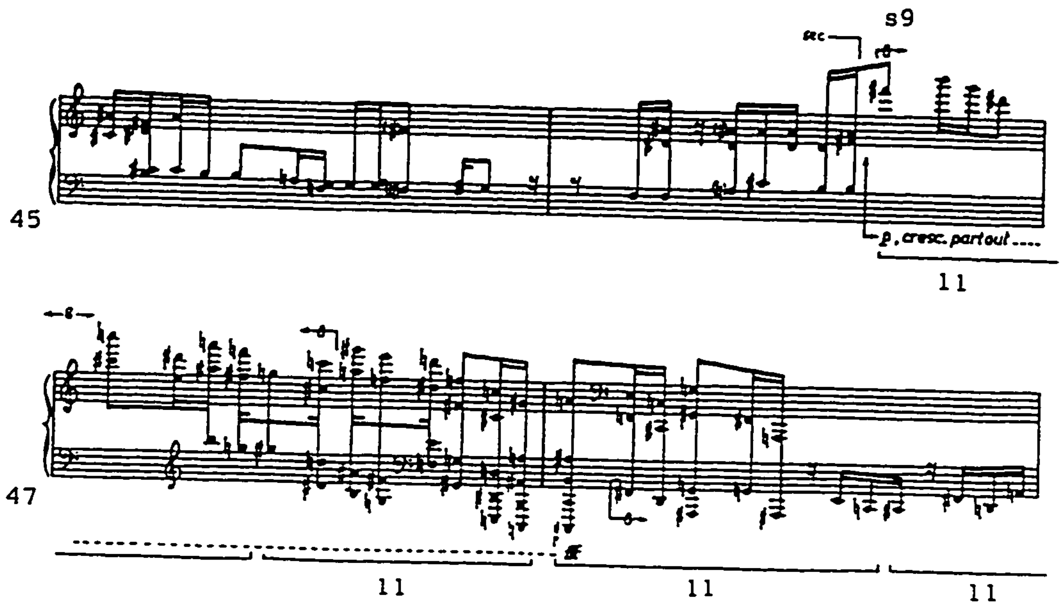


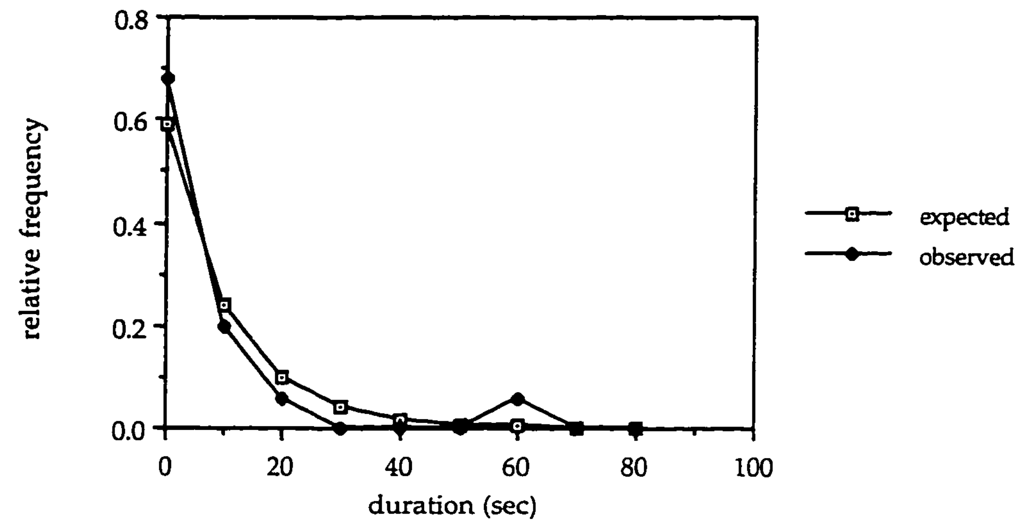
Figure 3.3: Tpseq INTs of period 11 in *Evryali*, segment 9, mm. 45-8
Iannis Xenakis, *Evryali* (Paris: Editions Salabert, 1974). Used with permission.

Table 3.2: Summary of Temporal Structure in *Evryali*

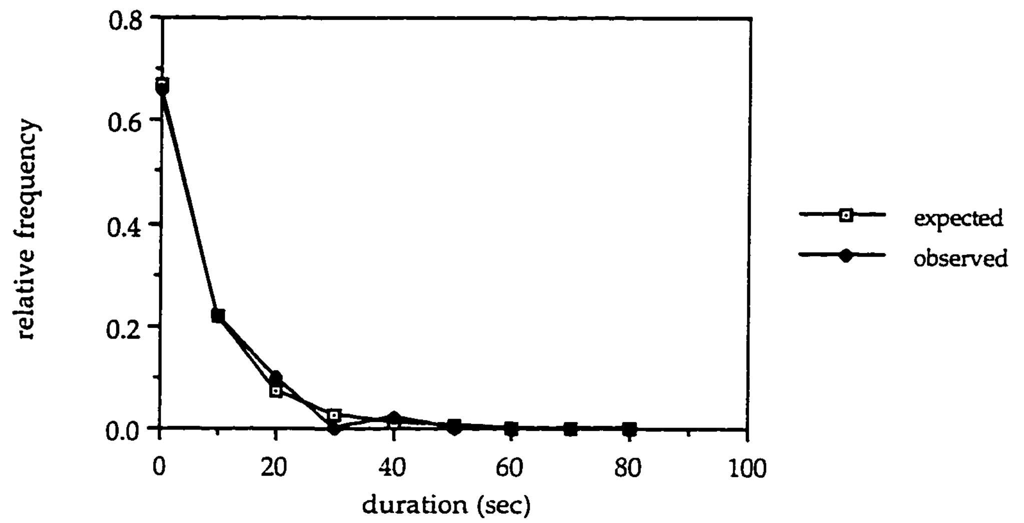
Total duration: 471"

part:				1					
segments:				1-22					
duration:				280.125" (.6)*					
supersection:		A						B	
segments:		1-11						12-22	
duration:		140" (.5)						140.125" (.5)	
section:	1		2			3			4
segments:	1-6		7-11			12-19			20-22
duration:	70" (.5)		70" (.5)			72.125" (.51)			68" (.49)
subsection:					a		b		
segments:					12-14		15-19		
duration:					43.75" (.61)		28.375" (.39)		
part:				2					
segments:				23-50					
duration:				190.875" (.4)					
section:			5			6			
segments:			23-36			37-50			
durations:			111.875" (.59)			79" (.41)			

*The quantities in parentheses indicate the proportion of the next higher level in the temporal structure that is occupied by the given duration.



a. ∂ = 0.09



b. ∂ = 0.11

Figure 3.4: Comparative histograms showing the distribution of (a) segment durations and (b) time-point intervals between the initiations of segments in *Evryali*

Table 3.3: Segment groups in *Evryali*

segment group	segment(s)	measures	seconds
1	1	1-4	0.000-8.000
2	2-6	5-35	8.000-70.000
3	7	36-40	70.000-78.125
4	8	40-46	78.125-91.250
5	9	46-60	91.250-119.875
6	10	60-64	119.875-128.000
7	11	65	128.000-140.000
8	12	66-69	140.000-147.125
9	13	69-74	147.125-158.000
10	14	75-87	158.000-183.750
11	15	87-92	183.750-192.500
12	16	92-95	192.500-198.250
13	17	95-97	198.250-202.625
14	18	97-100	202.625-208.250
15	19	100-102	208.250-212.125
16	20-22	102-136	212.125-280.125
17	23	136-146	280.125-302.000
18	24-34	147-179	302.000-366.750
19	35	179-188	366.750-385.125
20	36	188-189	385.125-392.000
21	37-46	190-206	392.000-426.000
22	47-48	207-212	426.000-451.000
23	49	212	451.000-461.000
24	50	213	461.000-471.000

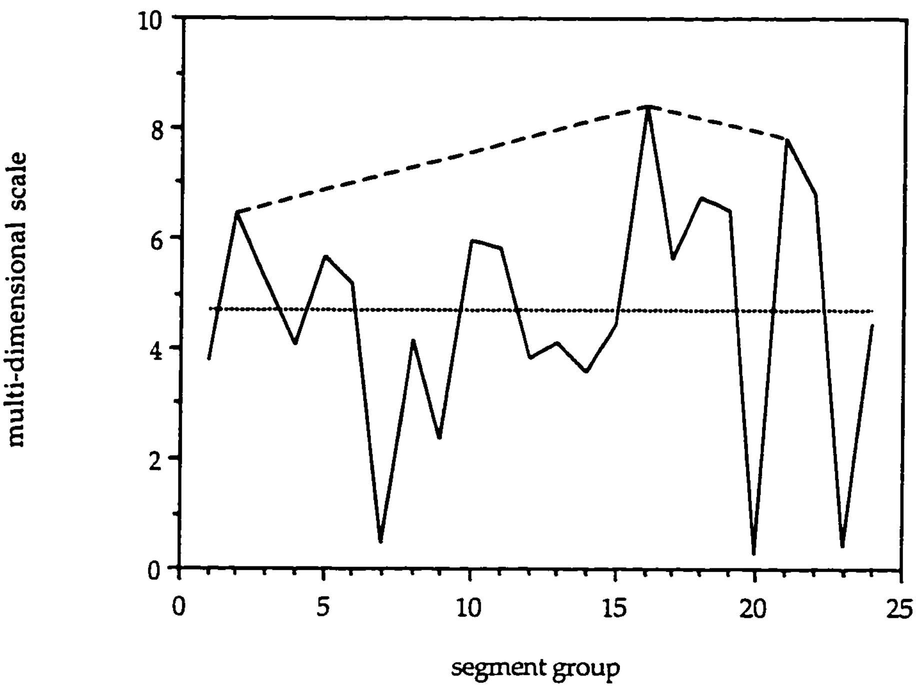


Figure 3.5: Graph of MDS for segment groups in *Evryali*

Table 3.4: Psets in *Evryali*

Model A: period = 20 < 1 1 1 1 2 2 1 2 1 1 1 2 2 1 1 >

{-39 -38 -37 -36 -35 -33 -31 -30 -28 -27 -26 -25 -23 -21 -20 -19 -18 -17 -16 -15 -13 -11 -10 -8 -7 -6 -5 -3 -1 0 1 2 3 4 5 7 9 10 12 13 14 15 17 19 20 21 22 23 24 25 27 29 30 32 33 34 35 37 39 40 41 42 43 44 45 47}

< 1 1 1 1 2 2 1 2 1 1 1 2 2 1 1 1 1 1 1 2 2 1 2 1 1 1 2 2 1 1 1 1 1 1 2 2 1 2 1 1 1 2 2 1 1 1 1 1 1 2 2 1 2 1 1 1 2 2 1 1 1 1 1 1 2 >

segment 1 (mm. 1-4, 0-8", TPS)

{0 2 3 4 5 7 9}

< 2 1 1 1 2 2 >

segment 2 (mm. 5-35, 8-70", ST)

{-38 -37 -36 -35 *-34* -31 -30 *-29* -27 -26 -25 -23 -21 -20 -19 -18 -17 -15 -13 -11 -10 -8 -7 -6 -5 -3 -1 0 1 2 3 4 5 7 9 10 12 13 14 15 17 19 20 22 23 24 25 27 *28* 29 30 32 33 35 37 39 40 42 43}

< 1 1 1 *1* *3* 1 *1* 2 1 1 2 2 1 1 1 1 1 2 2 1 2 1 1 1 2 2 1 1 1 1 1 2 2 1 2 1 1 1 2 2 1 1 1 1 1 2 *1* *1* 1 2 1 2 2 2 1 2 1 >

segment 3 (mm. 16-8, 31.75-36", TPS)

{32 34 37}

< 2 3 >

segment 4 (mm. 25-8, 49.75-54.5", TPS)

{4 12 13 14 15 17}

< 8 1 1 1 2 >

segment 5 (mm. 28-31, 55.25-62", TPS)

{*-14* *-12* *-9* -7 -6}

< *2* *3* *2* 1 >

but {-14 -12 -10 -9 -7 -6} = T1{-15 -13 -11 -10 -8 -7} (< 2 2 1 2 1 >)

segment 6 (mm. 33-5, 65.75-70", TPS)

{-35 -30 -25}

< 5 10 >

segment 7 (mm. 36-40, 70-78.125", ST)

{-37 -36 -33 *-29* -28 -27 -26 -23 -18 -17 -16 -15 -13 -8 -7 -6 -5 -3 1 2 3 4 5 *6* 7 13 14 15 17 22 24 27 32 34 37 42 44 47}

< 1 3 *4* *1* 1 1 3 5 1 1 1 2 5 1 1 1 2 4 1 1 1 1 *1* *1* 6 1 1 2 5 2 3 5 2 3 5 2 3 >

Table 3.4: Pets in *Evryali*, cont.

(Model A, cont.)

segment 8 (mm. 40-6, 78.125-91.25", TPS)

{-7 -5 -3 1 3 *6* 10}

<2 2 2 2 2 *3* *4*>

Model B: period = 11 <2 1 1 2 2 1 2>

{-38 -36 -35 -34 -32 -30 -29 -27 -25 -24 -23 -21 -19 -18 -16 -14 -13 -12 -10 -8 -7 -5 -3 -2 -1 1 3 4 6 8 9 10 12 14 15 17 19 20 21 23 25 26 28 30 31 32 34 36 37 39 41 42 43 45 47 48}

<2 1 1 2 2 1 2 2 1 1 2 2 1 2 2 1 1 2 2 1 2 2 1 1 2 2 1 2 2 1 1 2 2 1 2 2 1 1 2 2 1 2 2 1 1 2 2 1 2 2 1 1 2 2 1>

segment 8 (mm. 40-6, 78.125-91.25", TPS)

{-7 -5 -3 1 3 6 10}

<2 2 2 2 2 3 4>

segment 9 (mm. 46-60, 91.25-119.875", A)

{-38 -36 -35 -34 -32 *-31* -30 -29 -27 -25 -24 -23 -21 -19 -18 -16 -14 -13 -12 -10 -8 -7 -5 -3 -2 -1 1 3 4 *5* 6 8 9 10 12 *13* 14 15 17 *18* 19 20 21 23 25 26 28 30 31 32 34 36 37 39 41 42 43 45 47 48}

<2 1 1 2 *1 1* 1 2 2 1 1 2 2 1 2 2 1 1 2 2 1 2 2 1 1 2 2 1 *1 1* 2 1 1 2 *1 1* 1 2 *1 1* 1 1 2 2 1 2 2 1 1 2 2 1 2 2 1 1 2 2 1>

segment 10 (mm. 60-4, 119.875-128", TPS)

{*11 13* 14 *16 18* 19 20 *27*}

<*2 1 2 2 1 1 7*>

but {11 13 14 16 18 19 20 22 24 25 27} = T-1{12 14 15 17 19 20 21 23 25 26 28} (<2 1 2 2 1 1 2 2 1 2>)

Model C: period = 13 <2 3 2 1 1 2 2>

{-38 -36 -33 -31 -30 -29 -27 -25 -23 -20 -18 -17 -16 -14 -12 -10 -7 -5 -4 -3 -1 1 3 6 8 9 10 12 14 16 19 21 22 23 25 27 29 32 34 35 36 38 40 42 45 47 48}

<2 3 2 1 1 2 2 2 3 2 1 1 2 2 2 3 2 1 1 2 2 2 3 2 1 1 2 2 2 3 2 1 1 2 2 2 3 2 1 1 2 2 2 3 2 1>

segment 12 (mm. 92-5, 192.5-198.25", A)

{-38 *-37* -36 -33 -31 -30 -29 -27 -25 -23 -20 -18 -17 -16 -14 -12 -10 -7 -5 -4 -3 -1 1 3 6 8 9 10 12 14 16 19 21 22 23 25 27 29 32 34 35 36 *37* 40 42 45 48}

<*1 1* 3 2 1 1 2 2 2 3 2 1 1 2 2 2 3 2 1 1 2 2 2 3 2 1 1 2 2 2 3 2 1 1 2 2 2 3 2 1 1 *1 3* 2 3 3>

Table 3.4: Psets in *Evryali*, cont.

A ∪ B ∪ C:

{-39 -38 -37 -36 -35 -34 -33 -32 -31 -30 -29 -28 -27 -26 -25 -24 -23 -21 -20 -19 -18 -17 -16 -15 -14 -13 -12 -11 -10 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 12 13 14 15 16 17 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 47 48}

<1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1>

segments 1, 2, 3, 4, 5 (except for pitch -9), 6, 7, 8, 9, 10 (except for pitches 11 and 18), 12 (see models A, B, and C)

segment 13 (mm. 69-74, 147.125-158", TPS)

{30 33 37 39 41 *46*}

<3 4 2 2 *5*>

segment 15 (mm. 87-92, 183.75-192.5", TPS)

{0 1 4 5 20 23 26 27}

<1 3 1 15 3 3 1>

segment 17 (mm. 95-7, 198.25-202.625", TPS)

{-4 -3 -1 0 2 4 5 6}

<1 2 1 2 2 1 1>

segment 19 (mm. 100-2, 208.25-212.125", TPS)

{-1 0 2 3 5 7 8 9} = T3(segment 17)

<1 2 1 2 2 1 1>

segment 21 (mm. 107-9, 222-228", TPS)

{-31 -26 -23 -20}

<5 3 3>

segment 22 (mm. 113-4, 235.75-237.875", TPS)

{37 38 40 42 43 44 *46* 48}

<1 2 2 1 1 2 2>

segment 24 (mm. 147-79, 302-366.75", ST)

{-38 -31 -30 -29 -26 -25 -23 -20 -19 -16 -10 *-9* -7 -6 -5 -1 3 4 7 9 *11* 13 14 15 20 22 24 25 27 29 30 33 39 41 43 *46* 47 48}

<7 1 1 3 1 2 3 1 3 6 *1* 2 1 1 4 4 1 3 2 2 2 1 1 5 2 2 3 2 2 1 3 6 2 2 *3 1* 1>

Except for pitches -29, -9, 11, 46 and 48, this pset is included in model A.

Table 3.4: Psets in *Evryali*, cont.

(A ∪ B ∪ C, cont.)

segments 37-46 (mm. 190-206, 392-426", TPS)

{-39 -37 -34 -32 -31 -26 -24 -23 -21 -18 -17 -14 -12 -10 -7 -6 -5 -3 -2 1 2 3 4 5 6 8 9 10 12 13 19 24 25 26 27 31 32 34 39 40 48}

<2 3 2 1 5 2 1 2 3 1 3 2 2 3 1 1 2 1 3 1 1 1 1 1 2 1 1 2 1 6 5 1 1 1 4 1 2 5 1 8>

Model D: the set of all 88 pitches available on the standard piano keyboard

segment 14 (mm. 75-87, 158-183.75", A)

segment 16 (mm. 92-5, 192.5-198.25", A)

segment 18 (mm. 97-100, 202.625-208.25", A)

segment 20 (mm. 102-136, 212.125-280.125", A)

segment 23 (mm. 136-46, 280.125-302", ST)

segments 25-34 (mm. 148-71, 305.625-352", A)

segment 35 (mm. 179-88, 366.75-385.125", A)

segments 47-8 (mm. 207-12, 426-51", A)

segment 50 (m. 213, 461-471", A)

All of the segments listed contain either model D complete or subsets of it

Table 3.5: Segments in *Mists*

segment	measures	begin time* (sec)	duration (sec)	configuration type	density	intensity
1	1-7	0.000	34.375	RW	5.64	*f*
2	7-9	34.375	8.750	RW	6.74	*p*
3	9-11	43.125	9.063	RW	13.79	*fff*
4	11-13	52.187	12.813	R	0.00	-
5	14-16	65.000	11.250	A	3.73	*f*
6	16	76.250	0.625	R	0.00	-
7	16-18	76.875	8.500	RW	11.06	*fff*
8	18	85.375	3.375	R	0.00	-
9	18-22	88.750	17.500	RW	7.31	*p-ff* (c)†
10	22	106.250	0.625	R	0.00	-
11	22-24	106.875	9.375	A	4.48	*fff*
12	24	116.250	0.625	R	0.00	-
13	24-26	116.875	13.125	RW	7.31	*pp-fff* (c)
14	27-28	130.000	7.187	RW	13.77	*fff*
15	28	137.187	0.313	R	0.00	-
16	28-30	137.500	12.500	A	3.28	*ff*
17	31-32	150.000	5.833	RW	15.60	*pp*
18	32-33	155.833	3.333	RW	18.90	*fff*
19	33-34	159.166	2.917	RW	18.85	*p*
20	34-35	162.083	2.917	RW	18.85	*p*
21	35-36	165.000	2.917	RW	17.83	*p*
22	36-38	167.917	6.666	A	12.45	*fff*
23	38	174.583	2.084	R	0.00	-
24	39-40	176.667	8.000	RW	2.00	*fff*
25	41	184.667	4.062	ST	2.22	*p*
26	41-42	188.729	5.313	ST	4.71	*fff*
27	42-43	194.042	5.625	ST	2.31	*pp*
28	44-45	199.667	8.750	ST	7.77	*pp*
29	45-46	208.417	5.156	ST	11.25	*pp-fff* (c)
30	46-48	213.573	10.156	ST	9.06	*pp-fff* (c)
31	48-49	223.729	4.063	ST	6.89	*pp*
32	49-50	227.792	6.875	ST	4.22	*pp*
33	51-52	234.667	10.000	ST	5.80	*pp-fff* (c)
34	53	244.667	5.000	ST	2.80	*pp*
35	54	249.667	2.344	ST	9.39	*fff*
36	54-55	252.011	3.906	ST	8.45	*fff*
37	55-56	255.917	4.531	ST	7.28	*ppp*
38	56	260.448	2.656	ST	6.40	*fff*
39	56-57	263.104	1.719	ST	4.65	*p*
40	57	264.823	4.844	ST	7.02	*fff*
41	58-59	269.667	6.250	ST	9.60	*pp-fff* (c)
42	59	275.917	3.750	ST	10.67	*fff*

* refers to the location of the segments in the graphic transcription (Figure 3.7)
† c = continuous (gradual) changes in intensity; d = discrete changes in intensity
configuration types: RW = random walk; ST = stochastic; A = arborescence; R = rest

Table 3.5: Segments in *Mists*, cont.

segment	measures	begin time (sec)	duration (sec)	configuration type	density	intensity
43	60	279.667	5.000	ST	3.60	*fff*
44	61	284.667	3.437	ST	8.73	*pp*
45	61-63	288.104	7.500	ST	7.47	*fff*
46	63-64	295.604	6.407	ST	2.97	*p*
47	64-65	302.011	7.656	ST	8.36	*fff*
48	66	309.667	5.000	ST	14.00	*pp*
49	67	314.667	2.813	ST	2.13	*fff*
50	67-68	317.480	4.375	ST	11.66	*ppp*
51	68-71	321.855	13.437	ST	1.19	*fff*
52	71	335.292	2.500	ST	9.60	*ppp*
53	71-72	337.792	5.000	ST	1.00	*fff*
54	72-73	342.792	3.125	ST	1.28	*ppp*
55	73-74	345.917	8.750	ST	3.20	*ppp-fff* (c)
56	75	354.667	3.437	ST	0.87	*ppp*
57	75-76	358.104	5.313	ST	0.94	*pp-fff* (d)
58	76-77	363.417	5.156	ST	12.99	*fff*
59	77-78	368.573	4.531	ST	3.09	*fff*
60	78-79	373.104	6.563	ST	1.68	*fff*
61	80-83	379.667	18.437	A	13.29	*ppp-fff* (c)
62	83	398.104	0.313	R	0.00	-
63	83-84	398.417	4.687	ST	10.03	*pp*
64	84-85	403.104	3.125	ST	5.12	*ff*
65	85	406.229	3.438	ST	9.89	*ppp*
66	86-87	409.667	5.312	ST	5.27	*fff*
67	87-88	414.979	7.344	ST	3.54	*fff*
68	88-89	422.323	4.219	ST	1.42	*fff*
69	89-90	426.542	5.156	ST	1.55	*pp*
70	90-92	431.698	12.969	ST	2.93	*fff*
71	93	444.667	1.250	R	0.00	-
72	93-94	445.917	6.562	A	12.34	*ppp*
73	94	452.479	0.313	R	0.00	-
74	94-95	452.792	3.125	ST	1.60	*fff*
75	95-96	455.917	4.062	R	0.00	-
76	96	459.979	0.313	ST	19.17	*fff*
77	96-97	460.292	6.250	ST	7.20	*ppp*
78	97-99	466.542	11.250	ST	2.76	*fff*
79	99-100	477.792	2.813	ST	3.91	*fff*
80	100-101	480.605	7.813	ST	1.41	*pp-fff* (c, d)
81	101-102	488.418	3.281	ST	7.62	*ppp*
82	102	491.699	2.031	ST	0.98	*ff*
83	102-104	493.730	8.906	ST	2.47	*pp-fff* (c)
84	104-105	502.636	7.031	ST	2.56	*fff*
85	106-107	509.667	10.000	ST	4.10	*pp-fff* (c)
86	108-109	519.667	6.875	ST	4.51	*ppp*
87	109-110	526.542	6.562	A	13.87	*fff*
88	110	533.104	1.563	R	0.00	-
89	111	534.667	2.500	ST	4.80	*ppp*

Table 3.5: Segments in *Mists*, cont.

segment	measures	begin time (sec)	duration (sec)	configuration type	density	intensity
90	111	537.167	1.250	ST	4.80	*ppp*
91	111-112	538.417	5.781	ST	5.36	*ppp-fff* (c)
92	112-114	544.198	7.656	ST	6.40	*ppp-fff* (c)
93	114-115	551.854	3.438	R	0.00	-
94	115-116	555.292	7.500	A	12.93	*fff*
95	116	562.792	1.875	R	0.00	-
96	117-118	564.667	6.250	ST	4.16	*fff*
97	118	570.917	2.187	R	0.00	-
98	118	573.104	1.563	ST	4.48	*fff*
99	119	574.667	5.000	ST	1.40	*ppp*
100	120-121	579.667	10.000	ST	7.00	*ppp-fff* (c)
101	122-126	589.667	22.500	RW	14.89	*ppp-fff* (c)
102	126-127	612.167	4.375	RW	16.69	*ppp*
103	127-129	616.542	10.625	R	0.00	-
104	129-130	627.167	7.500	A	12.13	*mf-fff* (c)
105	131-132	634.667	10.000	R	0.00	-
106	133-134	644.667	7.187	A	13.91	*fff*
107	134	651.854	2.813	R	0.00	-

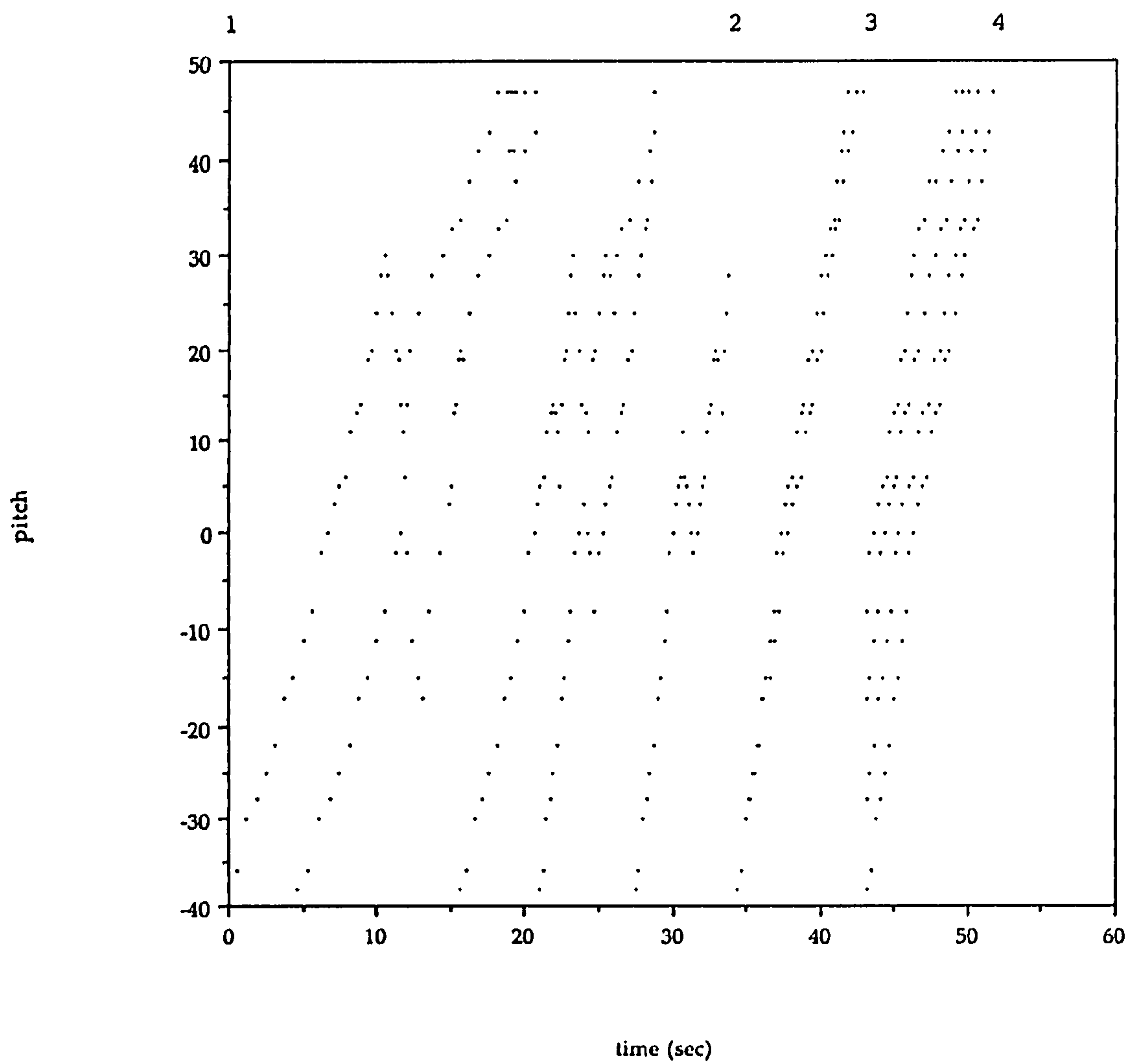


Figure 3.6: Graphic transcription of *Mists*

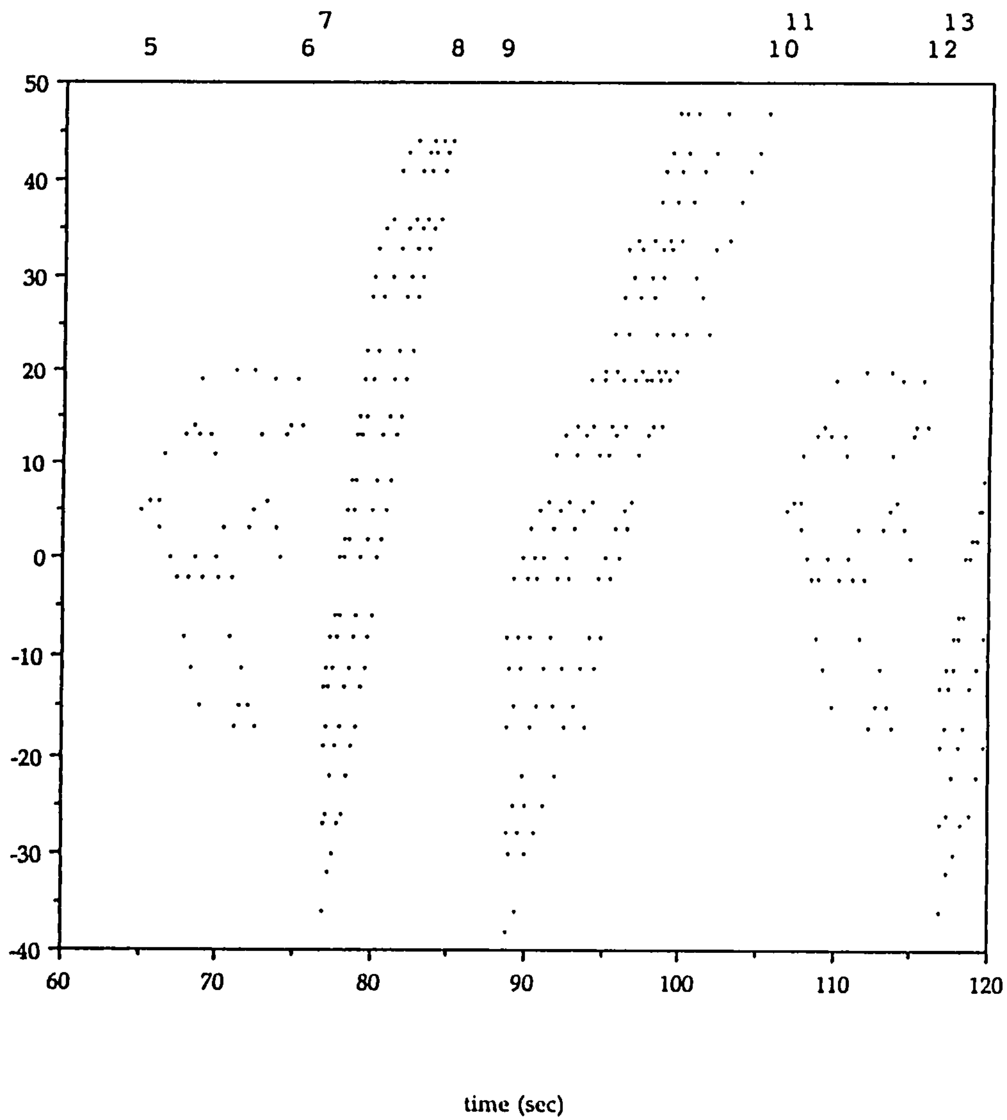


Figure 3.6: Graphic transcription of *Mists*, cont.

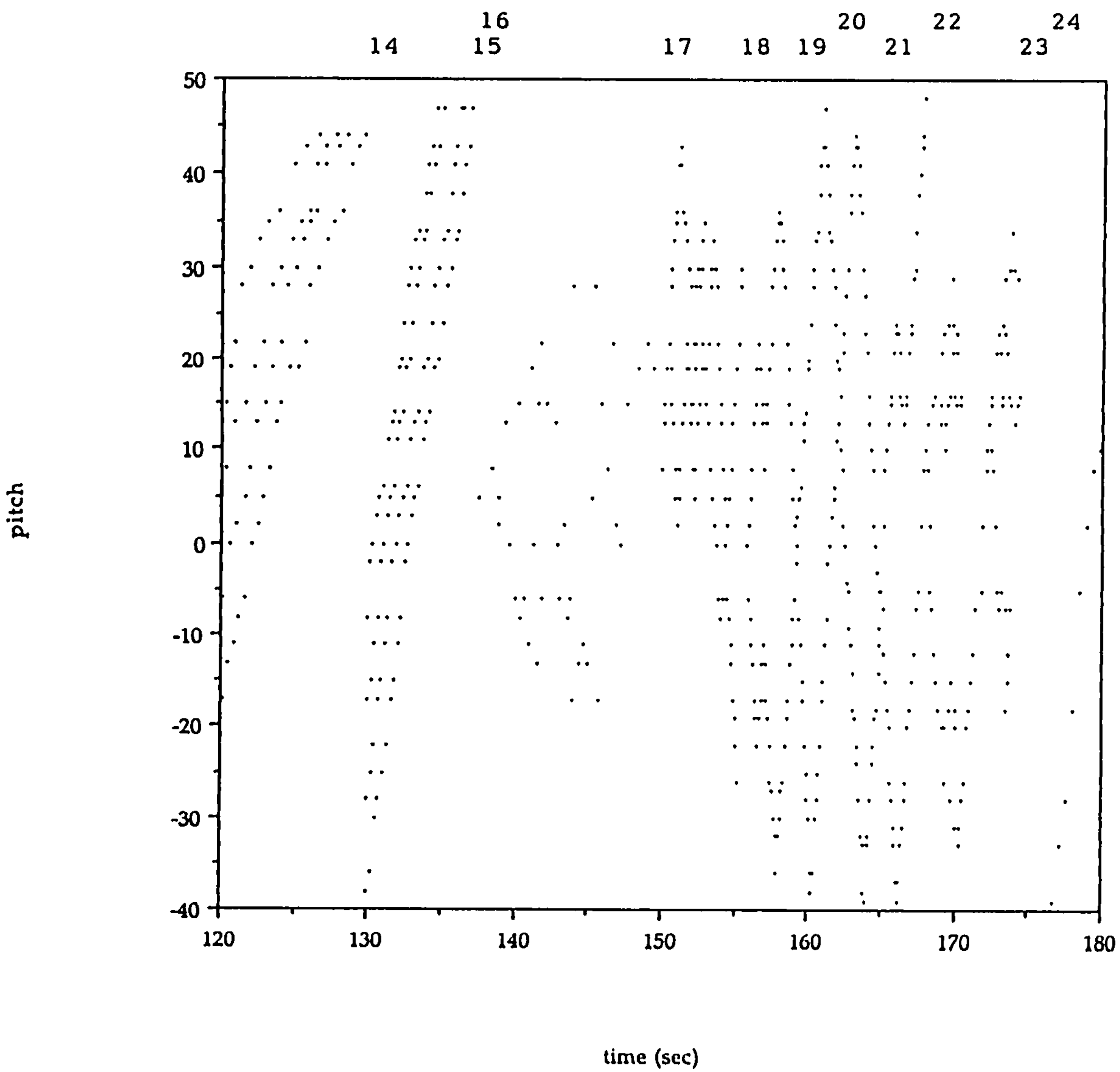


Figure 3.6: Graphic transcription of *Mists*, cont.

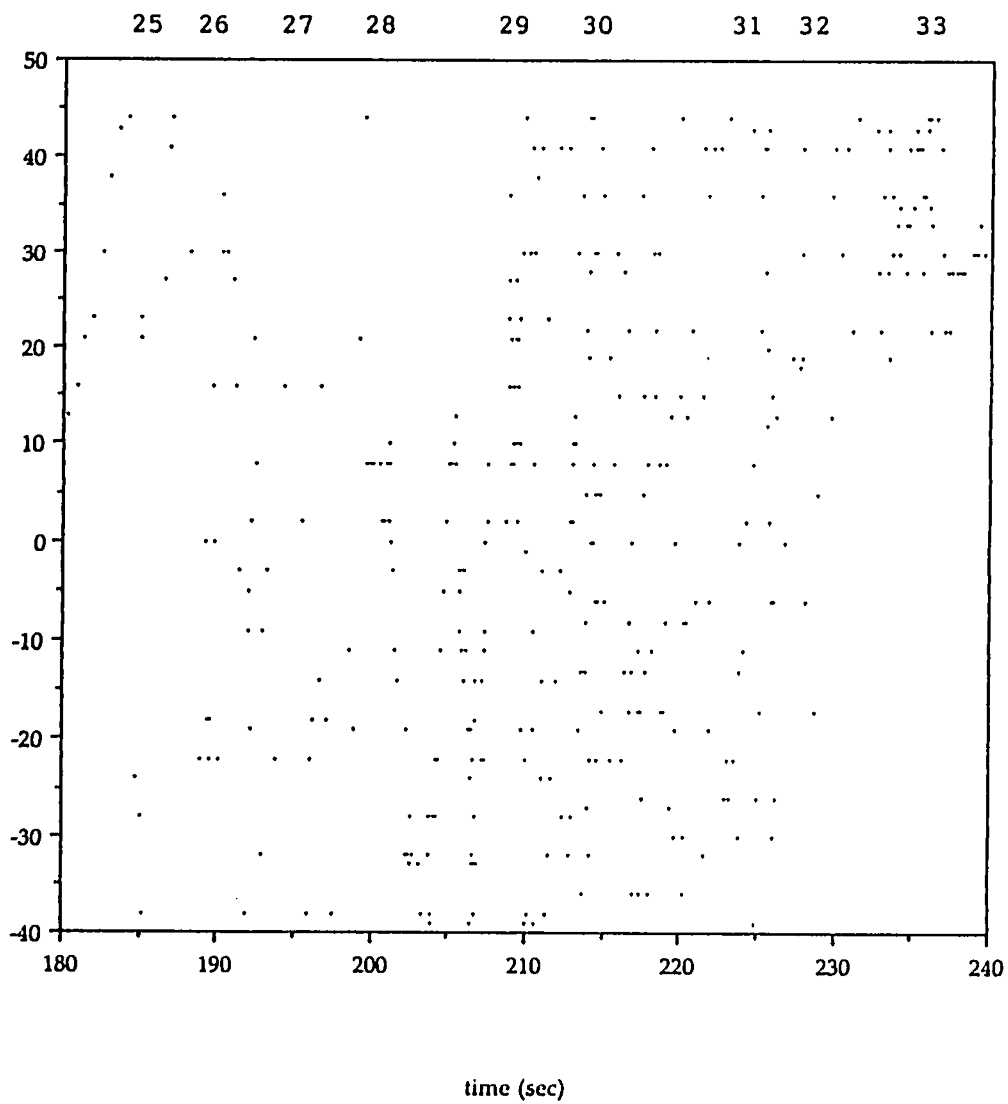


Figure 3.6: Graphic transcription of *Mists*, cont.

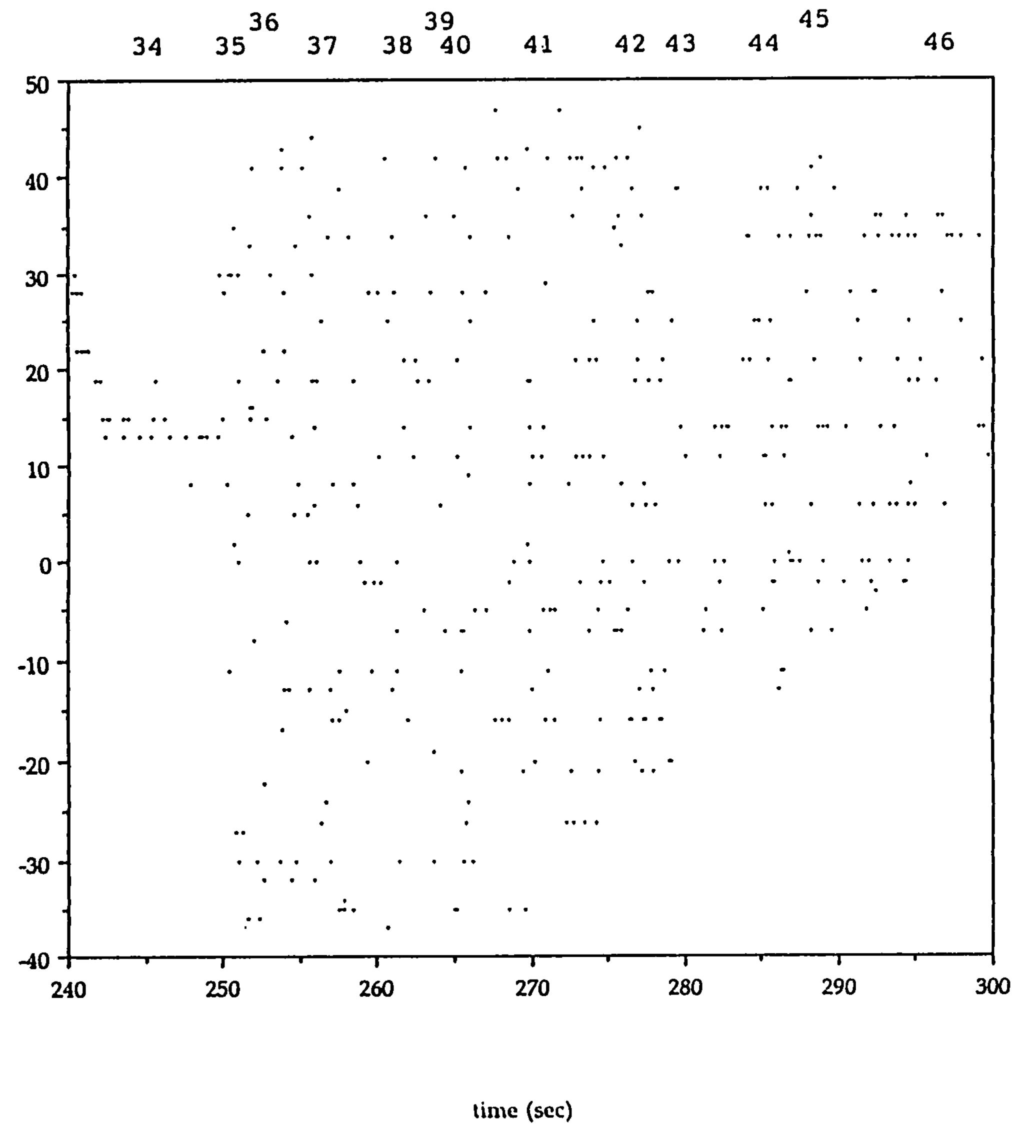


Figure 3.6: Graphic transcription of *Mists*, cont.

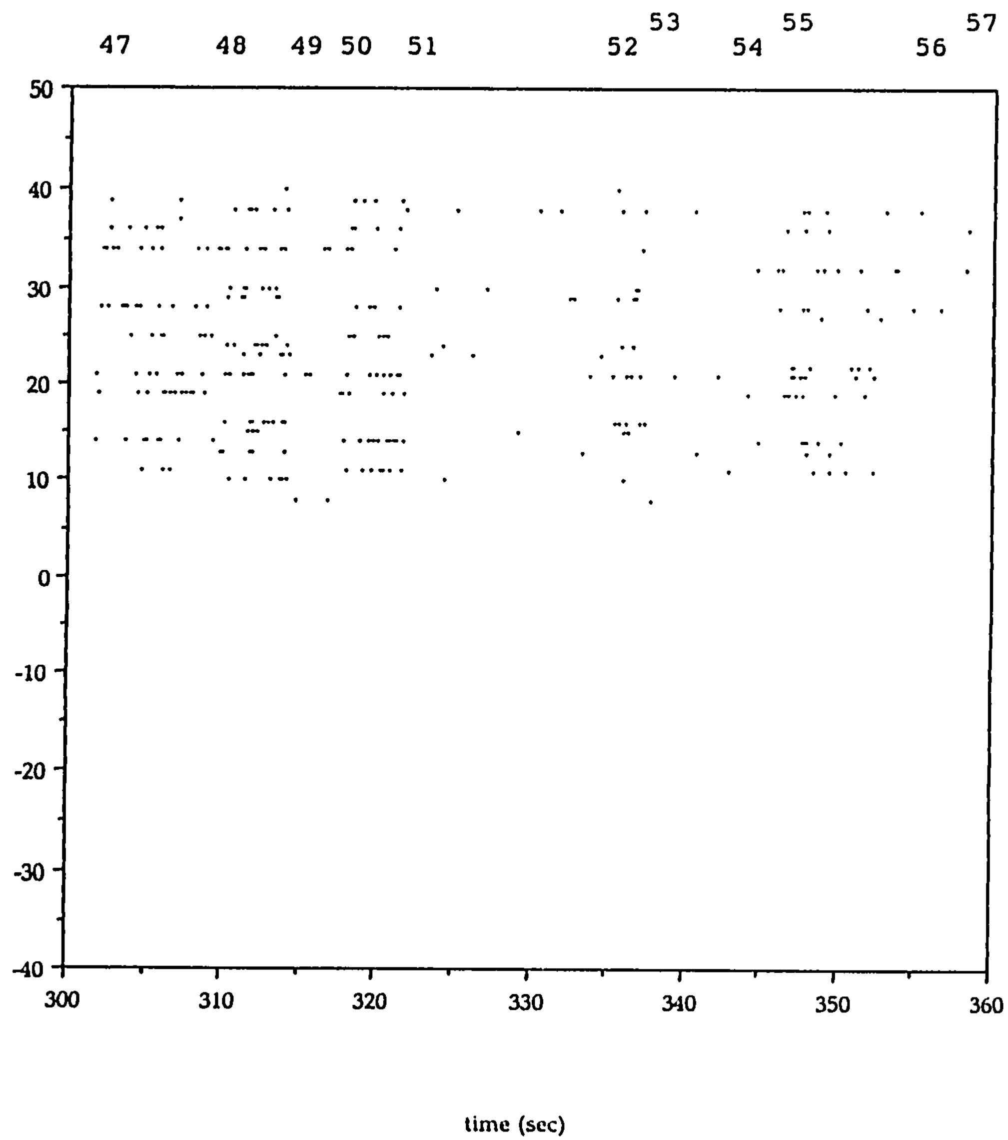


Figure 3.6: Graphic transcription of *Mists*, cont.

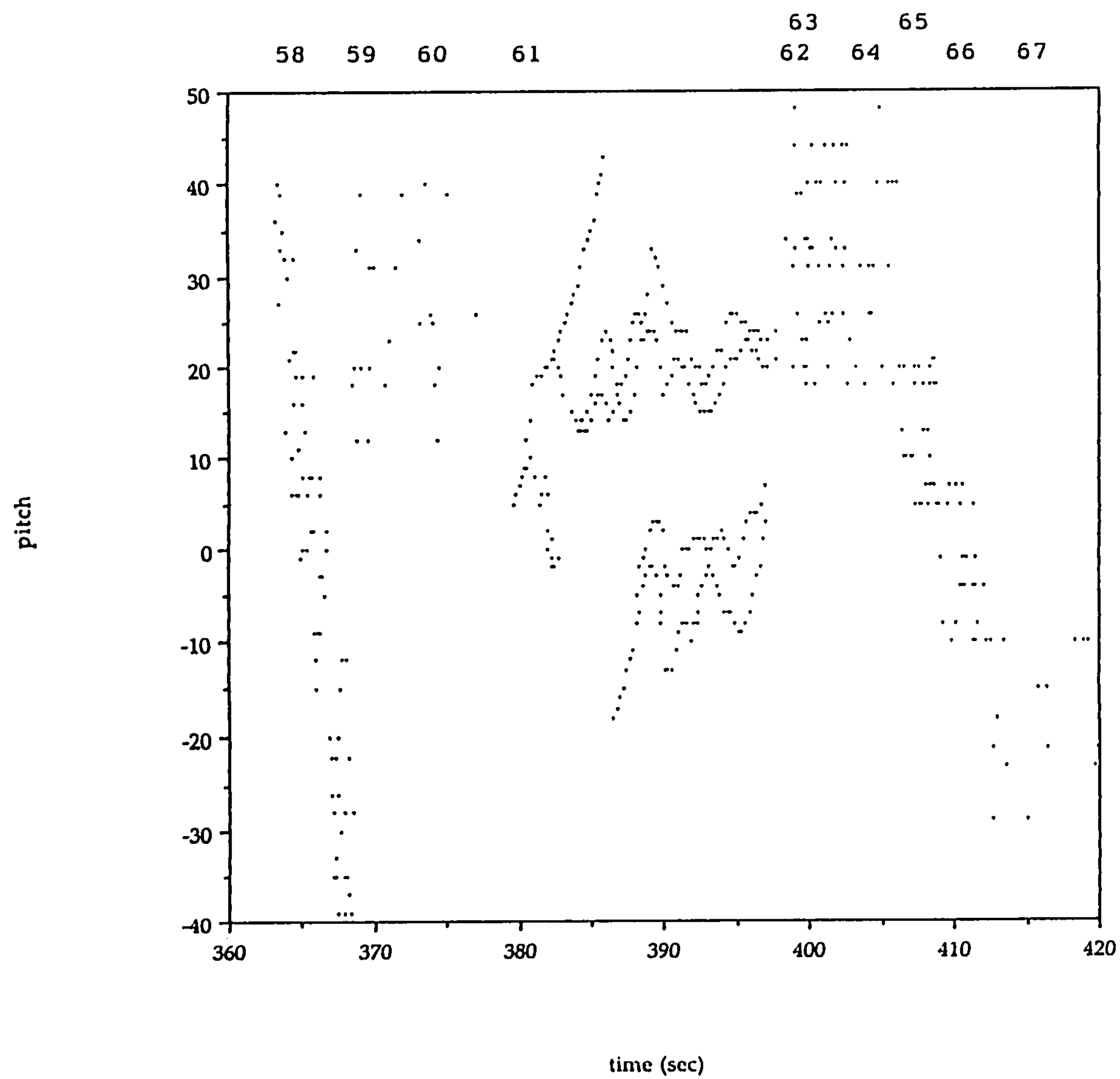


Figure 3.6: Graphic transcription of *Mists*, cont.

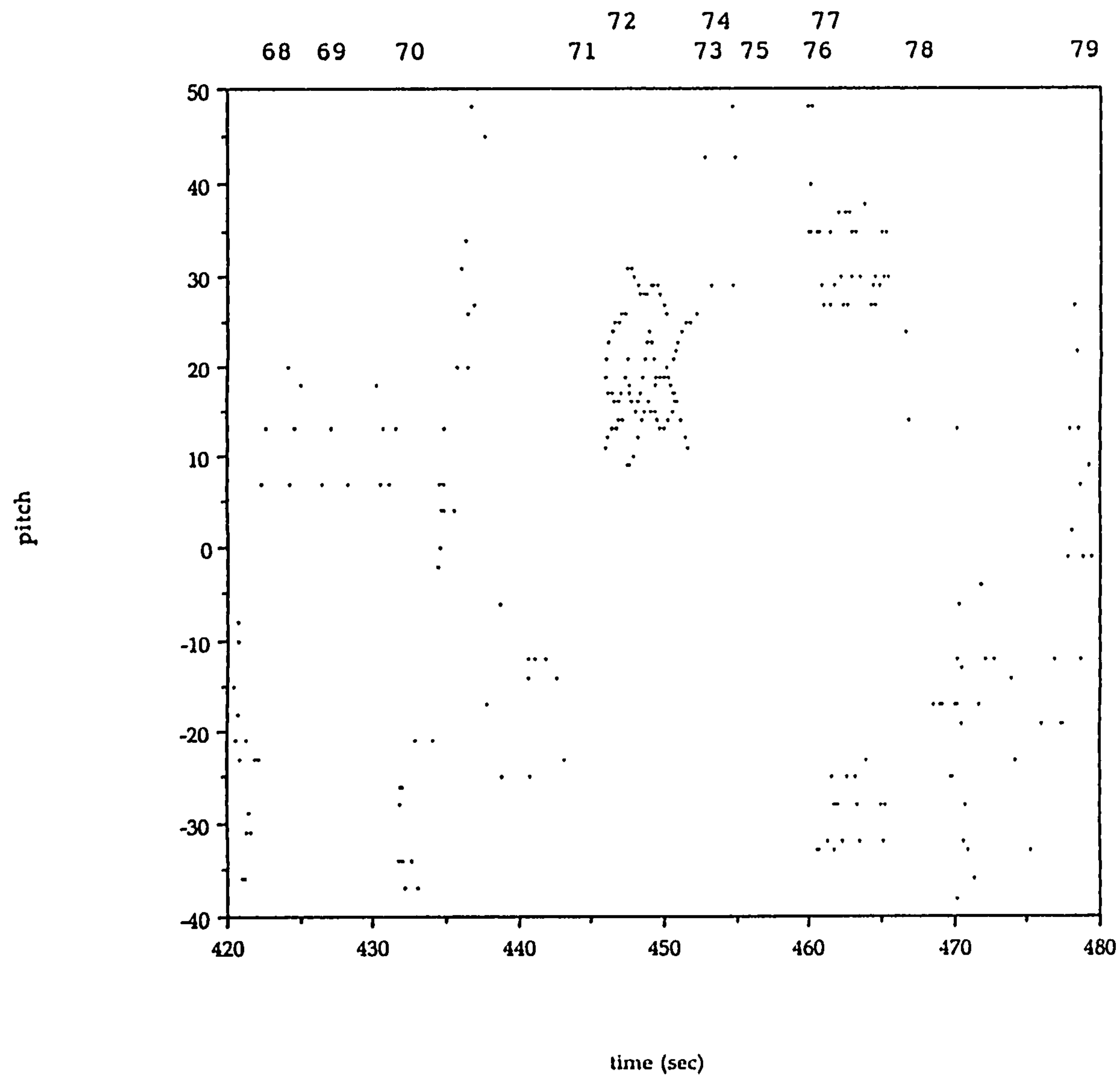


Figure 3.6: Graphic transcription of *Mists*, cont.

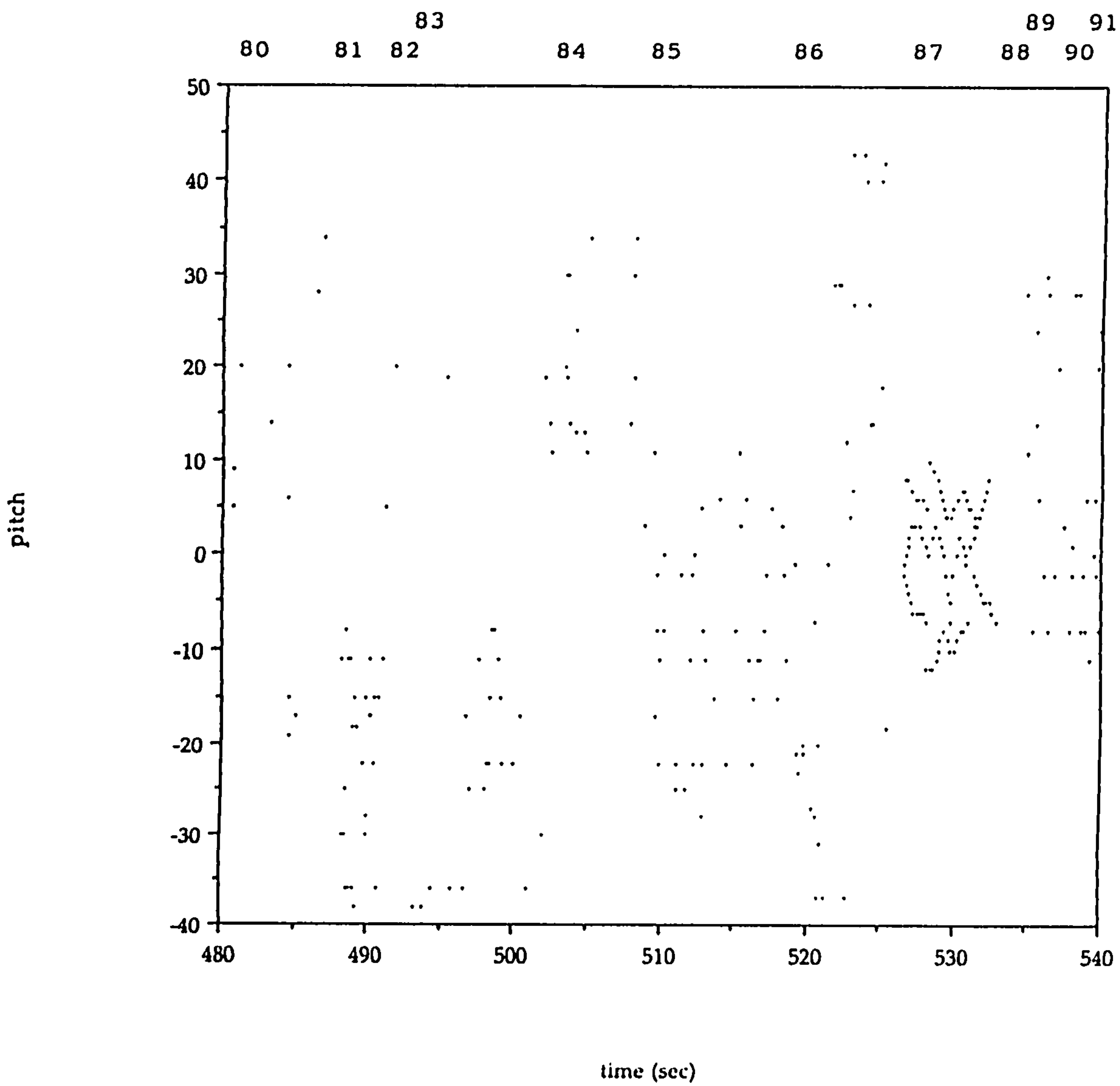


Figure 3.6: Graphic transcription of *Mists*, cont.

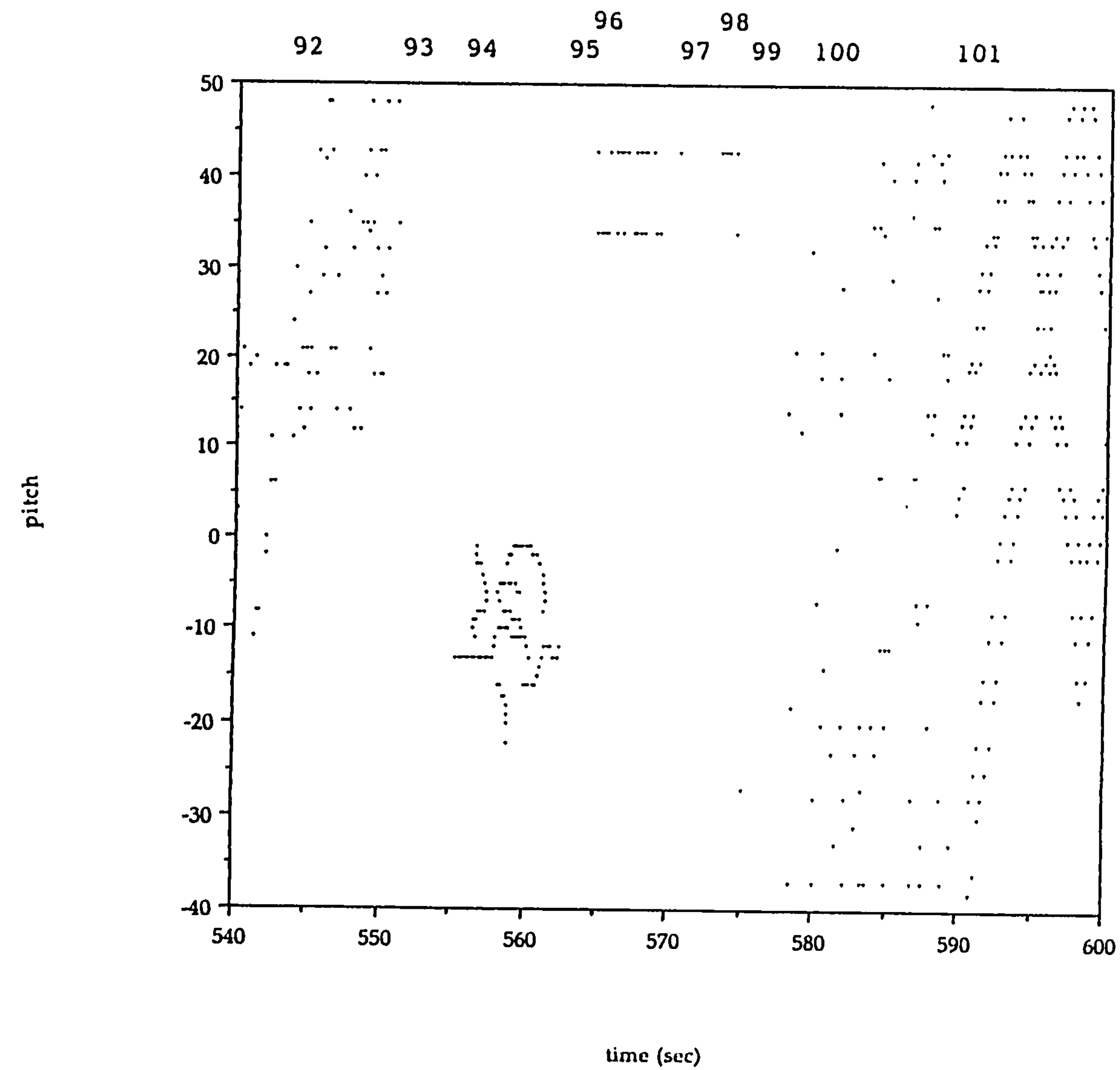


Figure 3.6: Graphic transcription of *Mists*, cont.

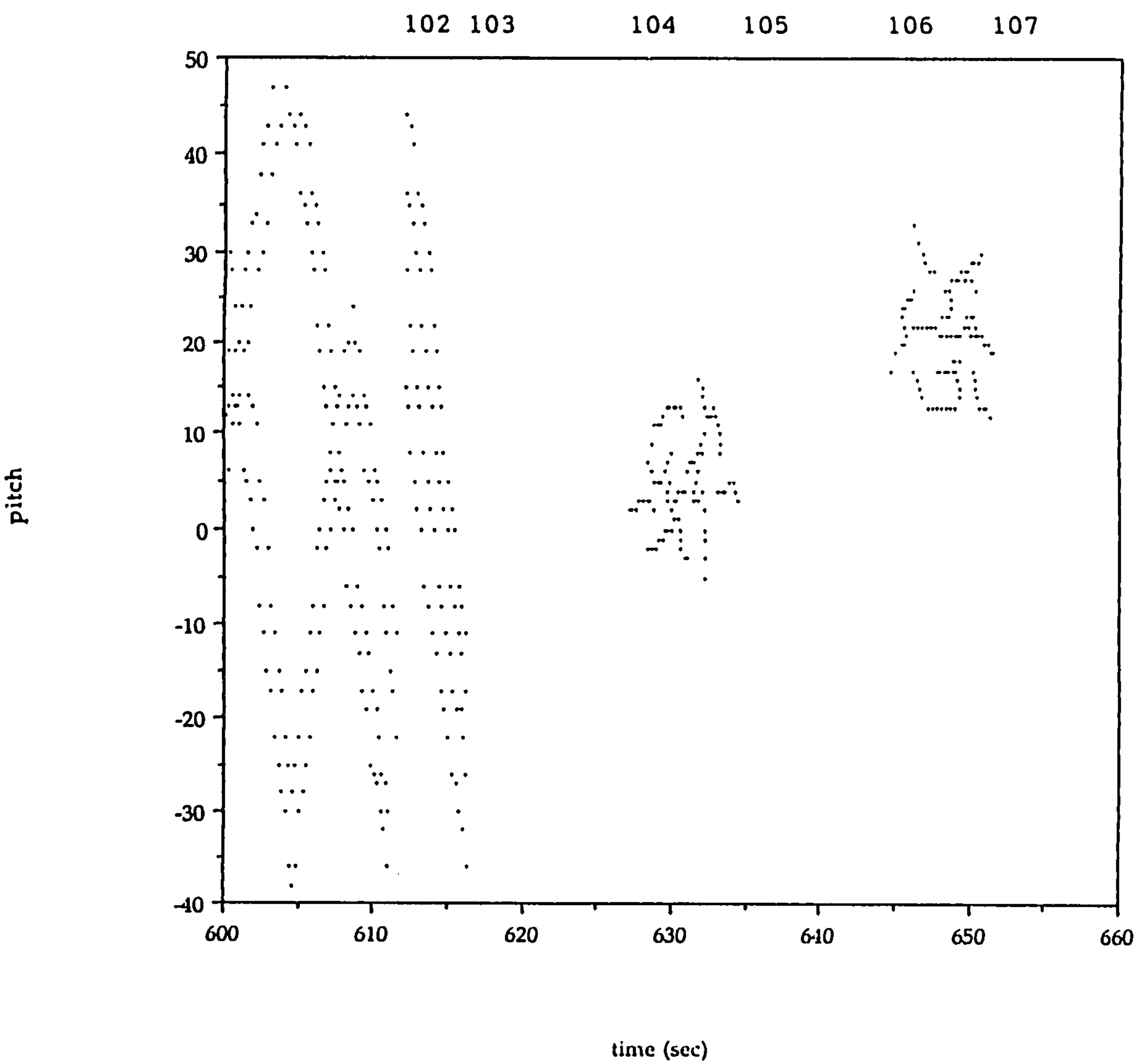


Figure 3.6: Graphic transcription of *Mists*, cont.

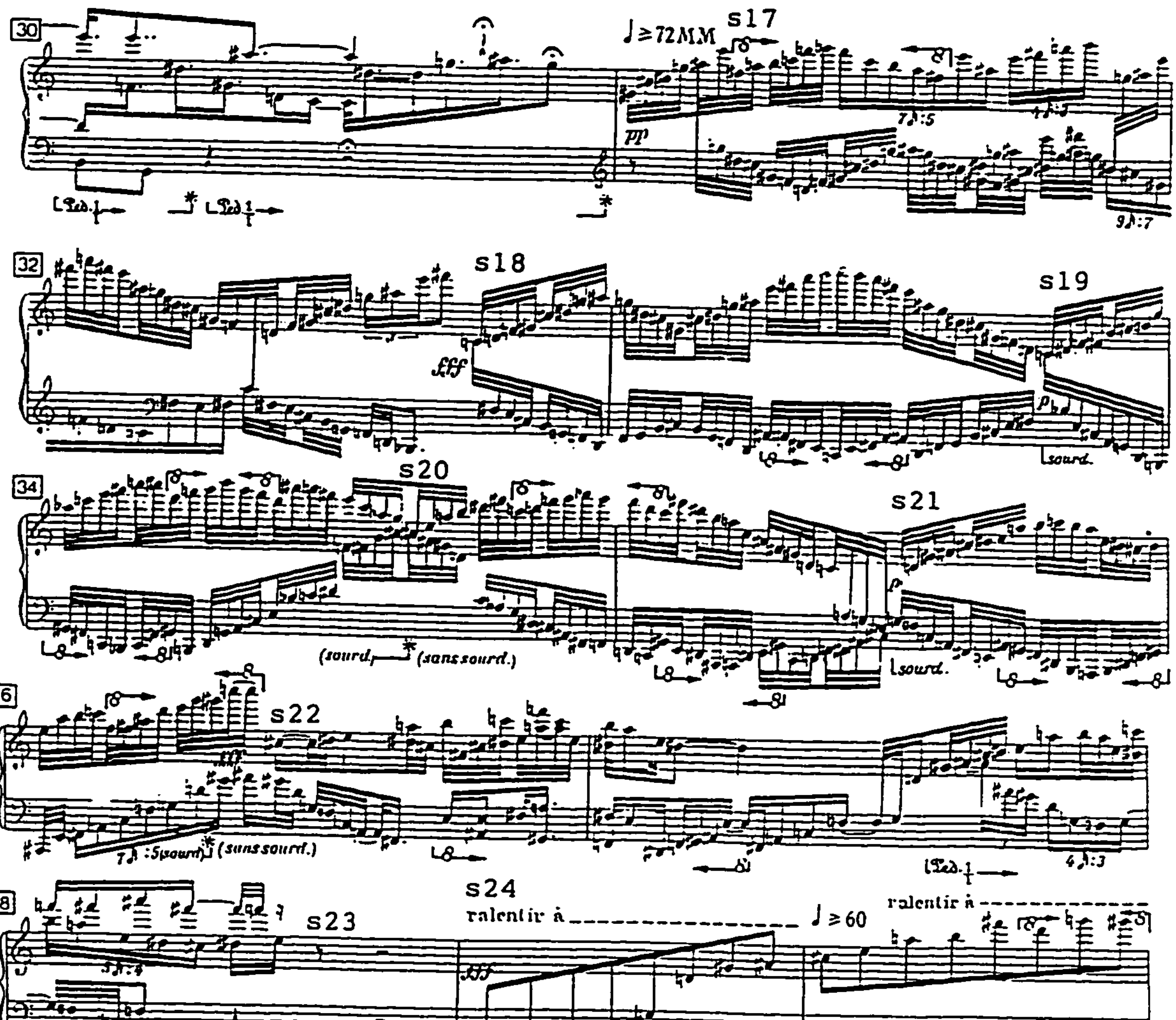


Figure 3.7: *Mists*, mm. 30-40
Iannis Xenakis, *Mists* (Paris: Editions Salabert, 1981). Used with permission.

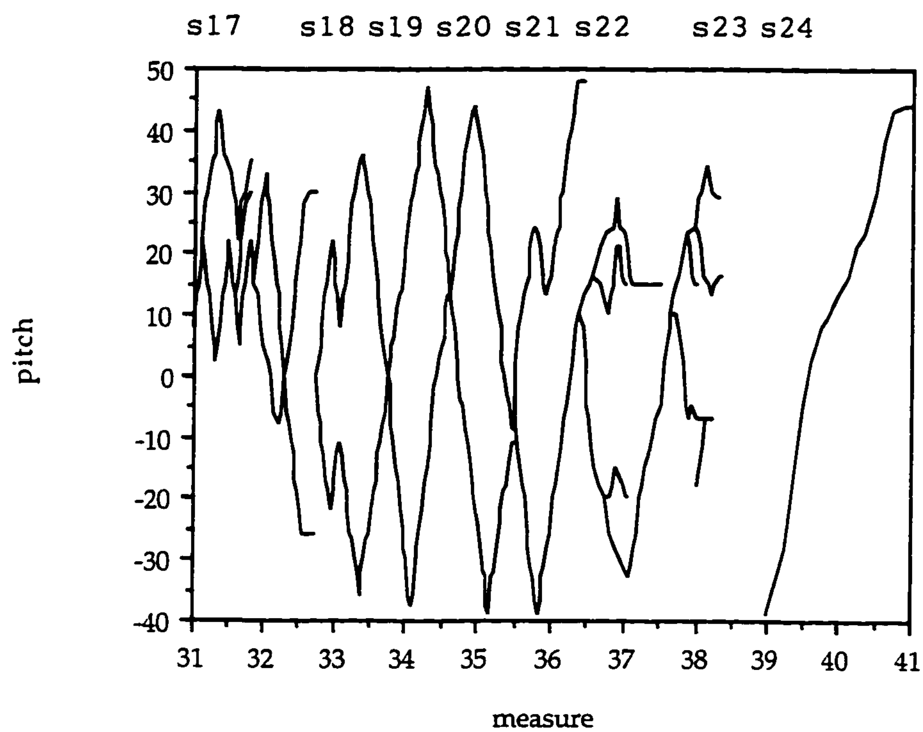


Figure 3.8: Graphic transcription of *Mists*, segments 17-24

Table 3.6: Summary of Temporal Structure in *Mists*

Total duration: 654.667"

part:	1			
segments:	1-24			
duration:	184.667" (.28)*			
section:	1		2	
segments:	1-16		17-24	
duration:	150" (.81)		34.667" (.19)	
subsection:	a	b		
segments:	1-4	5-16		
duration:	65" (.43)	85" (.57)		

part:	2			
segments:	25-100			
duration:	405" (.62)			
section:	3		4	
segments:	25-60		61-100	
duration:	195" (.48)		210" (.52)	
subsection:	a		b	
segments:	61-86		87-100	
duration:	146.875" (.7)		63.125" (.3)	
sub-subsection:	aa	ab	ba	bb
segments:	61-71	72-86	87-93	94-100
duration:	66.25" (.45)	80.625" (.55)	28.437" (.45)	34.688" (.55)

---

*The quantities in parentheses indicate the proportion of the next higher level in the temporal structure that is occupied by the given duration.

Table 3.6: Summary of Temporal Structure in *Mists*, cont.

part:		3	
segments:		101-107	
duration:		65" (.1)	
section:	5		6
segments:	101-103		104-107
duration:	37.5" (.58)		27.5" (.42)

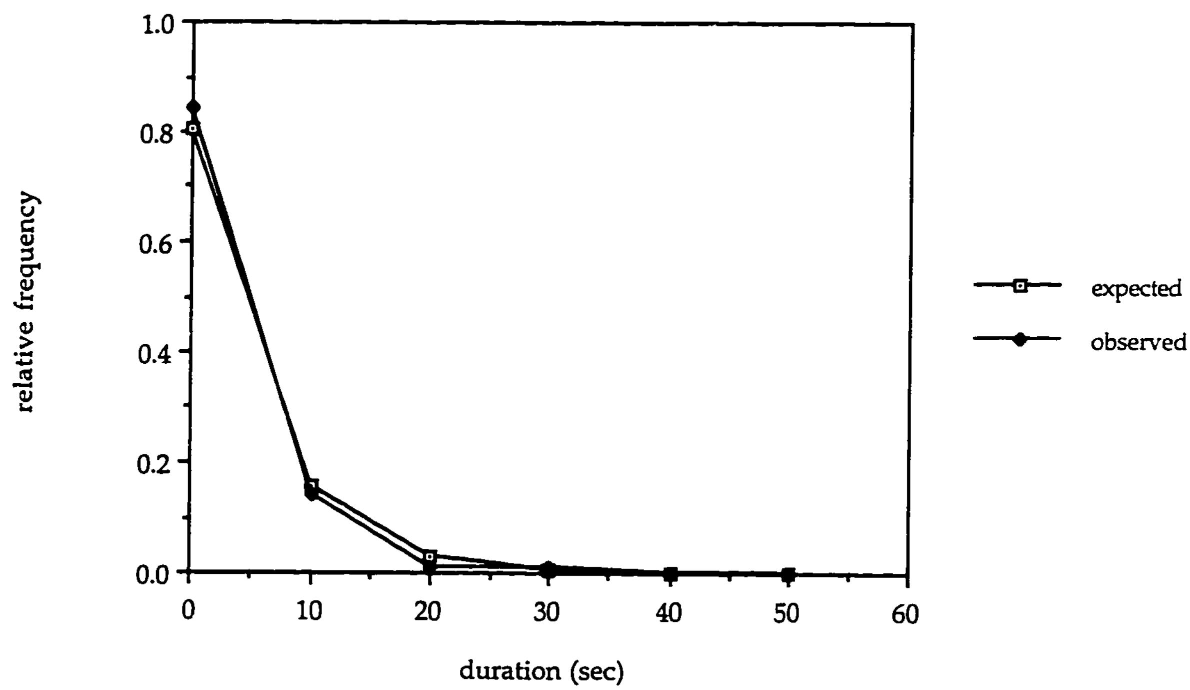


Figure 3.9: Comparative histogram showing the distribution of segment durations in *Mists* with the exponential distribution, ∂ = 0.163

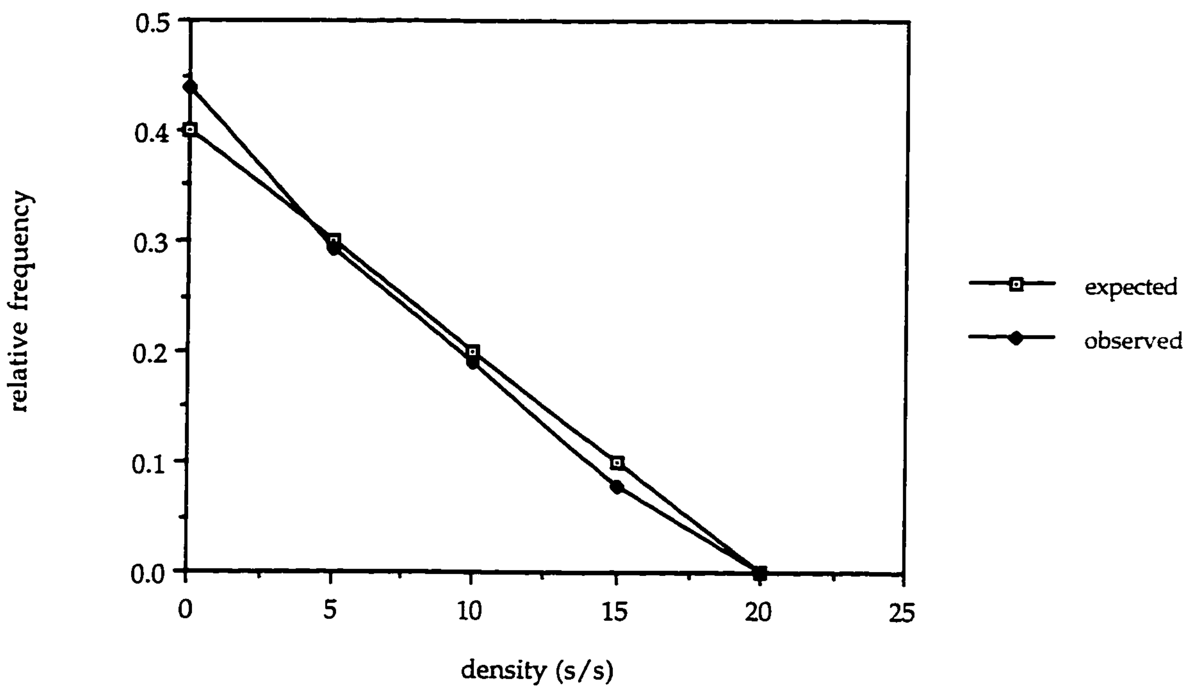


Figure 3.10: Comparative histogram showing the distribution of densities in the sounding segments of *Mists* versus the linear distribution

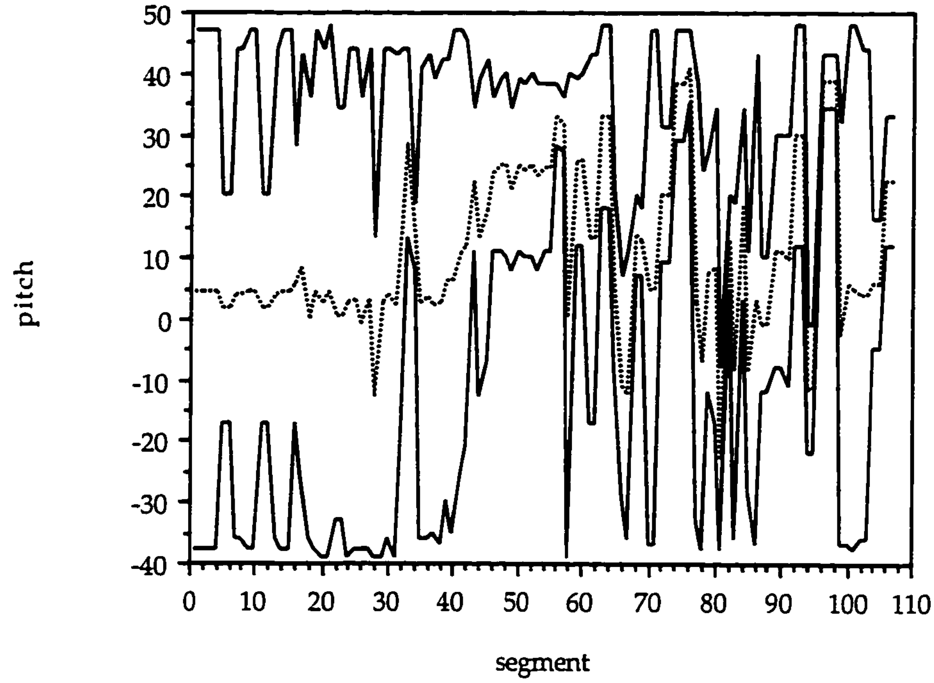


Figure 3.11: Registral boundaries within segments in *Mists*

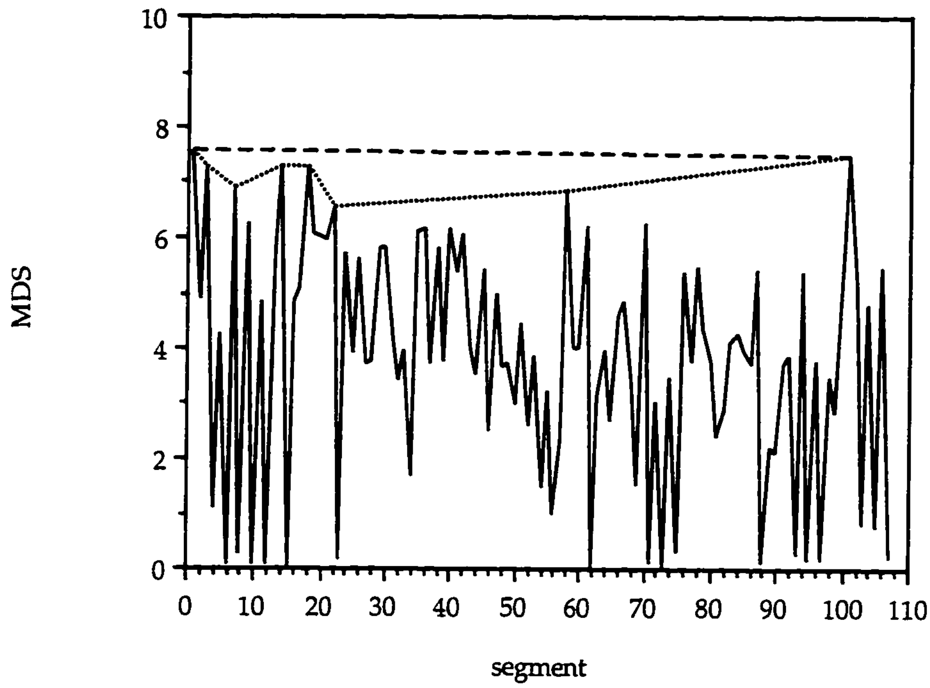


Figure 3.12: Graph of MDS values for segments in *Mists*

Table 3.7: Psets in *Mists*

Model A: period = 90
{-38 -36 -30 -28 -25 -22 -17 -15 -11 -8 -2 0 3 5 6 11 13 14 19 20 24 28 30 33 34 38 41 43 47 49 (52/-38)}
<2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 2 3 1 4 3 2 4 2 3>

Derivatives of Model A, listed in order of initial appearance:

A_0
{-38 -36 -30 -28 -25 -22 -17 -15 -11 -8 -2 0 3 5 6 11 13 14 19 20 24 28 30 33 34 38 41 43 47}
<2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 2 3 1 4 3 2 4 >
transposition operator: T0

A_1
{-36 -32 -30 -27 -26 -22 -19 -17 -13 -11 -8 -6 0 2 5 8 13 15 19 22 28 30 33 35 36 41 43 44}
<4 2 3 1 4 3 2 5 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 >
transposition operator: T30

A_2
{-39 -38 -33 -32 -28 -24 -22 -19 -18 -14 -11 -9 -5 -3 0 2 8 10 13 16 21 23 27 30 36 38 41 43 44}
<1 5 1 4 4 2 3 1 4 3 2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 >
transposition operator: T38

A_3
{-39 -37 -33 -31 -28 -26 -20 -18 -15 -12 -7 -5 -1 2 8 10 13 15 16 21 23 24 29 30 34 38 40 43 44 48}
<2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 2 3 1 4 >
transposition operator: T10

A_4
{-35 -34 -30 -26 -24 -21 -20 -16 -13 -11 -7 -5 -2 0 6 8 11 14 19 21 25 28 34 36 39 41 42 47}
<1 4 4 2 3 1 4 3 2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 >
transposition operator: T36

A_5
{-39 -35 -33 -30 -28 -22 -20 -17 -14 -9 -7 -3 0 6 8 11 13 14 19 21 22 27 28 32 36 38 41 42 46}
<4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 2 3 1 4 >
transposition operator: T8

A_6
{-37 -36 -32 -29 -27 -23 -21 -18 -16 -10 -8 -5 -2 3 5 9 12 18 20 23 25 26 31 33 34 39 40 44 48}
<1 4 3 2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 >
transposition operator: T20

A_7
{-36 -34 -31 -29 -23 -21 -18 -15 -10 -8 -4 -1 5 7 10 12 13 18 20 21 26 27 31 35 37 40 41 45 48}
<2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 2 3 1 4 3 >
transposition operator: T7

Table 3.7: Psets in *Mists*, cont.

A_8
{-37 -34 -32 -28 -26 -23 -21 -15 -13 -10 -7 -2 0 4 7 13 15 18 20 21 26 28 29 34 35 39 43 45 48}
<3 2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 2 3 >
transposition operator: T15

A_9
{-38 -36 -33 -32 -28 -25 -23 -19 -17 -14 -12 -6 -4 -1 2 7 9 13 16 22 24 27 29 30 35 37 38 43 44 48}
<2 3 1 4 3 2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 >
transposition operator: T24

A_{10}
{-37 -33 -31 -28 -27 -23 -20 -18 -14 -12 -9 -7 -1 1 4 7 12 14 18 21 27 29 32 34 35 40 42 43 48}
<4 2 3 1 4 3 2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 >
transposition operator: T29

Model B: the subset of the total chromatic collection contained within the registral boundaries of the piano

Table 3.8: Distribution of psets in *Mists*

A_0
s1, s2, s3, s5-, s9, s11-, s14, s19, s80-+(-13,14), s81-+(-18), s82-, s83-, s84-, s85-, s89-, s90-+(1), s91-+(21), s96-, s98-, s101+(12,48)

A_1
s7, s13, s16-, s17-, s18-, s30-, s31-+(-39,12,18,20), s32-, s33-, s34-, s35-+(16), s36-, s101+(12,48), s102

A_2
s20+(-4), s24-, s25-, s26-, s27-, s28-+(-7), s29-+(-1)

A_3
s21, s22-, s24-, s48-+(25), s49-, s51-, s52-, s53-, s58-+(33,35,39)

A_4
s37-+(-32,-15), s38-+(-37), s39-+(-19), s40-+(9), s41-+(2,29,33,35,43), s42-+(45), s43-, s44-+(1), s45-, s46-, s47-+(37), s49-, s50-

A_5
s53-, s54-, s55-, s56-, s57-, s58-+(33,35,39)

A_6
s59-, s60-, s63-, s64-

A_7
s65-, s66-, s67-, s68-, s69-

A_8
s67-, s68-, s69-, s70-, s74-, s76-+(40)

A_9
s70-, s74-, s76-+(40), s77-, s78-+(-13,14), s79-

A_{10}
s86-+(-21), s92-+(36), s96-, s98-, s99-, s100-+(28,36)

$A_0 \cup A_1$
s101+(12,48)

$A_0 \cap A_{10}$
s96-, s98-

$A_2 \cap A_3$
s24

Table 3.8: Distribution of psets in *Mists*, cont.

$A_3 \cap A_4$
s49-

$A_3 \cap A_5$
s53-

$A_3 \cup A_5$
s58-+(33,35,39)

$A_7 \cap A_8$
s68-, s69-

$A_8 \cup A_9$
s70-

$A_8 \cap A_9$
s74-, s76-+(40)

B
s61-, s72-, s87-, s94-, s104-, s106-

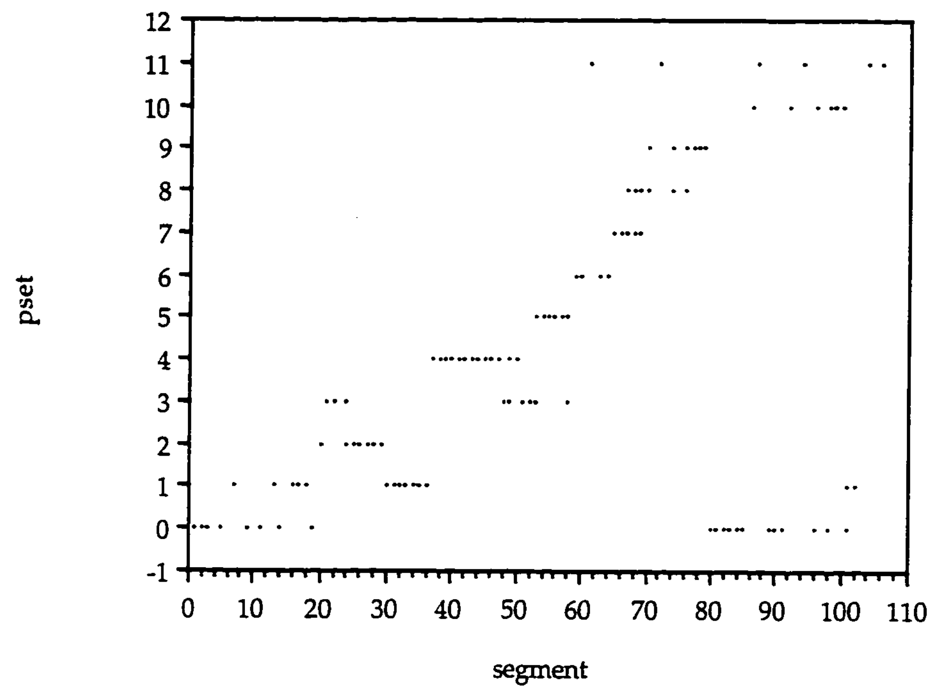
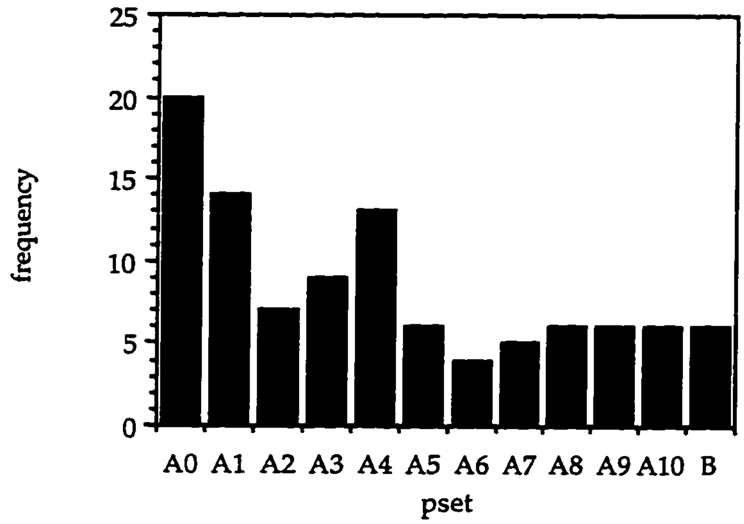
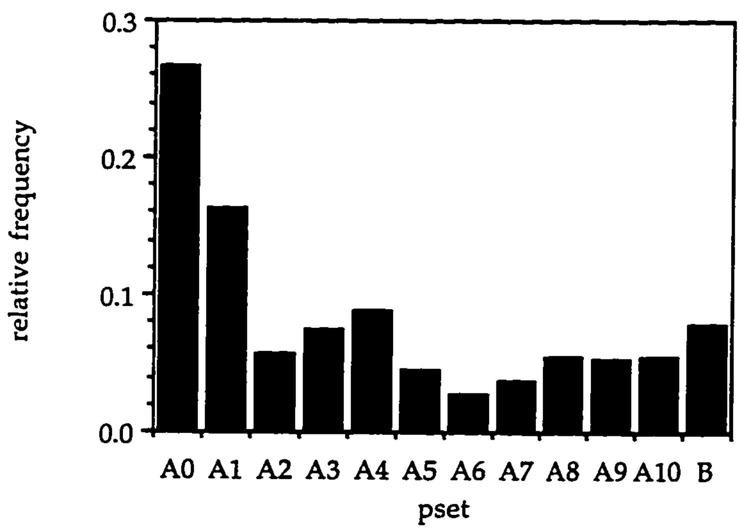


Figure 3.13: Pset distribution by segment in *Mists*



a. Frequency by segment



b. Relative frequency by duration

Figure 3.14: Frequency of psets in *Mists*

Table 3.9: Cardinalities of Intersections Among the Derivatives of Model A

	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
A_0	29	15	10	8	12	13	8	7	9	10	4
A_1		28	13	8	10	11	7	6	9	12	3
A_2			29	16	8	15	5	6	6	14	8
A_3				30	7	9	8	9	12	14	15
A_4					28	15	7	4	11	0	6
A_5						29	0	4	6	8	11
A_6							29	15	12	6	9
A_7								29	14	10	12
A_8									29	9	13
A_9										30	12
A_{10}											29

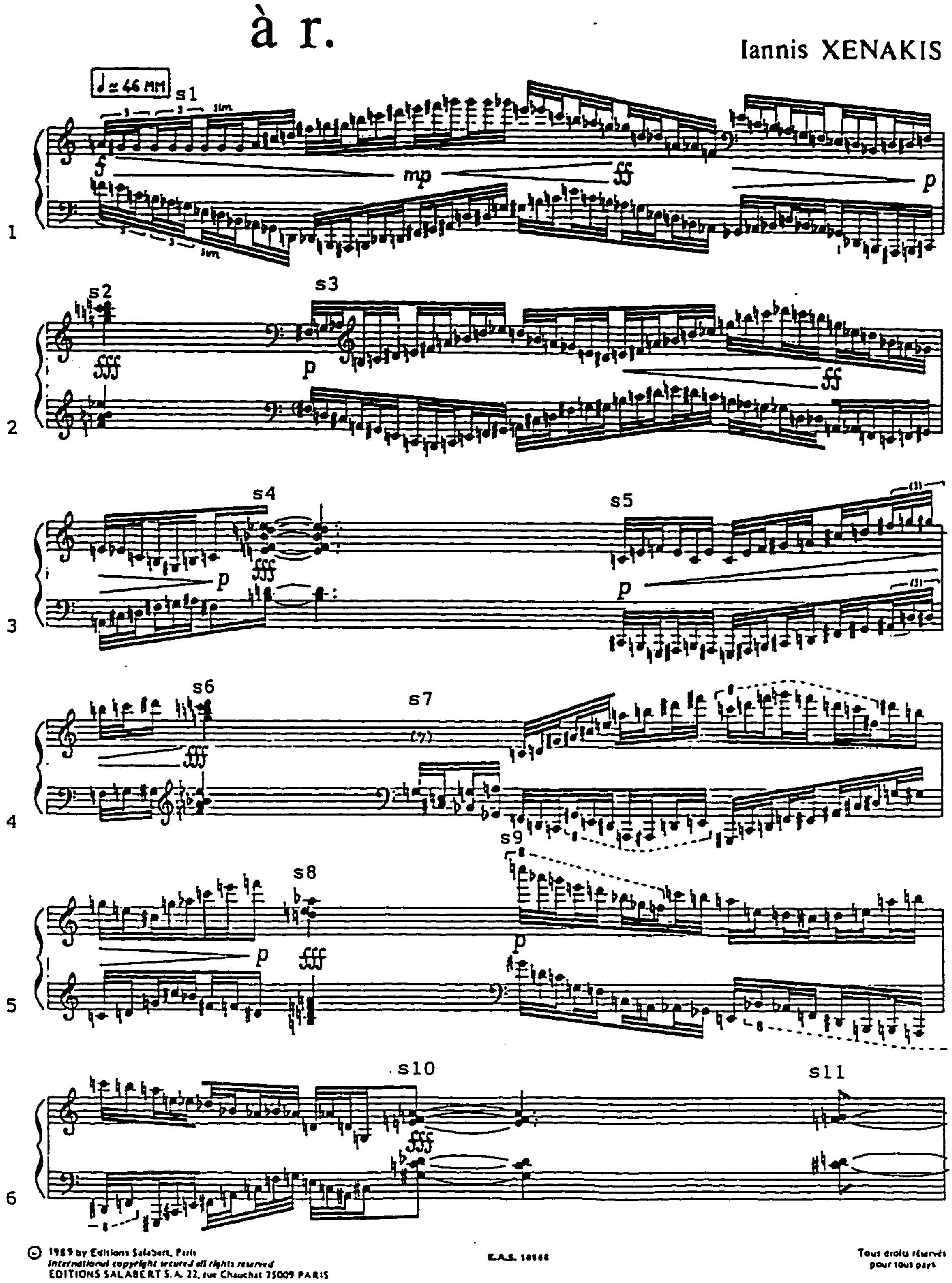


Figure 3.15: Annotated score of *à r.*
Iannis Xenakis, *à r.* (Paris: Editions Salabert, 1989). Used with permission.

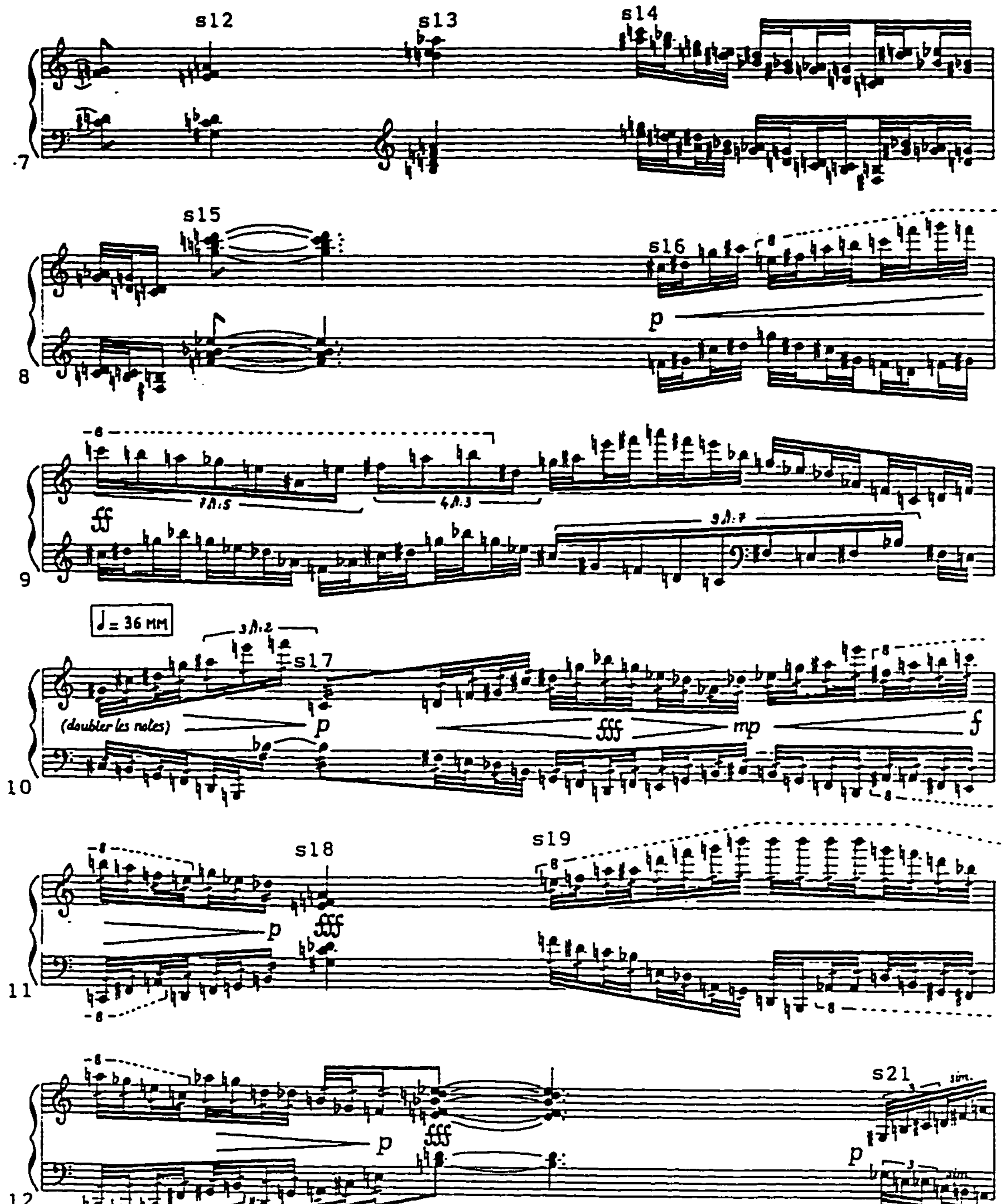


Figure 3.15: Annotated score of *à r.*, cont.
Iannis Xenakis, *à r.* (Paris: Editions Salabert, 1989). Used with permission.

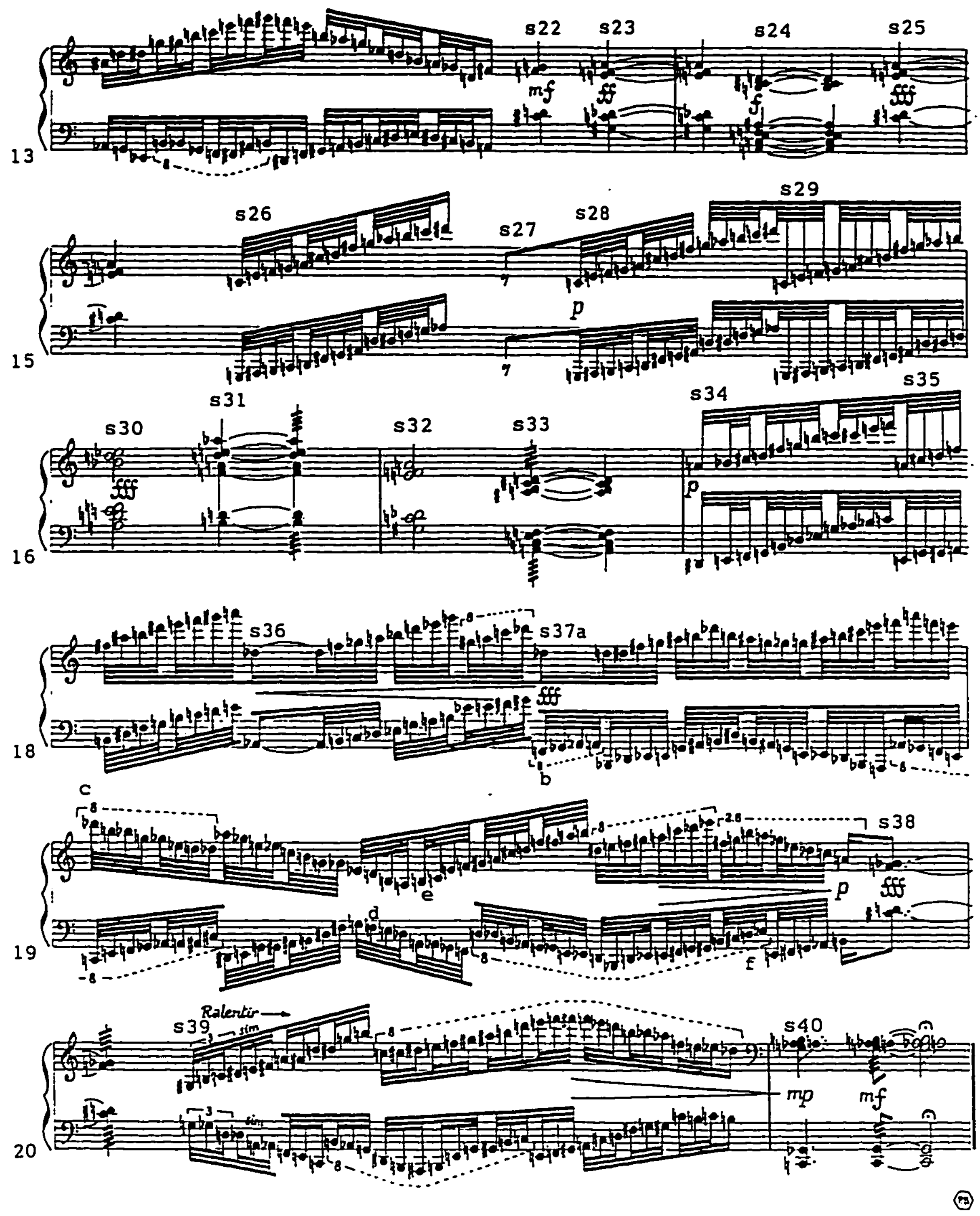


Figure 3.15: Annotated score of *à r.*, cont.
Iannis Xenakis, *à r.* (Paris: Editions Salabert, 1989). Used with permission.

Table 3.10: Segments in *à r.*

segment	measures	seconds*	duration (sec)	configuration type	density	intensity
1	1	0.000-5.216	5.216	RW	18.41	*p-ff* (c)[†]
2	2	5.216-6.520	1.304	S	6.14	*fff*
3	2-3	6.520-11.410	4.890	RW	17.59	*p-ff* (c)
4	3	11.410-13.692	2.282	S	3.07	*fff*
5	3-4	13.692-16.300	2.608	RW	14.57	*p-fff* (c)
6	4	16.300-17.604	1.304	S	6.14	*fff*
7	4-5	17.604-22.168	4.564	RW	13.80	*p-fff* (c)
8	5	22.168-23.472	1.304	S	5.37	*fff*
9	5-6	23.472-28.036	4.564	RW	14.02	*p*
10	6	28.036-30.644	2.608	S	2.30	*fff*
11	6-7	30.644-31.948	1.304	S	3.07	*fff*
12	7	31.948-33.252	1.304	S	4.60	*fff*
13	7	33.252-34.556	1.304	S	5.37	*fff*
14	7-8	34.556-37.164	2.608	RW/S(?)	23.01	*fff*
15	8	37.164-39.772	2.608	S	3.07	*fff*
16	8-10	39.772-48.610	8.838	RW	11.77	*p-ff* (c)
17	10-11	48.194-55.274	7.080	RW	18.64	*p-fff* (c)
18	18	55.274-56.940	1.666	S	3.60	*fff*
19	11-12	56.940-62.771	5.831	RW	19.21	*p-fff* (c)
20	12	62.771-66.104	3.333	S	2.10	*fff*
21	12-13	66.104-70.270	4.166	RW	13.44	*p*
22	13	70.270-71.936	1.666	S	2.40	*mf*
23	13-14	71.936-75.269	3.333	S	1.80	*ff*
24	14	75.269-78.602	3.333	S	2.40	*f*
25	14-15	78.602-81.935	3.333	S	1.50	*fff*
26	15	81.935-83.601	1.666	RW	15.61	*fff*
27	15	83.601-84.434	0.833	R	0.00	-
28	15	84.434-85.788	1.354	RW	19.20	*p*
29	15	85.788-86.933	1.145	RW	19.21	*p*
30	16	86.933-90.266	3.333	S	2.10	*fff*
31	16	90.266-93.600	3.334	S	33.60	*fff*
32	17	93.600-96.933	3.333	S	1.80	*fff*
33	17	96.933-100.266	3.333	S	38.40	*fff*
34	18	100.266-101.516	1.250	RW	17.60	*p*
35	18	101.516-102.766	1.250	RW	19.20	*p*
36	18	102.766-104.433	1.667	RW	14.40	*p-fff* (c)
37	18-19	104.433-112.348	7.915	RW	18.07	*p-fff* (c)
[37a	18	104.433-106.933	2.500	RW	8.40	*fff*]
[37b	18-19	104.433-108.702	4.269	RW	9.60	*fff*]
[37c	19	106.933-109.016	2.083	RW	9.60	*fff*]
[37d	19	108.599-111.410	2.811	RW	9.60	*mf-fff* (c)]
[37e	19	108.911-112.348	3.437	RW	8.73	*p-fff* (c)]
[37f	19	111.306-112.348	1.042	RW	6.72	*p-mf* (c)]

* refers to the location of the segments in the graphic transcriptions (Figures 3.16 and 3.17)
† d = discrete changes in intensity; c = continuous (gradual) changes in intensity
configuration types: RW = random walk; S= simultaneity; R = rest

Table 3.10: Segments in *à r.*, cont.

segment	measures	seconds	duration (sec)	configuration type	density	intensity
38	19-20	112.348-115.264	2.916	S	21.95	*fff*
39	20	115.264-120.263	4.999	RW	13.60	*mp-fff* (c)
40	21	120.263-126.929	6.666	S	8.10	*mp-mf* (d)

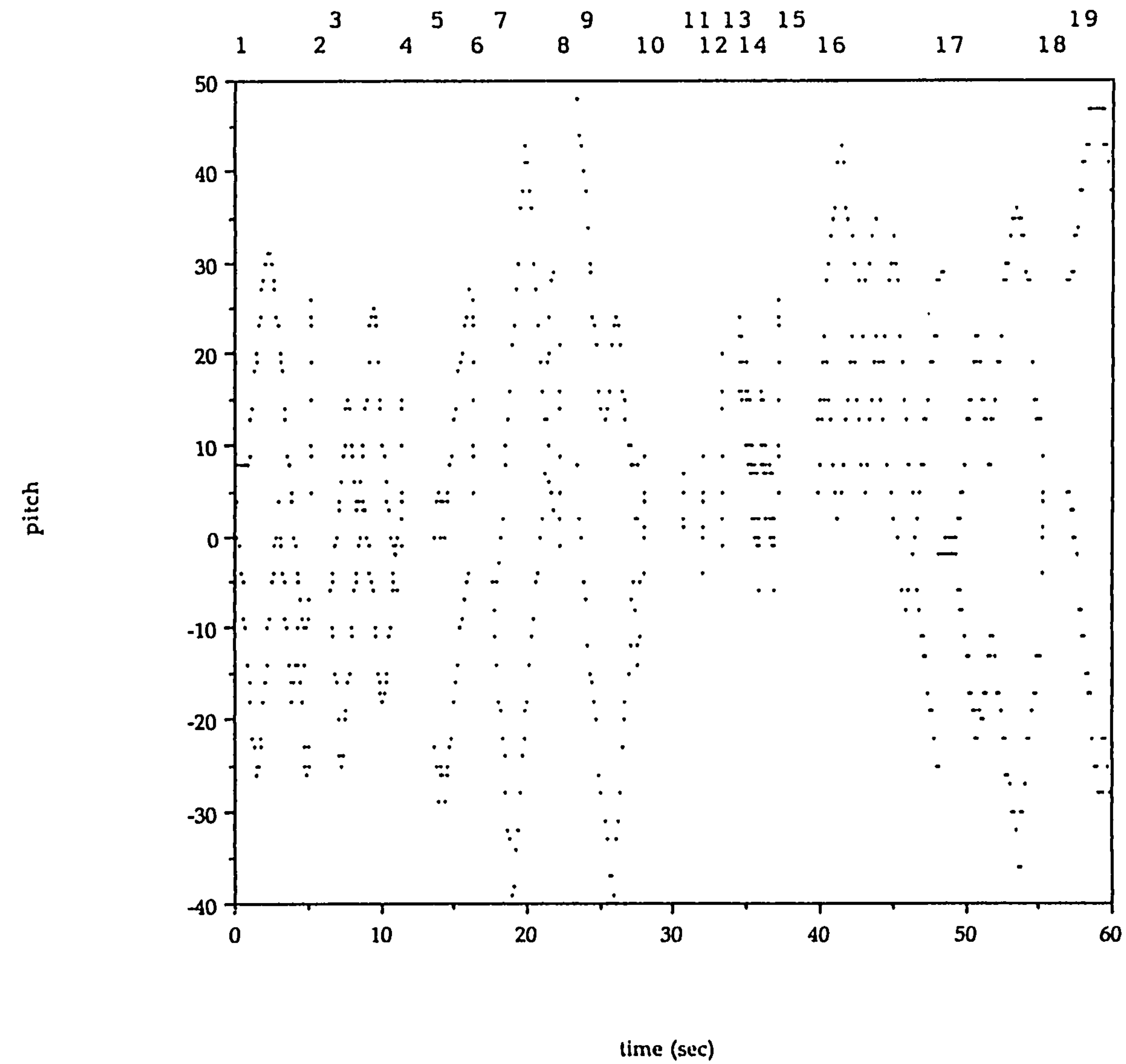


Figure 3.16: Time-point/pitch graphic transcription of *à r.*

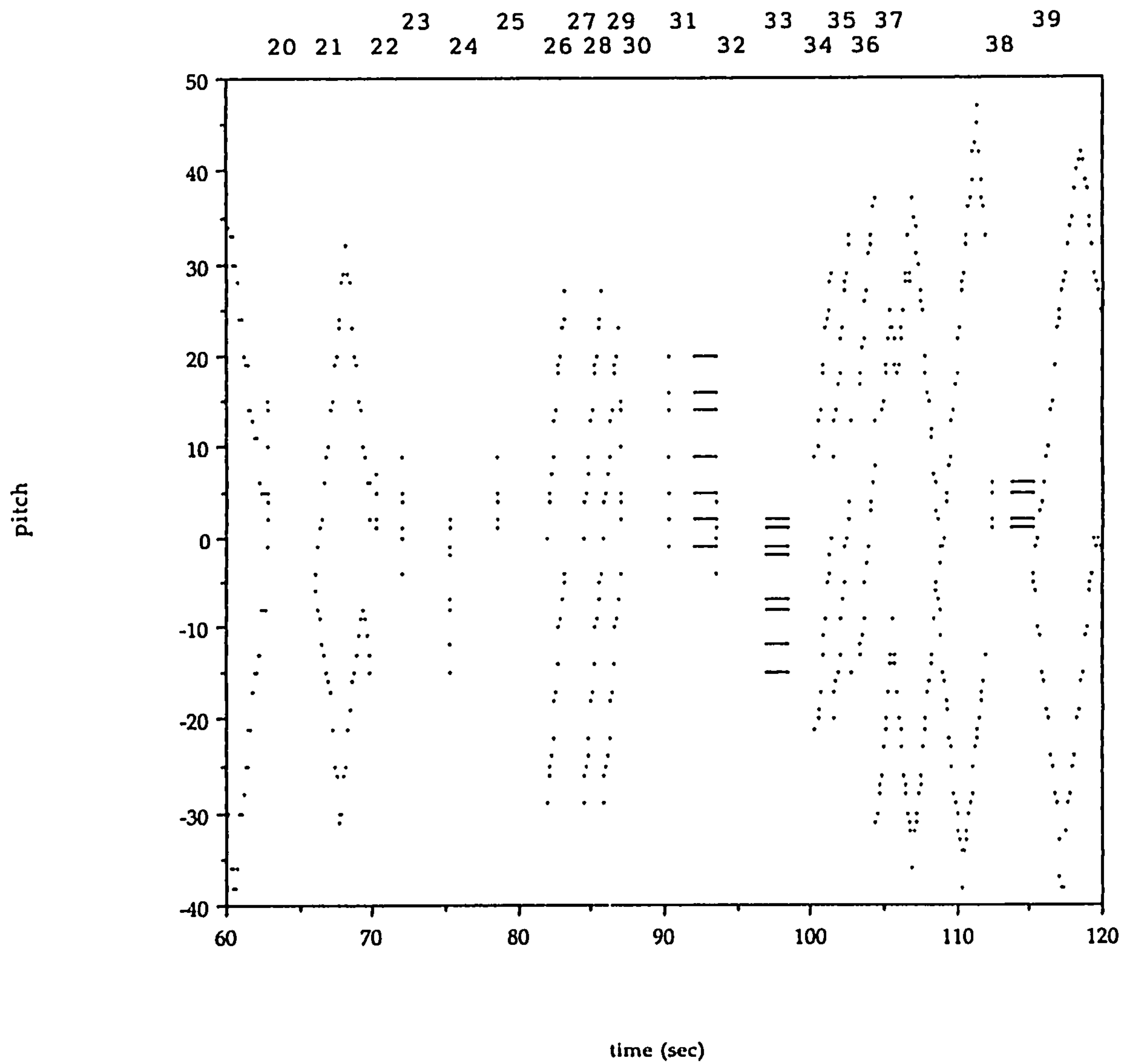


Figure 3.16: Time-point/pitch graphic transcription of *à r.*, cont.

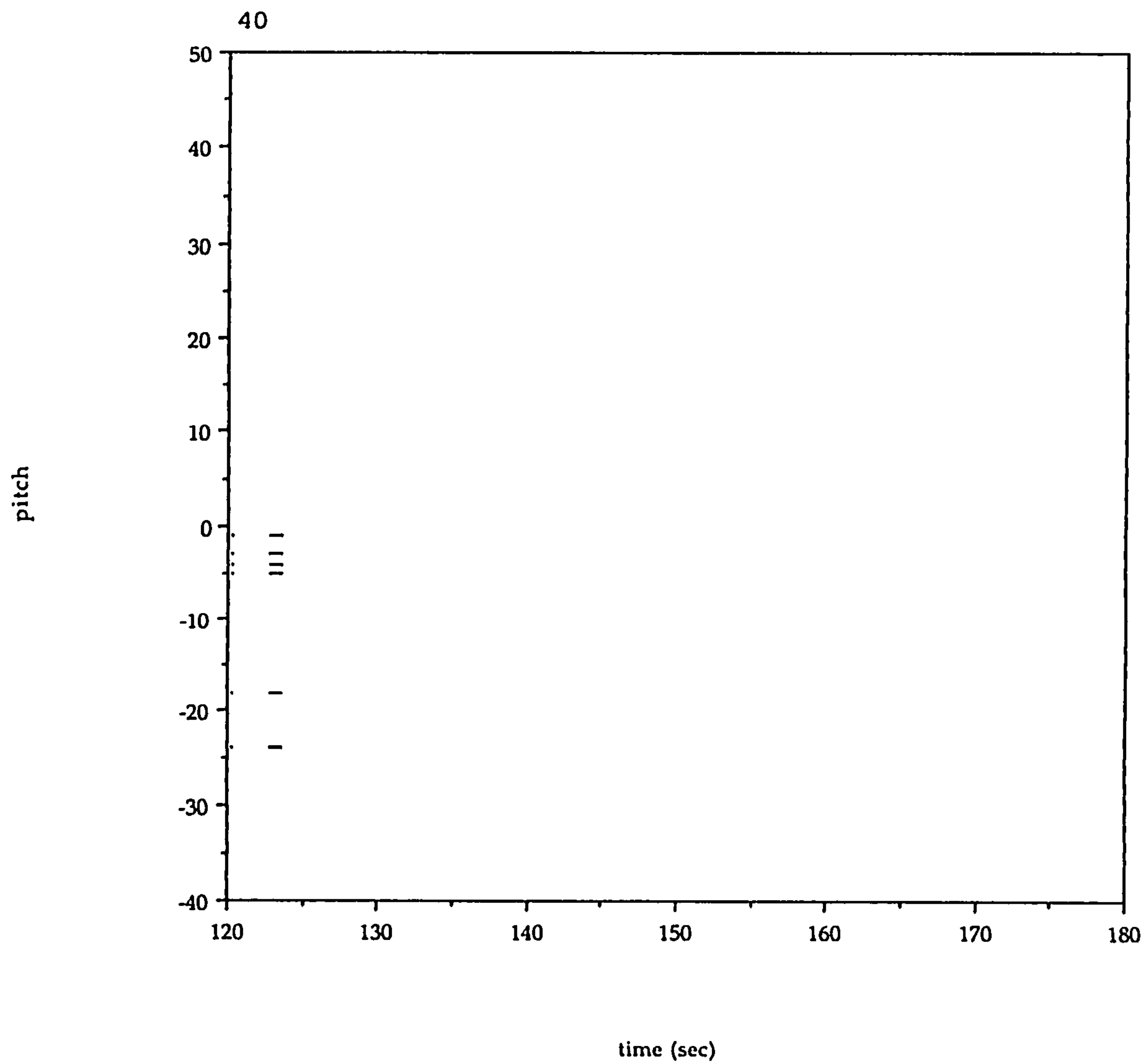


Figure 3.16: Time-point/pitch graphic transcription of *à r.*, cont.

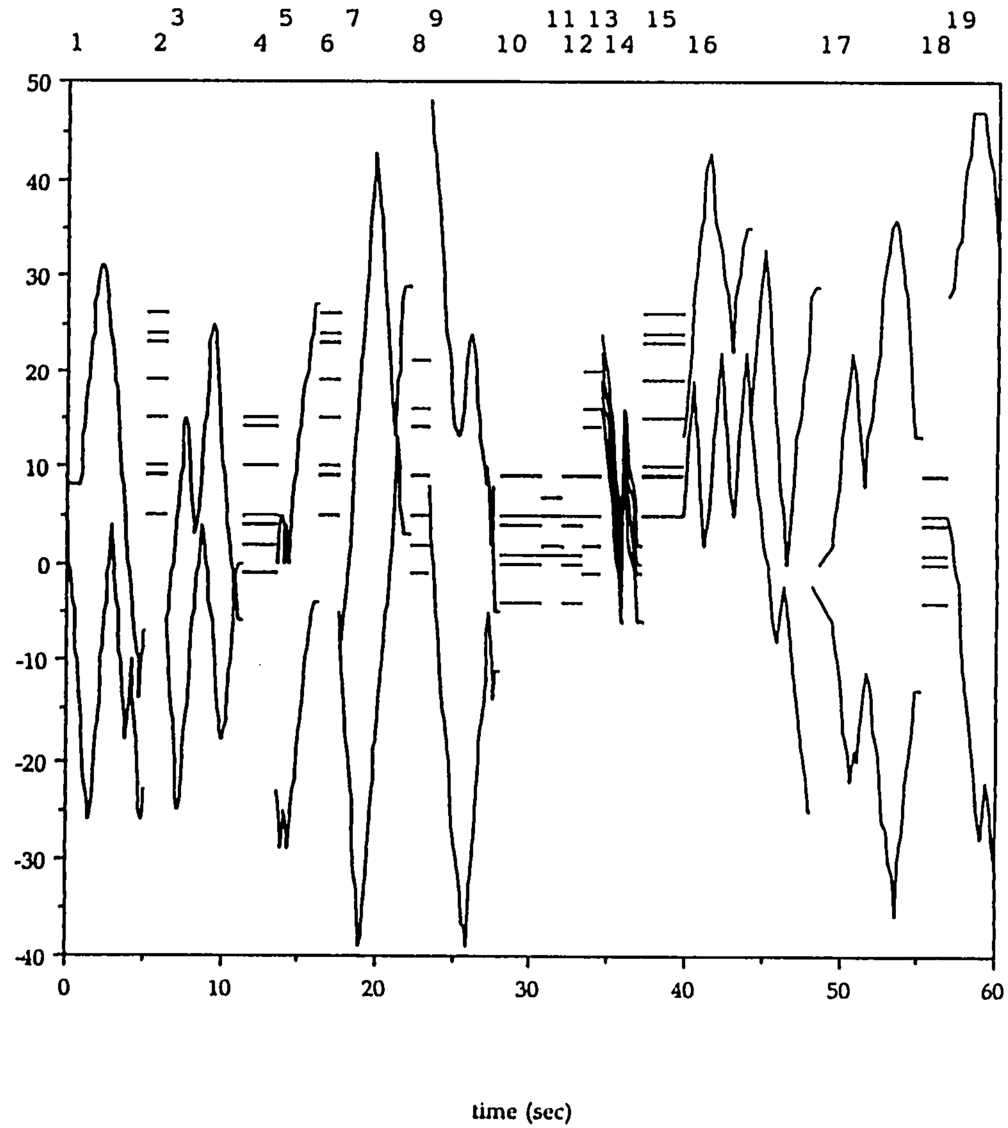


Figure 3.17: Continuous graphic transcription of *à r.*

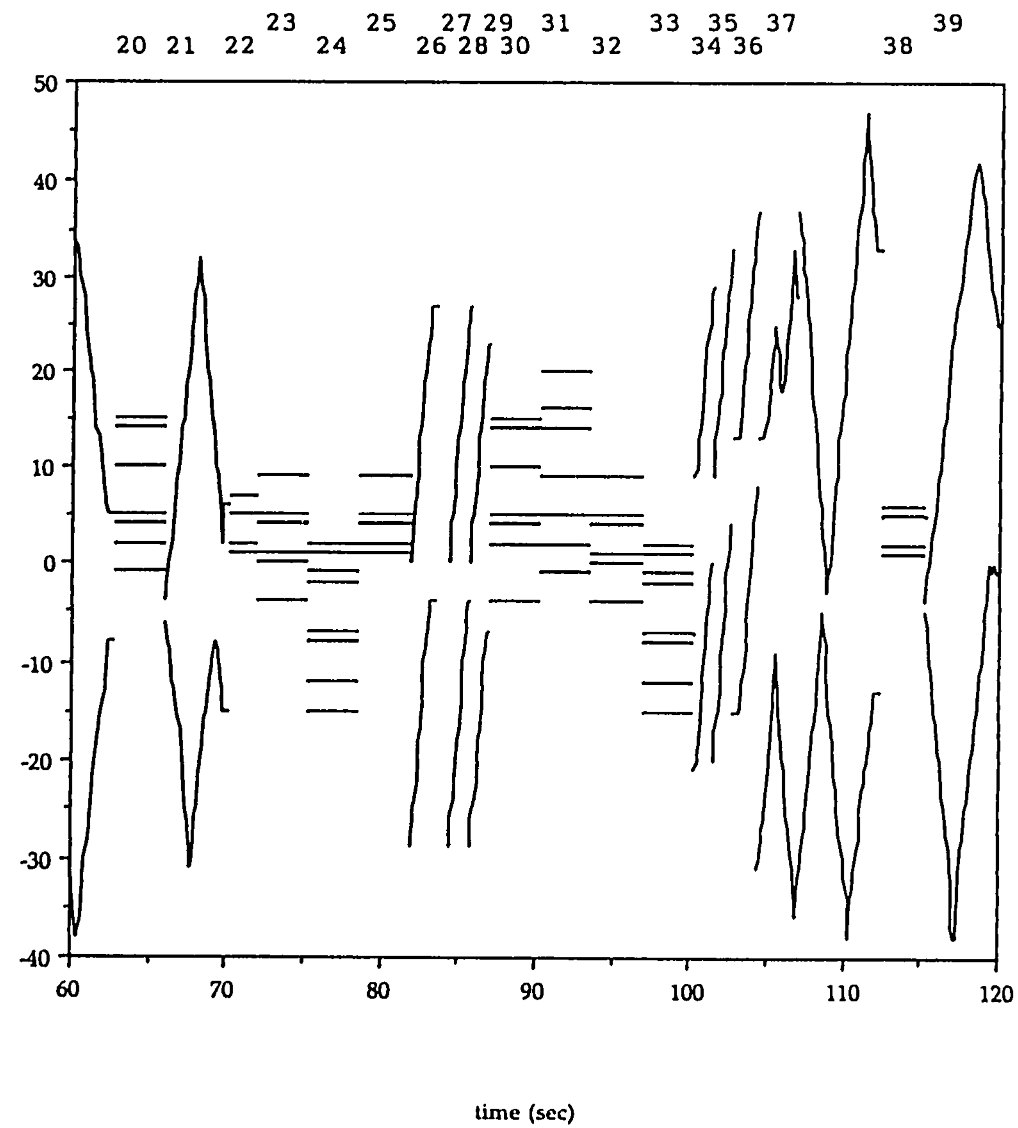


Figure 3.17: Continuous graphic transcription of *à r.*, cont.

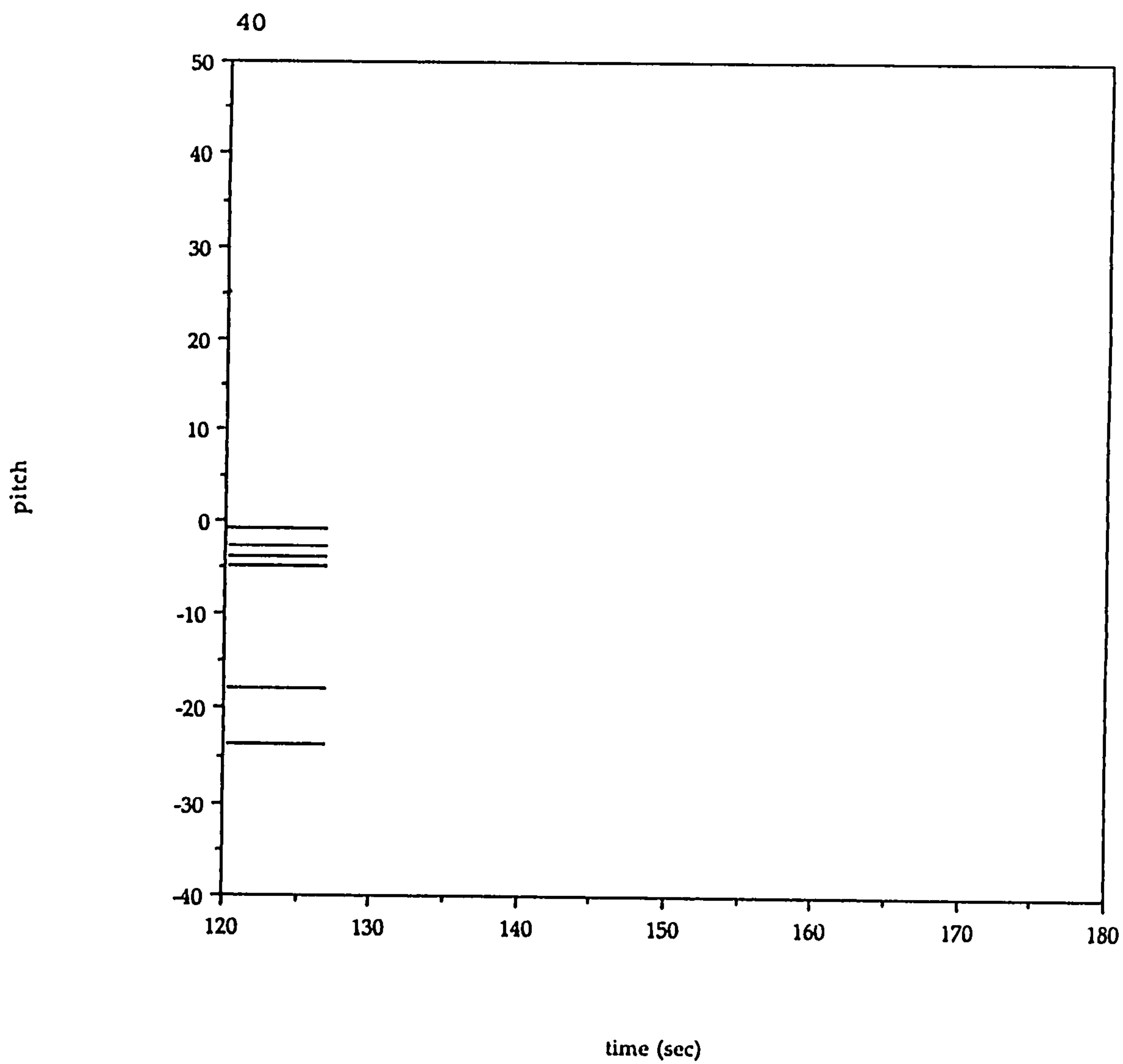


Figure 3.17: Continuous graphic transcription of *à r.*, cont.

Table 3.11: Sections in *à r.*

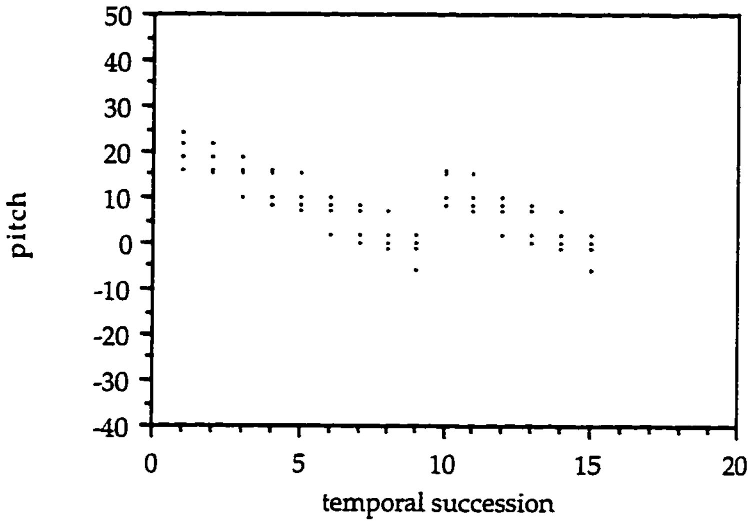
section	succession	pattern	segments	duration
1	ababababa	X_0	1-9	28.036"
2	bbbb?ba	Y_0	10-16	20.574"
3	ababa	X_1	17-21	21.660"
4	bbbbaraa	Y_1	22-29	16.663"
5	bbbbaaa	Y_2	30-36	17.500"
6	abab	X_2	37-40	22.496"

Durations and proportions

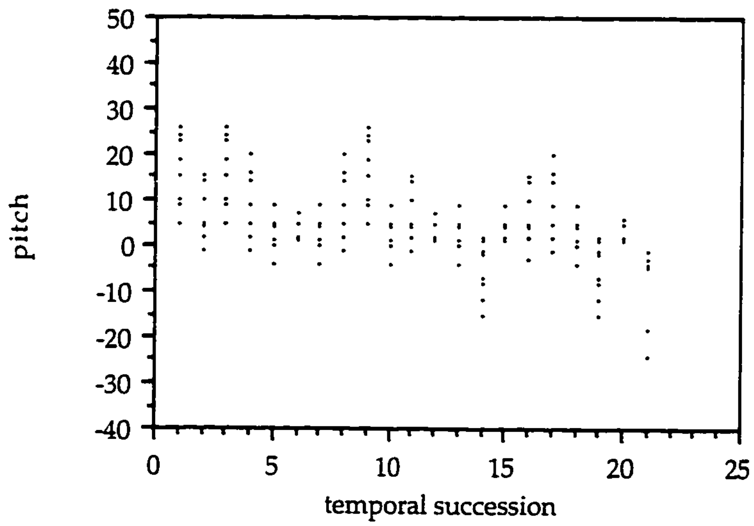
a) inside-time:

X_0/Y_0 = 28.036"/20.574" = 1.363
X_1/Y_1 = 21.660"/16.663" = 1.300
X_2/Y_2 = 17.500"/22.496" = 1.286

b) outside-time:
$X_{0\text{-}2}/Y_{0\text{-}2}$ = 72.192"/54.737" = 1.319
(a + ? + r)/b = 72.028"/54.901" = 1.312



a. Simultaneities in segment 14



b. Long-held simultaneities in whole work

Figure 3.18: Graphic comparison of successions of simultaneities in *à r*.

Table 3.12: Summary of Temporal Structure in *à r.*

Total duration: 126.929"

part:	1			
segments:	1-16			
duration:	48.61" (.383)*			
supersection:	A			
segments:	1-16			
duration:	48.61" (1.0)			
section:	1		2	
segments:	1-9		10-16	
duration:	28.036" (.577)		20.574" (.423)	

part:	2			
segments:	17-40			
duration:	78.319" (.617)			
supersection:	B		C	
segments:	17-29		30-36	
duration:	38.323" (.489)		39.996" (.511)	
section:	3	4	5	6
segments:	17-21	22-29	30-36	37-40
durations:	21.66" (.565)	16.663" (.434)	17.5" (.438)	22.496" (.562)

*The quantities in parentheses indicate the proportion of the next higher level in the temporal structure that is occupied by the given duration.

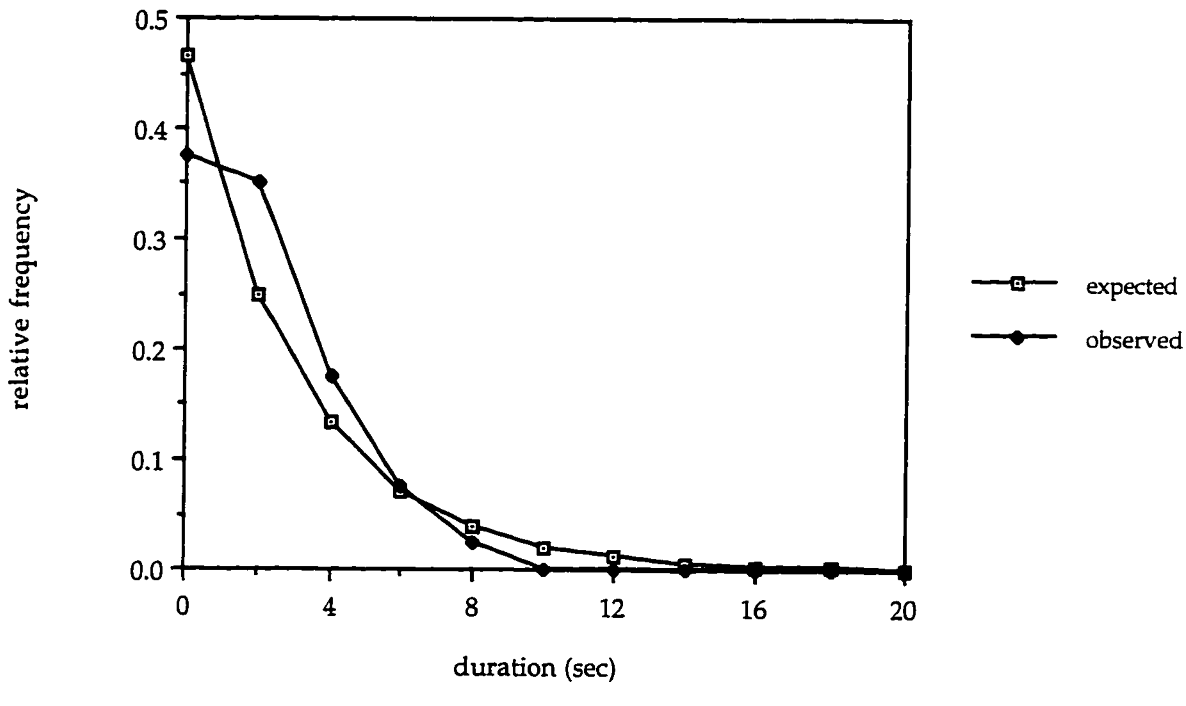


Figure 3.19: Comparative histogram showing the distribution of segment durations in *à r.* with the exponential distribution, ∂ = 0.315

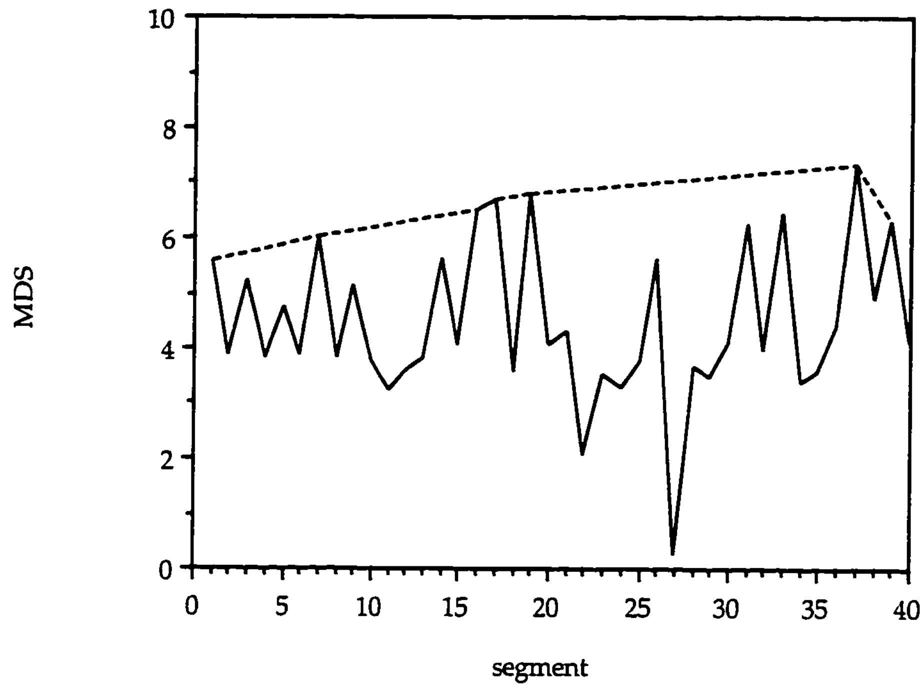


Figure 3.20: Graph of MDS values for segments in *à r*.

Table 3.13: Psets in the Random Walks in *à r.*

Model A: period = 88
{-39 -37 -36 -35 -32 -30 -29 -26 -25 -23 -22 -18 -16 -14 -10 -9 -7 -5 -4 -1 0 4 5 8 9 13 14 18 19 20 23 24 27
28 30 33 34 38 39 43 47 48 (49/-39)}
<2 1 1 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 4 1 4 4 1 1>

Derivatives of Model A, listed in order of initial appearance, with subsets:

A_0
{-39 -37 -36 -35 -32 -30 -29 -26 -25 -23 -22 -18 -16 -14 -10 -9 -7 -5 -4 -1 0 4 5 8 9 13 14 18 19 20 23 24 27
28 30 33 34 38 39 43 47 48}
<2 1 1 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 4 1 4 4 1>
transposition operator: T0

subsets:

segment 1
{-26 -25 -23 -22 -18 -16 -14 -10 -9 -7 -5 -4 -1 0 4 5 8 9 13 14 18 19 20 23 24 27 28 30 *31*}
<1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 *1*>

segment 5
{-29 -26 -25 -23 -22 -18 -16 -14 -10 -9 -7 -5 -4 0 4 5 8 9 13 14 18 19 20 23 24 27}
<3 1 2 1 4 2 2 4 1 2 2 1 4 4 1 3 1 4 1 4 1 1 3 1 3>

segment 26
{-29 -26 -25 *-24* -22 -18 *-17* -14 -10 -9 -7 -5 -4 0 4 5 *7* 9 13 14 18 19 20 23 24 27}
<3 1 *1* 2 4 *1 3* 4 1 2 2 1 4 4 1 2 2 4 1 4 1 1 3 1 3>

segment 28
{-29 -26 -25 *-24* -22 -18 *-17* -14 -10 -9 -7 -5 -4 0 4 5 *7* 9 13 14 18 19 20 23 24 27}
<3 1 *1* 2 4 *1 3* 4 1 2 2 1 4 4 1 2 2 4 1 4 1 1 3 1 3>

segment 29
{-29 -26 -25 *-24* -22 -18 *-17* -14 -10 -9 -7 0 4 5 *7* 9 13 14 18 19 20 23}
<3 1 *1* 2 4 *1 3* 4 1 2 7 4 1 2 2 4 1 4 1 1 3>

A_1
{-38 -34 -33 -31 -29 -28 -25 -24 -20 -19 -16 -15 -11 -10 -6 -5 -4 -1 0 3 4 6 9 10 14 15 19 23 24 25 27 28 29
32 34 35 38 39 41 42 46 48}
<4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 4 1 4 4 1 1 2 1 1 3 2 1 3 1 2 1 4 2>
transposition operator: T64

subsets:

segment 3
{-25 -24 -20 -19 -16 -15 -11 -10 -6 -5 -4 -2 -1 0 3 4 6 9 10 14 15 19 23 24 25}
<1 4 1 3 1 4 1 4 1 1 2 *1* 1 3 1 2 3 1 4 1 4 4 1 1>

Table 3.13: Psets in the Random Walks in *à r.*, cont.

segment 39
{-38 -37 -33 -32 -29 -28 -25 -24 -20 -19 -16 -15 -11 -10 -6 -5 -4 -1 0 3 4 6 9 10 14 15 19 23 24 25 27 28 29 32 34 35 38 39 40 41 42}
<1 4 1 3 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 4 1 4 4 1 1 2 1 1 3 2 1 3 1 1 1 1>

A_2
{-37 -36 -34 -31 -30 -26 -25 -21 -17 -16 -15 -13 -12 -11 -8 -6 -5 -2 -1 1 2 6 8 10 14 15 17 19 20 23 24 28 29 32 33 37 38 42 43 44 47 48}
<1 2 3 1 4 1 4 4 1 1 2 1 1 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1>
transposition operator: T24

subset:

segment 21
{-31 -30 -26 -25 -21 -19 -17 -16 -15 -13 -12 -11 -9 -8 -6 -4 -1 1 2 6 9 10 14 15 19 20 23 24 28 29 32}
<1 4 1 4 2 2 1 1 2 1 1 2 1 2 2 3 2 1 4 3 1 4 1 4 1 3 1 4 1 3>

A_3
{-36 -35 -34 -32 -31 -30 -27 -25 -24 -21 -20 -18 -17 -13 -11 -9 -5 -4 -2 0 1 4 5 9 10 13 14 18 19 23 24 25 28 29 32 33 35 38 39 43 44 48}
<1 1 2 1 1 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 4 1 4>
transposition operator: T5

subset:

segment 34
{-21 -20 -19 -17 -13 -11 -9 -5 -4 -2 0 9 10 13 14 18 19 23 24 25 28 29}
<1 1 2 4 2 2 4 1 2 2 9 1 3 1 4 1 4 1 1 3 1>

A_4
{-36 -32 -31 -30 -28 -27 -26 -23 -21 -20 -17 -16 -14 -13 -9 -7 -5 -1 0 2 4 5 8 9 13 14 17 18 22 23 27 28 29 32 33 36 37 39 42 43 47 48}
<4 1 1 2 1 1 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 4 1>
transposition operator: T9

subsets:

segment 35
{-20 -17 -16 -15 -13 -9 -7 -5 -1 0 2 4 9 13 14 17 18 22 23 27 28 29 32 33}
<3 1 1 2 4 2 2 4 1 2 2 5 4 1 3 1 4 1 4 1 1 3 1>

segment 37b
{-36 -32 -31 -30 -28 -27 -26 -23 -21 -20 -17 -16 -14 -13 -9 -6 -5}
<4 1 1 2 1 1 3 2 1 3 1 2 1 4 3 1>

segment 37e
{-1 0 4 5 8 9 13 14 17 18 22 23 27 28 29 32 33 36 37 39 42 43 45 47}
<1 4 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 2 2>

Table 3.13: Psets in the Random Walks in *à r.*, cont.

A_5
{-37 -36 -32 -28 -27 -26 -24 -23 -22 -19 -17 -16 -13 -12 -10 -9 -5 -3 -1 3 4 6 8 9 12 13 17 18 21 22 26 27 31 32 33 36 37 40 41 43 46 47}
< 1 4 4 1 1 2 1 1 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 >
transposition operator: T13

subset:

segment 36
{-16 -13 -12 *-11* -9 -5 -3 -1 3 4 6 8 13 17 18 21 22 26 27 31 32 33 36 37}
< 3 1 *1* 2 4 2 2 4 1 2 2 5 4 1 3 1 4 1 4 1 1 3 1>

A_6
{-37 -35 -34 -31 -30 -28 -27 -23 -21 -19 -15 -14 -12 -10 -9 -6 -5 -1 0 3 4 8 9 13 14 15 18 19 22 23 25 28 29 33 34 38 42 43 44 46 47 48}
< 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 4 1 4 4 1 1 2 1 1 >
transposition operator: T83

segment 37a
{13 14 15 18 19 22 23 25 28 29 33}
< 1 1 3 1 3 1 2 3 1 4 >

A_7
{-38 -34 -33 -32 -30 -29 -28 -25 -23 -22 -19 -18 -16 -15 -11 -9 -7 -3 -2 0 2 3 6 7 11 12 15 16 20 21 25 26 27 30 31 34 35 37 40 41 45 46}
< 4 1 1 2 1 1 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 4 1 >
transposition operator: T7

segment 37c
{-3 *-1* 2 3 6 7 11 12 15 16 *18* 20 25 26 27 30 31 34 35 37}
< 2 *3* 1 3 1 4 1 3 1 2 2 5 1 1 3 1 3 1 2 >

segment 37d
{-38 -34 -33 -32 -30 -29 -28 -25 -23 -22 -19 -18 -16 -15 -11 *-8* -7}
< 4 1 1 2 1 1 3 2 1 3 1 2 1 4 *3 1* >

A_8
{-39 -36 -35 -31 -30 -26 -22 -21 -20 -18 -17 -16 -13 -11 -10 -7 -6 -4 -3 1 3 5 9 10 12 14 15 18 19 23 24 27 28 32 33 37 38 39 42 43 46 47}
< 3 1 4 1 4 4 1 1 2 1 1 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 >
transposition operator: T19

subset:

segment 37f
{-22 -21 -20 -18 -17 -16 -13}
< 1 1 2 1 1 3 >

Table 3.13: Psets in the Random Walks in *à r.*, cont.

Model B: period = 90
{-39 -38 -33 -32 -28 -24 -22 -19 -18 -14 -11 -9 -5 -3 0 2 8 10 13 16 21 23 27 30 36 38 41 43 44 49 (51/-39)}
< 1 5 1 4 4 2 3 1 4 3 2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 >

Derivatives of Model B, listed in order of initial appearance, with subsets:

B_0
{-39 -38 -33 -32 -28 -24 -22 -19 -18 -14 -11 -9 -5 -3 0 2 8 10 13 16 21 23 27 30 36 38 41 43 44}
< 1 5 1 4 4 2 3 1 4 3 2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 >
transposition operator: T0

subset:

segment 7
{-39 -38 -*34* -33 -32 -28 -24 -22 -10 -19 -18 -14 -11 -9 -*8* -5 -*4* -3 0 2 *3 5 6 7* 8 10 13 16 *19 20* 21 23 *24* 27 *28 29* 30 36 38 41 43}
< 1 *4 1* 1 4 4 2 3 1 4 3 2 *1 3 1 1* 3 2 *1* 2 *1 1 1* 2 3 3 *3 1 1* 2 *1 3 1 1 1* 6 2 3 2 >

B_1
{-39 -37 -33 -31 -28 -26 -20 -18 -15 -12 -7 -5 -1 2 8 10 13 15 16 21 23 24 29 30 34 38 40 43 44 48}
< 2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 2 3 1 4 >
transposition operator: T62

subset:

segment 9
{-39 -37 -33 -31 -28 -26 -*23* -20 -18 -*16* -15 -*14* -12 -*11* -*8* -*7* -5 *0* 2 8 10 13 *14* 15 16 21 23 24 29 30 34 38 40 43 44 48}
< 2 4 2 3 2 *3 3* 2 *2 1 1* 2 *1 3 1* 2 *5* 2 6 2 3 *1 1* 1 5 2 1 5 1 4 4 2 3 1 4 >

B_2
{-36 -32 -30 -27 -26 -22 -19 -17 -13 -11 -8 -6 0 2 5 8 13 15 19 22 28 30 33 35 36 41 43 44}
< 4 4 2 3 1 4 3 2 4 2 3 2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 >
transposition operator: T82

subsets:

segment 16
{-*25* -22 -19 -17 -13 -11 -8 -6 -2 0 2 5 8 13 15 19 22 28 *29* 30 33 35 36 41 43}
< *3* 3 2 4 2 3 2 *4* 2 2 3 3 5 2 4 3 6 *1 1* 3 2 1 5 2 >

segment 17
{-36 -32 -30 -27 -26 -22 -*20* -19 -17 -13 -11 -8 -6 -2 0 2 5 8 13 15 19 22 28 *29* 30 33 35 36}
< 4 2 3 1 4 2 *1* 2 4 2 3 2 *4* 2 2 3 3 5 2 4 3 6 *1 1* 3 2 1 >

Table 3.13: Psets in the Random Walks in *à r.*, cont.

B_3

{-38 -36 -30 -28 -25 -22 -17 -15 -11 -8 -2 0 3 5 6 11 13 14 19 20 24 28 30 33 34 38 41 43 47 49}

<2 6 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 2 3 1 4 3 2 4 2>

transposition operator: T52

subset:

segment 19

{-38 -36 -30 -28 -25 -22 *-21* -17 -15 -11 -8 -2 0 3 5 6 11 13 14 19 20 24 28 *29* 30 33 34 38 41 43 47}

<2 6 2 3 3 *1 4* 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 *1 1* 3 1 4 3 2 4>

Table 3.14: Psets in the Simultaneities in *à r.*

1) Simultaneities related by inclusion to $A_9 \cup B_4$

A9
{-39 -36 -34 -33 -30 -29 -27 -26 -22 -20 -18 -14 -13 -11 -9 -8 -5 -4 0 1 4 5 9 14 15 16 19 20 23 24 26 29 30 34 35 39 43 44 45 47 48}
< 3 2 1 3 1 2 1 4 2 2 4 1 2 2 1 3 1 4 1 3 1 4 1 4 1 1 3 1 3 1 2 3 1 4 1 4 4 1 1 2 1 >
transposition operator: T84

B4
{-34 -32 -29 -26 -21 -19 -15 -12 -6 -4 -1 1 2 7 9 10 15 16 20 24 26 29 30 34 37 39 43 45 48}
< 2 3 3 5 2 4 3 6 2 3 2 1 5 2 1 5 1 4 4 2 3 1 4 3 2 4 2 3 >
transposition operator: T48

$A_9 \cup B_4$
{-39 -36 -34 -33 -32 -30 -29 -27 -26 -22 -21 -20 -19 -18 -15 -14 -13 -12 -11 -9 -8 -6 -5 -4 -1 0 1 2 4 5 7 9 10 14 15 16 19 20 23 24 26 29 30 34 35 37 39 43 44 45 47 48}
< 3 2 1 1 2 1 2 1 4 1 1 1 1 3 1 1 1 1 2 1 2 1 1 3 1 1 1 2 1 2 2 1 4 1 1 3 1 3 1 2 3 1 4 1 2 2 4 1 1 2 1 >

subset of $A_9 \cup B_4$
{-4 -1 0 1 2 4 5 7 9 10 14 15 16 19 20 23 24 26}
< 3 1 1 1 2 1 2 2 2 1 4 1 1 3 1 3 1 2 >

segments 2, 6, 15
{5 9 10 15 19 23 24 26}
< 4 1 5 4 4 1 2 >

segments 4, 20
{-1 2 4 5 10 14 15}
< 3 2 1 5 4 1 >

segments 8, 13, 31
{-1 2 5 9 14 16 20}
< 3 3 4 5 2 4 >

segments 10, 12, 18, 23, 32
{-4 0 1 4 5 9}
< 4 1 3 1 4 >

segments 11, 22
{1 2 5 7}
< 1 3 2 >

segment 25
{1 2 4 5 9}
< 1 2 1 4 >

Table 3.14: Psets in the Simultaneities in *à r.*, cont.

2) Simultaneities related by inclusion to unions involving at least one derivative found in preceding or following random walk

segment	config. type	pset contents	derivative(s)
1	RW		A_0
2	S	{5 9 10 15 19 23 24 26}	$A_0 \cup B_4$
3	RW		A_1
4	S	{-1 2 4 5 10 14 15}	$A_1 \cup A_4$
5	RW		A_0
6	S	{5 9 10 15 19 23 24 26}	$A_0 \cup B_4$, $A_9 \cup B_0$
7	RW		B_0
8	S	{-1 2 5 9 14 16 20}	$A_0 \cup B_0$, $A_0 \cup B_1$
9	RW		B_1
10	S	{-4 0 1 4 5 9}	$A_9 \cup B_1$, $A_9 \cup B_2$
11	S	{1 2 5 7}	$B_2 \cup B_4$
12	S	{-4 0 1 4 5 9}	$A_9 \cup B_1$, $A_9 \cup B_2$
13	S	{-1 2 5 9 14 16 20}	$A_0 \cup B_1$
14	?	{-6 -1 0 2 7 8 10 15 16 19 22 24}	$B_2 \cup B_4$
15	S	{5 9 10 15 19 23 24 26}	$A_9 \cup B_1$
16	RW		B_2
17	RW		B_2
18	S	{-4 0 1 4 5 9}	$A_9 \cup B_2$, $A_9 \cup B_3$
19	RW		B_3
20	S	{-1 2 4 5 10 14 15}	$A_0 \cup A_2$
21	RW		A_2
22	S	{1 2 5 7}	$A_0 \cup B_4$
23	S	{-4 0 1 4 5 9}	$A_0 \cup A_2$
24	S	{-15 -12 -8 -7 -2 -1 1 2}	$A_0 \cup A_2$
25	S	{1 2 4 5 9}	$A_0 \cup A_2$
26	RW		A_0
27	R		
28	RW		A_0
29	RW		A_0
30	S	{-3 2 4 5 10 14 15}	$A_3 \cup A_7$
31	S	{-1 2 5 9 14 16 20}	$A_0 \cup A_7$, $A_3 \cup B_4$
32	S	{-4 0 1 4 5 9}	$A_0 \cup A_3$
33	S	{-15 -12 -8 -7 -2 -1 1 2}	$A_0 \cup A_2$
34	RW		A_3
35	RW		A_4
36	RW		A_5
37	RW		$A_4 \cup A_6 \cup A_7 \cup A_8$
38	S	{1 2 5 6}	$A_7 \cup A_8$
39	RW		A_1
40	S	{-24 -18 -5 -4 -3 -1}	$A_1 \cup A_7$, $A_1 \cup A_8$

Figure 4.1: Annotated score of *Mikka*
Iannis Xenakis, *Mikka* (Paris: Editions Salabert, 1972). Used with permission.

Figure 4.1: Annotated score of *Mikka*, cont.
Iannis Xenakis, *Mikka* (Paris: Editions Salabert, 1972). Used with permission.

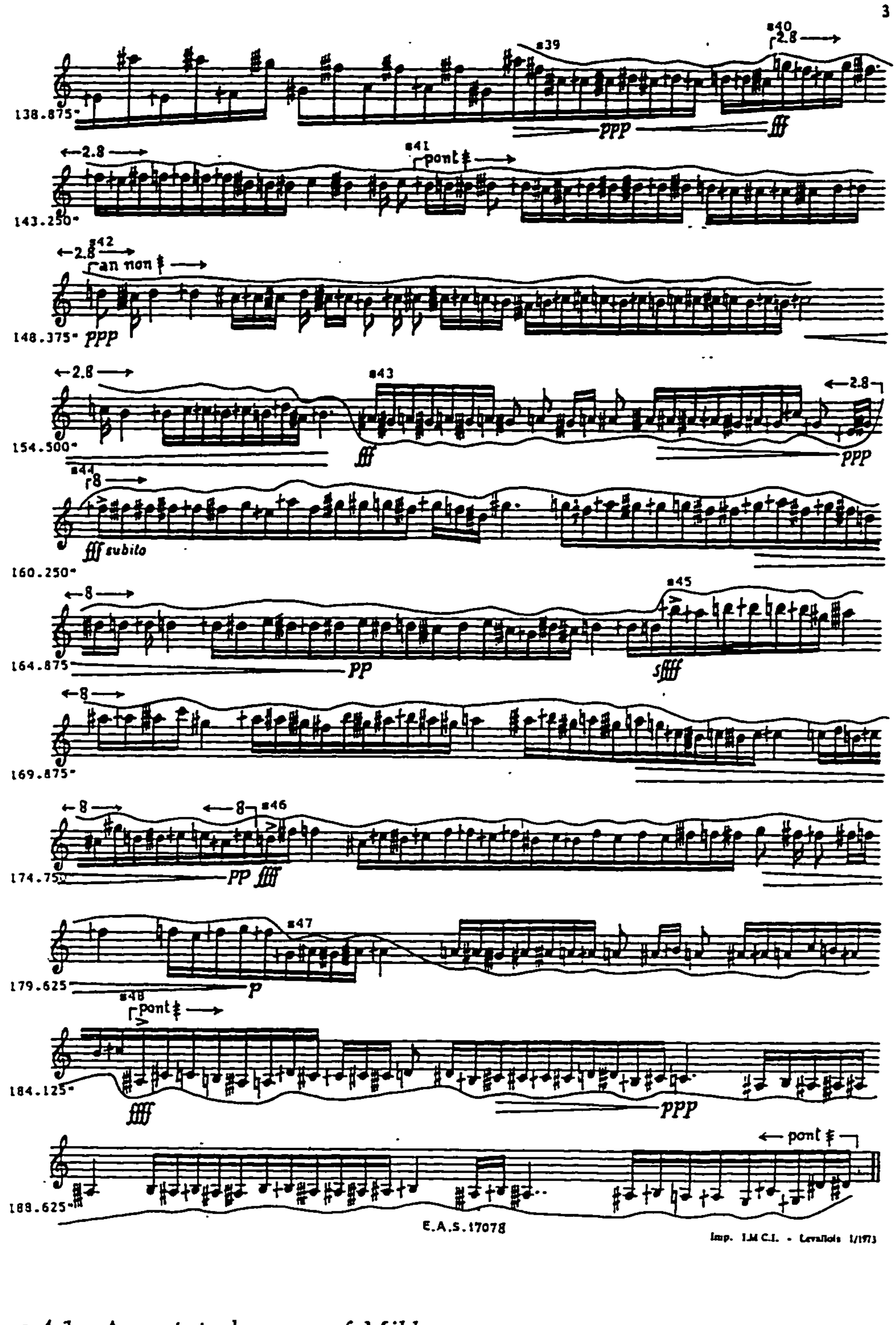


Figure 4.1: Annotated score of *Mikka*, cont.
Iannis Xenakis, *Mikka* (Paris: Editions Salabert, 1972). Used with permission.

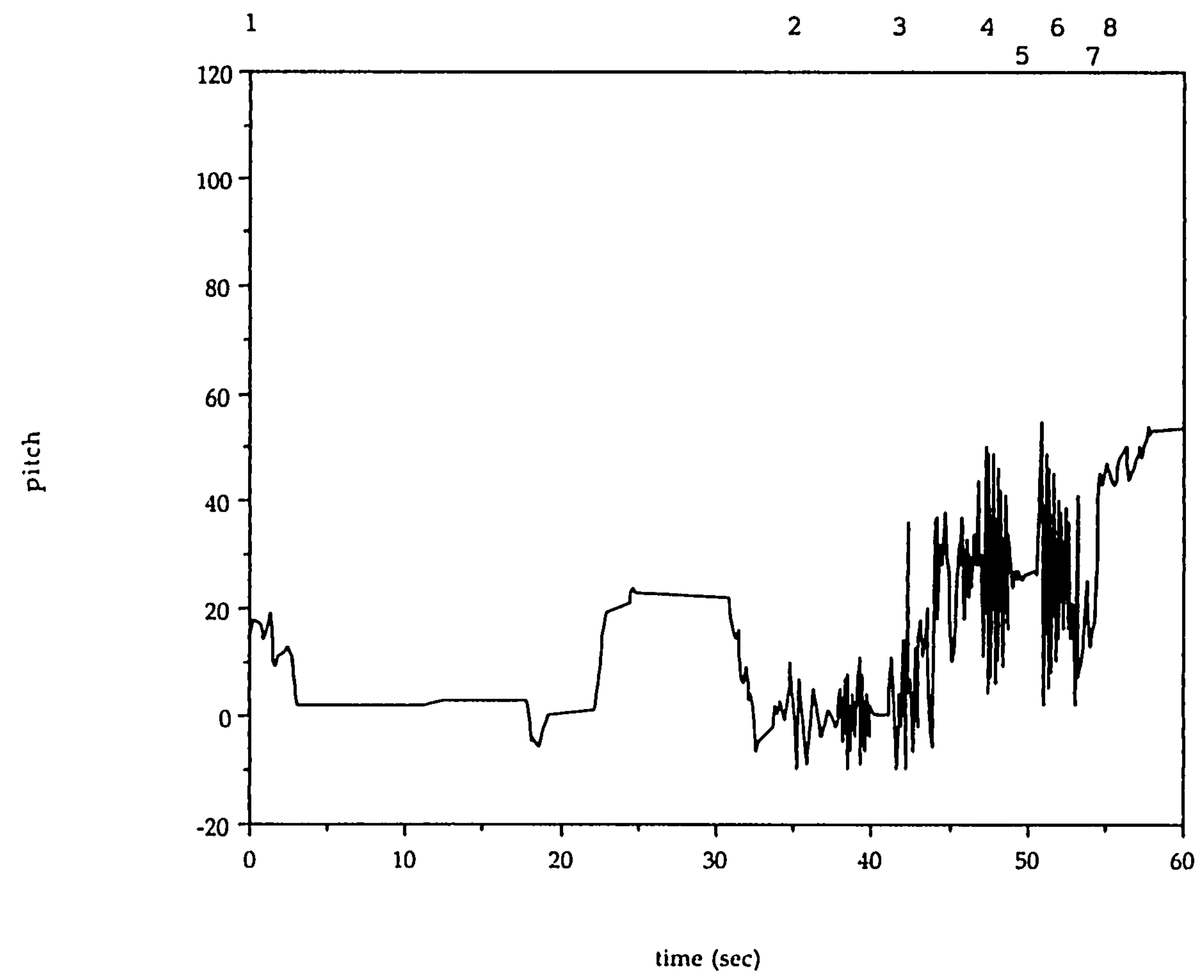


Figure 4.2: Graphic transcription of *Mikka*

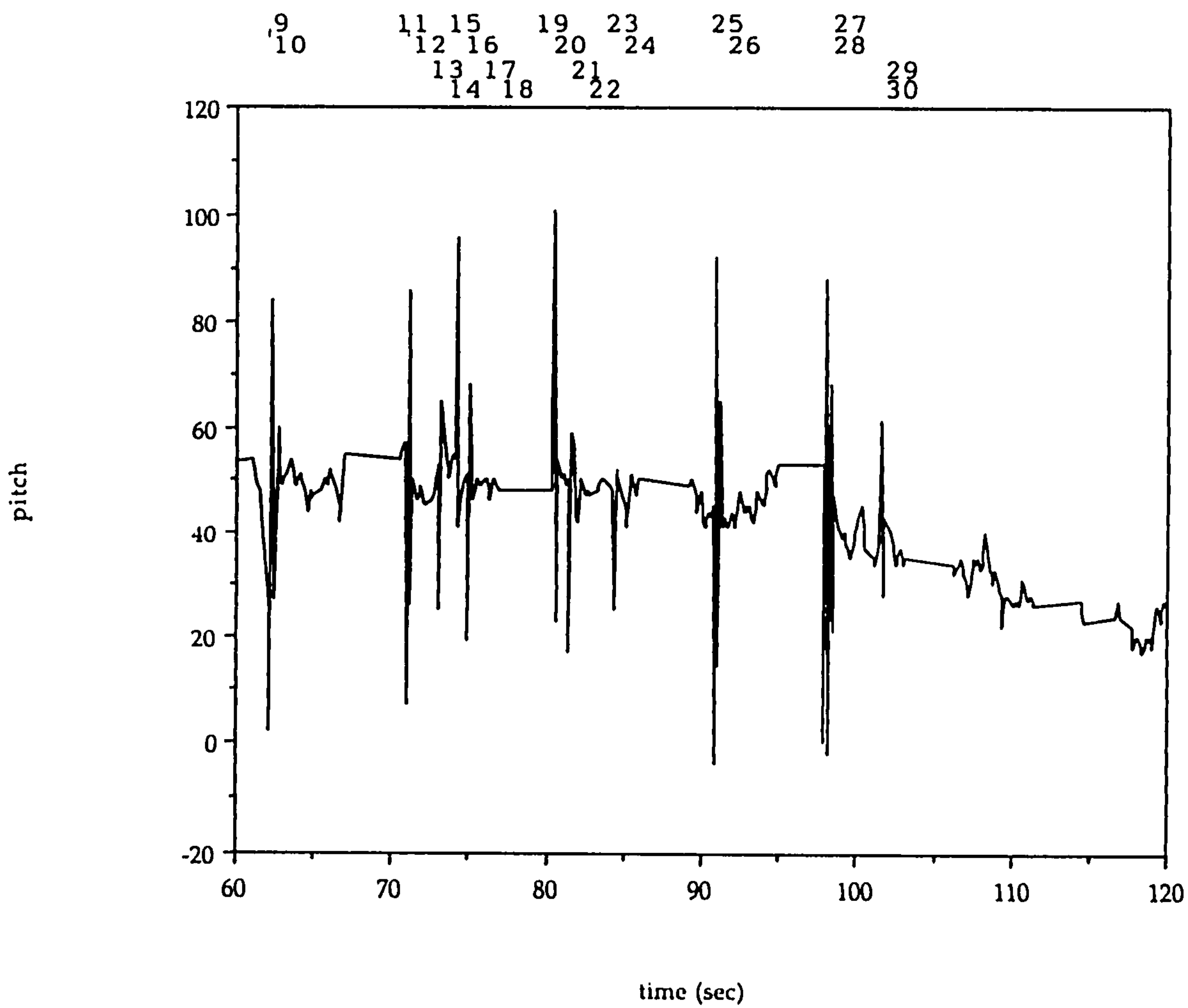


Figure 4.2: Graphic transcription of *Mikka*, cont.

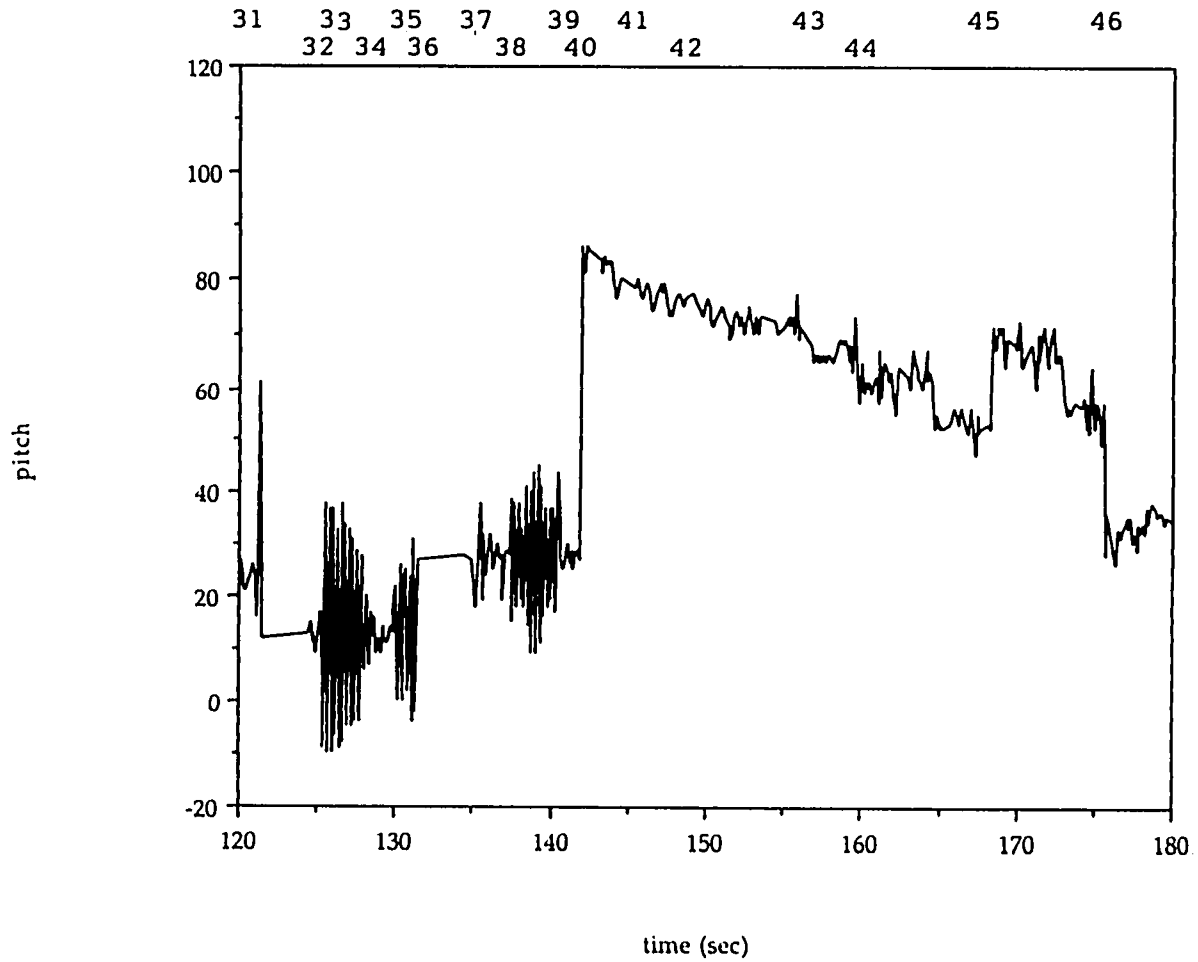


Figure 4.2: Graphic transcription of *Mikka*, cont.

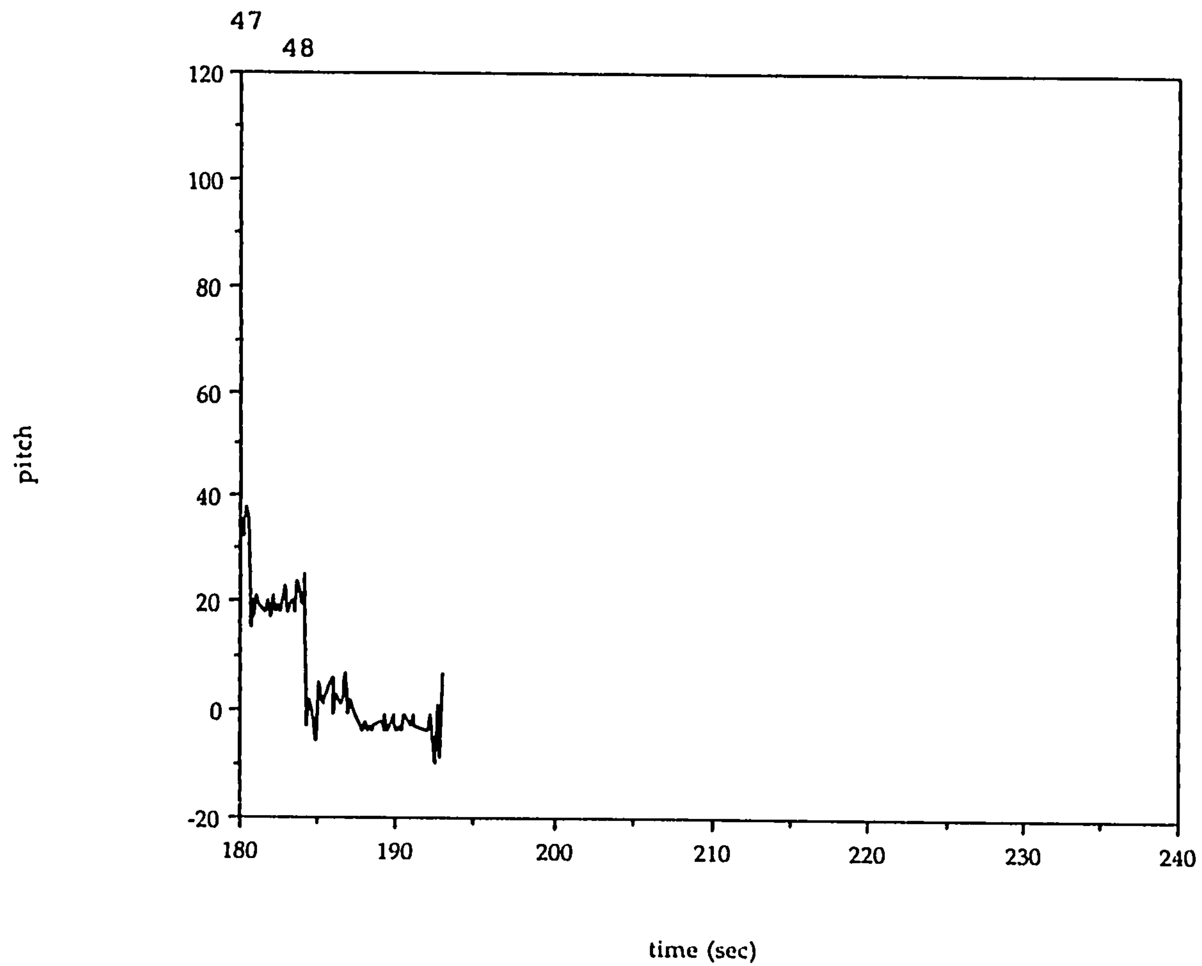


Figure 4.2: Graphic transcription of *Mikka*, cont.

Table 4.1: Segments in *Mikka*

segment	seconds*	duration	density	contour	intensity	articulation†
1	0.000	34.875	1.80	smooth	*mf*	an
2	34.875	6.375	4.86	smooth	*mf*	an
3	41.250	6.000	8.00	smooth	*mf-fff*	an
4	47.250	1.625	8.62	jagged	*fff*	an
5	48.875	2.125	5.18	smooth	*pp-fff*	an
6	51.000	2.500	8.00	jagged	*fff*	an
7	53.500	1.125	7.20	smooth	*fff*	an
8	54.625	7.500	4.00	smooth	*ppp*	sp
9	62.125	0.500	8.00	jagged	*fff*	an
10	62.625	8.250	4.12	smooth	*ppp-ff*	sp
11	70.875	0.500	8.00	jagged	*ff*	an
12	71.875	1.625	6.15	smooth	*ppp*	sp
13	73.000	0.375	8.00	jagged	*fff*	an
14	73.375	0.750	8.00	smooth	*fff*	an
15	74.125	0.375	8.00	jagged	*fff*	an
16	74.500	0.375	8.00	smooth	*fff*	an
17	74.875	0.500	8.00	jagged	*fff*	an
18	75.375	5.000	2.00	smooth	*fff*	an
19	80.375	0.500	8.00	jagged	*fff*	an
20	80.875	0.500	8.00	smooth	*fff*	an
21	81.375	0.375	8.00	jagged	*fff*	an
22	81.750	2.500	4.80	smooth	*ppp*	sp
23	84.250	0.375	8.00	jagged	*ppp*	sp
24	84.625	6.125	3.76	smooth	*ppp-fff*	sp
25	90.750	0.750	8.00	jagged	*fff*	an
26	91.500	6.500	4.46	smooth	*fff*	an
27	98.000	0.625	8.00	jagged	*fff*	an
28	98.625	2.875	6.96	smooth	*ppp*	sp trem
29	101.500	0.500	8.00	jagged	*ppp*	sp trem
30	102.000	19.125	4.34	smooth	*ppp-fff*	sp trem, sp, an
31	121.125	3.375	1.19	jagged	*fff*	an
32	124.500	1.000	8.00	smooth	*ppp-fff*	an
33	125.500	3.000	8.00	jagged	*ppp-fff*	an
34	128.500	1.750	8.00	smooth	*pp-fff*	an
35	130.250	1.250	8.00	jagged	*pp*	an
36	131.500	3.500	0.57	smooth	*pp*	sp
37	135.000	2.500	8.00	smooth	*fff*	sp
38	137.500	3.125	8.00	jagged	*fff*	sp
39	140.625	1.375	8.00	smooth	*ppp-fff*	sp
40	142.000	3.875	4.90	smooth	*fff*	sp
41	145.875	2.500	7.60	smooth	*fff*	sp trem
42	148.375	8.500	5.17	smooth	*ppp-fff*	an
43	156.875	3.375	6.82	smooth	*ppp-fff*	an

* Times are given for the beginnings of segments only, since all of the segments in *Mikka* follow one another in direct succession.

† an = *arco normale*, sp = *sul ponticello*, sp trem = *sul ponticello tremolo*

Table 4.1: Segments in *Mikka*, cont.

segment	seconds	duration	density	contour	intensity	articulation
44	160.250	8.250	6.67	smooth	*pp-fff*	an
45	168.500	7.250	6.76	smooth	*pp-ffff*	an
46	175.750	5.000	6.40	smooth	*p-ffff*	an
47	180.750	3.625	6.62	smooth	*p*	an
48	184.375	7.000	5.97	smooth	*ppp-ffff*	sp trem

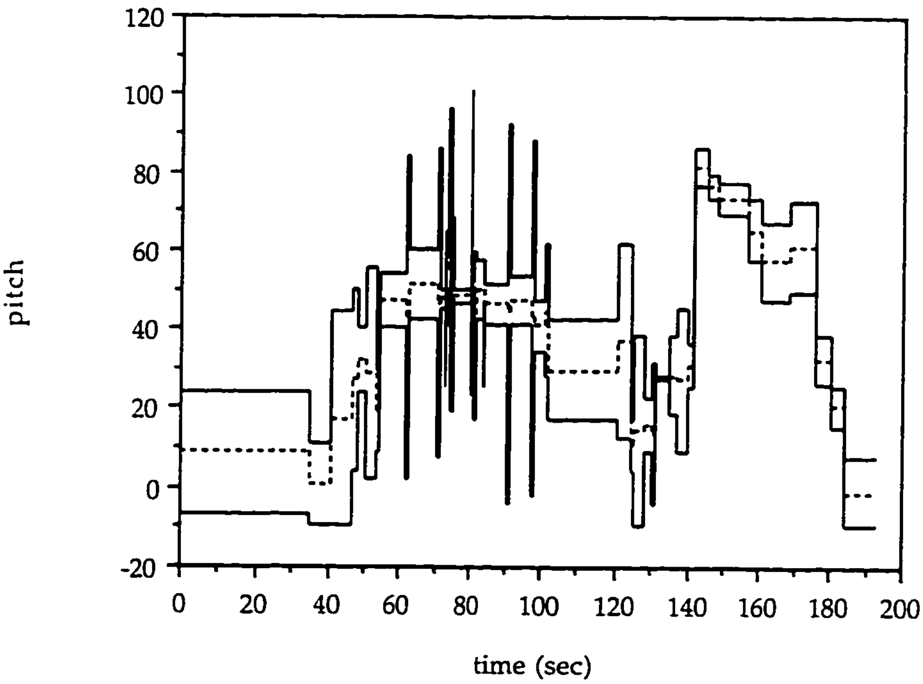


Figure 4.3: Minimum, maximum, and mean pitch levels per segment in *Mikka*

Table 4.2: Summary of Temporal Structure of *Mikka* (inside-time)

Duration of whole work: 193.25"

part:	1		
duration:	98.625" (.51)*		
section:	1		2
segments:	1-7		8-27
duration:	54.625" (.55)		44" (.45)

part:	2		
duration:	94.625" (.49)		
supersection:	A		B
segments:	28-39		40-48
duration:	43.375" (.46)		51.25" (.54)
section:	3	4	5
segments:	28-30	31-9	40-48
duration:	22.5" (.52)	20.875" (.48)	51.25" (1)

* The quantities in parentheses indicate the proportion of the whole, part, or supersection that is occupied by the given duration.

Table 4.3: Summary of Temporal Structure of *Mikka* (outside-time)

Duration of whole work: 193.25"

contour:	smooth	alternating
sections:*	1, 3, 5	2, 4
segments:	1-3, 28-30, 40-8	4-27, 31-9
duration:	121" (.626)	72.25" (.374)

articulation:	an	sp & sp trem	
duration:	119.375" (.618)	73.875" (.382)	
articulation:		sp	sp trem
duration:		48.25" (.65)	25.625" (.35)

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* Grouping of segments into sections follows secondary segmentation (see text).

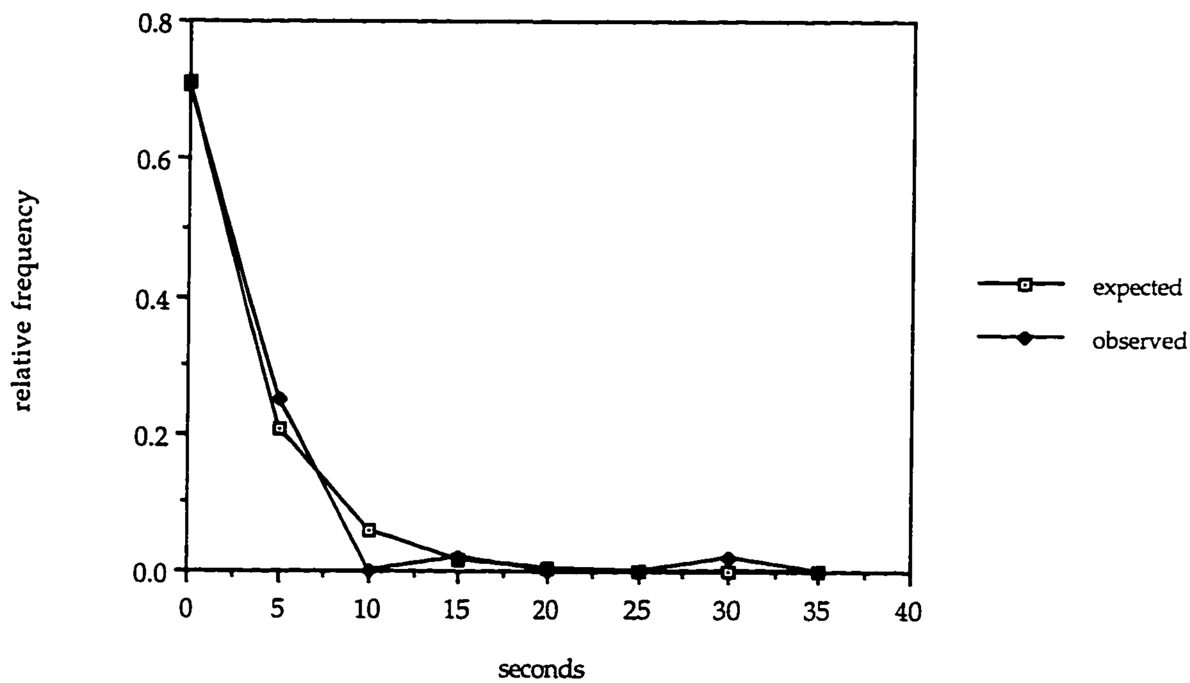

Figure 4.4: Comparative histogram showing distribution of segment durations in *Mikka* versus exponential distribution, $\partial = 0.248$

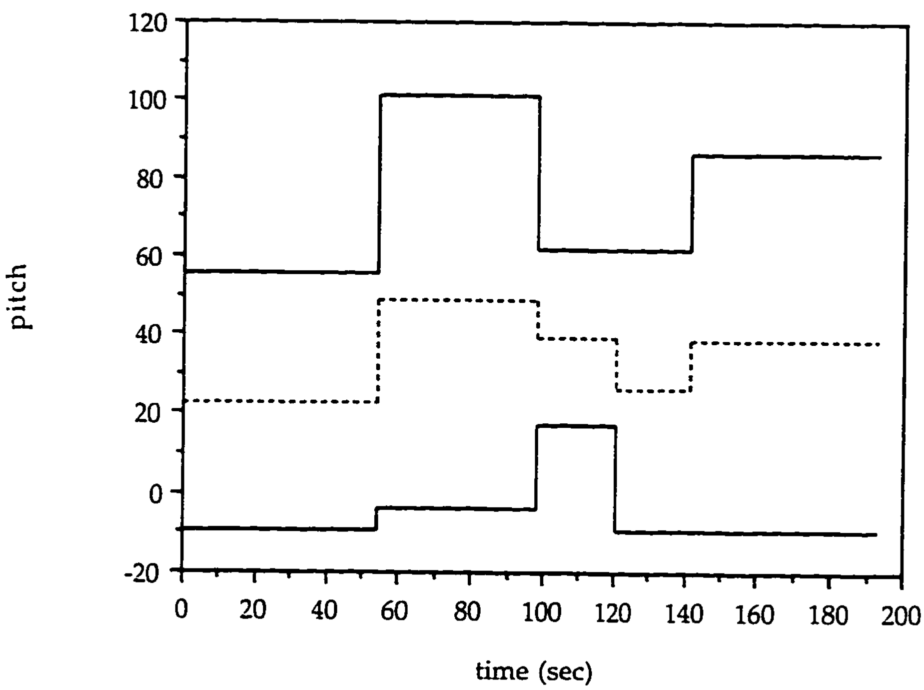

Figure 4.5: Minimum, maximum, and mean pitch levels per section in *Mikka*

Figure 4.6: Annotated score of *Mikka "S"*
Iannis Xenakis, *Mikka "S"* (Paris: Editions Salabert, 1976). Used with permission.

Figure 4.6: Annotated score of *Mikka "S"*, cont.
Iannis Xenakis, *Mikka "S"* (Paris: Editions Salabert, 1976). Used with permission.

Figure 4.6: Annotated score of *Mikka "S"*, cont.
Iannis Xenakis, *Mikka "S"* (Paris: Editions Salabert, 1976). Used with permission.

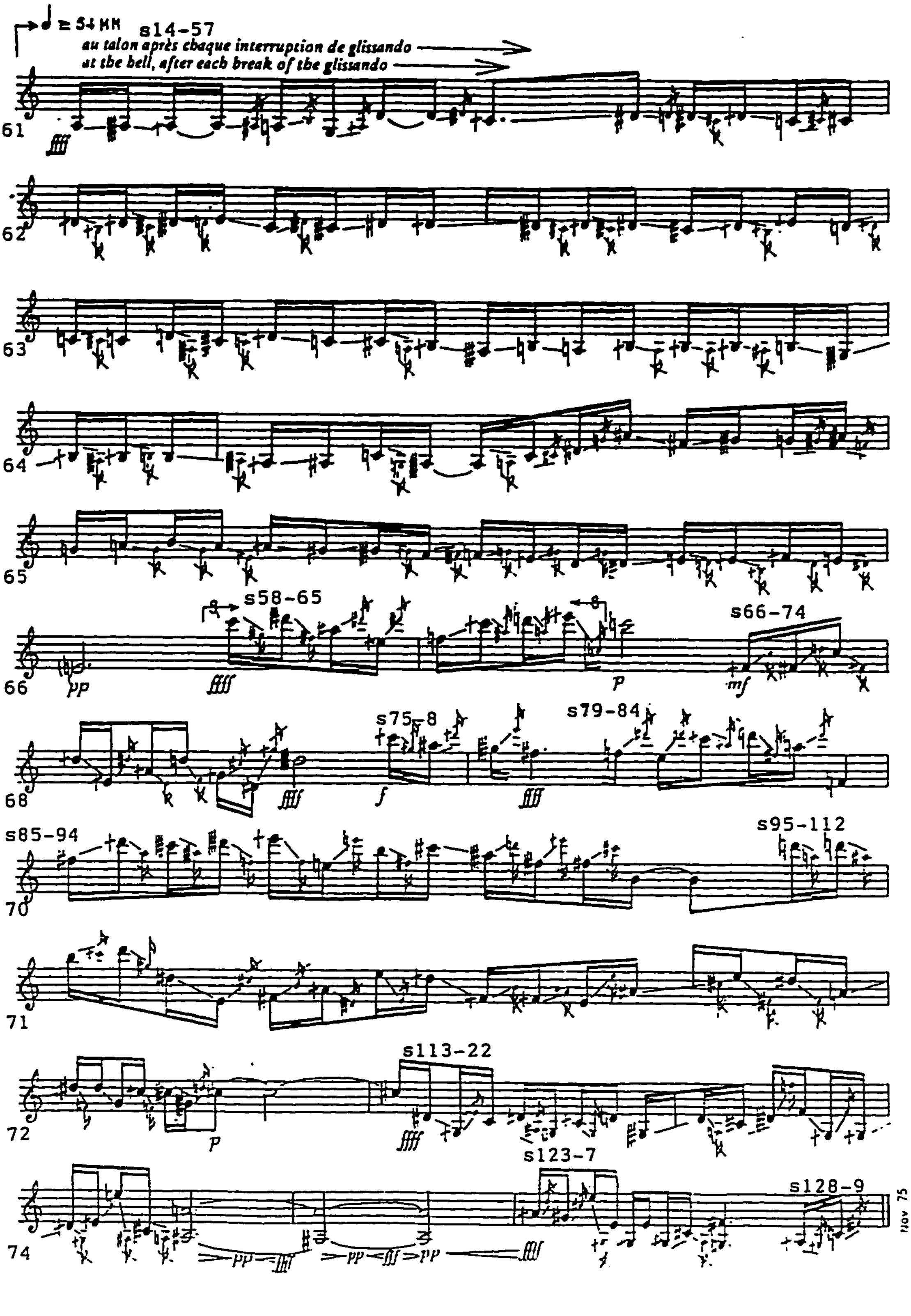

Figure 4.6: Annotated score of *Mikka "S"*, cont.
Iannis Xenakis, *Mikka "S"* (Paris: Editions Salabert, 1976). Used with permission.

Table 4.4: Segments in *Mikka "S"*

segment	measures	seconds	duration	intensity
1	1-3	0.000-5.278	5.278	*ppp*
2	2-4	2.778-8.750	5.972	*ppp*
3	4-6	7.639-12.917	5.278	*ppp*
4	5-18	10.278-38.889	28.611	*ppp-fff*
5	7-45	13.889-100.000	86.111	*ppp-fff*
6	23-26	49.028-57.361	8.333	*p-fff*
7	27-50	59.444-109.861	50.417	*p-fff*
8	47-50	103.611-110.833	7.222	*p-fff*
9	50-52	109.861-114.583	4.722	*p-fff*
10	51-53	111.667-117.778	6.111	*p-fff*
11	52-54	114.861-119.861	5.000	*p-fff*
12	54-60	118.056-133.333	15.278	*p-fff*
13	55-60	120.000-133.333	13.333	*fff*
14	61	133.333-134.444	1.111	*ffff*
15	61	134.444-134.722	0.278	*ffff*
16	61	134.722-135.000	0.278	*ffff*
17	61	135.000-135.556	0.556	*ffff*
18	61	135.556-136.667	1.111	*ffff*
19	61	136.667-136.944	0.278	*ffff*
20	61	136.944-137.500	0.556	*ffff*
21	61-62	137.500-138.056	0.556	*ffff*
22	62	138.056-138.333	0.278	*ffff*
23	62	138.333-138.611	0.278	*ffff*
24	62	138.611-139.167	0.556	*ffff*
25	62	139.167-140.556	1.389	*ffff*
26	62	140.556-140.833	0.278	*ffff*
27	62	140.833-141.667	0.833	*ffff*
28	62	141.667-142.222	0.556	*ffff*
29	63	142.222-142.500	0.278	*ffff*
30	63	142.500-143.056	0.556	*ffff*
31	63	143.056-143.333	0.278	*ffff*
32	63	143.333-144.167	0.833	*ffff*
33	63	144.167-145.556	1.389	*ffff*
34	63	145.556-145.833	0.278	*ffff*
35	63	145.833-146.111	0.278	*ffff*
36	63-64	146.111-146.944	0.833	*ffff*
37	64	146.944-147.222	0.278	*ffff*
38	64	147.222-147.778	0.556	*ffff*
39	64	147.778-148.611	0.833	*ffff*
40	64	148.611-149.167	0.556	*ffff*
41	64	149.167-149.444	0.278	*ffff*
42	64	149.444-149.722	0.278	*ffff*
43	64	149.722-150.833	1.111	*ffff*
44	64	150.833-151.111	0.278	*ffff*
45	65	151.111-151.667	0.556	*ffff*
46	65	151.667-151.944	0.278	*ffff*
47	65	151.944-152.222	0.278	*ffff*
48	65	152.222-153.056	0.833	*ffff*

Table 4.4: Segments in *Mikka "S"*, cont.

segment	measures	seconds	duration	intensity
49	65	153.056-153.333	0.278	*ffff*
50	65	153.333-153.611	0.278	*ffff*
51	65	153.611-153.889	0.278	*ffff*
52	65	153.889-154.167	0.278	*ffff*
53	65	154.167-154.722	0.556	*ffff*
54	65	154.722-155.000	0.278	*ffff*
55	65	155.000-155.278	0.278	*ffff*
56	65	155.278-155.556	0.278	*ffff*
57	66	155.556-158.889	3.333	*pp*
58	66	158.889-159.167	0.278	*ffff*
59	66	159.167-159.444	0.278	*ffff*
60	66	159.444-159.722	0.278	*ffff*
61	66	159.722-160.000	0.278	*ffff*
62	67	160.000-160.556	0.556	*ffff*
63	67	160.556-160.833	0.278	*ffff*
64	67	160.833-161.389	0.556	*ffff*
65	67	161.389-163.611	2.222	*p*
66	67	163.611-163.889	0.278	*mf*
67	67	163.889-164.167	0.278	*mf*
68	67	164.167-164.444	0.278	*mf*
69	68	164.444-165.000	0.556	*mf*
70	68	165.000-165.278	0.278	*mf*
71	68	165.278-165.556	0.278	*mf*
72	68	165.556-165.833	0.278	*mf*
73	68	165.833-166.111	0.278	*mf*
74	68	166.111-168.333	2.222	*ffff*
75	68	168.333-168.611	0.278	*f*
76	68	168.611-168.889	0.278	*f*
77	69	168.889-169.167	0.278	*f*
78	69	169.167-170.833	1.667	*ffff*
79	69	170.833-171.111	0.278	*ffff*
80	69	171.111-171.389	0.278	*ffff*
81	69	171.389-171.667	0.278	*ffff*
82	69	171.667-171.944	0.278	*ffff*
83	69	171.944-172.222	0.278	*ffff*
84	69	172.222-173.333	1.111	*ffff*
85	70	173.333-173.889	0.556	*ffff*
86	70	173.889-174.167	0.278	*ffff*
87	70	174.167-174.444	0.278	*ffff*
88	70	174.444-174.722	0.278	*ffff*
89	70	174.722-175.000	0.278	*ffff*
90	70	175.000-175.278	0.278	*ffff*
91	70	175.278-175.833	0.556	*ffff*
92	70	175.833-176.111	0.278	*ffff*
93	70	176.111-176.389	0.278	*ffff*
94	70	176.389-172.222	0.833	*ffff*
95	70	177.222-177.500	0.278	*ffff*
96	70	177.500-177.778	0.278	*ffff*

Table 4.4: Segments in *Mikka "S"*, cont.

segment	measures	seconds	duration	intensity
97	71	177.778-178.056	0.278	*ffff*
98	71	178.056-178.333	0.278	*ffff*
99	71	178.333-178.889	0.556	*ffff*
100	71	178.889-179.167	0.278	*ffff*
101	71	179.167-179.444	0.278	*ffff*
102	71	179.444-179.722	0.278	*ffff*
103	71	179.722-180.278	0.556	*ffff*
104	71	180.278-180.556	0.278	*ffff*
105	71	180.556-180.833	0.278	*ffff*
106	71	180.833-181.389	0.556	*ffff*
107	71	181.389-181.667	0.278	*ffff*
108	71	181.667-181.944	0.278	*ffff*
109	71-72	181.944-182.500	0.556	*ffff*
110	72	182.500-183.333	0.833	*ffff*
111	72	183.333-183.889	0.556	*ffff*
112	72-73	183.889-186.944	3.056	*p*
113	73	186.944-187.500	0.556	*ffff*
114	73	187.500-188.056	0.556	*ffff*
115	73	188.056-188.611	0.556	*ffff*
116	73	188.611-189.722	1.111	*ffff*
117	73	189.722-190.278	0.556	*ffff*
118	73	190.278-190.833	0.556	*ffff*
119	73-74	190.833-191.389	0.556	*ffff*
120	74	191.389-191.944	0.556	*ffff*
121	74	191.944-192.222	0.278	*ffff*
122	74-75	192.222-200.000	7.778	*pp-ffff*
123	76	200.000-100.278	0.278	*ffff*
124	76	200.278-200.556	0.278	*ffff*
125	76	200.566-200.833	0.278	*ffff*
126	76	200.883-202.222	1.389	*ffff*
127	76	202.222-203.889	1.667	*ffff*
128	76	203.889-204.167	0.278	*ffff*
129	76	204.167-204.444	0.278	*ffff*

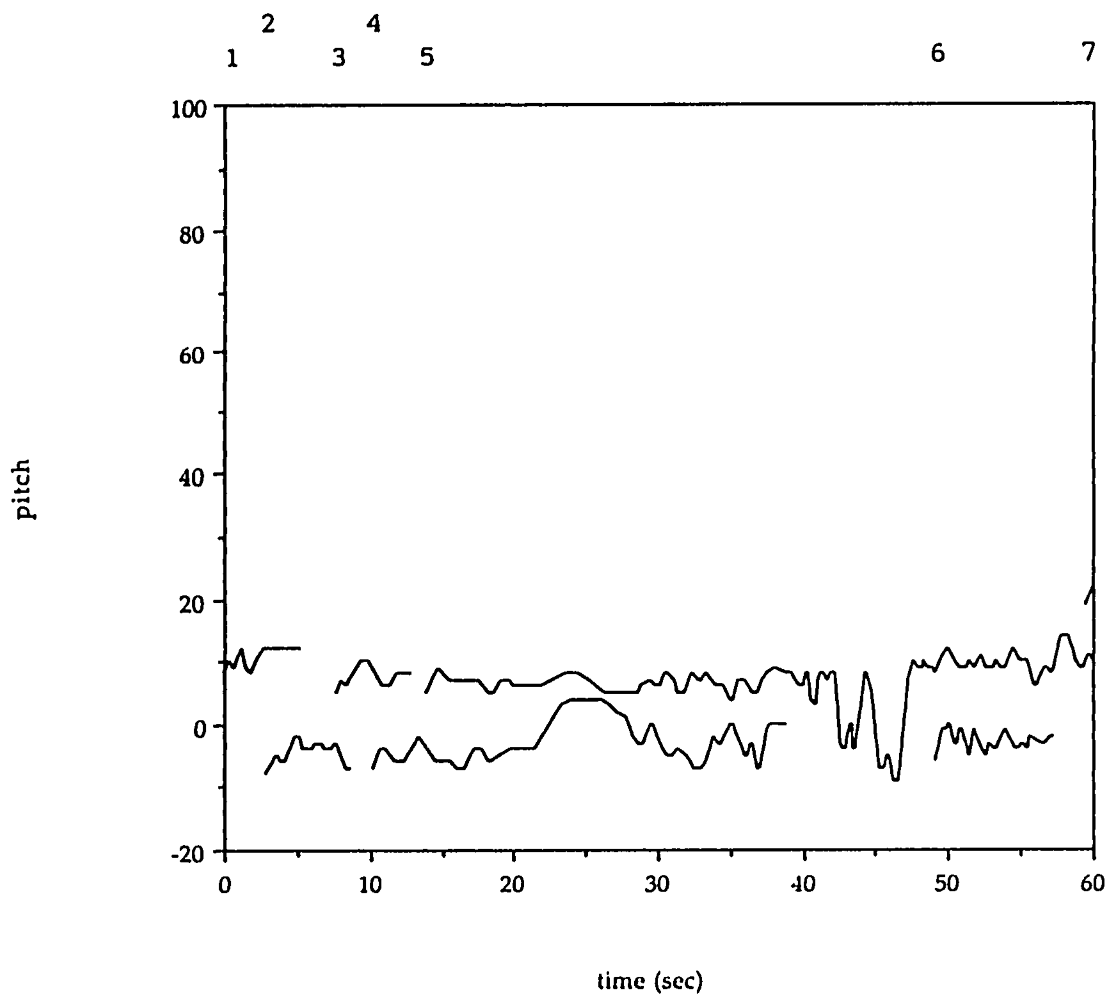

Figure 4.7: Graphic transcription of *Mikka "S"*

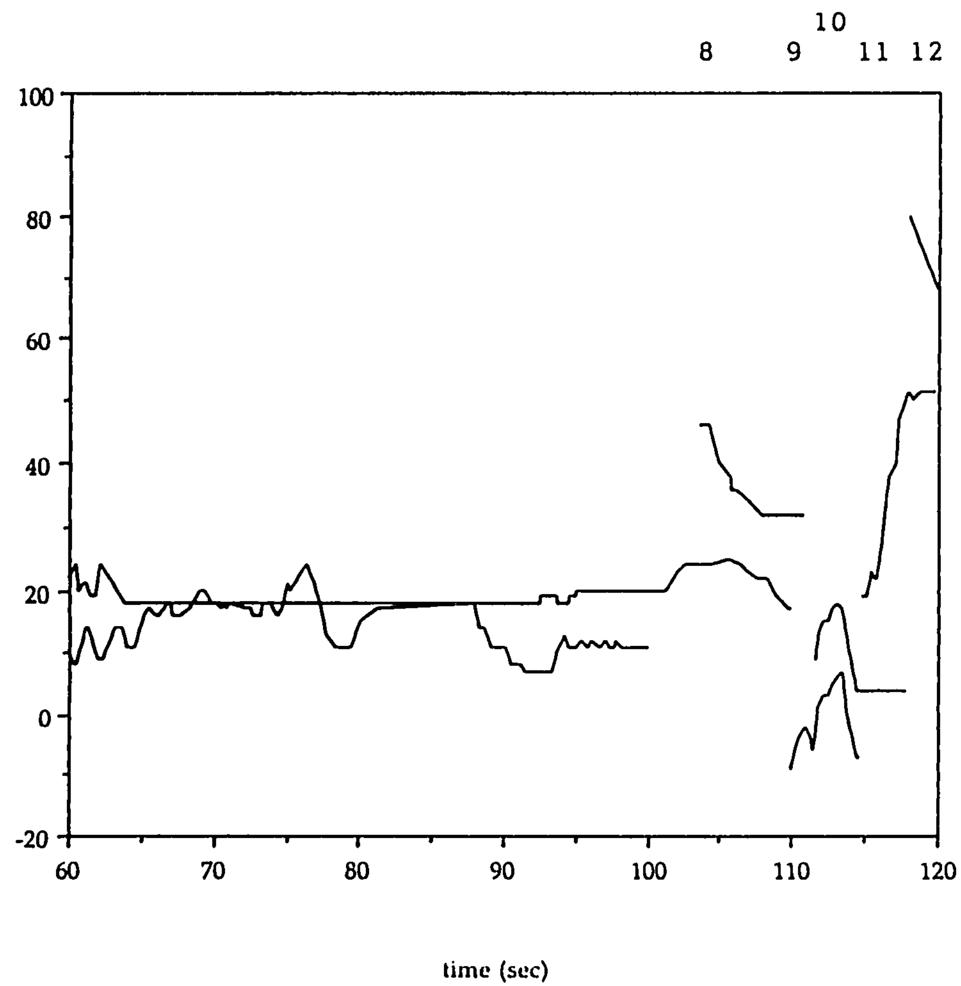

Figure 4.7: Graphic transcription of *Mikka "S"*, cont.

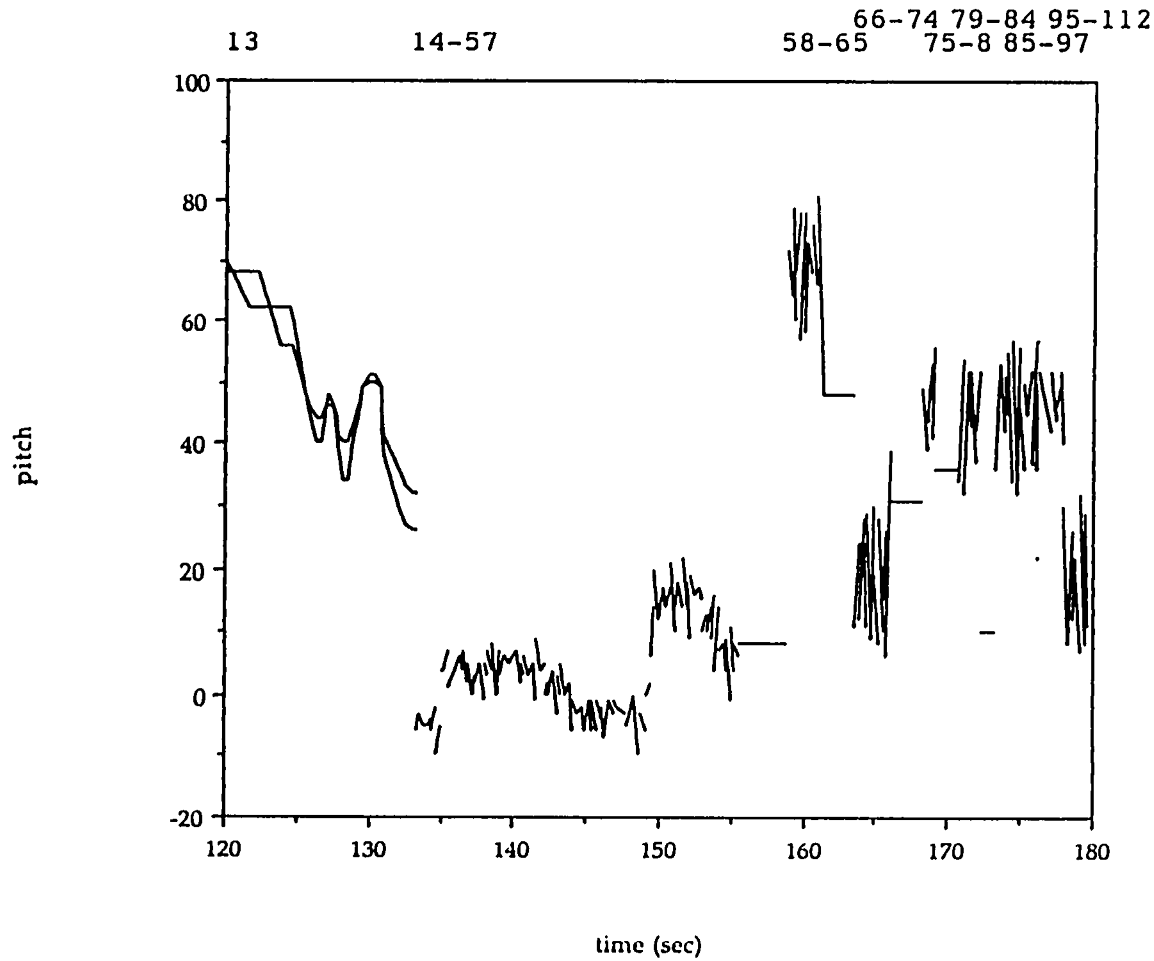

Figure 4.7: Graphic transcription of *Mikka "S"*, cont.

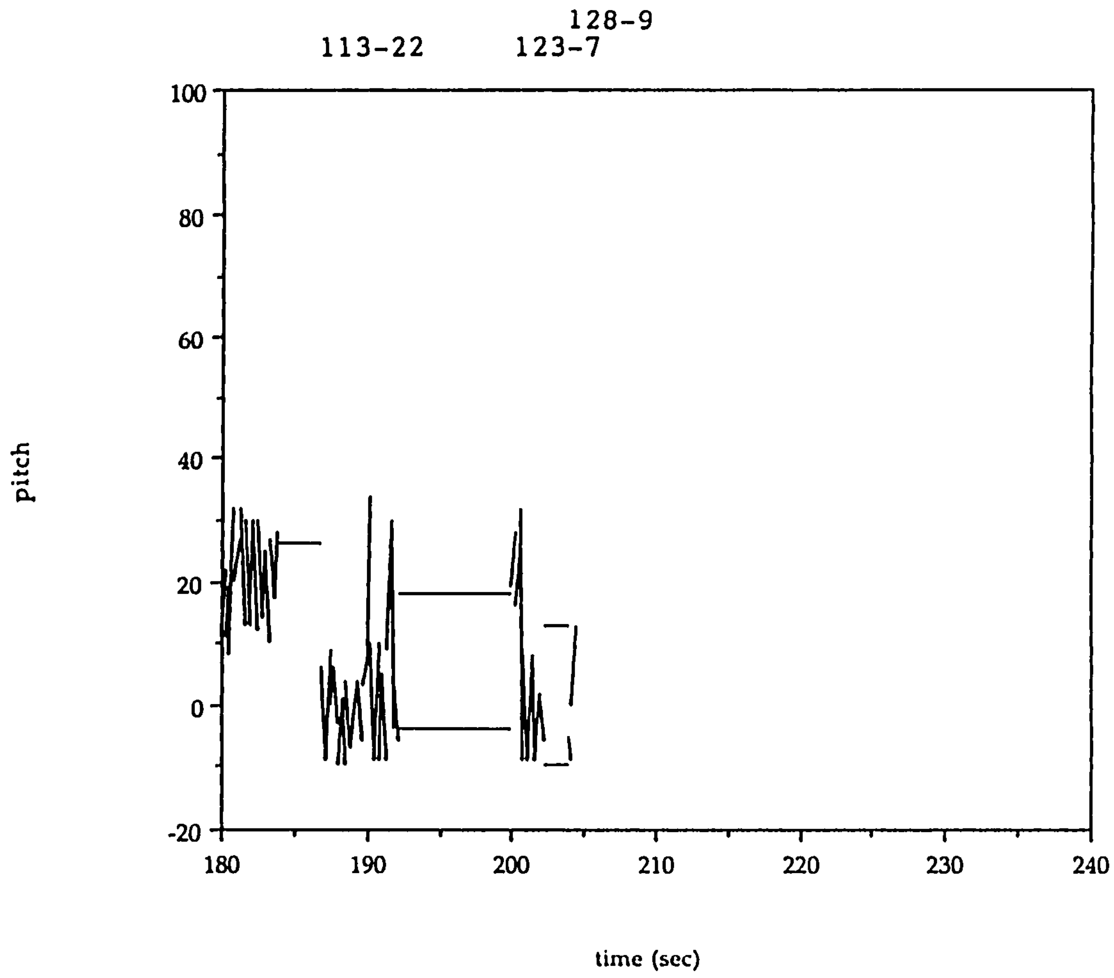

Figure 4.7: Graphic transcription of *Mikka "S"*, cont.

Table 4.5: Durations between beginning times of segments in *Mikka "S"*

segment	begins in measure	begins at time (sec)	time (sec) until start of next segment
1	1	0.000	2.778
2	2	2.778	4.861
3	4	7.639	2.639
4	5	10.278	3.611
5	7	13.889	35.139
6	23	49.028	10.416
7	27	59.444	44.167
8	47	103.611	6.250
9	50	109.861	1.806
10	51	111.667	3.194
11	52	114.861	3.195
12	54	118.056	1.944
13	55	120.000	13.333
14	61	133.333	1.111
15	61	134.444	0.278
16	61	134.722	0.278
17	61	135.000	0.556
18	61	135.556	1.111
19	61	136.667	0.278
20	61	136.944	0.556
21	61	137.500	0.556
22	62	138.056	0.278
23	62	138.333	0.278
24	62	138.611	0.556
25	62	139.167	1.389
26	62	140.556	0.278
27	62	140.833	0.833
28	62	141.667	0.556
29	63	142.222	0.278
30	63	142.500	0.556
31	63	143.056	0.278
32	63	143.333	0.833
33	63	144.167	1.389
34	63	145.556	0.278
35	63	145.833	0.278
36	63	146.111	0.833
37	64	146.944	0.278
38	64	147.222	0.556
39	64	147.778	0.833
40	64	148.611	0.556
41	64	149.167	0.278
42	64	149.444	0.278
43	64	149.722	1.111
44	64	150.833	0.278
45	65	151.111	0.556
46	65	151.667	0.278
47	65	151.944	0.278

Table 4.5: Durations between beginning times of segments in *Mikka "S"*, cont.

segment	begins in measure	begins at time (sec)	time (sec) until start of next segment
48	65	152.222	0.833
49	65	153.056	0.278
50	65	153.333	0.278
51	65	153.611	0.278
52	65	153.889	0.278
53	65	154.167	0.556
54	65	154.722	0.278
55	65	155.000	0.278
56	65	155.278	0.278
57	66	155.556	3.333
58	66	158.889	0.278
59	66	159.167	0.278
60	66	159.444	0.278
61	66	159.722	0.278
62	67	160.000	0.556
63	67	160.556	0.278
64	67	160.833	0.556
65	67	161.389	2.222
66	67	163.611	0.278
67	67	163.889	0.278
68	67	164.167	0.278
69	68	164.444	0.556
70	68	165.000	0.278
71	68	165.278	0.278
72	68	165.556	0.278
73	68	165.833	0.278
74	68	166.111	2.222
75	68	163.333	0.278
76	68	168.611	0.278
77	69	168.889	0.278
78	69	169.167	1.667
79	69	170.833	0.278
80	69	171.111	0.278
81	69	171.389	0.278
82	69	171.667	0.278
83	69	171.944	0.278
84	69	172.222	1.111
85	70	173.333	0.556
86	70	173.889	0.278
87	70	174.167	0.278
88	70	174.444	0.278
89	70	174.722	0.278
90	70	175.000	0.278
91	70	175.278	0.556
92	70	175.833	0.278
93	70	176.111	0.278
94	70	176.389	0.833

Table 4.5: Durations between beginning times of segments in *Mikka "S"*, cont.

segment	begins in measure	begins at time (sec)	time (sec) until start of next segment
95	70	177.222	0.278
96	70	177.500	0.278
97	71	177.778	0.278
98	71	178.056	0.278
99	71	178.333	0.556
100	71	178.889	0.278
101	71	179.167	0.278
102	71	179.444	0.278
103	71	179.722	0.556
104	71	180.278	0.278
105	71	180.556	0.278
106	71	180.833	0.556
107	71	181.389	0.278
108	71	181.667	0.278
109	71	181.944	0.556
110	72	182.500	0.833
111	72	183.333	0.556
112	72	183.889	3.056
113	73	186.944	0.556
114	73	187.500	0.556
115	73	188.056	0.556
116	73	188.611	1.111
117	73	189.722	0.556
118	73	190.278	0.556
119	73	190.833	0.556
120	74	191.389	0.556
121	74	191.944	0.278
122	74	192.222	7.778
123	76	200.000	0.278
124	76	200.278	0.278
125	76	200.566	0.278
126	76	200.883	1.389
127	76	202.222	1.667
128	76	203.889	0.278
129	76	204.167	0.278

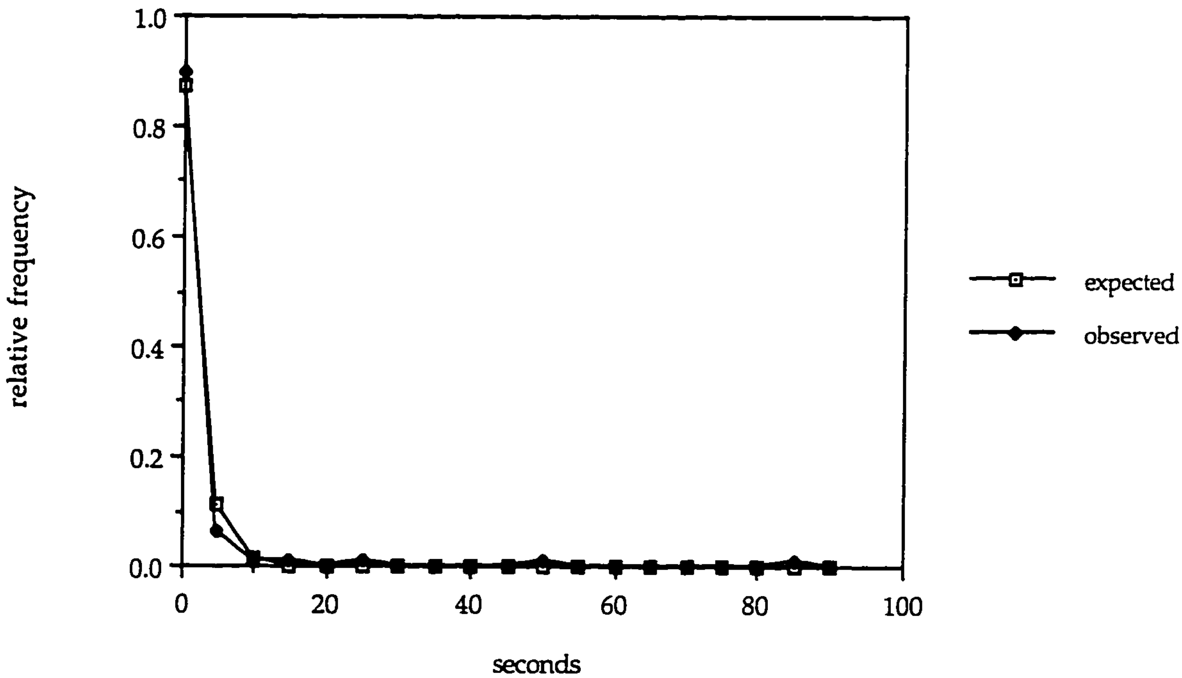

Figure 4.8: Comparative histogram showing distribution of segment durations in *Mikka* "S" versus exponential distribution, ∂ = 0.412

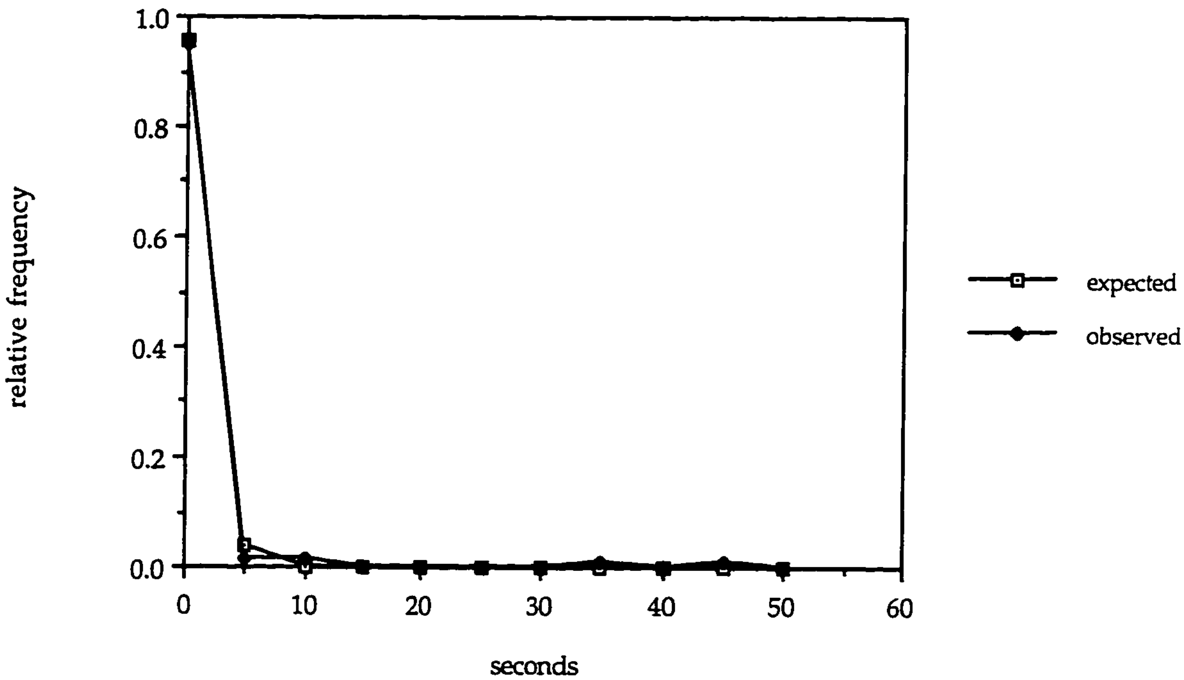

Figure 4.9: Comparative histogram showing distribution of time-point intervals between initiations of segments in *Mikka "S"* versus exponential distribution, $\partial = 0.631$

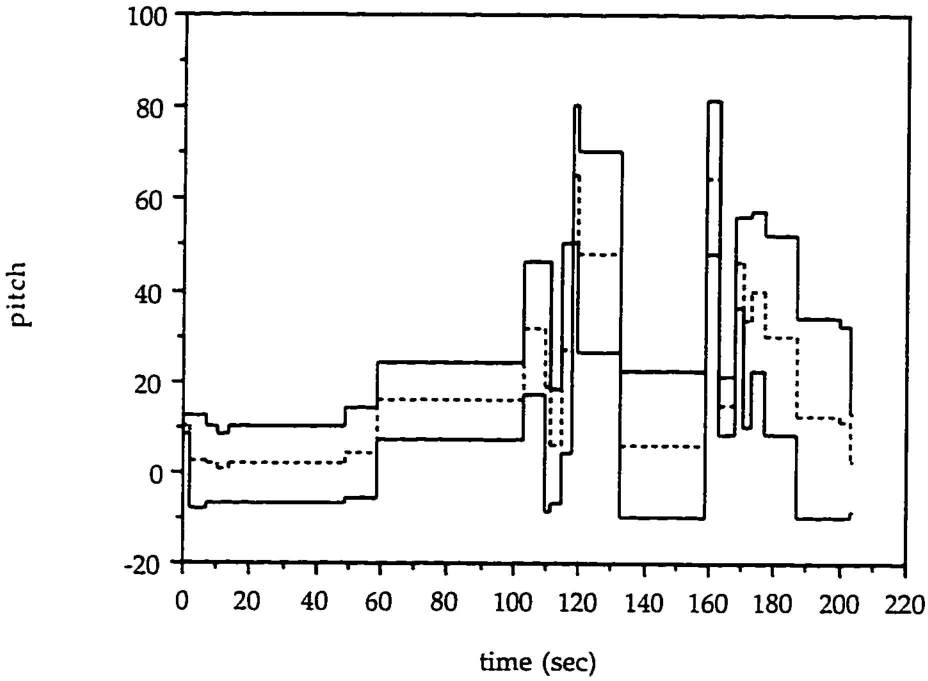

Figure 4.10: Minimum, maximum, and mean pitch levels for segments 1-13 and for groups of segments from segments 14-129 in *Mikka "S"*

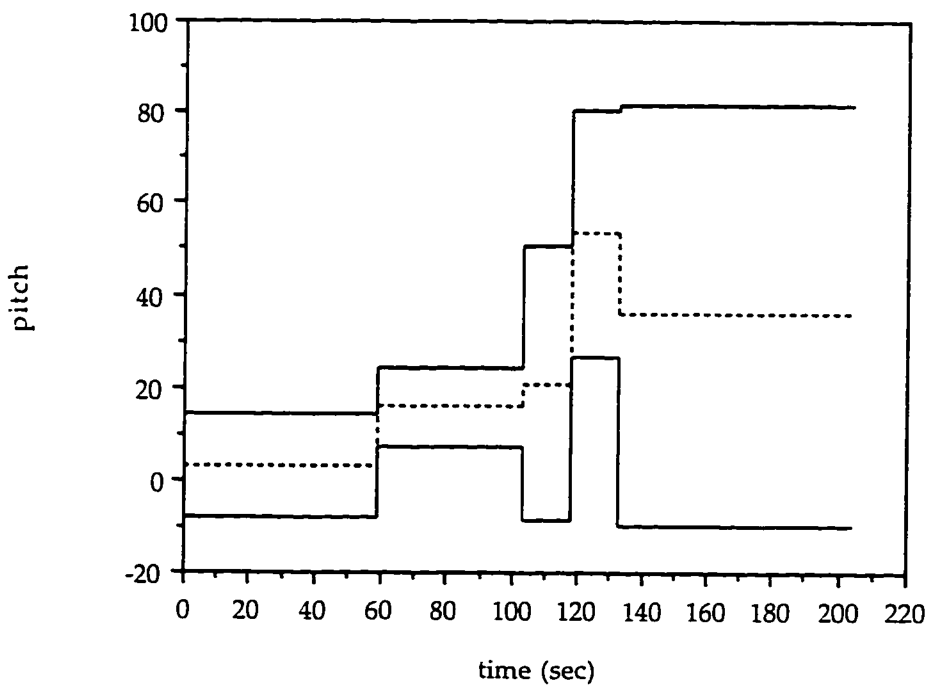

Figure 4.11: Minimum, maximum, and mean pitch levels per section in *Mikka "S"*

Table 4.6: Summary of Temporal Structure of *Mikka "S"*

Duration of whole work: 204.444"

part:	1			
duration:	133.333" (.65)*			
section:	1	2	3	4
segments:	1-6	7	8-11	12-13
duration:	59.444" (.45)	44.167" (.33)	14.445" (.11)	15.277" (.11)

part:	2	
duration:	71.111" (.35)	
section:	5	
segments:	14-129	
duration:	71.111" (1.00)	
section:	a	b
segments:	14-57	58-129
duration:	25.556" (.36)	45.555" (.64)

* The quantities in parentheses indicate the proportion of the whole, part, or section that is occupied by the given duration.

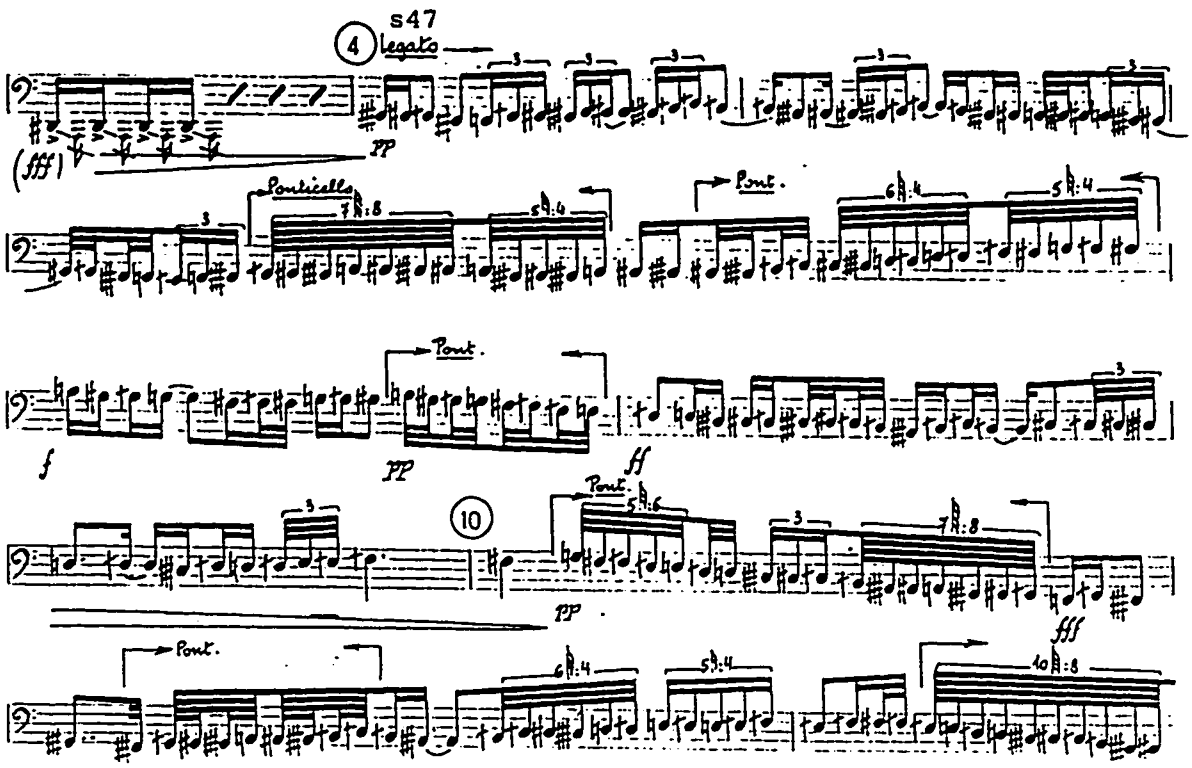

Figure 4.12: *Theraps*, mm. 3-10
Iannis Xenakis, *Theraps* (Paris: Editions Salabert, 1976). Used with permission.

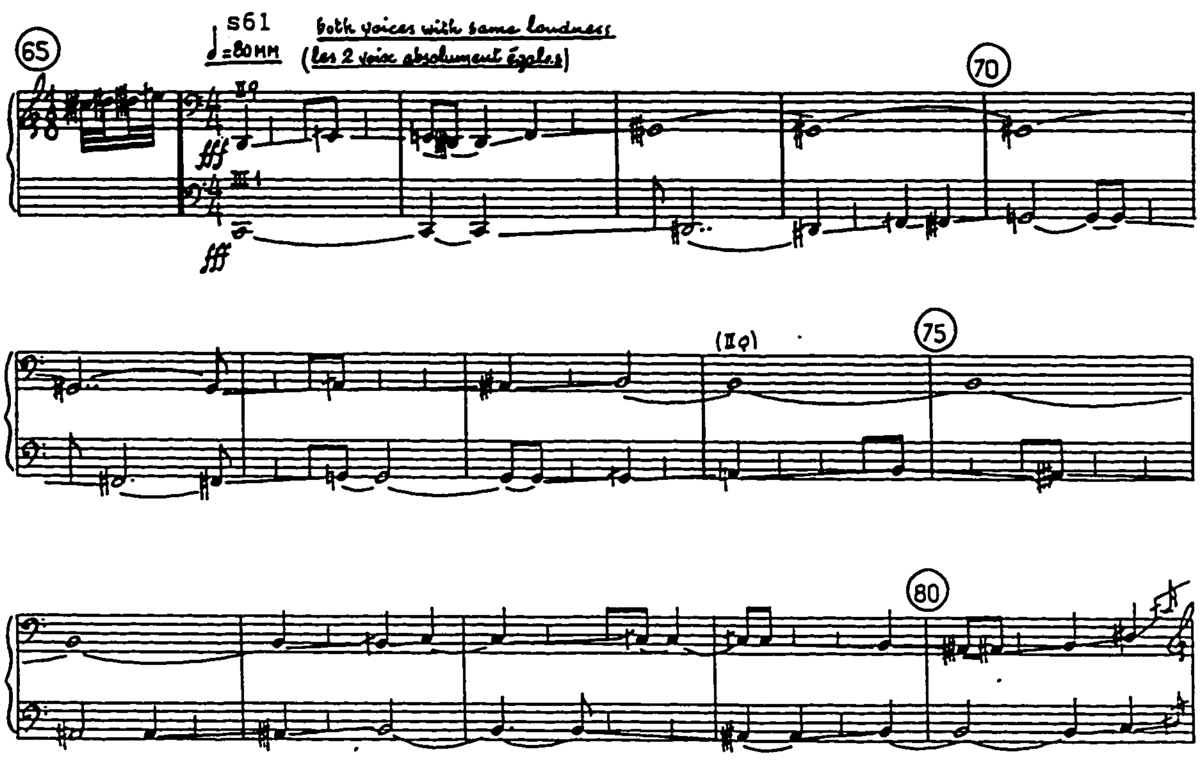

Figure 4.13: *Theraps*, mm. 65-80
Iannis Xenakis, *Theraps* (Paris: Editions Salabert, 1976). Used with permission.

Table 4.7: Segments in *Theraps*

segment	measures	seconds*	duration (sec)	configuration type	intensity
1	1	0.000-0.259	0.259	SG	*fff*
2	1	0.259-0.517	0.259	SG	*fff*
3	1	0.517-0.776	0.259	SG	*fff*
4	1	0.776-1.034	0.259	SG	*fff*
5	1	1.034-1.293	0.259	SG	*fff*
6	1	1.293-1.552	0.259	SG	*fff*
7	1	1.552-1.810	0.259	SG	*fff*
8	1	1.810-2.069	0.259	SG	*fff*
9	1	2.069-2.328	0.259	SG	*fff*
10	1	2.328-2.586	0.259	SG	*fff*
11	1	2.586-2.845	0.259	SG	*fff*
12	1	2.845-3.103	0.259	SG	*fff*
13	1	3.103-3.362	0.259	SG	*fff*
14	1	3.362-3.621	0.259	SG	*fff*
15	1	3.621-3.879	0.259	SG	*fff*
16	1	3.879-4.138	0.259	SG	*fff*
17	2	4.138-4.397	0.259	SG	*fff*
18	2	4.397-4.655	0.259	SG	*fff*
19	2	4.655-4.914	0.259	SG	*fff*
20	2	4.914-5.172	0.259	SG	*fff*
21	2	5.172-5.948	0.776	?	*fff*
22	2	5.948-6.207	0.259	SG	*fff*
23	2	6.207-6.466	0.259	SG	*fff*
24	2	6.466-6.724	0.259	SG	*fff*
25	2	6.724-6.983	0.259	SG	*fff*
26	2	6.983-7.241	0.259	SG	*fff*
27	2	7.241-7.500	0.259	SG	*fff*
28	2	7.500-7.759	0.259	SG	*fff*
29	2	7.759-8.017	0.259	SG	*fff*
30	2	8.017-8.276	0.259	SG	*fff*
31	3	8.276-8.534	0.259	SG	*fff*
32	3	8.534-8.793	0.259	SG	*fff*
33	3	8.793-9.052	0.259	SG	*fff*
34	3	9.052-9.310	0.259	SG	*ff*
35	3	9.310-9.569	0.259	SG	*ff*
36	3	9.569-9.828	0.259	SG	*ff*
37	3	9.828-10.086	0.259	SG	*f*
38	3	10.086-10.345	0.259	SG	*f*
39	3	10.345-10.603	0.259	SG	*f*
40	3	10.603-10.862	0.259	SG	*mf*
41	3	10.862-11.121	0.259	SG	*mf*
42	3	11.121-11.379	0.259	SG	*mf*

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* refers to the location of the segments in the graphic transcription (Figure 4.18)

† d = discrete changes in intensity; c = continuous (gradual) changes in intensity

configuration types: SG = short glissando; RW = random walk; H = sustained harmonics; TVG = two-voice glissando

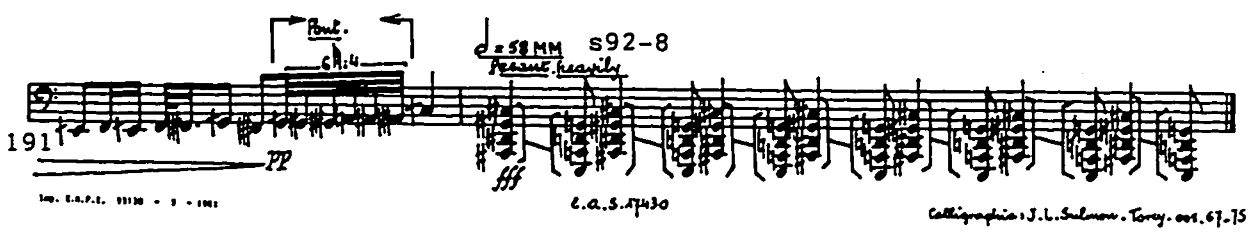
Table 4.7: Segments in *Theraps*, cont.

segment	measures	seconds*	duration (sec)	configuration type	intensity
43	3	11.397-11.638	0.259	SG	*mp*
44	3	11.638-11.897	0.259	SG	*mp*
45	3	11.897-12.155	0.259	SG	*p*
46	3	12.155-12.414	0.259	SG	*p*
47	4-16	12.414-65.172	52.758	RW	*pp, f, ff, fff* (d, c)†
48	16-19	65.172-77.586	12.414	H	*p*
49	18-20	72.414-80.690	8.276	H	*p*
50	19-21	77.586-84.828	7.241	H	*p*
51	20-23	80.690-95.172	14.483	H	*p*
52	21-26	84.828-106.552	21.724	H	*p*
53	24-26	95.172-106.552	11.379	H	*p*
54	26-35	106.552-141.724	35.172	RW	*pp, p, ff, fff* (d, c)
55	35-37	141.724-151.034	9.310	H	*p*
56	36-40	144.828-163.448	18.621	H	*p*
57	37-39	151.034-160.345	9.310	H	*p*
58	39-42	160.345-170.172	9.828	H	*p*
59	40-42	163.448-170.172	6.724	H	*p*
60	42-65	170.172-263.276	93.104	RW	*pp, p, mf, f, ff, fff* (d, c)
61	66-80	263.276-308.250	44.974	TVG	*fff*
62	81-94	308.250-363.077	54.827	RW	*pp, p, f, ff, fff* (d, c)
63	94-137	363.077-494.327	131.250	TVG	*fff*
64	138-140	494.327-504.672	10.345	H	*p*
65	139	498.465-502.603	4.138	H	*p*
66	140-141	502.603-508.810	6.207	H	*p*
67	140-142	504.672-512.948	8.276	H	*p*
68	141-143	508.810-518.637	9.828	H	*p*
69	142-146	512.948-529.499	16.552	H	*p*
70	143-145	518.637-524.327	5.690	H	*p*
71	145-147	524.327-535.706	11.379	H	*p*
72	146-149	529.499-541.913	12.414	H	*p*
73	148-150	535.706-546.051	10.345	H	*p*
74	149-151	541.913-552.258	10.345	H	*p*
75	150-151	546.051-552.258	6.207	H	*p*
76	152-153	552.258-558.258	6.000	TVG	*fff*
77	154-160	558.258-587.224	28.966	RW	*pp, p, mf, f, ff* (c)
78	161-162	587.224-593.224	6.000	TVG	*fff*
79	163-165	593.224-603.569	10.345	H	*p*
80	163-168	593.224-617.017	23.793	H	*p*
81	165-167	603.569-611.845	8.276	H	*p*
82	167-169	611.845-622.190	10.345	H	*p*
83	168-171	617.017-630.465	13.448	H	*p*
84	170-173	622.190-636.672	14.483	H	*p*
85	172-174	630.465-640.810	10.345	H	*p*
86	173-177	636.672-652.190	15.517	H	*p*
87	174-175	640.810-647.017	6.207	H	*p*
88	176-179	647.017-663.569	16.552	H	*p*
89	177-179	652.190-663.569	11.379	H	*p*
90	180-186	663.569-683.819	20.250	TVG	*fff*

Table 4.7: Segments in *Theraps*, cont.

segment	measures	seconds*	duration (sec)	configuration type	intensity
91	186-191	683.819-708.129	24.310	RW	*p*, *fff* (d, c)
92	192	708.129-708.905	0.776	SG	*fff*
93	192	708.905-709.681	0.776	SG	*fff*
94	192	709.681-710.457	0.776	SG	*fff*
95	192	710.457-711.232	0.776	SG	*fff*
96	192	711.232-712.008	0.776	SG	*fff*
97	192	712.008-712.784	0.776	SG	*fff*
98	192	712.784-713.560	0.776	SG	*fff*

a. mm. 1-5



b. mm. 191-2

Figure 4.14: *Theraps*, (a) mm. 1-5 and (b) mm. 191-2
Iannis Xenakis, *Theraps* (Paris: Editions Salabert, 1976). Used with permission.

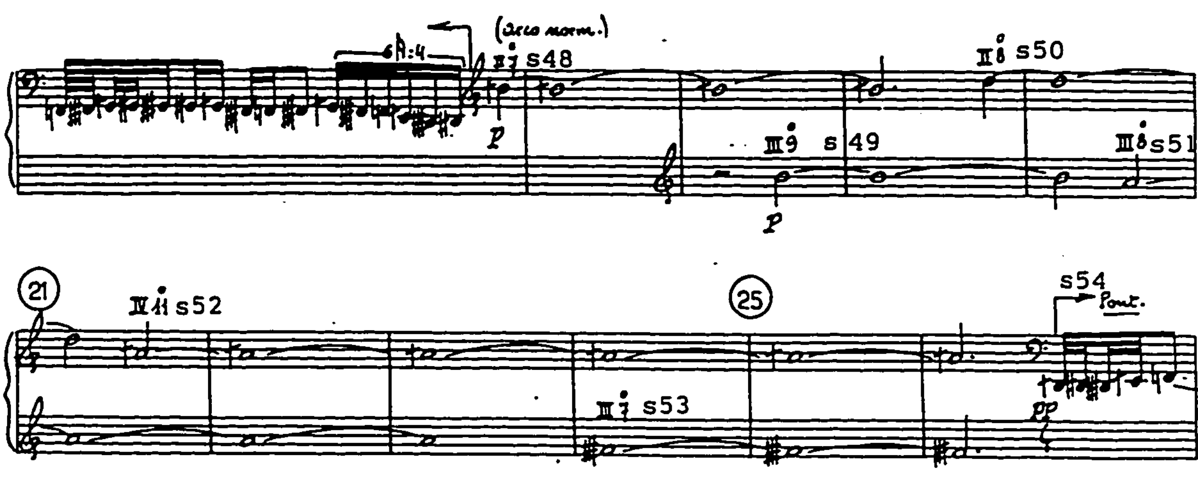


Figure 4.15: *Theraps*, mm. 16-26
Iannis Xenakis, *Theraps* (Paris: Editions Salabert, 1976). Used with permission.

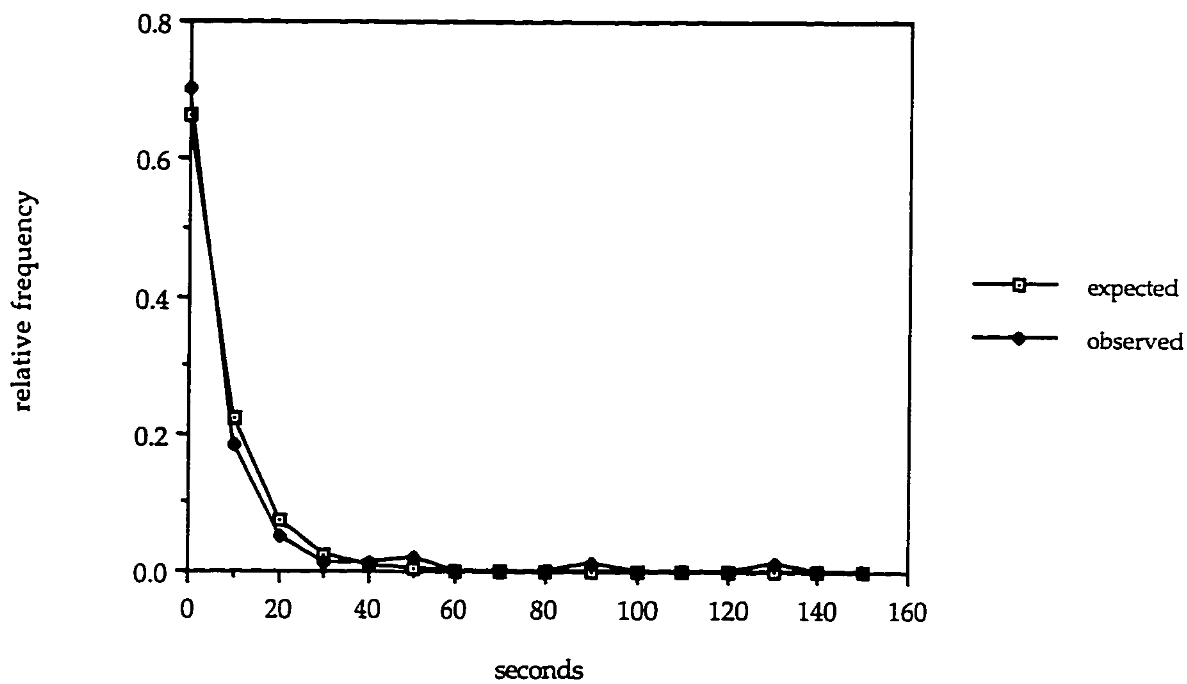


Figure 4.16: Comparative histogram showing distribution of segment durations in *Theraps* versus exponential distribution, $\partial = 0.109$

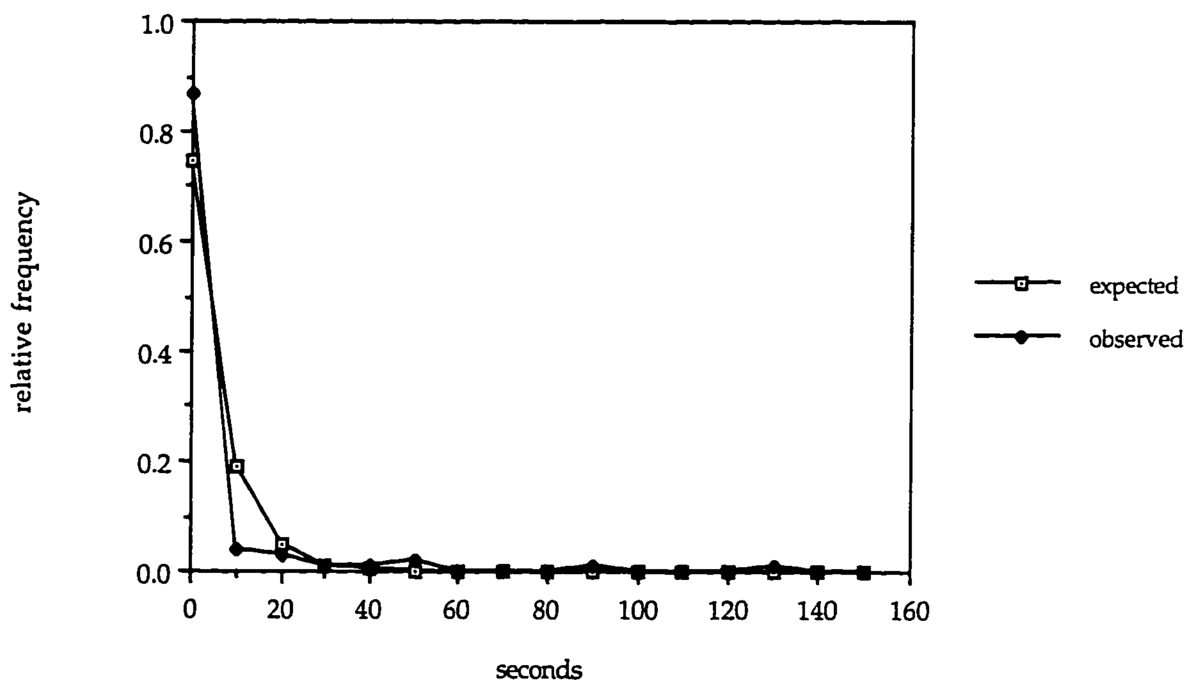


Figure 4.17: Comparative histogram showing distribution of intervals between initiations of segments in *Theraps* versus exponential distribution, $\partial = 0.137$

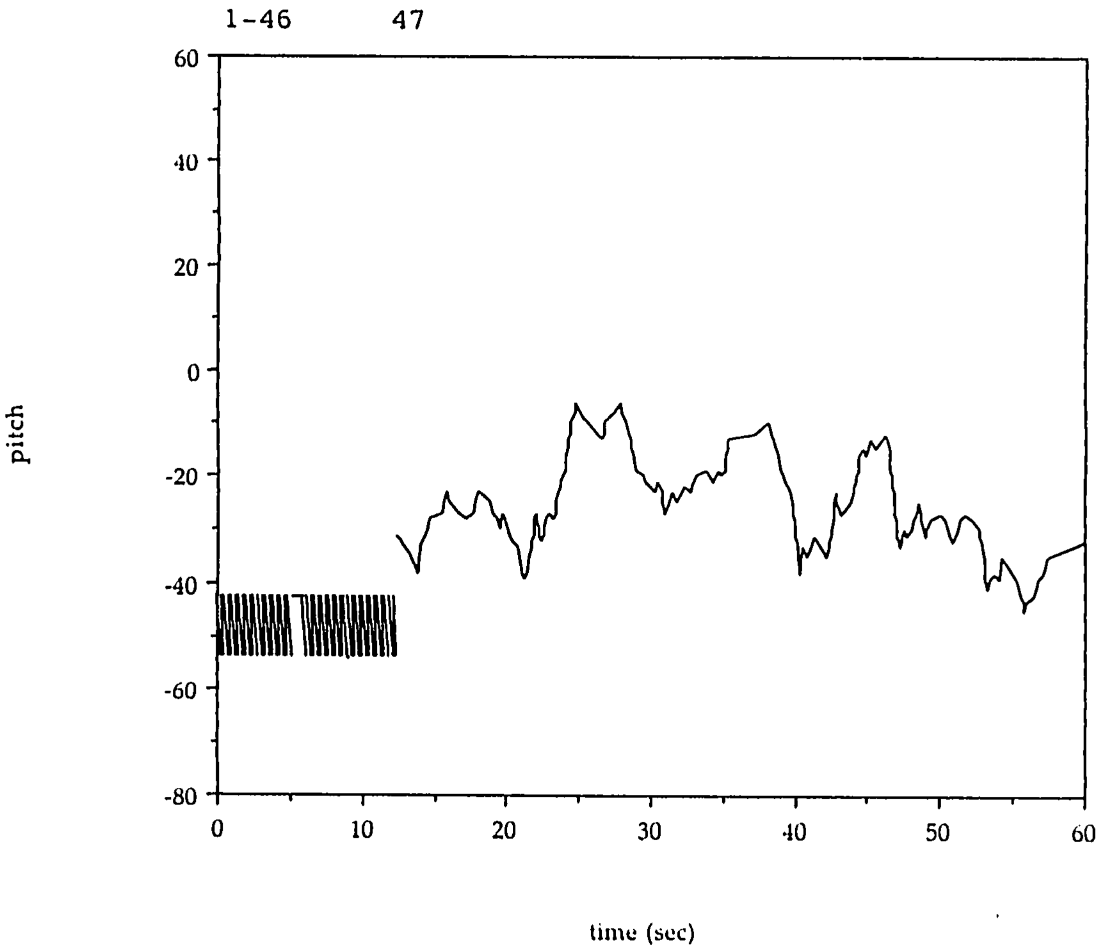


Figure 4.18: Graphic transcription of *Theraps*

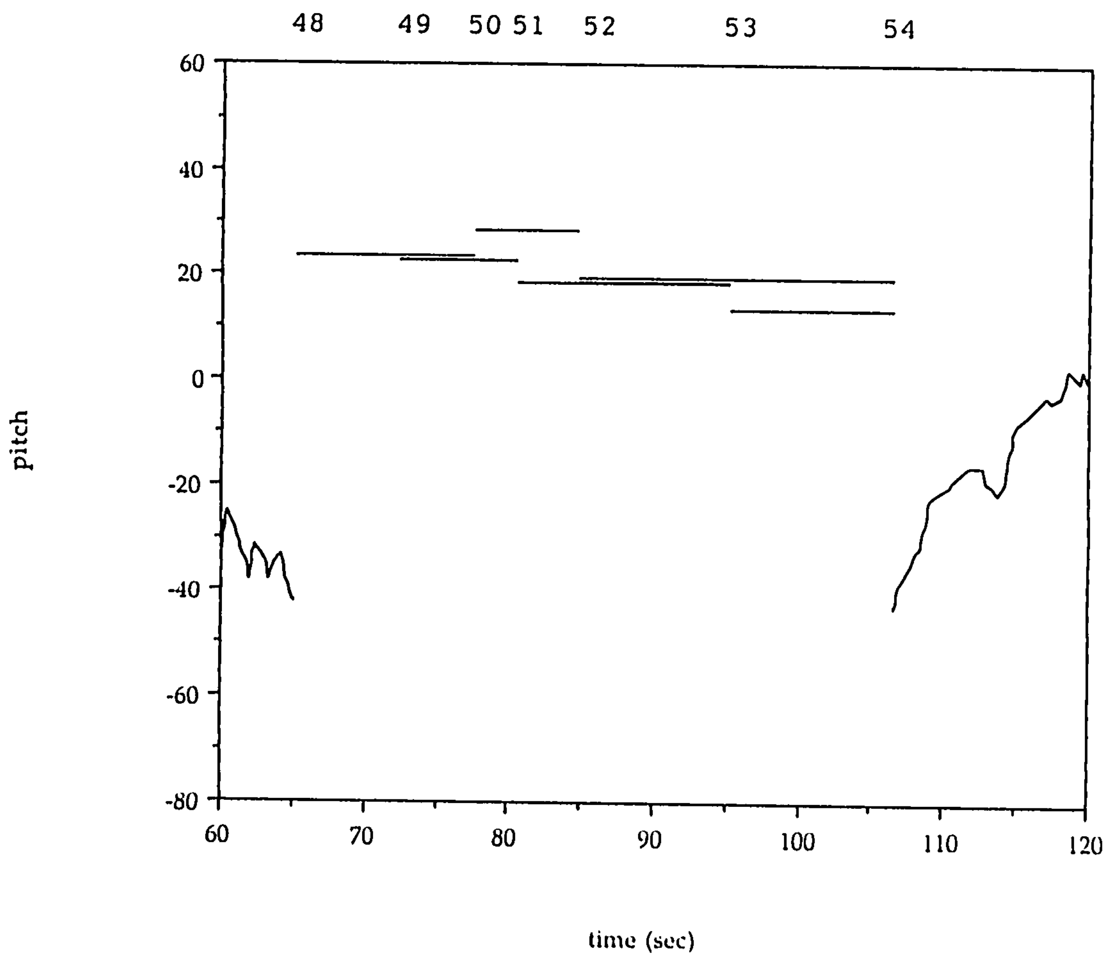


Figure 4.18: Graphic transcription of *Theraps*, cont.

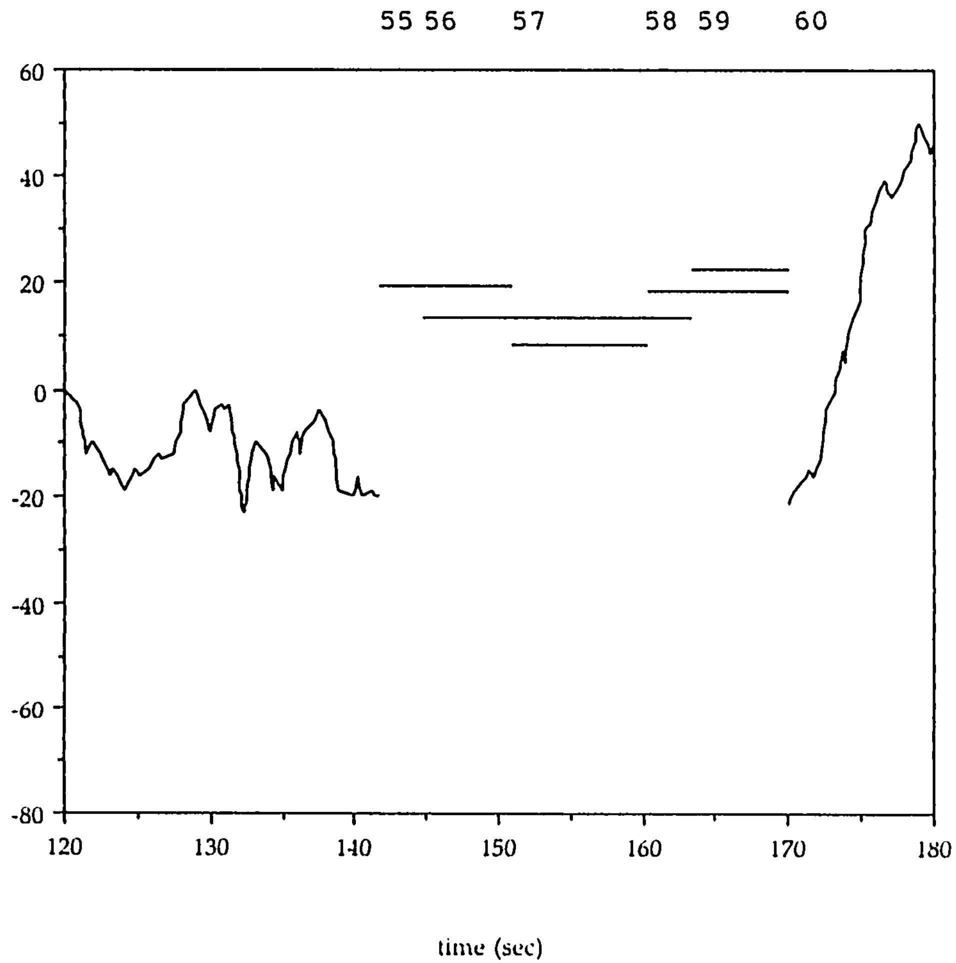


Figure 4.18: Graphic transcription of *Theraps*, cont.

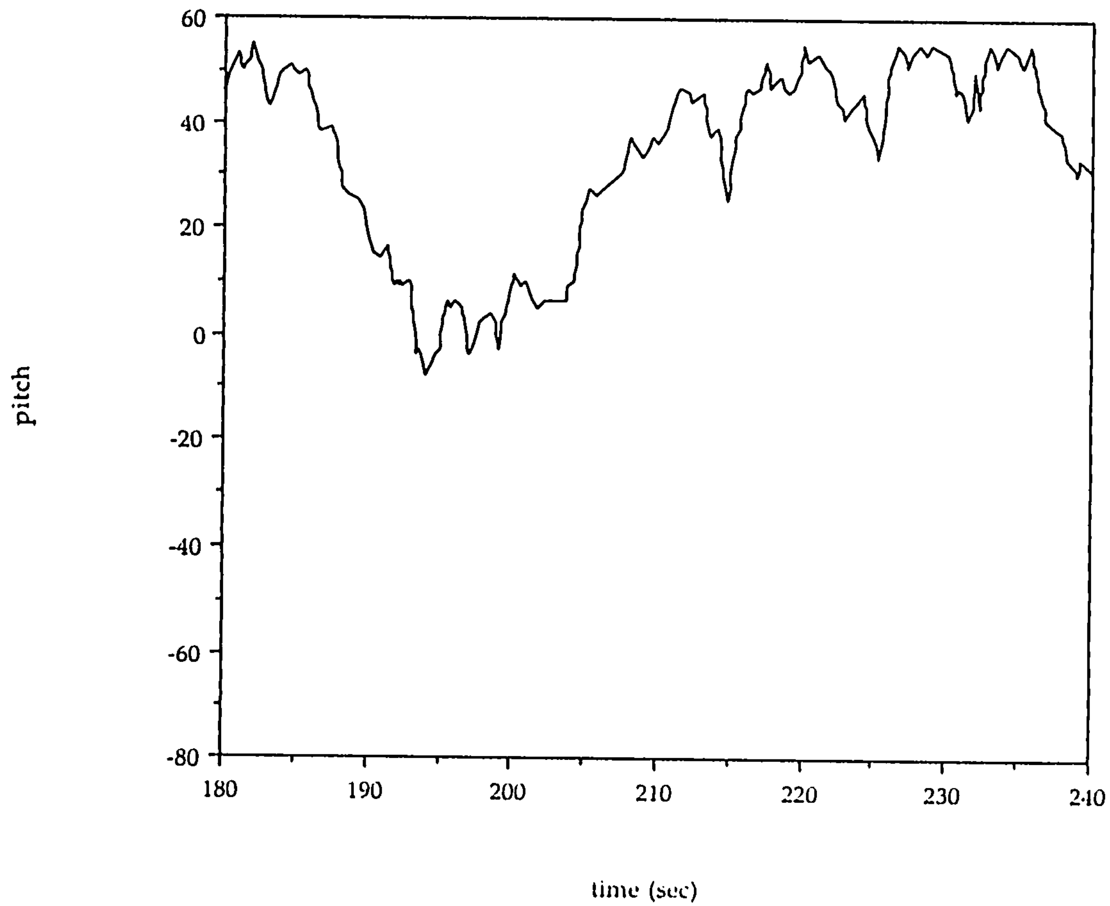


Figure 4.18: Graphic transcription of *Theraps*, cont.

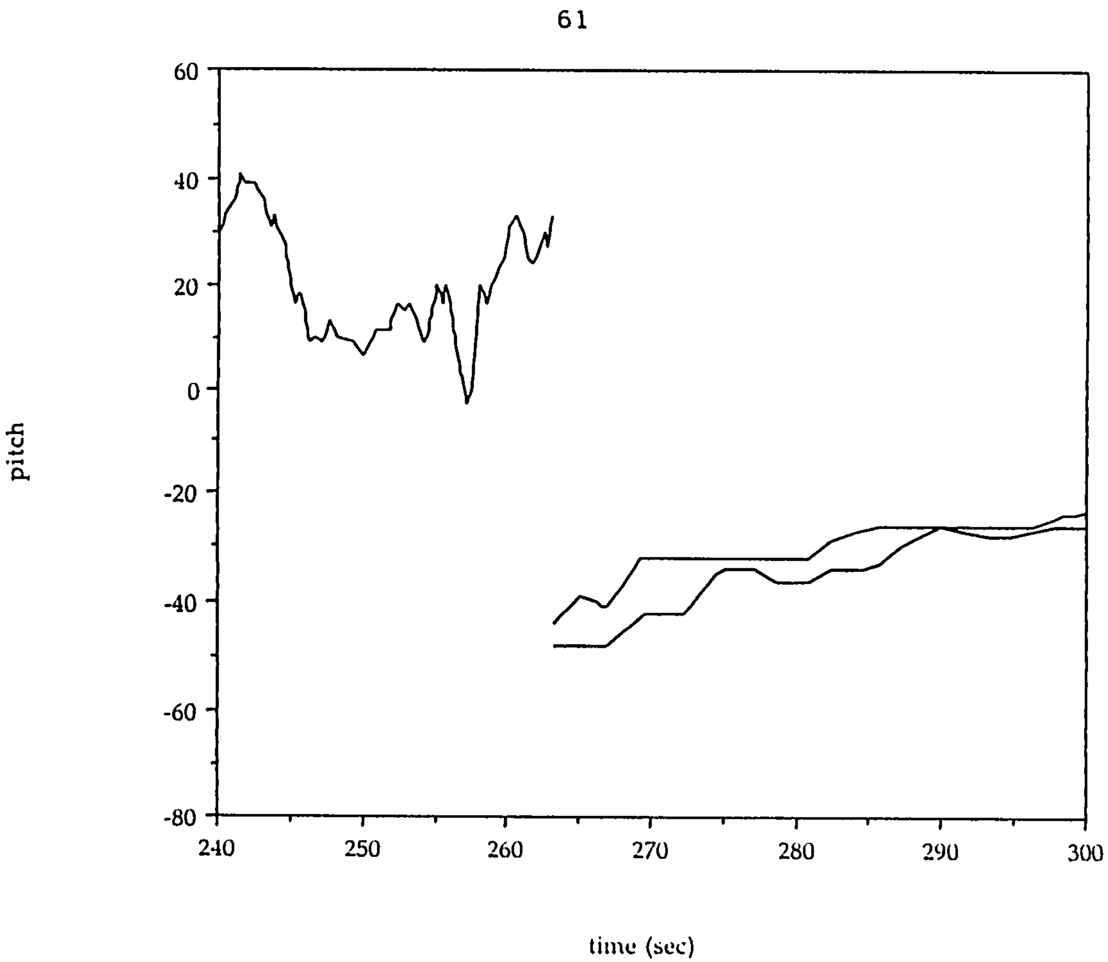


Figure 4.18: Graphic transcription of *Theraps*, cont.

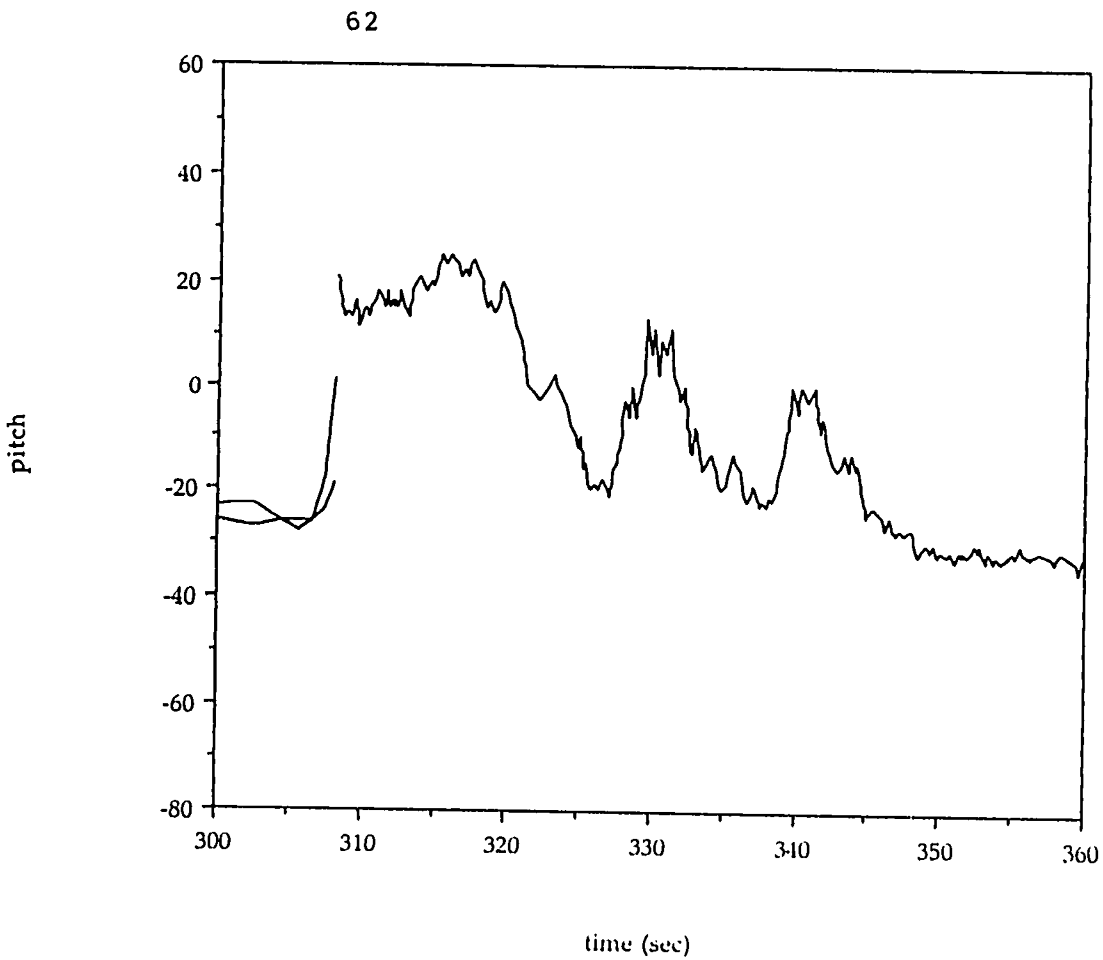


Figure 4.18: Graphic transcription of *Theraps*, cont.

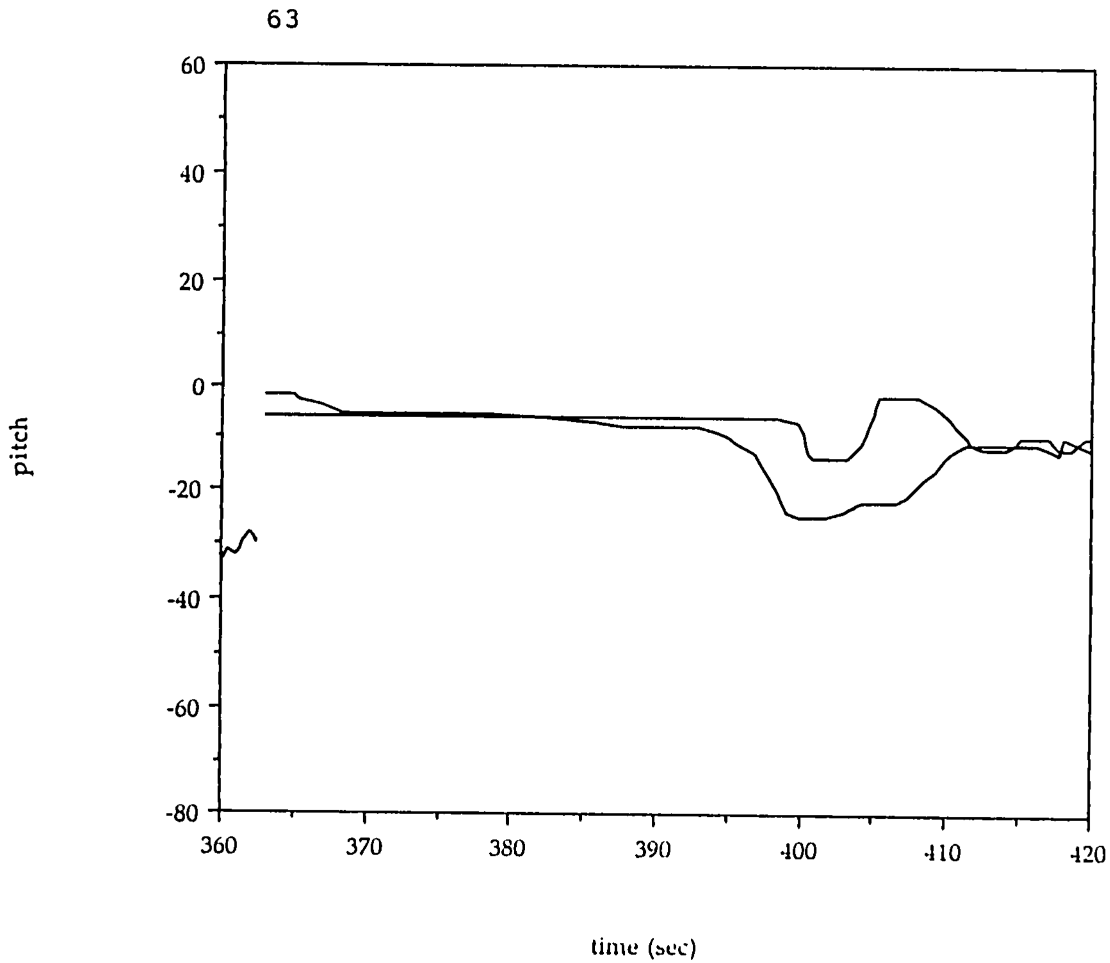


Figure 4.18: Graphic transcription of *Theraps*, cont.

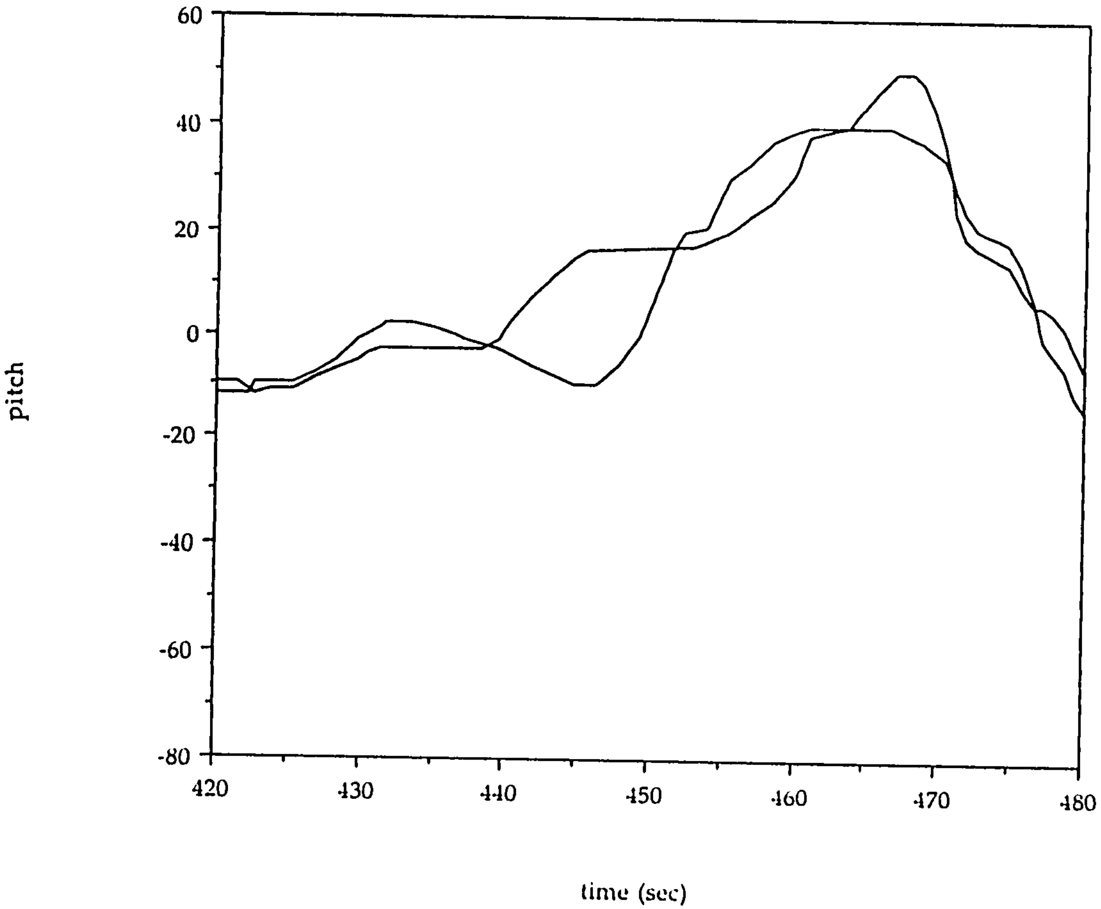


Figure 4.18: Graphic transcription of *Theraps*, cont.

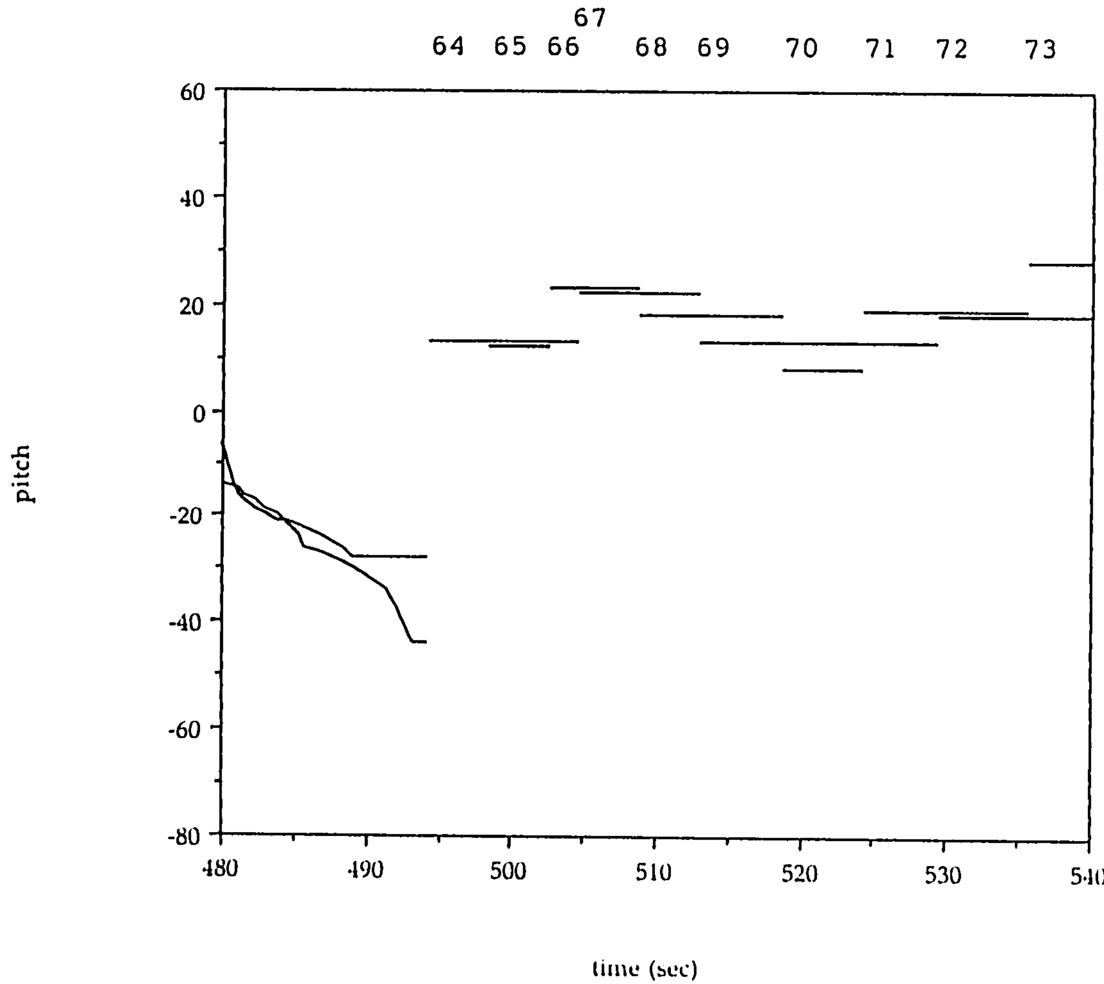


Figure 4.18: Graphic transcription of *Theraps*, cont.

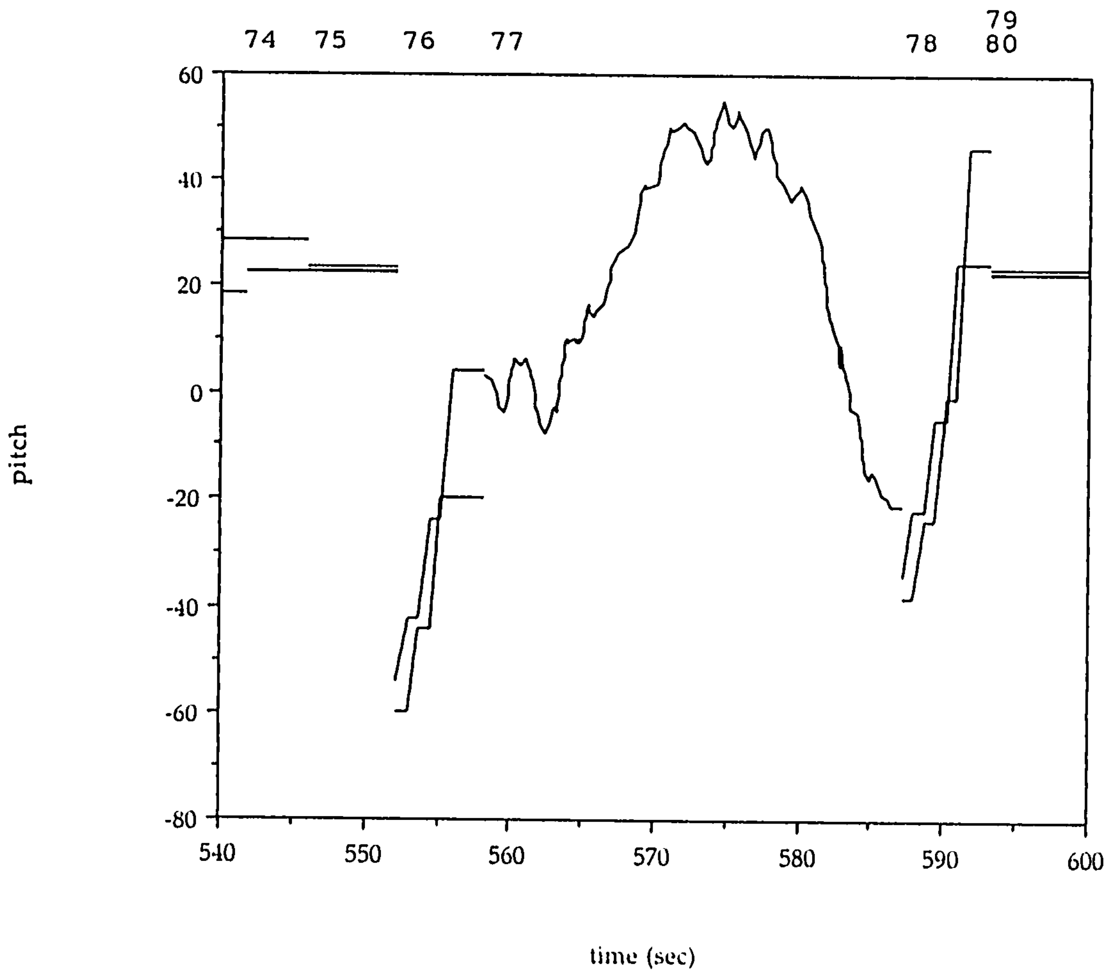


Figure 4.18: Graphic transcription of *Theraps*, cont.

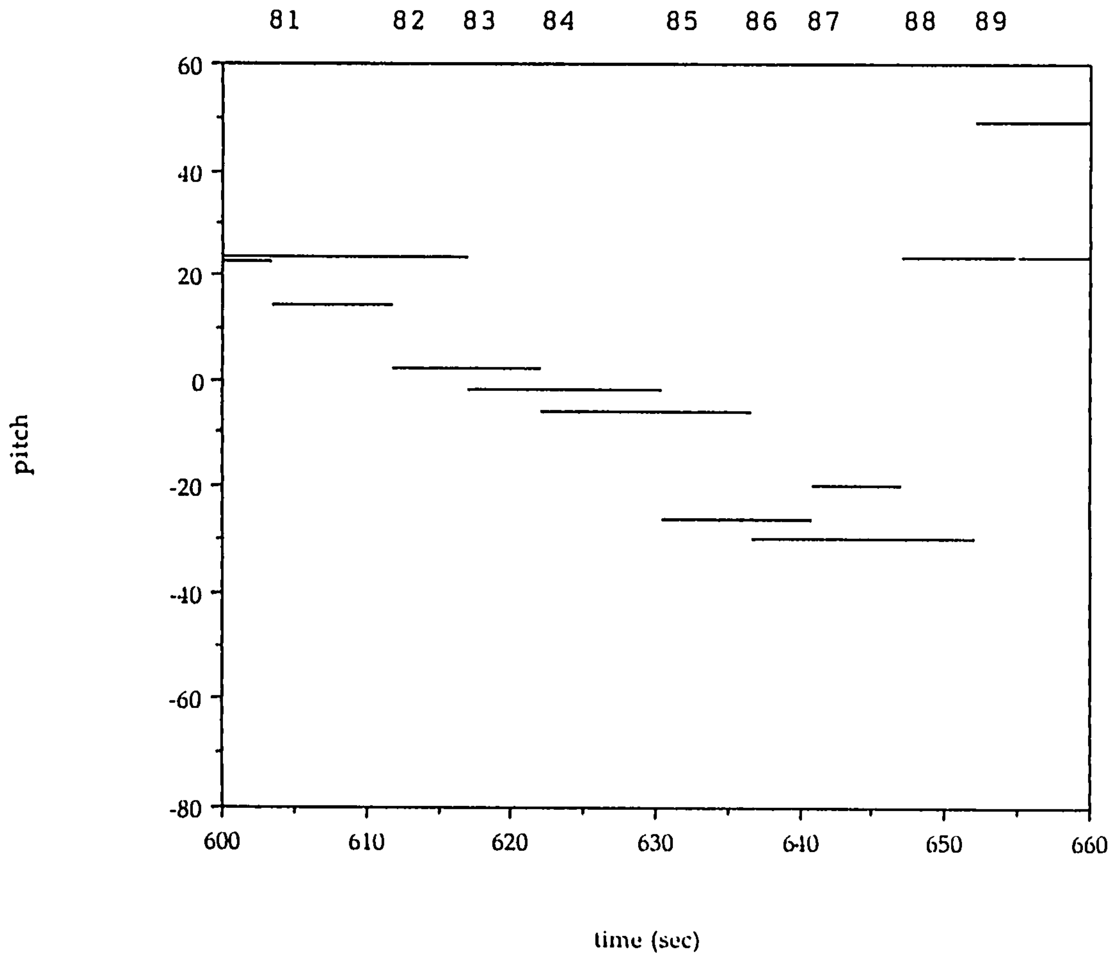


Figure 4.18: Graphic transcription of *Theraps*, cont.

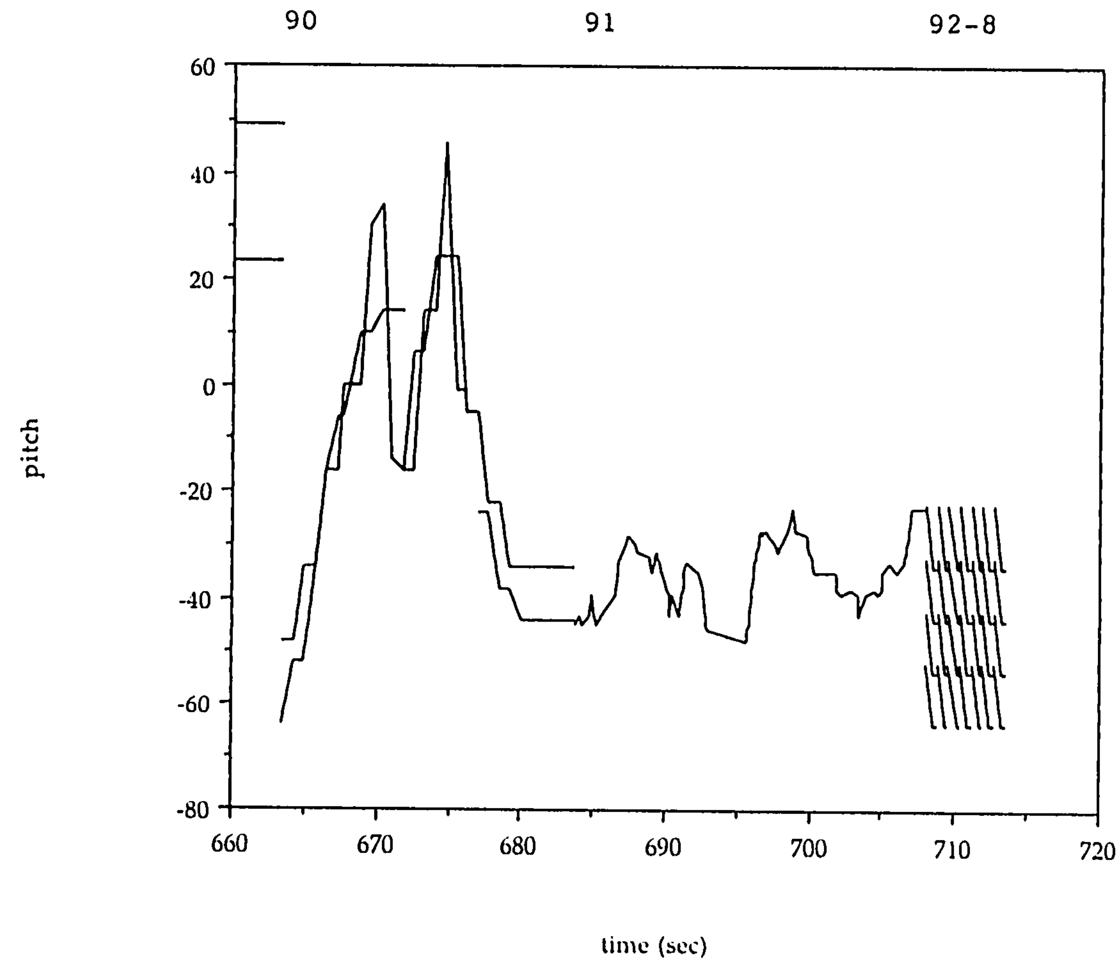


Figure 4.18: Graphic transcription of *Theraps*, cont.

Table 4.8: Summary of Temporal Structure in *Theraps*

Total duration: 713.56"

part:			1			
segments:			1-60			
duration:			263.276" (.369)*			
supersection:	A				B	
segments:	1-46				47-60	
duration:	12.414" (.047)				250.862" (.953)	
section:	1		2			3
segments:	1-46		47-59			60
duration:	12.414" (1.0)		157.758" (.629)			93.104" (.371)
subsection:	a	b	c	d	e	f
segments:	1-46	47	48-53	54	55-59	60
duration:	12.414" (1.0)	52.758" (.334)	41.379" (.262)	35.172" (.223)	28.448" (.18)	93.104" (1.0)

*The quantities in parentheses indicate the proportion of the next higher level in the temporal structure that is occupied by the given duration.

Table 4.8: Summary of Temporal Structure in *Theraps*, cont.

part:							2
segments:							61-98
duration:							450.284" (.631)
supersection:					C		
segments:					61-91		
duration:					444.853" (.988)		
section:		4					
segments:		61-75					
duration:		288.982" (.65)					
subsection:	g	h	i	j	k	l	
segments:	61	62	63	64-75	76	77	
duration:	44.974" (.156)	54.827" (.19)	131.25" (.454)	57.931" (.2)	6" (.038)	28.966" (.186)	

Table 4.8: Summary of Temporal Structure in *Theraps*, cont.

part:	2				
segments:	61-98				
duration:	450.284" (.631)				
supersection:					D
segments:					92-98
duration:					5.431" (.012)
section:	5				6
segments:	76-91				92-98
duration:	155.871" (.35)				5.431" (1.0)
subsection:	m	n	o	p	q
segments:	78	79-89	90	91	92-98
duration:	6" (.038)	70.345" (.451)	20.25" (.13)	24.31" (.156)	5.431" (1.0)

Table 4.9: Classification of Subsections by Configuration Type

Total duration: 713.56"

short glissando

a	12.414"	
q	5.431"	
	17.845"	(.025)

random walk

b	52.759"	
d	35.172"	
f	93.104"	
h	54.827"	
l	28.966"	
p	24.310"	
	289.138"	(.405)

harmonics

c	41.379"	
e	28.448"	
j	57.931"	
n	70.345"	
	198.103"	(.278)

two-voice glissando

g	44.974"	
i	131.250"	
k	6.000"	
m	6.000"	
o	20.250"	
	208.474"	(.291)

summary:

random walk = .405
everything else = .595
.405 : .595 ≈ 2:3

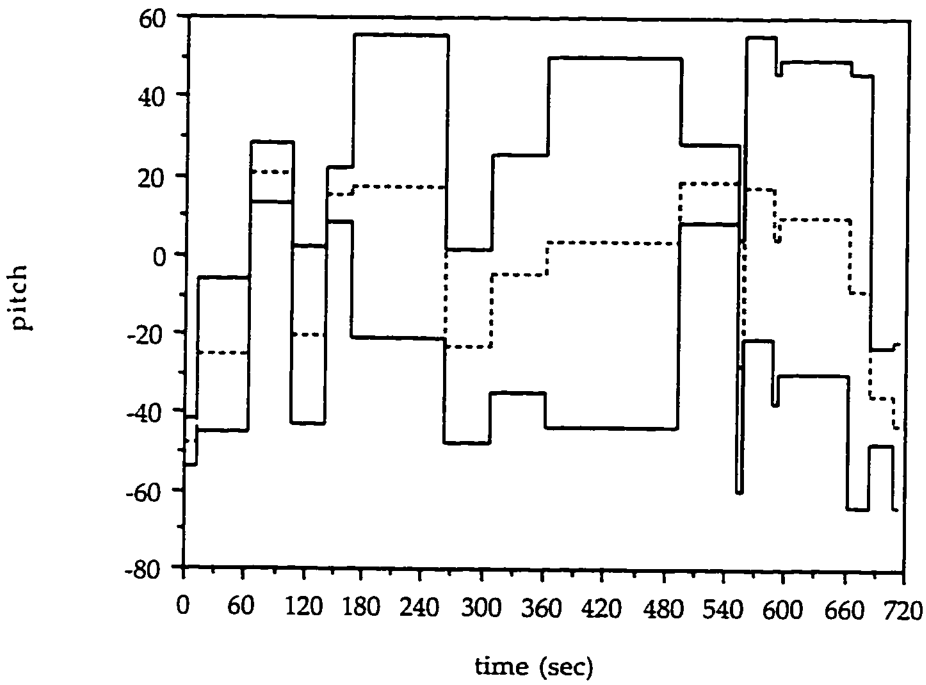


Figure 4.19: Minimum, maximum, and mean pitch levels per subsection in *Theraps*

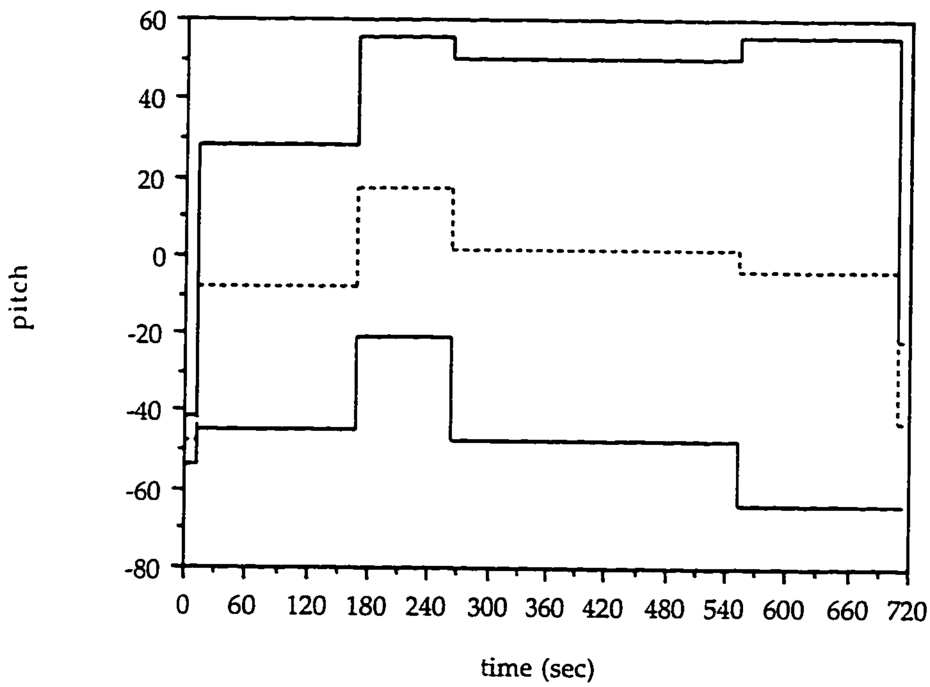


Figure 4.20: Minimum, maximum, and mean pitch levels per section in *Theraps*

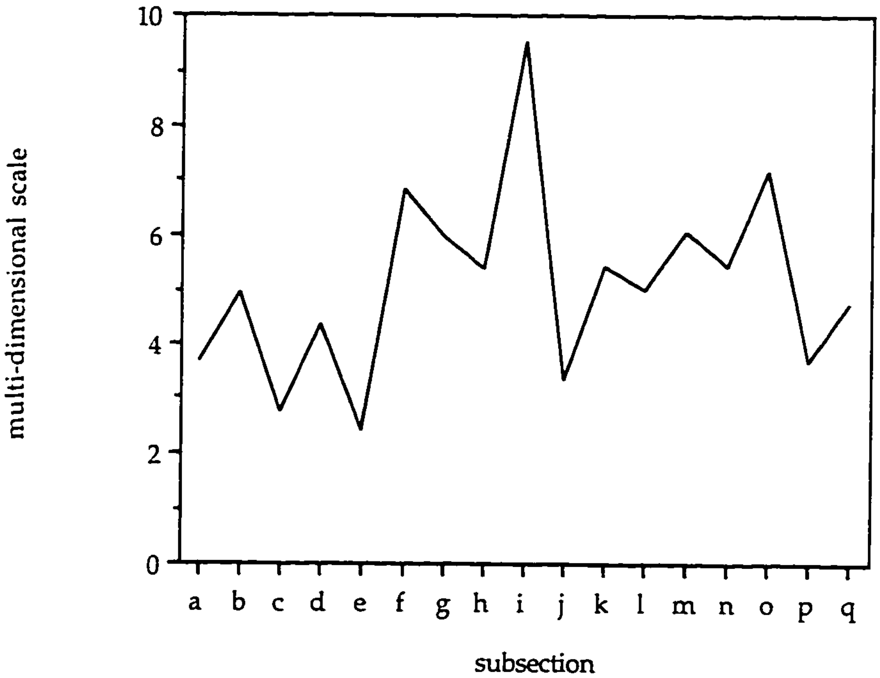


Figure 4.21: MDS for *Theraps*

Table 4.10: Psets in *Theraps*

Model A (random walks)

{-48 -46 -45 -43 -42 -41 -39 -38 -35 -33 -32 -31 -30 -28 -27 -25 -23 -22 -21 -20 -19 -16 -15 -13 -12 -10 -9 -8 -6 -4 -3 0 2 3 4 5 6 9 10 11 12 13 14 15 16 18 20 21 23 24 25 26 27 30 31 33 36 37 38 39 41 43 44 46 47 49 50 51 52 53 55}
<2 1 2 1 1 2 1 3 2 1 1 1 2 1 2 2 1 1 1 1 3 1 2 1 2 1 1 2 2 1 3 2 1 1 1 1 3 1 1 1 1 1 1 2 2 1 2 1 1 1 1 3 1 2 3 1 1 1 2 2 1 2 1 2 1 1 1 1 2>

Subsets of Model A, listed by subsection:

subsection b
{-45 -43 -42 -41 -39 -38 -35 -33 -32 -31 -30 -28 -27 -25 -23 -22 -21 -20 -19 -16 -15 -13 -12 -10 -9 -8 -6}
<2 1 1 2 1 3 2 1 1 1 2 1 2 2 1 1 1 1 3 1 2 1 2 1 1 2>

subsection d
{-43 -42 -41 -39 -38 -35 -33 -32 -31 -30 -28 -27 -25 -23 -22 -21 -20 -19 -16 -15 -13 -12 -10 -9 -8 -6 -4 -3 0 2}
<1 1 2 1 3 2 1 1 1 2 1 2 2 1 1 1 1 3 1 2 1 2 1 1 2 2 1 3 2>

subsection f
{-21 -20 -19 -16 -15 -13 -12 -10 -9 -8 -6 -4 -3 0 2 3 4 5 6 *7* 9 10 11 12 13 14 15 16 18 20 21 23 24 25 26 27 30 31 33 36 37 38 39 41 43 44 46 47 49 50 51 52 53 55}
<1 1 3 1 2 1 2 1 1 2 2 1 3 2 1 1 1 1 *1* 2 1 1 1 1 1 1 2 2 1 2 1 1 1 1 3 1 2 3 1 1 1 2 2 1 2 1 2 1 1 1 1 2>

subsection h
{-35 -33 -32 -31 -30 -28 -27 -25 -23 -22 -21 -20 -19 *-18* -16 -15 -13 -12 -10 -9 -8 -6 *-5* -4 -3 0 2 3 4 5 6 9 10 11 12 13 14 15 16 18 *19* 20 21 *22* 23 24 25}
<2 1 1 1 2 1 2 2 1 1 1 1 *1* 2 1 2 1 2 1 1 2 *1 1* 1 3 2 1 1 1 3 1 1 1 1 1 1 2 *1 1* 1 *1 1* 1 1>

subsection l (cf. subsection f)
{-21 -20 -19 -16 -15 -13 -12 -10 -9 -8 -6 -4 -3 0 2 3 4 5 6 9 10 11 13 14 15 16 18 20 21 23 24 25 26 27 30 31 33 36 37 38 39 41 43 44 46 47 49 50 51 52 53 55}
<1 1 3 1 2 1 2 1 1 2 2 1 3 2 1 1 1 1 3 1 1 2 1 1 1 2 2 1 2 1 1 1 1 3 1 2 3 1 1 1 2 2 1 2 1 2 1 1 1 1 2>

subsection p
{-48 -46 -45 -43 -39 -38 -35 -33 -32 -31 -30 -28 -27 -23}
<2 1 2 4 1 3 2 1 1 1 2 1 4>

Table 4.10: Psets in *Theraps*, cont.

Model B (harmonics)
{-30 -26 -20 -6 -2 2 8 13 18 19 22 23 28 49}
< 4 6 14 4 4 6 5 5 1 3 1 5 21 >

Subsets of model B, listed by subsection:

subsection c
{13 18 19 22 23 28}
< 5 1 3 1 5 >

subsection e
{8 13 18 19 22}
< 5 5 1 3 >

subsection j
{8 *12* 13 18 19 22 23 28}
< *4* *1* 5 1 3 1 5 >

subsection n
{-30 -26 -20 -6 -2 2 *14* 22 23 49}
< 4 6 14 4 4 *12* *8* 1 26 >

Pitch space in short glissandi and two-voice glissandi is not consistently differentiated.

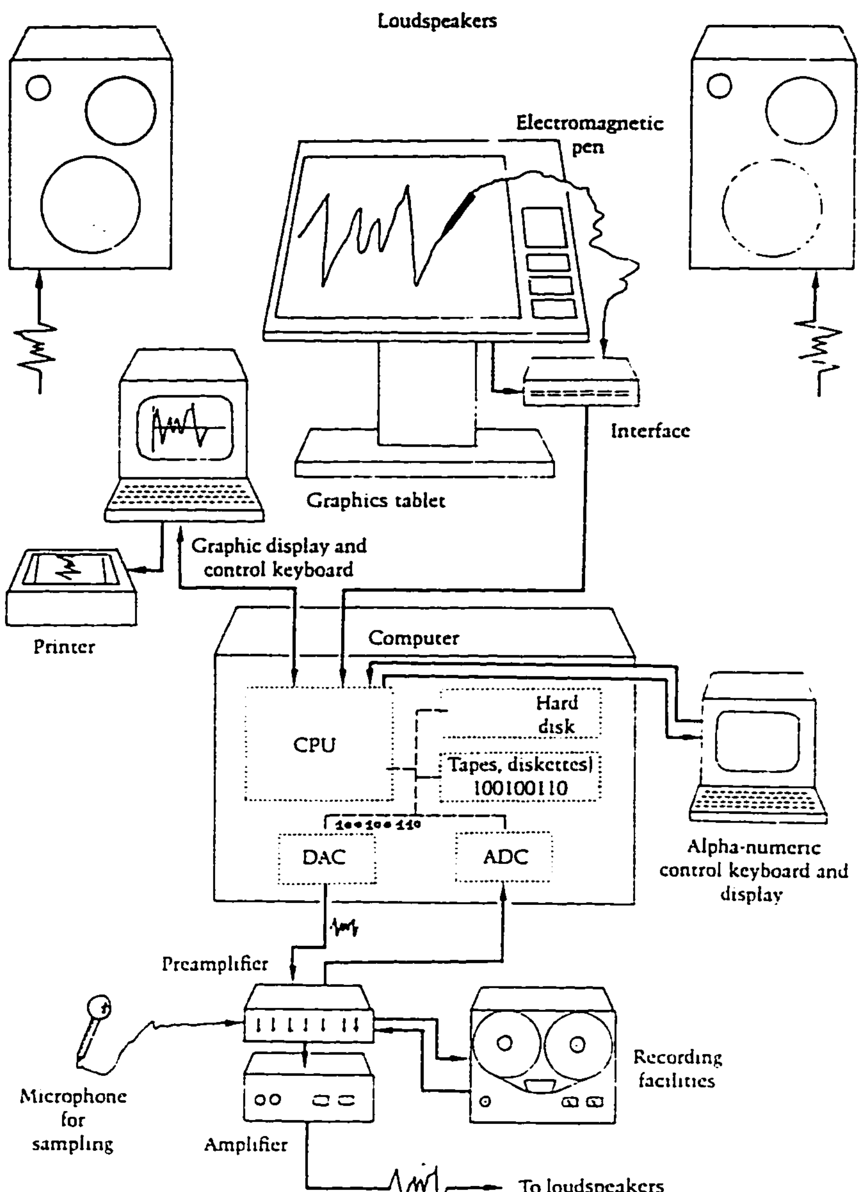


Figure 5.1: Schematic of an early version of the UPIC system
(Schematic by Alain Depres, reprinted from Henning Lohner, "The UPIC Systerm: A User's Report." *Computer Music Journal* 10/4 [1986]: 42-9.)

MYCENAE-ALPHA

1978

by

IANNIS XENAKIS

For mono tape, to be projected onto either two or four sound sources around the audience.

Composed on the UPIC graphic/computer system at the CEMAMu (Centre d'Etudes de Mathématique et Automatique Musicales), Paris, France.

World premiere in 1978 at the "Polytope of Mycenae" of Xenakis, festival of lights, movement, and music in the surrounding area of the Mycenae Acropolis in Greece. French premiere in 1978 in the "Homage to Messiaen" concert, part of the festival "Cycle Olivier Messiaen" in Paris.

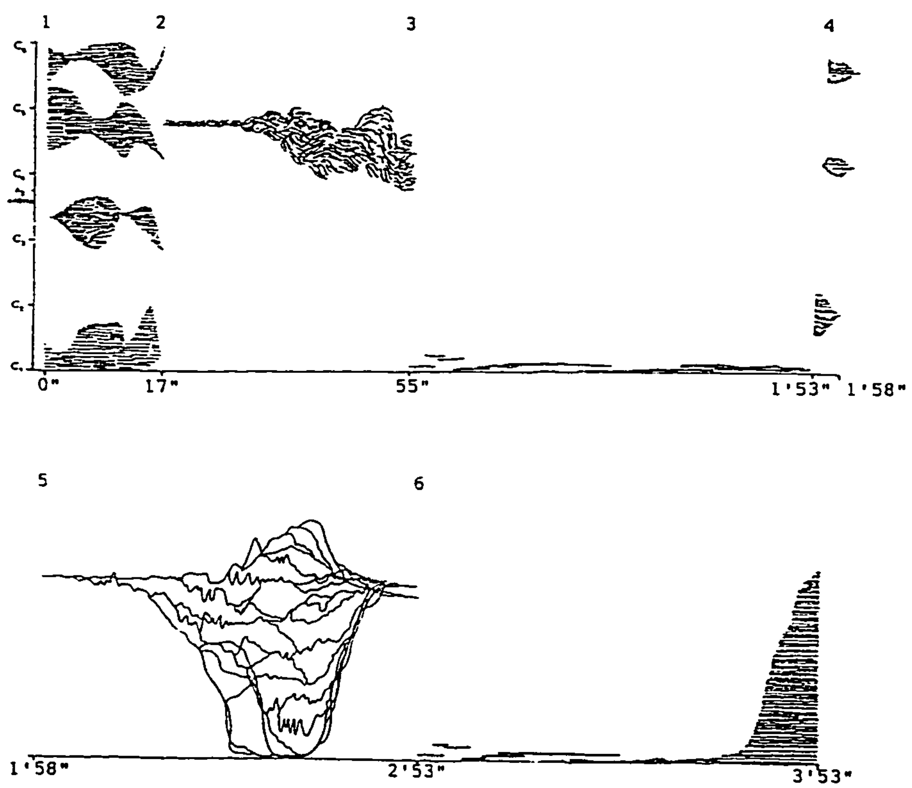


Figure 5.2: Annotated graphic score of *Mycenae-Alpha*
Iannis Xenakis, *Mycenae-Alpha* (Paris: Editions Salabert, 1978). Used with permission.

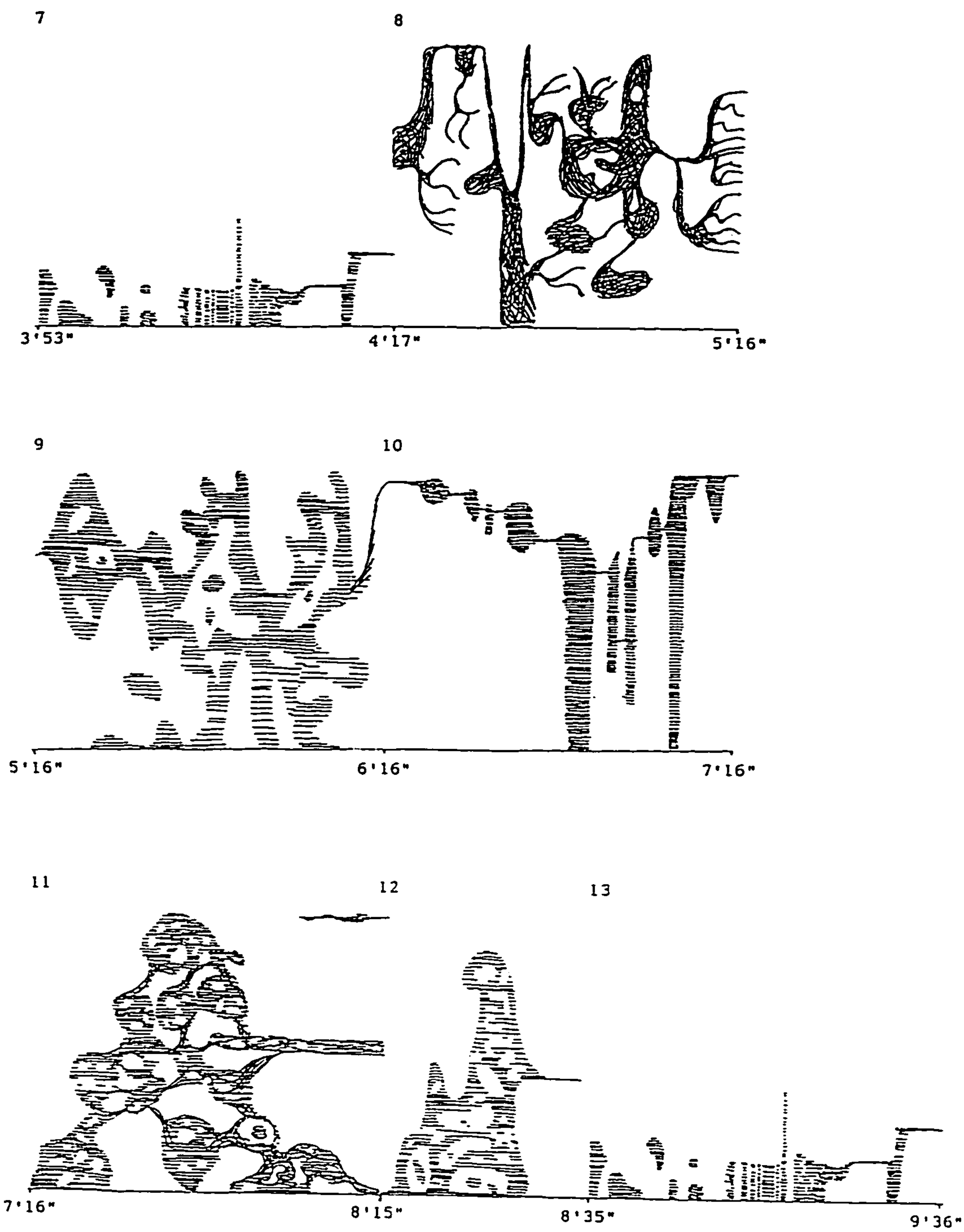


Figure 5.2: Annotated graphic score of *Mycenae-Alpha*, cont.
Iannis Xenakis, *Mycenae-Alpha* (Paris: Editions Salabert, 1978). Used with permission.

Table 5.1: Segments in *Mycenae-Alpha*

segment	begin time	duration (sec)	arc orientation
1	0"	17	horizontal
2	17"	38	curved
3	55"	58	horizontal
4	1'53"	5	horizontal
5	1'58"	55	curved
6	2'53"	60	horizontal
7	3'53"	24	horizontal
8	4'17"	59	curved
9	5'16"	60	horizontal
10	6'16"	60	horizontal
11	7'16"	59	horizontal, curved
12	8'15"	20	horizontal
13	8'35"	61	horizontal

Table 5.2: Summary of Temporal Structure of *Mycenae-Alpha*

Duration of whole work: 576"

1) Inside-time structure:

part:	1	2
segments:	1-6	7-13
duration:	233" (.405)*	343" (.595)

part 1/part 2 = 0.679 ≈ 2:3

in segment 6, ascent through p-space begins at 220"

ascent/whole = 220"/576" = .382 (1 - GS)
ascent/remainder = 220"/356" = .618 (GS)
remainder/whole = 356"/576" = .618 (GS)

2) Outside-time structure:

Categories of segments by arc orientation

horizontal		curved†	
segment	duration (sec)	segment	duration (sec)
1	17	2	38
3	58	5	55
4	5	8	59
6	60	11	59
7	24	total	211
9	60		
10	60		
12	20		
13	61		
total	365		

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* The quantities in parentheses indicate the proportion of the whole, part, or supersection that is occupied by the given duration.
† Includes segment 11, which contains a mixture of horizontal and curved arcs.

Table 5.2: Summary of Temporal Structure of *Mycenae-Alpha*, cont.

within *Mycenae-Alpha*:

curved/total = 211"/576" = .366
horizontal/total = 365"/576" = .634
curved/horizontal ≈ 2:3

within part 1:

curved/part 1 = 93"/233" = .4
horizontal/part 1 = 140"/233" = .6
curved/horizontal = 2:3

within part 2:

curved/part 2 = 118"/343" = .344
horzontal/part 2 = 225"/343" = .656

or

curved + 6"/part 2 = 124"/343" = .362
horizontal - 6"/part 2 = 219"/343" = .638
curved/horizontal ≈ 2:3