PART FOUR: Cryptographic Algorithms

CHAPTER 20

SYMMETRIC ENCRYPTION AND MESSAGE CONFIDENTIALITY

LEARNING OBJECTIVES

After studying this chapter, you should be able to:

- Explain the basic principles of symmetric encryption.
- Understand the significance of the Feistel cipher structure.
- Describe the structure and function of DES.
- Distinguish between two-key and three-key triple DES.
- Describe the structure and function of AES.
- Compare and contrast stream encryption and block cipher encryption.
- Distinguish among the major block cipher modes of operation.
- Discuss the issues involved in key distribution.

Symmetric encryption, also referred to as conventional encryption, secret-key, or single-key encryption, was the only type of encryption in use prior to the development of public-key encryption in the late 1970s.[1] It remains by far the most widely used of the two types of encryption.

This chapter begins with a look at a general model for the symmetric encryption process; this will enable us to understand the context within which the algorithms are used. Then we look at three important block encryption algorithms: DES, triple DES, and AES. Next, the chapter introduces symmetric stream encryption and describes the widely used stream cipher RC4. We then examine the application of these algorithms to achieve confidentiality.

20.1 SYMMETRIC ENCRYPTION PRINCIPLES

At this point the reader should review Section 2.1. Recall that a symmetric encryption scheme has five ingredients (Figure 2.1):

- **Plaintext:** This is the original message or data that is fed into the algorithm as input.
- **Encryption algorithm:** The encryption algorithm performs various substitutions and transformations on the plaintext.
- **Secret key:** The secret key is also input to the algorithm. The exact substitutions and transformations performed by the algorithm depend on the key.
- **Ciphertext:** This is the scrambled message produced as output. It depends on the plaintext and the secret key. For a given message, two different keys will produce two different ciphertexts.
- **Decryption algorithm:** This is essentially the encryption algorithm run in reverse. It takes the ciphertext and the same secret key and produces the original plaintext.

[1]Public-key encryption was first described in the open literature in 1976; the National Security Agency (NSA) claims to have discovered it some years earlier.

Cryptography

Cryptographic systems are generically classified along three independent dimensions:

1. **The type of operations used for transforming plaintext to ciphertext.** All encryption algorithms are based on two general principles: substitution, in which each element in the plaintext (bit, letter, group of bits or letters) is mapped into another element, and transposition, in which elements in the plaintext are rearranged. The fundamental requirement is that no information be lost (i.e., that all operations be reversible). Most systems, referred to as product systems, involve multiple stages of substitutions and transpositions.
2. **The number of keys used.** If both sender and receiver use the same key, the system is referred to as symmetric, single-key, secret-key, or conventional encryption. If the sender and receiver each use a different key, the system is referred to as asymmetric, two-key, or public-key encryption.
3. **The way in which the plaintext is processed.** A *block cipher* processes the input one block of elements at a time, producing an output block for each input block. A *stream cipher* processes the input elements continuously, producing output one element at a time, as it goes along.

Cryptanalysis

The process of attempting to discover the plaintext or key is known as **cryptanalysis**. The strategy used by the cryptanalyst depends on the nature of the encryption scheme and the information available to the cryptanalyst.

Table 20.1 summarizes the various types of cryptanalytic attacks, based on the amount of information known to the cryptanalyst. The most difficult problem is presented when all that is available is the *ciphertext only*. In some cases, not even the encryption algorithm is known, but in general we can assume that the opponent does know the algorithm used for encryption. One possible attack under these circumstances is the brute-force approach of trying all possible keys. If the key space is very large, this becomes impractical. Thus, the opponent must rely on an analysis of the ciphertext itself, generally applying various statistical tests to it. To use this approach, the opponent must have some general idea of the type of plaintext that is concealed, such as English or French text, an EXE file, a Java source listing, an accounting file, and so on.

The ciphertext-only attack is the easiest to defend against because the opponent has the least amount of information to work with. In many cases, however, the analyst has more information. The analyst may be able to capture one or more plaintext messages as well as their encryptions. Or the analyst may know that certain plaintext patterns will appear in a message. For example, a file that is encoded in the Postscript format always begins with the same pattern, or there may be a standardized header or banner to an electronic funds transfer message, and so on. All these are examples of *known plaintext*. With this knowledge, the analyst may be able to deduce the key on the basis of the way in which the known plaintext is transformed.

Table 20.1 Types of Attacks on Encrypted Messages

Type of Attack	Known to Cryptanalyst
Ciphertext only	• Encryption algorithm • Ciphertext to be decoded
Known plaintext	• Encryption algorithm • Ciphertext to be decoded • One or more plaintext-ciphertext pairs formed with the secret key
Chosen plaintext	• Encryption algorithm • Ciphertext to be decoded • Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key
Chosen ciphertext	• Encryption algorithm • Ciphertext to be decoded • Purported ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key
Chosen text	• Encryption algorithm • Ciphertext to be decoded • Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key • Purported ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key

Closely related to the known-plaintext attack is what might be referred to as a probable-word attack. If the opponent is working with the encryption of some general prose message, he or she may have little knowledge of what is in the message. However, if the opponent is after some very specific information, then parts of the message may be known. For example, if an entire accounting file is being transmitted, the opponent may know the placement of certain key words in the header of the file. As another example, the source code for a program developed by a corporation might include a copyright statement in some standardized position.

If the analyst is able somehow to get the source system to insert into the system a message chosen by the analyst, then a *chosen-plaintext* attack is possible. In general, if the analyst is able to choose the messages to encrypt, the analyst may deliberately pick patterns that can be expected to reveal the structure of the key.

Table 20.1 lists two other types of attack: chosen ciphertext and chosen text. These are less commonly employed as cryptanalytic techniques but are nevertheless possible avenues of attack.

Only relatively weak algorithms fail to withstand a ciphertext-only attack. Generally, an encryption algorithm is designed to withstand a known-plaintext attack.

An encryption scheme is **computationally secure** if the ciphertext generated by the scheme meets one or both of the following criteria:

- The cost of breaking the cipher exceeds the value of the encrypted information.
- The time required to break the cipher exceeds the useful lifetime of the information.

Unfortunately, it is very difficult to estimate the amount of effort required to cryptanalyze ciphertext successfully. However, assuming there are no inherent mathematical weaknesses in the algorithm, then a brute-force approach is indicated, and here we can make some reasonable estimates about costs and time.

A brute-force approach involves trying every possible key until an intelligible translation of the ciphertext into plaintext is obtained. On average, half of all possible keys must be tried to achieve success. This type of attack is discussed in Section 2.1.

Feistel Cipher Structure

Many symmetric block encryption algorithms, including DES, have a structure first described by Horst Feistel of IBM in 1973 [FEIS73] and shown in Figure 20.1. The inputs to the encryption algorithm are a plaintext block of length $2w$ bits and a key K. The plaintext block is divided into two halves, L_0 and R_0. The two halves of the data pass through n rounds of processing and then combine to produce the ciphertext block. Each round i has as inputs L_{i-1} and R_{i-1}, derived from the previous round, as well as a subkey K_i, derived from the overall K. In general, the subkeys K_i are different from K and from each other and are generated from the key by a subkey generation algorithm.

All rounds have the same structure. A substitution is performed on the left half of the data. This is done by applying a *round function* F to the right half of the data and then taking the exclusive-OR (XOR) of the output of that function and the left half of the data. The round function has the same general structure for each round but is parameterized by the round subkey K_i. Following this substitution, a permutation is performed that consists of the interchange of the two halves of the data.

The Feistel structure is a particular example of the more general structure used by all symmetric block ciphers. In general, a symmetric block cipher consists of a sequence of rounds, with each round performing substitutions and permutations conditioned by a secret key value. The exact realization of a symmetric block cipher depends on the choice of the following parameters and design features:

- **Block size:** Larger block sizes mean greater security (all other things being equal) but reduced encryption/decryption speed. A block size of 128 bits is a reasonable tradeoff and is nearly universal among recent block cipher designs.
- **Key size:** Larger key size means greater security but may decrease encryption/decryption speed. The most common key length in modern algorithms is 128 bits.
- **Number of rounds:** The essence of a symmetric block cipher is that a single round offers inadequate security but that multiple rounds offer increasing security. A typical size is 16 rounds.
- **Subkey generation algorithm:** Greater complexity in this algorithm should lead to greater difficulty of cryptanalysis.
- **Round function:** Again, greater complexity generally means greater resistance to cryptanalysis.

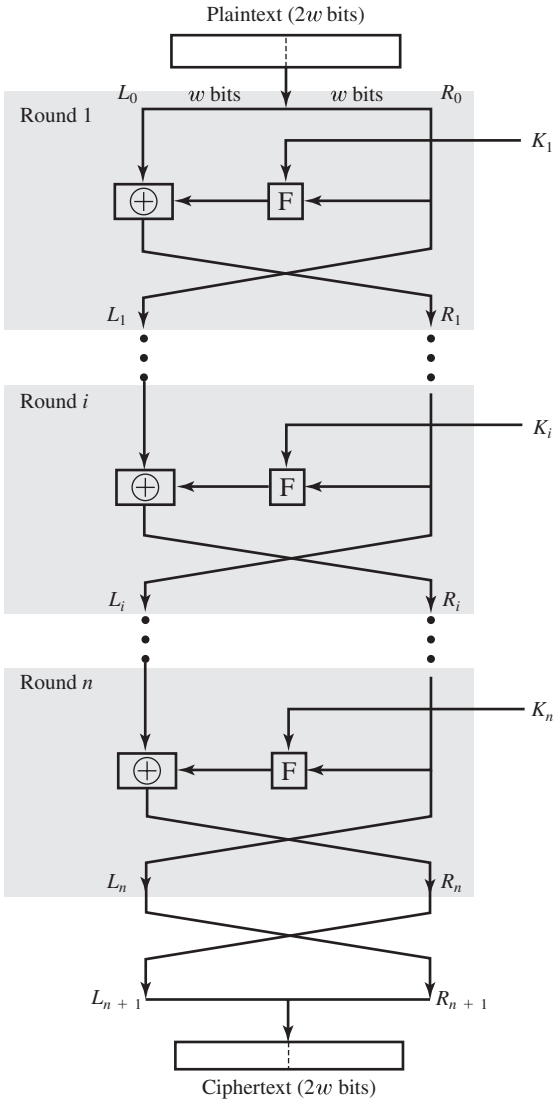

Figure 20.1 Classical Feistel Network

There are two other considerations in the design of a symmetric block cipher:

- **Fast software encryption/decryption:** In many cases, encryption is embedded in applications or utility functions in such a way as to preclude a hardware implementation. Accordingly, the speed of execution of the algorithm becomes a concern.
- **Ease of analysis:** Although we would like to make our algorithm as difficult as possible to cryptanalyze, there is great benefit in making the algorithm easy to analyze. That is, if the algorithm can be concisely and clearly explained, it is easier to analyze that algorithm for cryptanalytic vulnerabilities and therefore

develop a higher level of assurance as to its strength. DES, for example, does not have an easily analyzed functionality.

Decryption with a symmetric block cipher is essentially the same as the encryption process. The rule is as follows: Use the ciphertext as input to the algorithm, but use the subkeys K_i in reverse order. That is, use K_n in the first round, K_{n-1} in the second round, and so on until K_1 is used in the last round. This is a nice feature because it means we need not implement two different algorithms, one for encryption and one for decryption.

20.2 DATA ENCRYPTION STANDARD

The most commonly used symmetric encryption algorithms are block ciphers. A block cipher processes the plaintext input in fixed-size blocks and produces a block of ciphertext of equal size for each plaintext block. This section and the next focus on the three most important symmetric block ciphers: the Data Encryption Standard (DES), triple DES (3DES), and the Advanced Encryption Standard (AES).

Data Encryption Standard

The most widely used encryption scheme is based on the Data Encryption Standard (DES) adopted in 1977 by the National Bureau of Standards, now the National Institute of Standards and Technology (NIST), as Federal Information Processing Standard 46 (FIPS PUB 46). The algorithm itself is referred to as the Data Encryption Algorithm (DEA).[2]

The DES algorithm can be described as follows. The plaintext is 64 bits in length and the key is 56 bits in length; longer plaintext amounts are processed in 64-bit blocks. The DES structure is a minor variation of the Feistel network shown in Figure 20.1. There are 16 rounds of processing. From the original 56-bit key, 16 subkeys are generated, one of which is used for each round.

The process of decryption with DES is essentially the same as the encryption process. The rule is as follows: Use the ciphertext as input to the DES algorithm, but use the subkeys K_i in reverse order. That is, use K_{16} on the first iteration, K_{15} on the second iteration, and so on until K_1 is used on the sixteenth and last iteration.

Triple DES

Triple DES (3DES) was first standardized for use in financial applications in ANSI standard X9.17 in 1985. 3DES was incorporated as part of the Data Encryption Standard in 1999, with the publication of FIPS PUB 46-3.

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[2]The terminology is a bit confusing. Until recently, the terms *DES* and *DEA* could be used interchangeably. However, the most recent edition of the DES document includes a specification of the DEA described here plus the triple DEA (3DES) described subsequently. Both DEA and 3DES are part of the Data Encryption Standard. Further, until the recent adoption of the official term 3*DES*, the triple DEA algorithm was typically referred to as *triple DES* and written as 3DES. For the sake of convenience, we will use 3DES.

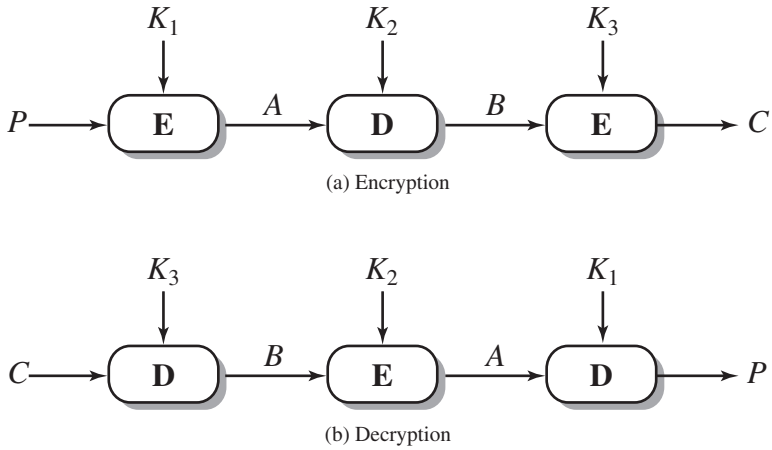


Figure 20.2 **Triple DES**

3DES uses three keys and three executions of the DES algorithm. The function follows an encrypt-decrypt-encrypt (EDE) sequence (Figure 20.2a):

$$C = \mathrm{E}(K_3, \mathrm{D}(K_2, \mathrm{E}(K_1, p)))$$

where

C = ciphertext
P = plaintext
$\mathrm{E}[K, X]$ = encryption of X using key K
$\mathrm{D}[K, Y]$ = decryption of Y using key K

Decryption is simply the same operation with the keys reversed (Figure 20.2b):

$$P = \mathrm{D}(K_1, \mathrm{E}(K_2, \mathrm{D}(K_3, \mathrm{C})))$$

There is no cryptographic significance to the use of decryption for the second stage of 3DES encryption. Its only advantage is that it allows users of 3DES to decrypt data encrypted by users of the older single DES:

$$C = \mathrm{E}(K_1, \mathrm{D}(K_1, \mathrm{E}(K_1, P))) = \mathrm{E}[K, P]$$

With three distinct keys, 3DES has an effective key length of 168 bits. FIPS 46-3 also allows for the use of two keys, with $K_1 = K_3$; this provides for a key length of 112 bits. FIPS 46-3 includes the following guidelines for 3DES:

- 3DES is the FIPS approved symmetric encryption algorithm of choice.
- The original DES, which uses a single 56-bit key, is permitted under the standard for legacy systems only. New procurements should support 3DES.
- Government organizations with legacy DES systems are encouraged to transition to 3DES.
- It is anticipated that 3DES and the Advanced Encryption Standard (AES) will coexist as FIPS-approved algorithms, allowing for a gradual transition to AES.

It is easy to see that 3DES is a formidable algorithm. Because the underlying cryptographic algorithm is DEA, 3DES can claim the same resistance to cryptanalysis based on the algorithm as is claimed for DEA. Further, with a 168-bit key length, brute-force attacks are effectively impossible.

Ultimately, AES is intended to replace 3DES, but this process will take a number of years. NIST anticipates that 3DES will remain an approved algorithm (for U.S. government use) for the foreseeable future.

20.3 ADVANCED ENCRYPTION STANDARD

The Advanced Encryption Standard (AES) was issued as a federal information processing standard (FIPS 197). It is intended to replace DES and triple DES with an algorithm that is more secure and efficient.

Overview of the Algorithm

AES uses a block length of 128 bits and a key length that can be 128, 192, or 256 bits. In the description of this section, we assume a key length of 128 bits, which is likely to be the one most commonly implemented.

Figure 20.3 shows the overall structure of AES. The input to the encryption and decryption algorithms is a single 128-bit block. In FIPS PUB 197, this block is depicted as a square matrix of bytes. This block is copied into the **State** array, which is modified at each stage of encryption or decryption. After the final stage, **State** is copied to an output matrix. Similarly, the 128-bit key is depicted as a square matrix of bytes. This key is then expanded into an array of key schedule words; each word is 4 bytes and the total key schedule is 44 words for the 128-bit key. The ordering of bytes within a matrix is by column. So, for example, the first 4 bytes of a 128-bit plaintext input to the encryption cipher occupy the first column of the **in** matrix, the second 4 bytes occupy the second column, and so on. Similarly, the first 4 bytes of the expanded key, which form a word, occupy the first column of the **w** matrix.

The following comments give some insight into AES:

1. One noteworthy feature of this structure is that it is not a Feistel structure. Recall that in the classic Feistel structure, half of the data block is used to modify the other half of the data block, and then the halves are swapped. AES does not use a Feistel structure but processes the entire data block in parallel during each round using substitutions and permutation.
2. The key that is provided as input is expanded into an array of forty-four 32-bit words, **w**[*i*]. Four distinct words (128 bits) serve as a round key for each round.
3. Four different stages are used, one of permutation and three of substitution:

   - **Substitute Bytes:** Uses a table, referred to as an S-box,[3] to perform a byte-by-byte substitution of the block
   - **Shift Rows:** A simple permutation that is performed row by row

[3]The term *S-box*, or substitution box, is commonly used in the description of symmetric ciphers to refer to a table used for a table-lookup type of substitution mechanism.

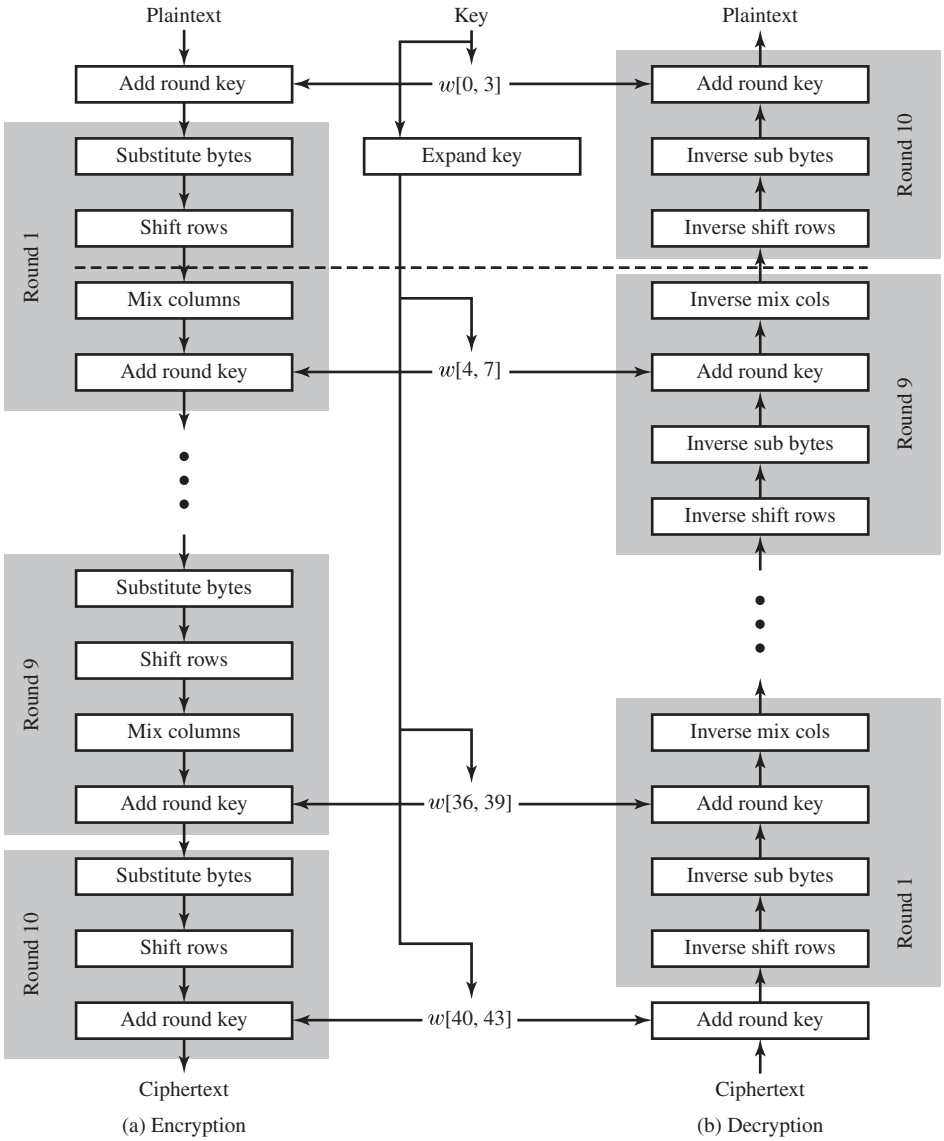


Figure 20.3 **AES Encryption and Decryption**

- **Mix Columns:** A substitution that alters each byte in a column as a function of all of the bytes in the column
- **Add Round key:** A simple bitwise XOR of the current block with a portion of the expanded key

4. The structure is quite simple. For both encryption and decryption, the cipher begins with an Add Round Key stage, followed by nine rounds that each includes all four stages, followed by a tenth round of three stages. Figure 20.4 depicts the structure of a full encryption round.

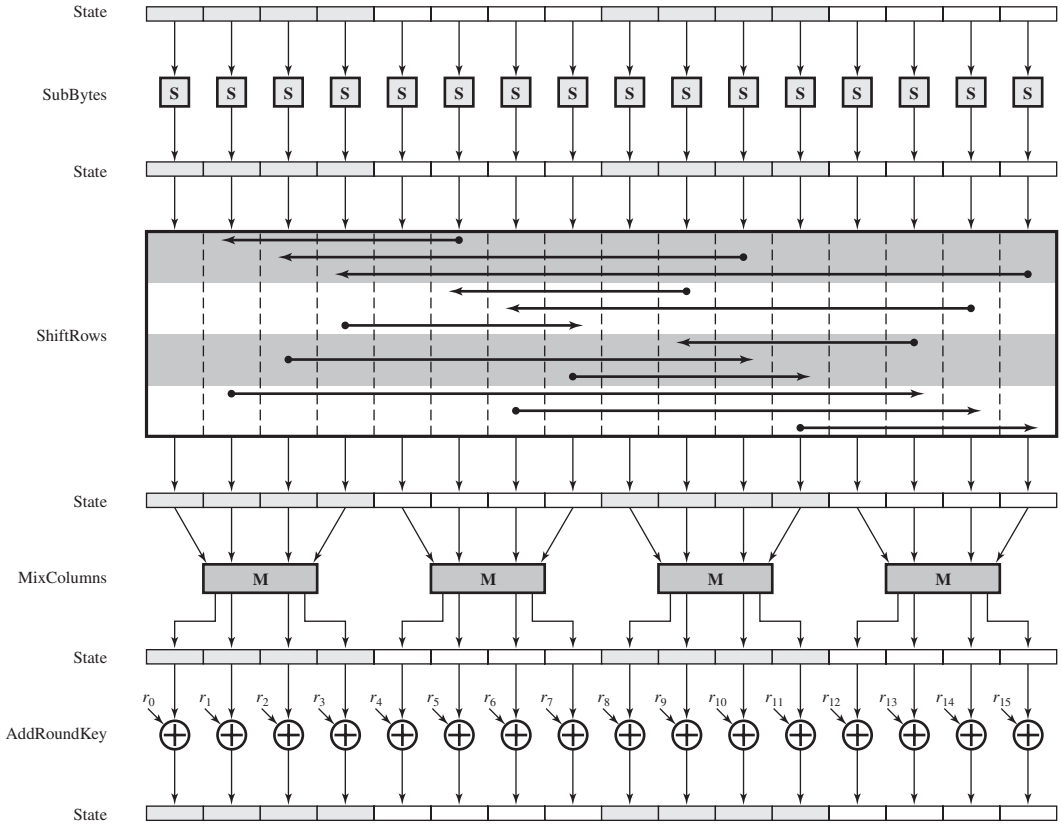


Figure 20.4 **AES Encryption Round**

5. Only the Add Round Key stage makes use of the key. For this reason, the cipher begins and ends with an Add Round Key stage. Any other stage, applied at the beginning or end, is reversible without knowledge of the key and so would add no security.
6. The Add Round Key stage by itself would not be formidable. The other three stages together scramble the bits, but by themselves would provide no security because they do not use the key. We can view the cipher as alternating operations of XOR encryption (Add Round Key) of a block, followed by scrambling of the block (the other three stages), followed by XOR encryption, and so on. This scheme is both efficient and highly secure.
7. Each stage is easily reversible. For the Substitute Byte, Shift Row, and Mix Columns stages, an inverse function is used in the decryption algorithm. For the Add Round Key stage, the inverse is achieved by XORing the same round key to the block, using the result that $A \oplus A \oplus B = B$.
8. As with most block ciphers, the decryption algorithm makes use of the expanded key in reverse order. However, the decryption algorithm is not identical to the encryption algorithm. This is a consequence of the particular structure of AES.

9. Once it is established that all four stages are reversible, it is easy to verify that decryption does recover the plaintext. Figure 20.3 lays out encryption and decryption going in opposite vertical directions. At each horizontal point (e.g., the dashed line in the figure), **State** is the same for both encryption and decryption.
10. The final round of both encryption and decryption consists of only three stages. Again, this is a consequence of the particular structure of AES and is required to make the cipher reversible.

Algorithm Details

We now look briefly at the principal elements of AES in more detail.

SUBSTITUTE BYTES TRANSFORMATION The **forward substitute byte transformation**, called SubBytes, is a simple table lookup. AES defines a 16·16 matrix of byte values, called an S-box (Table 20.2a), that contains a permutation of all possible 256 8-bit values. Each individual byte of **State** is mapped into a new byte in the following way: The leftmost 4 bits of the byte are used as a row value and the rightmost 4 bits are used as a column value. These row and column values serve as indexes into the S-box to select a unique 8-bit output value. For example, the hexadecimal value[4] {95} references row 9, column 5 of the S-box, which contains the value {2A}. Accordingly, the value {95} is mapped into the value {2A}.

Here is an example of the SubBytes transformation:

The S-box is constructed using properties of finite fields. The topic of finite fields is beyond the scope of this book; it is discussed in detail in [STAL14a].

EA	04	65	85
83	45	5D	96
5C	33	98	B0
F0	2D	AD	C5

→

87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

The **inverse substitute byte transformation**, called InvSubBytes, makes use of the inverse S-box shown in Table 20.2b. Note, for example, that the input {2A} produces the output {95}, and the input {95} to the S-box produces {2A}.

The S-box is designed to be resistant to known cryptanalytic attacks. Specifically, the AES developers sought a design that has a low correlation between input bits and output bits and the property that the output cannot be described as a simple mathematical function of the input.

SHIFT ROW TRANSFORMATION For the **forward shift row transformation**, called ShiftRows, the first row of **State** is not altered. For the second row, a 1-byte circular

[4]In FIPS PUB 197, a hexadecimal number is indicated by enclosing it in curly brackets. We use that convention.

Table 20.2 AES S-Boxes

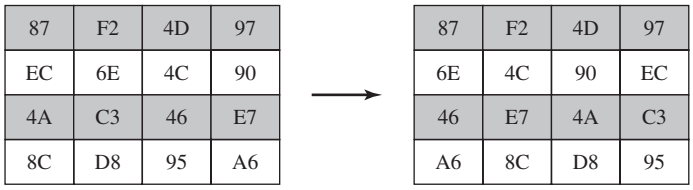
(a) S-box

		y															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	BI	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

(b) Inverse S-box

		y															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
	1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
	2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
	3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
	4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
	5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
	6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
	7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
	8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
	9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
	A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
	B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	FA
	C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
	D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
	E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
	F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

left shift is performed. For the third row, a 2-byte circular left shift is performed. For the third row, a 3-byte circular left shift is performed. The following is an example of ShiftRows:



87	F2	4D	97
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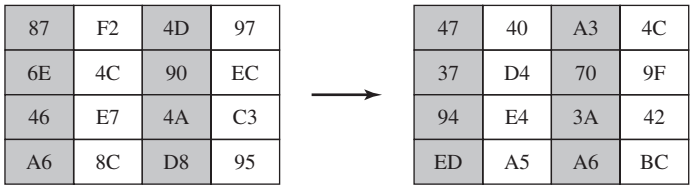
87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

The **inverse shift row transformation**, called InvShiftRows, performs the circular shifts in the opposite direction for each of the last three rows, with a 1-byte circular right shift for the second row, and so on.

The shift row transformation is more substantial than it may first appear. This is because the **State**, as well as the cipher input and output, is treated as an array of four 4-byte columns. Thus, on encryption, the first 4 bytes of the plaintext are copied to the first column of **State**, and so on. Further, as will be seen, the round key is applied to **State** column by column. Thus, a row shift moves an individual byte from one column to another, which is a linear distance of a multiple of 4 bytes. Also note that the transformation ensures that the 4 bytes of one column are spread out to four different columns.

MIX COLUMN TRANSFORMATION The **forward mix column transformation**, called MixColumns, operates on each column individually. Each byte of a column is mapped into a new value that is a function of all 4 bytes in the column. The mapping makes use of equations over finite fields. The following is an example of MixColumns:



87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

The mapping is designed to provide a good mixing among the bytes of each column. The mix column transformation combined with the shift row transformation ensures that after a few rounds, all output bits depend on all input bits.

ADD ROUND KEY TRANSFORMATION In the **forward add round key transformation**, called AddRoundKey, the 128 bits of **State** are bitwise XORed with the 128 bits of the round key. The operation is viewed as a column-wise

operation between the four bytes of a **State** column and one word of the round key; it can also be viewed as a byte-level operation. The following is an example of AddRoundKey:

47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

$\oplus$

AC	19	28	57
77	FA	D1	5C
66	DC	29	00
ED	A5	A6	BC

=

EB	59	8B	1B
40	2E	A1	C3
F2	38	13	42
1E	84	E7	D2

The first matrix is **State**, and the second matrix is the round key.

The **inverse add round key transformation** is identical to the forward add round key transformation, because the XOR operation is its own inverse.

The add round key transformation is as simple as possible and affects every bit of **State**. The complexity of the round key expansion, plus the complexity of the other stages of AES, ensure security.

AES Key Expansion The AES key expansion algorithm takes as input a 4-word (16-byte) key and produces a linear array of 44 words (156 bytes). This is sufficient to provide a 4-word round key for the initial Add Round Key stage and each of the 10 rounds of the cipher.

The key is copied into the first four words of the expanded key. The remainder of the expanded key is filled in four words at a time. Each added word **w**[i] depends on the immediately preceding word, **w**[i − 1], and the word four positions back, **w**[i − 4]. A complex finite-field algorithm is used in generating the expanded key.

20.4 STREAM CIPHERS AND RC4

A **block cipher** processes the input one block of elements at a time, producing an output block for each input block. A **stream cipher** processes the input elements continuously, producing output one element at a time, as it goes along. Although block ciphers are far more common, there are certain applications in which a stream cipher is more appropriate. Examples are given subsequently in this book. In this section, we look at perhaps the most popular symmetric stream cipher, RC4. We begin with an overview of stream cipher structure and then examine RC4.

Stream Cipher Structure

A typical stream cipher encrypts plaintext 1 byte at a time, although a stream cipher may be designed to operate on 1 bit at a time or on units larger than a byte at a time. Figure 2.3b is a representative diagram of stream cipher structure. In this

structure a key is input to a pseudorandom bit generator that produces a stream of 8-bit numbers that are apparently random. A pseudorandom stream is one that is unpredictable without knowledge of the input key and that has an apparently random character. The output of the generator, called a **keystream**, is combined 1 byte at a time with the plaintext stream using the bitwise exclusive-OR (XOR) operation. For example, if the next byte generated by the generator is 01101100 and the next plaintext byte is 11001100, then the resulting ciphertext byte is:

```
  11001100    plaintext
⊕ 01101100    key stream
  10100000    ciphertext
```

Decryption requires the use of the same pseudorandom sequence:

```
  10100000    ciphertext
⊕ 01101100    key stream
  11001100    plaintext
```

With a properly designed pseudorandom number generator, a stream cipher can be as secure as block cipher of comparable key length. The primary advantage of a stream cipher is that stream ciphers are almost always faster and use far less code than do block ciphers. The example in this section, RC4, can be implemented in just a few lines of code. Table 20.3 compares execution times of RC4 with three well-known symmetric block ciphers. The advantage of a block cipher is that you can reuse keys. However, if two plaintexts are encrypted with the same key using a stream cipher, then cryptanalysis is often quite simple [DAWS96]. If the two ciphertext streams are XORed together, the result is the XOR of the original plaintexts. If the plaintexts are text strings, credit card numbers, or other byte streams with known properties, then cryptanalysis may be successful.

For applications that require encryption/decryption of a stream of data, such as over a data communications channel or a browser/Web link, a stream cipher might be the better alternative. For applications that deal with blocks of data, such as file transfer, e-mail, and database, block ciphers may be more appropriate. However, either type of cipher can be used in virtually any application.

Table 20.3 Speed Comparisons of Symmetric Ciphers on a Pentium 4

Cipher	Key Length	Speed (Mbps)
DES	56	21
3DES	168	10
AES	128	61
RC4	Variable	113

Source: http://www.cryptopp.com/benchmarks.html

The RC4 Algorithm

RC4 is a stream cipher designed in 1987 by Ron Rivest for RSA Security. It is a variable-key-size stream cipher with byte-oriented operations. The algorithm is based on the use of a random permutation. Analysis shows that the period of the cipher is overwhelmingly likely to be greater than 10^{100} [ROBS95]. Eight to sixteen machine operations are required per output byte, and the cipher can be expected to run very quickly in software. RC4 is used in the SSL/TLS (Secure Sockets Layer/Transport Layer Security) standards that have been defined for communication between Web browsers and servers. It is also used in the WEP (Wired Equivalent Privacy) protocol and the newer WiFi Protected Access (WPA) protocol that are part of the IEEE 802.11 wireless LAN standard. RC4 was kept as a trade secret by RSA Security. In September 1994, the RC4 algorithm was anonymously posted on the Internet on the Cypherpunks anonymous remailers list.

The RC4 algorithm is remarkably simple and quite easy to explain. A variable-length key of from 1 to 256 bytes (8 to 2048 bits) is used to initialize a 256-byte state vector **S**, with elements **S**[0], **S**[1], . . . , **S**[255]. At all times, **S** contains a permutation of all 8-bit numbers from 0 through 255. For encryption and decryption, a byte *k* (see Figure 2.3b) is generated from **S** by selecting one of the 255 entries in a systematic fashion. As each value of *k* is generated, the entries in **S** are once again permuted.

INITIALIZATION OF S To begin, the entries of **S** are set equal to the values from 0 through 255 in ascending order; that is, **S**[0] = 0, **S**[1] = 1, . . . , **S**[255] = 255. A temporary vector, T, is also created. If the length of the key K is 256 bytes, then K is transferred to T. Otherwise, for a key of length *keylen* bytes, the first *keylen* elements of T are copied from K and then K is repeated as many times as necessary to fill out T. These preliminary operations can be summarized as follows:

```
/* Initialization */
for i = 0 to 255 do
S[i] =i;
T[i] = K[i mod keylen];
```

Next we use T to produce the initial permutation of S. This involves starting with S[0] and going through to S[255], and, for each **S**[i], swapping **S**[i] with another byte in **S** according to a scheme dictated by T[i]:

```
/* Initial Permutation of S */
j = 0;
for i = 0 to 255 do
   j = (j + S[i] + T[i]) mod 256;
   Swap (S[i], S[j]);
```

Because the only operation on **S** is a swap, the only effect is a permutation. **S** still contains all the numbers from 0 through 255.

STREAM GENERATION Once the **S** vector is initialized, the input key is no longer used. Stream generation involves cycling through all the elements of **S**[i], and, for each **S**[i], swapping **S**[i] with another byte in **S** according to a scheme dictated by the current configuration of S. After S[255] is reached, the process continues, starting over again at S[0]:

```
/* Stream Generation */
i, j = 0;
while (true)
    i = (i + 1) mod 256;
    j = (j + S[i]) mod 256;
    Swap (S[i], S[j]);
    t = (S[i] + S[j]) mod 256;
    k = S[t];
```

To encrypt, XOR the value *k* with the next byte of plaintext. To decrypt, XOR the value *k* with the next byte of ciphertext.

Figure 20.5 illustrates the RC4 logic.

STRENGTH OF RC4 A number of papers have been published analyzing methods of attacking RC4. None of these approaches is practical against RC4 with a reasonable key length, such as 128 bits. A more serious problem is reported in [FLUH01]. The

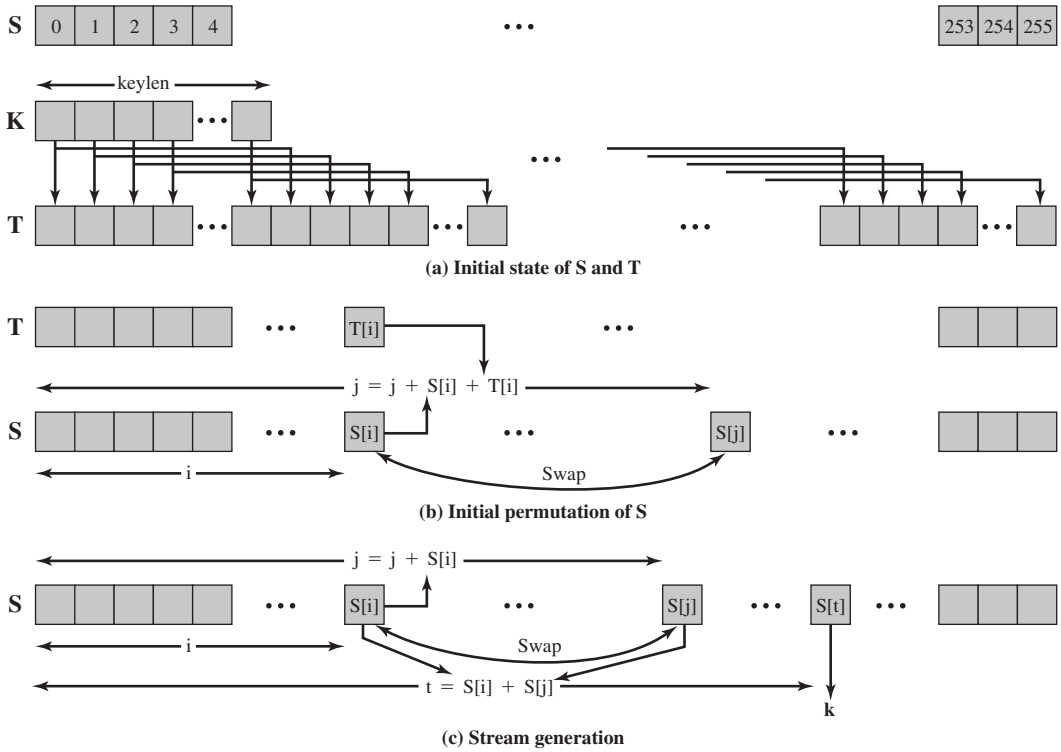


Figure 20.5 RC4

authors demonstrate that the WEP protocol, intended to provide confidentiality on 802.11 wireless LAN networks, is vulnerable to a particular attack approach. In essence, the problem is not with RC4 itself but the way in which keys are generated for use as input to RC4. This particular problem does not appear to be relevant to other applications using RC4 and can be remedied in WEP by changing the way in which keys are generated. This problem points out the difficulty in designing a secure system that involves both cryptographic functions and protocols that make use of them.

20.5 CIPHER BLOCK MODES OF OPERATION

A symmetric block cipher processes one block of data at a time. In the case of DES and 3DES, the block length is 64 bits. For longer amounts of plaintext, it is necessary to break the plaintext into 64-bit blocks (padding the last block if necessary). To apply a block cipher in a variety of applications, five **modes of operation** have been defined by NIST (Special Publication 800-38A). The five modes are intended to cover virtually all the possible applications of encryption for which a block cipher could be used. These modes are intended for use with any symmetric block cipher, including triple DES and AES. The modes are summarized in Table 20.4, and the most important are described briefly in the remainder of this section.

Electronic Codebook Mode

The simplest way to proceed is what is known as electronic codebook (ECB) mode, in which plaintext is handled b bits at a time and each block of plaintext is encrypted using the same key (Figure 2.3a). The term *codebook* is used because, for a given

Table 20.4 Block Cipher Modes of Operation

Mode	Description	Typical Application
Electronic Code book (ECB)	Each block of 64 plaintext bits is encoded independently using the same key.	• Secure transmission of single values (e.g., an encryption key)
Cipher Block Chaining (CBC)	The input to the encryption algorithm is the XOR of the next 64 bits of plaintext and the preceding 64 bits of ciphertext.	• General-purpose block-oriented transmission • Authentication
Cipher Feedback (CFB)	Input is processed s bits at a time. Preceding ciphertext is used as input to the encryption algorithm to produce pseudorandom output, which is XORed with plaintext to produce next unit of ciphertext.	• General-purpose stream-oriented transmission • Authentication
Output Feedback (OFB)	Similar to CFB, except that the input to the encryption algorithm is the preceding DES output.	• Stream-oriented transmission over noisy channel (e.g., satellite communication)
Counter (CTR)	Each block of plaintext is XORed with an encrypted counter. The counter is incremented for each subsequent block.	• General-purpose block-oriented transmission • Useful for high-speed requirements

key, there is a unique ciphertext for every b-bit block of plaintext. Therefore, one can imagine a gigantic codebook in which there is an entry for every possible b-bit plaintext pattern showing its corresponding ciphertext.

With ECB, if the same b-bit block of plaintext appears more than once in the message, it always produces the same ciphertext. Because of this, for lengthy messages, the ECB mode may not be secure. If the message is highly structured, it may be possible for a cryptanalyst to exploit these regularities. For example, if it is known that the message always starts out with certain predefined fields, then the cryptanalyst may have a number of known plaintext-ciphertext pairs to work with. If the message has repetitive elements, with a period of repetition a multiple of b-bits, then these elements can be identified by the analyst. This may help in the analysis or may provide an opportunity for substituting or rearranging blocks.

To overcome the security deficiencies of ECB, we would like a technique in which the same plaintext block, if repeated, produces different ciphertext blocks.

Cipher Block Chaining Mode

In the cipher block chaining (CBC) mode (Figure 20.6), the input to the encryption algorithm is the XOR of the current plaintext block and the preceding ciphertext block; the same key is used for each block. In effect, we have chained together the processing of the sequence of plaintext blocks. The input to the encryption function for each plaintext block bears no fixed relationship to the plaintext block. Therefore, repeating patterns of b-bits are not exposed.

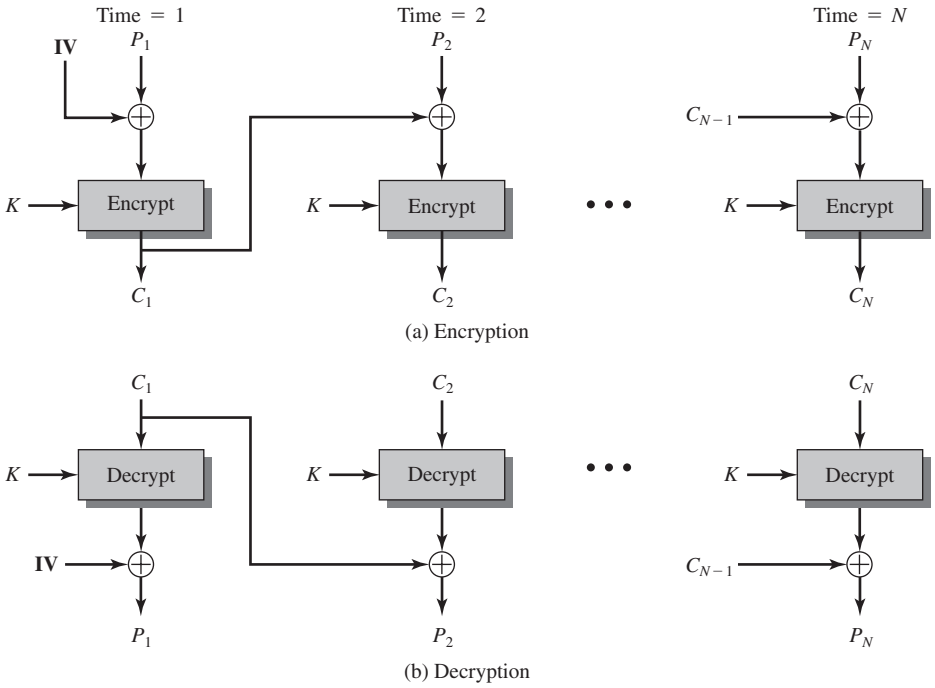


Figure 20.6 Cipher Block Chaining (CBC) Mode

For decryption, each cipher block is passed through the decryption algorithm. The result is XORed with the preceding ciphertext block to produce the plaintext block. To see that this works, we can write

$$C_j = \mathrm{E}(K, [C_{j-1} \oplus P_j])$$

where $\mathrm{E}[K, X]$ is the encryption of plaintext X using key K, and $\oplus$ is the exclusive-OR operation. Then

$$\mathrm{D}(K, C_j) = \mathrm{D}(K, \mathrm{E}(K, [C_{j-i} \oplus P_j]))$$
$$\mathrm{D}(K, C_j) = C_{j-1} \oplus P_j$$
$$C_{j-1} \oplus \mathrm{D}(K, C_j) = C_{j-1} \oplus C_{j-1} \oplus P_j = P_j$$

which verifies Figure 20.6b.

To produce the first block of ciphertext, an initialization vector (IV) is XORed with the first block of plaintext. On decryption, the IV is XORed with the output of the decryption algorithm to recover the first block of plaintext.

The IV must be known to both the sender and receiver. For maximum security, the IV should be protected as well as the key. This could be done by sending the IV using ECB encryption. One reason for protecting the IV is as follows: If an opponent is able to fool the receiver into using a different value for IV, then the opponent is able to invert selected bits in the first block of plaintext. To see this, consider the following:

$$C_1 = \mathrm{E}(K, [\mathrm{IV} \oplus P_1])$$
$$P_1 = \mathrm{IV} \oplus \mathrm{D}(K, C_1)$$

Now use the notation that $X[j]$ denotes the jth bit of the b-bit quantity X. Then

$$P_1[i] = \mathrm{IV}[i] \oplus \mathrm{D}(K, C_1)[i]$$

Then, using the properties of XOR, we can state

$$P_1[i]' = \mathrm{IV}[i]' \oplus \mathrm{D}(K, C_1)[i]$$

where the prime notation denotes bit complementation. This means that if an opponent can predictably change bits in IV, the corresponding bits of the received value of P_1 can be changed.

Cipher Feedback Mode

It is possible to convert any block cipher into a stream cipher by using the cipher feedback (CFB) mode. A stream cipher eliminates the need to pad a message to be an integral number of blocks. It also can operate in real time. Thus, if a character stream is being transmitted, each character can be encrypted and transmitted immediately using a character-oriented stream cipher.

One desirable property of a stream cipher is that the ciphertext be of the same length as the plaintext. Thus, if 8-bit characters are being transmitted, each character

should be encrypted using 8 bits. If more than 8 bits are used, transmission capacity is wasted.

Figure 20.7 depicts the CFB scheme. In the figure, it is assumed that the unit of transmission is s bits; a common value is $s = 8$. As with CBC, the units of plaintext are chained together, so that the ciphertext of any plaintext unit is a function of all the preceding plaintext.

First, consider encryption. The input to the encryption function is a b-bit shift register that is initially set to some initialization vector (IV). The leftmost (most significant) s bits of the output of the encryption function are XORed with the first unit of plaintext P_1 to produce the first unit of ciphertext C_1, which is then transmitted. In addition, the contents of the shift register are shifted left by s bits and C_1 is placed in the rightmost (least significant) s bits of the shift register. This process continues until all plaintext units have been encrypted.

For decryption, the same scheme is used, except that the received ciphertext unit is XORed with the output of the encryption function to produce the plaintext unit. Note that it is the *encryption* function that is used, not the decryption

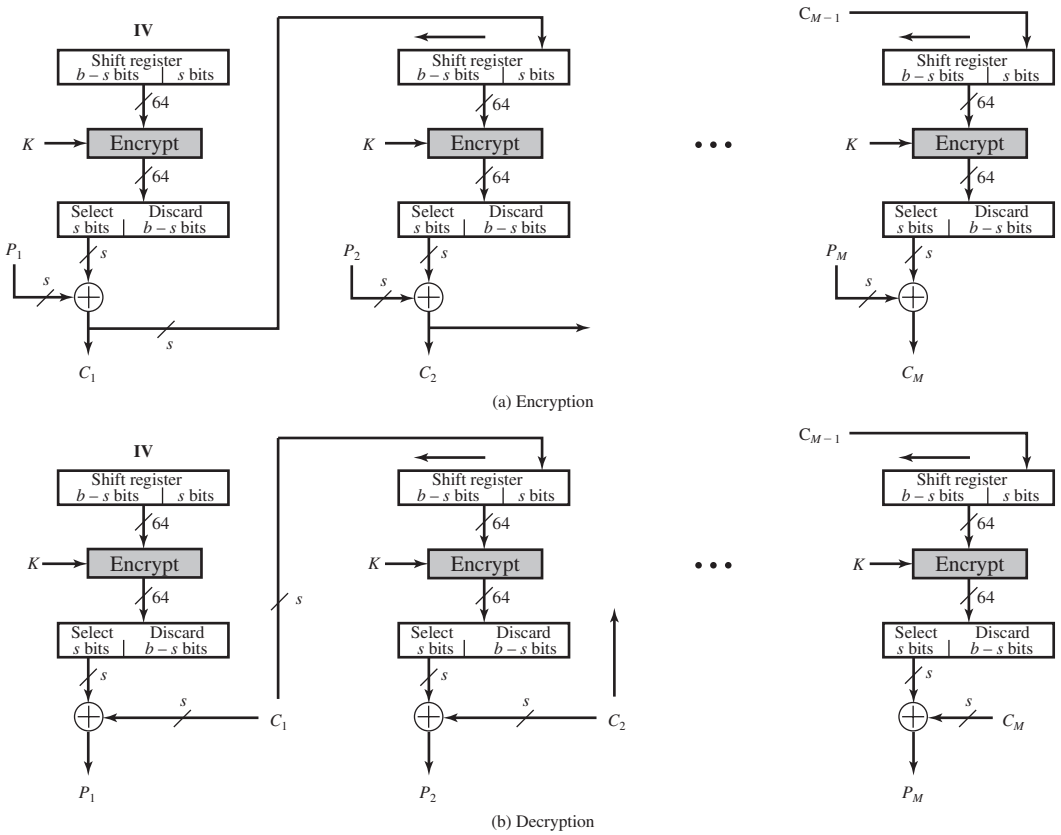


Figure 20.7 ***s*-bit Cipher Feedback (CFB) Mode**

function. This is easily explained. Let $S_s(X)$ be defined as the most significant s bits of X. Then

$$C_1 = P_1 \oplus S_s[E(K, IV)]$$

Therefore,

$$P_1 = C_1 \oplus S_s[E(K, IV)]$$

The same reasoning holds for subsequent steps in the process.

Counter Mode

Although interest in the counter mode (CTR) has increased recently, with applications to ATM (asynchronous transfer mode) network security and IPSec (IP security), this mode was proposed early on (e.g., [DIFF79]).

Figure 20.8 depicts the CTR mode. A counter equal to the plaintext block size is used. The only requirement stated in SP 800-38A is that the counter value must be different for each plaintext block that is encrypted. Typically, the counter is initialized to some value and then incremented by 1 for each subsequent block (modulo 2^b, where b is the block size). For encryption, the counter is encrypted and then XORed with the plaintext block to produce the ciphertext block; there is no

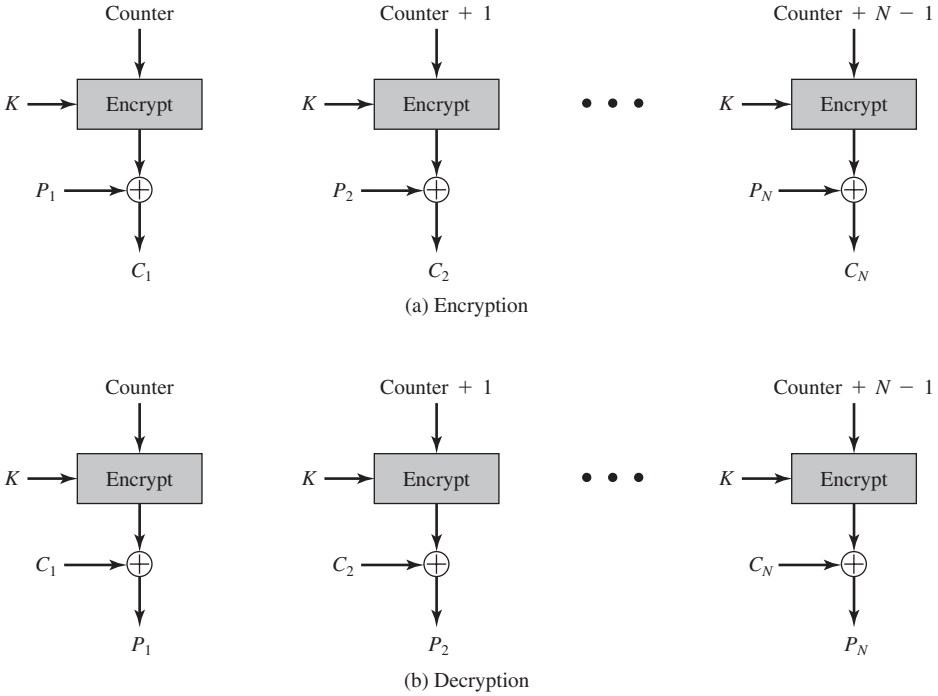


Figure 20.8 Counter (CTR) Mode

chaining. For decryption, the same sequence of counter values is used, with each encrypted counter XORed with a ciphertext block to recover the corresponding plaintext block.

[LIPM00] lists the following advantages of CTR mode:

- **Hardware efficiency:** Unlike the three chaining modes, encryption (or decryption) in CTR mode can be done in parallel on multiple blocks of plaintext or ciphertext. For the chaining modes, the algorithm must complete the computation on one block before beginning on the next block. This limits the maximum throughput of the algorithm to the reciprocal of the time for one execution of block encryption or decryption. In CTR mode, the throughput is only limited by the amount of parallelism that is achieved.
- **Software efficiency:** Similarly, because of the opportunities for parallel execution in CTR mode, processors that support parallel features, such as aggressive pipelining, multiple instruction dispatch per clock cycle, a large number of registers, and SIMD instructions, can be effectively utilized.
- **Preprocessing:** The execution of the underlying encryption algorithm does not depend on input of the plaintext or ciphertext. Therefore, if sufficient memory is available and security is maintained, preprocessing can be used to prepare the output of the encryption boxes that feed into the XOR functions in Figure 20.8. When the plaintext or ciphertext input is presented, then the only computation is a series of XORs. Such a strategy greatly enhances throughput.
- **Random access:** The ith block of plaintext or ciphertext can be processed in random access fashion. With the chaining modes, block C_i cannot be computed until the $i - 1$ prior block are computed. There may be applications in which a ciphertext is stored and it is desired to decrypt just one block; for such applications, the random access feature is attractive.
- **Provable security:** It can be shown that CTR is at least as secure as the other modes discussed in this section.
- **Simplicity:** Unlike ECB and CBC modes, CTR mode requires only the implementation of the encryption algorithm and not the decryption algorithm. This matters most when the decryption algorithm differs substantially from the encryption algorithm, as it does for AES. In addition, the decryption key scheduling need not be implemented.

20.6 LOCATION OF SYMMETRIC ENCRYPTION DEVICES

The most powerful, and most common, approach to countering the threats to network security is encryption. In using encryption, we need to decide what to encrypt and where the encryption gear should be located. There are two fundamental alternatives: link encryption and end-to-end encryption; these are illustrated in use over a frame network in Figure 20.9.

With link encryption, each vulnerable communications link is equipped on both ends with an encryption device. Thus, all traffic over all communications links

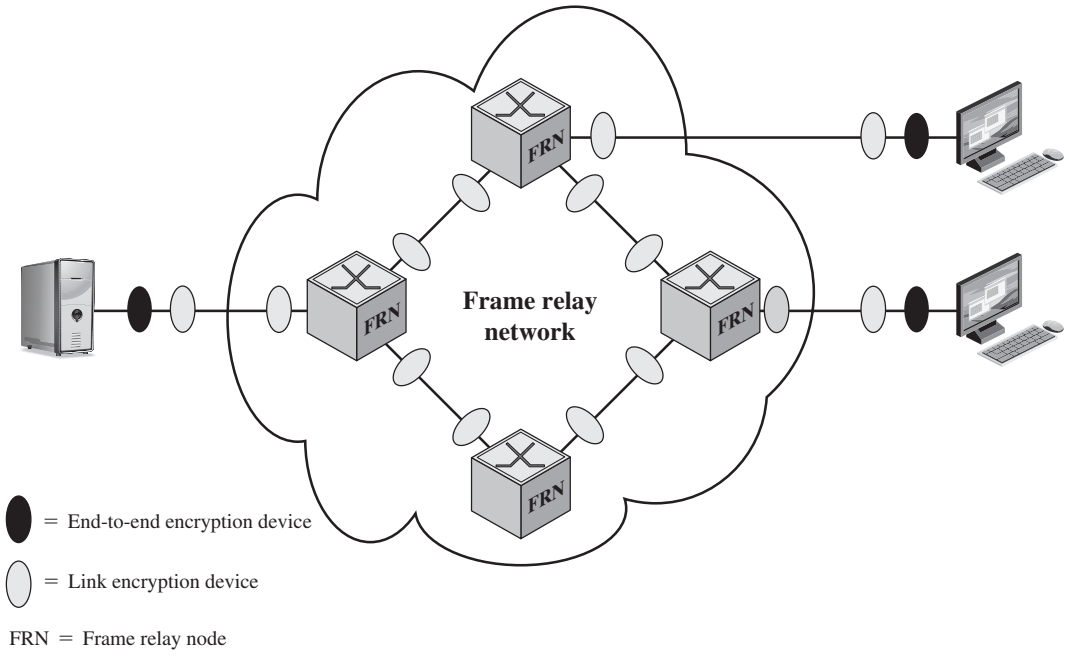


Figure 20.9 **Encryption Across a Frame Relay Network**

is secured. Although this requires a lot of encryption devices in a large network, it provides a high level of security. One disadvantage of this approach is that the message must be decrypted each time it enters a frame switch; this is necessary because the switch must read the address (connection identifier) in the frame header to route the frame. Thus, the message is vulnerable at each switch. If this is a public frame-relay network, the user has no control over the security of the nodes.

With end-to-end encryption, the encryption process is carried out at the two end systems. The source host or terminal encrypts the data. The data, in encrypted form, are then transmitted unaltered across the network to the destination terminal or host. The destination shares a key with the source and so is able to decrypt the data. This approach would seem to secure the transmission against attacks on the network links or switches. There is, however, still a weak spot.

Consider the following situation. A host connects to a frame relay network, sets up a logical data link connection to another host, and is prepared to transfer data to that other host using end-to-end encryption. Data are transmitted over such a network in the form of frames, consisting of a header and some user data. What part of each frame will the host encrypt? Suppose that the host encrypts the entire frame, including the header. This will not work because, remember, only the other host can perform the decryption. The frame relay node will receive an encrypted frame and be unable to read the header. Therefore, it will not be able to route the frame. It follows that the host may only encrypt the user data portion of the frame and must leave the header in the clear, so that it can be read by the network.

Thus, with end-to-end encryption, the user data are secure. However, the traffic pattern is not, because frame headers are transmitted in the clear. To achieve greater security, both link and end-to-end encryption are needed, as shown in Figure 20.9.

To summarize, when both forms are employed, the host encrypts the user data portion of a frame using an end-to-end encryption key. The entire frame is then encrypted using a link encryption key. As the frame traverses the network, each switch decrypts the frame using a link encryption key to read the header and then encrypts the entire frame again for sending it out on the next link. Now the entire frame is secure except for the time that the frame is actually in the memory of a frame switch, at which time the frame header is in the clear.

20.7 KEY DISTRIBUTION

For symmetric encryption to work, the two parties to an exchange must share the same key, and that key must be protected from access by others. Furthermore, frequent key changes are usually desirable to limit the amount of data compromised if an attacker learns the key. Therefore, the strength of any cryptographic system rests with the key distribution technique, a term that refers to the means of delivering a key to two parties that wish to exchange data, without allowing others to see the key. Key distribution can be achieved in a number of ways. For two parties A and B:

1. A key could be selected by A and physically delivered to B.
2. A third party could select the key and physically deliver it to A and B.
3. If A and B have previously and recently used a key, one party could transmit the new key to the other, encrypted using the old key.
4. If A and B each have an encrypted connection to a third party C, C could deliver a key on the encrypted links to A and B.

Options 1 and 2 call for manual delivery of a key. For link encryption, this is a reasonable requirement, because each link encryption device is only going to be exchanging data with its partner on the other end of the link. However, for end-to-end encryption, manual delivery is awkward. In a distributed system, any given host or terminal may need to engage in exchanges with many other hosts and terminals over time. Thus, each device needs a number of keys, supplied dynamically. The problem is especially difficult in a wide area distributed system.

Option 3 is a possibility for either link encryption or end-to-end encryption, but if an attacker ever succeeds in gaining access to one key, then all subsequent keys are revealed. Even if frequent changes are made to the link encryption keys, these should be done manually. To provide keys for end-to-end encryption, option 4 is preferable.

Figure 20.10 illustrates an implementation that satisfies option 4 for end-to-end encryption. In the figure, link encryption is ignored. This can be added, or not, as required. For this scheme, two kinds of keys are identified:

- **Session key:** When two end systems (hosts, terminals, etc.) wish to communicate, they establish a logical connection (e.g., virtual circuit). For the duration

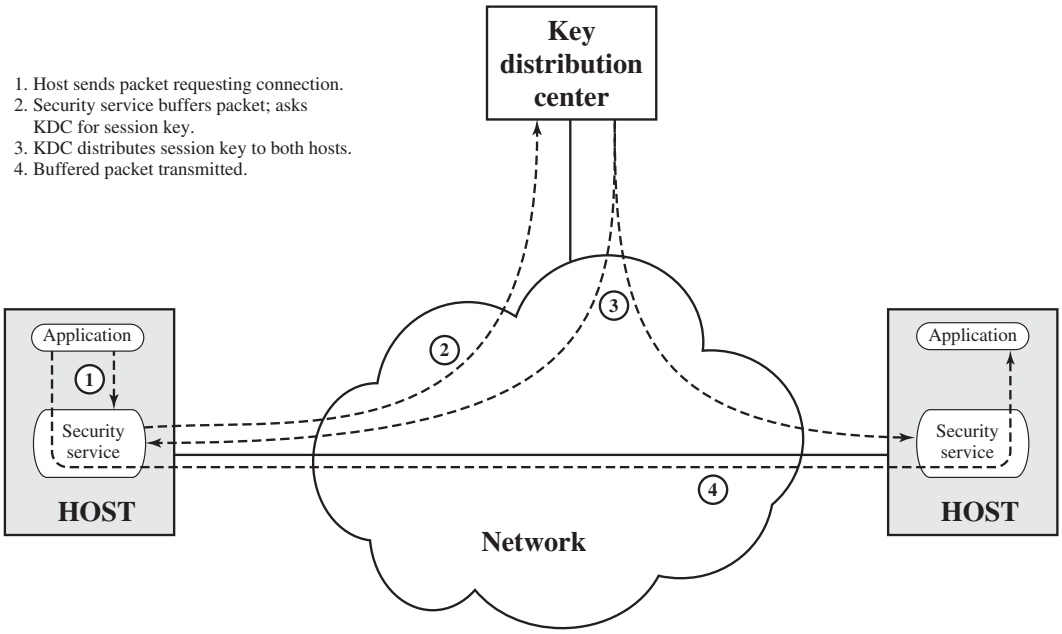


Figure 20.10 **Automatic Key Distribution for Connection-Oriented Protocd**

of that logical connection, all user data are encrypted with a one-time session key. At the conclusion of the session, or connection, the session key is destroyed.

- **Permanent key:** A permanent key is a key used between entities for the purpose of distributing session keys.

The configuration consists of the following elements:

- **Key distribution center:** The key distribution center (KDC) determines which systems are allowed to communicate with each other. When permission is granted for two systems to establish a connection, the KDC provides a one-time session key for that connection.
- **Security service module (SSM):** This module, which may consist of functionality at one protocol layer, performs end-to-end encryption and obtains session keys on behalf of users.

The steps involved in establishing a connection are shown in Figure 20.10. When one host wishes to set up a connection to another host, it transmits a connection-request packet (step 1). The SSM saves that packet and applies to the KDC for permission to establish the connection (step 2). The communication between the SSM and the KDC is encrypted using a master key shared only by this SSM and the KDC. If the KDC approves the connection request, it generates the session key and delivers it to the two appropriate SSMs, using a unique permanent key for each SSM (step 3). The requesting SSM can now release the connection request packet, and a connection is set up between the two end systems (step 4). All user data exchanged between the two end systems are encrypted by their respective SSMs using the one-time session key.

The automated key distribution approach provides the flexibility and dynamic characteristics needed to allow a number of terminal users to access a number of hosts and for the hosts to exchange data with each other.

Another approach to key distribution uses public-key encryption, which is discussed in Chapter 21.

20.8 RECOMMENDED READING

The topics in this chapter are covered in greater detail in [STAL14a].

STAL14a Stallings, W. *Cryptography and Network Security: Principles and Practice, Sixth Edition.* Upper Saddle River, NJ: Prentice Hall, 2014.

20.9 KEY TERMS, REVIEW QUESTIONS, AND PROBLEMS

Key Terms

Advanced Encryption Standard (AES)	cryptanalysis	keystream
block cipher	cryptography	link encryption
brute-force attack	Data Encryption Standard (DES)	modes of operation
computationally secure	decryption	plaintext
cipher block chaining (CBC) mode	electronic codebook (ECB) mode	RC4
cipher feedback (CFB) mode	encryption	session key
ciphertext	end-to-end encryption	stream cipher
counter mode	Feistel cipher	subkey
	key distribution	symmetric encryption
		triple DES (3DES)

Review Questions

20.1 What are the essential ingredients of a symmetric cipher?
20.2 What are the two basic functions used in encryption algorithms?
20.3 How many keys are required for two people to communicate via a symmetric cipher?
20.4 What is the difference between a block cipher and a stream cipher?
20.5 What are the two general approaches to attacking a cipher?
20.6 Why do some block cipher modes of operation only use encryption while others use both encryption and decryption?
20.7 What is triple encryption?
20.8 Why is the middle portion of 3DES a decryption rather than an encryption?
20.9 What is the difference between link and end-to-end encryption?
20.10 List ways in which secret keys can be distributed to two communicating parties.
20.11 What is the difference between a session key and a master key?
20.12 What is a key distribution center?

Problems

20.1 Show that Feistel decryption is the inverse of Feistel encryption.

20.2 Consider a Feistel cipher composed of 16 rounds with block length 128 bits and key length 128 bits. Suppose that, for a given k, the key scheduling algorithm determines values for the first 8 round keys, $k_1, k_2, \ldots k_8$, and then sets

$$k_9 = k_8, k_{10} = k_7, k_{11} = k_6, \ldots, k_{16} = k_1$$

Suppose you have a ciphertext c. Explain how, with access to an encryption oracle, you can decrypt c and determine m using just a single oracle query. This shows that such a cipher is vulnerable to a chosen plaintext attack. (An encryption oracle can be thought of as a device that, when given a plaintext, returns the corresponding ciphertext. The internal details of the device are not known to you and you cannot break open the device. You can only gain information from the oracle by making queries to it and observing its responses.)

20.3 For any block cipher, the fact that it is a nonlinear function is crucial to its security. To see this, suppose that we have a linear block cipher EL that encrypts 128-bit blocks of plaintext into 128-bit blocks of ciphertext. Let EL(k, m) denote the encryption of a 128-bit message m under a key k (the actual bit length of k is irrelevant). Thus

$$\text{EL}(k, [m_1 \oplus m_2]) = \text{EL}(k, m_1) \oplus \text{EL}(k, m_1) \text{ for all 128-bit patterns } m_1, m_2$$

Describe how, with 128 chosen ciphertexts, an adversary can decrypt any ciphertext without knowledge of the secret key k. (A "chosen ciphertext" means that an adversary has the ability to choose a ciphertext and then obtain its decryption. Here, you have 128 plaintext/ciphertext pairs to work with and you have the ability to chose the value of the ciphertexts.)

20.4 What RC4 key value will leave S unchanged during initialization? That is, after the initial permutation of S, the entries of S will be equal to the values from 0 through 255 in ascending order.

20.5 RC4 has a secret internal state which is a permutation of all the possible values of the vector **S** and the two indices i and j.

a. Using a straightforward scheme to store the internal state, how many bits are used?

b. Suppose we think of it from the point of view of how much information is represented by the state. In that case, we need to determine how may different states there are, then take the log to the base 2 to find out how many bits of information this represents. Using this approach, how many bits would be needed to represent the state?

20.6 With the ECB mode, if there is an error in a block of the transmitted ciphertext, only the corresponding plaintext block is affected. However, in the CBC mode, this error propagates. For example, an error in the transmitted C_1 (Figure 20.6) obviously corrupts P_1 and P_{21}.

a. Are any blocks beyond P_2 affected?

b. Suppose that there is a bit error in the source version of P_1. Through how many ciphertext blocks is this error propagated? What is the effect at the receiver?

20.7 Suppose an error occurs in a block of ciphertext on transmission using CBC. What effect is produced on the recovered plaintext blocks?

20.8 You want to build a hardware device to do block encryption in the cipher block chaining (CBC) mode using an algorithm stronger than DES. 3DES is a good candidate. Figure 20.11 shows two possibilities, both of which follow from the definition of CBC. Which of the two would you choose:

a. For security?

b. For performance?

20.9 Can you suggest a security improvement to either option in Figure 20.11, using only three DES chips and some number of XOR functions? Assume you are still limited to two keys.

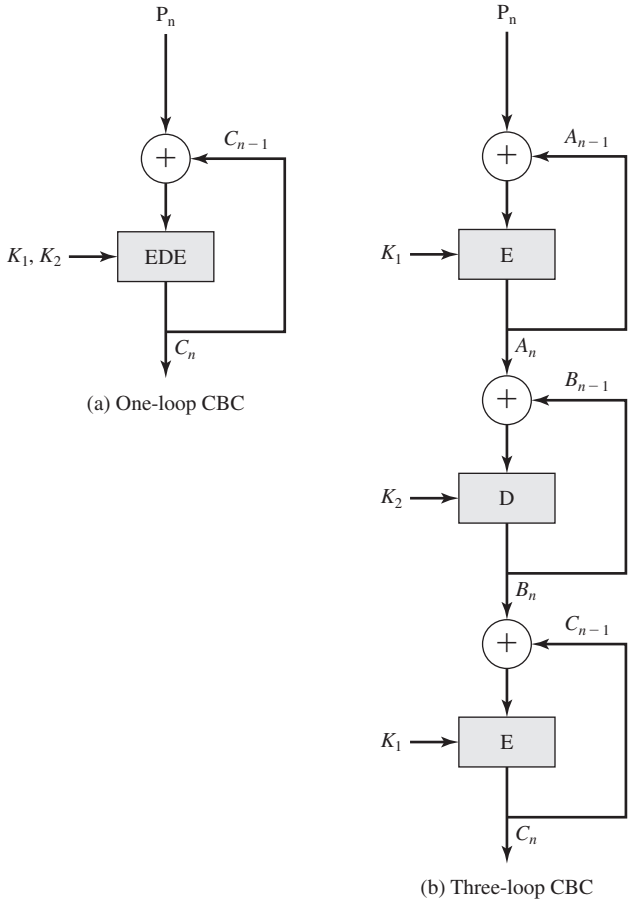


Figure 20.11 **Use of Triple DES in CBC Mode**

20.10 Fill in the remainder of this table:

Mode	Encrypt	Decrypt
ECB	$C_j = E(K, P_j) \quad j = 1, \ldots, N$	$P_j = D(K, C_j) \quad j = 1, \ldots, N$
CBC	$C_1 = E(K, [P_1 \oplus IV])$ $C_j = E(K, [P_j \oplus C_{j-1}]) \quad j = 2, \ldots, N$	$P_1 = D(K, C_1) \oplus IV$ $P_j = D(K, C_j) \oplus C_{j-1} \quad j = 2, \ldots, N$
CFB		
CTR		

20.11 CBC-Pad is a block cipher mode of operation used in the RC5 block cipher, but it could be used in any block cipher. CBC-Pad handles plaintext of any length. The ciphertext is longer then the plaintext by at most the size of a single block. Padding is used to assure that the plaintext input is a multiple of the block length. It is assumed that the original plaintext is an integer number of bytes. This plaintext is padded at the end by from 1 to *bb* bytes, where *bb* equals the block size in bytes. The pad bytes

are all the same and set to a byte that represents the number of bytes of padding. For example, if there are 8 bytes of padding, each byte has the bit pattern 00001000. Why not allow zero bytes of padding? That is, if the original plaintext is an integer multiple of the block size, why not refrain from padding?

20.12 Padding may not always be appropriate. For example, one might wish to store the encrypted data in the same memory buffer that originally contained the plaintext. In that case, the ciphertext must be the same length as the original plaintext. A mode for that purpose is the ciphertext stealing (CTS) mode. Figure 20.12a shows an implementation of this mode.

a. Explain how it works.

b. Describe how to decrypt C_{n-1} and C_n.

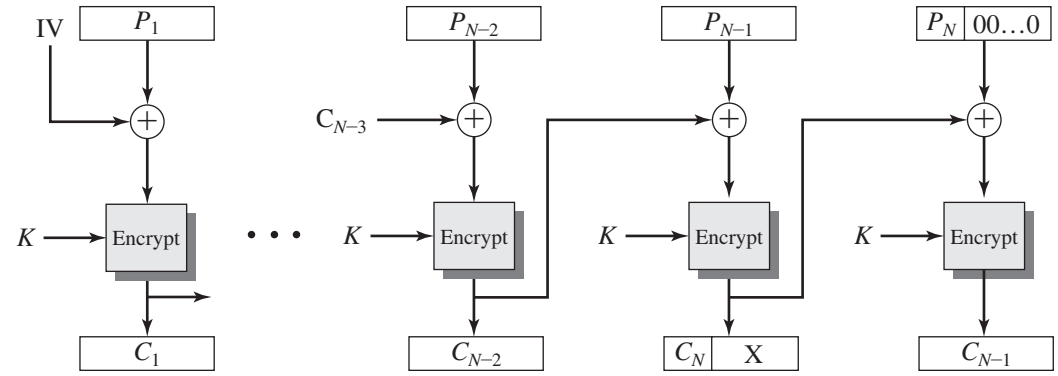
20.13 Figure 20.12b shows an alternative to CTS for producing ciphertext of equal length to the plaintext when the plaintext is not an integer multiple of the block size.

a. Explain the algorithm.

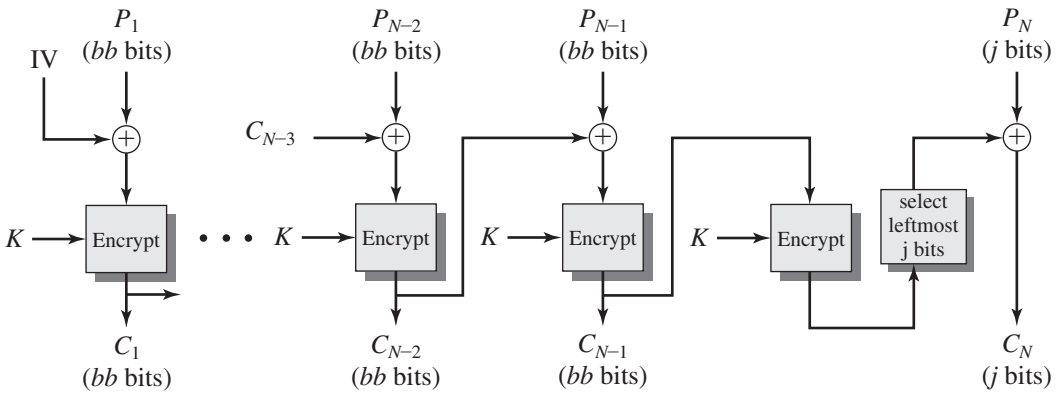
b. Explain why CTS is preferable to this approach illustrated in Figure 20.12b.

20.14 If a bit error occurs in the transmission of a ciphertext character in 8-bit CFB mode, how far does the error propagate?

20.15 One of the most widely used message authentication codes (MACs), referred to as the Data Authentication Algorithm, is based on DES. The algorithm is both a FIPS



(a) Cipheretext stealing mode



(b) Alternative method

Figure 20.12 Block Cipher Modes for Plaintext Not a Multiple of Block Size

publication (FIPS PUB 113) and an ANSI standard (X9.17). The algorithm can be defined as using the cipher block chaining (CBC) mode of operation of DES with an initialization vector of zero (Figure 20.6). The data (e.g., message, record, file, or program) to be authenticated are grouped into contiguous 64-bit blocks: $P_1, P_2, \ldots, P_N$. If necessary, the final block is padded on the right with 0s to form a full 64-bit block. The MAC consists of either the entire ciphertext block C_N or the leftmost M bits of the block, with $16 \leq M \leq 64$. Show that the same result can be produced using the cipher feedback mode.

20.16 Key distribution schemes using an access control center and/or a key distribution center have central points vulnerable to attack. Discuss the security implications of such centralization.

20.17 Suppose that someone suggests the following way to confirm that the two of you are both in possession of the same secret key. You create a random bit string the length of the key, XOR it with the key, and send the result over the channel. Your partner XORs the incoming block with the key (which should be the same as your key) and sends it back. You check, and if what you receive is your original random string, you have verified that your partner has the same secret key, yet neither of you has ever transmitted the key. Is there a flaw in this scheme?

CHAPTER 21

PUBLIC-KEY CRYPTOGRAPHY AND MESSAGE AUTHENTICATION

LEARNING OBJECTIVES

After studying this chapter, you should be able to:

- Understand the operation of SHA-1 and SHA-2.
- Present an overview of the use of HMAC for message authentication.
- Describe the RSA algorithm.
- Describe the Diffie-Hellman algorithm.

This chapter provides technical detail on the topics introduced in Sections 2.2 through 2.4.

21.1 SECURE HASH FUNCTIONS

The one-way hash function, or secure hash function, is important not only in message authentication but also in digital signatures. The requirements for and security of secure hash functions are discussed in Section 2.2. Here, we look at several hash functions, concentrating on perhaps the most widely used family of hash functions: Secure Hash Algorithm (SHA).

Simple Hash Functions

All hash functions operate using the following general principles. The input (message, file, etc.) is viewed as a sequence of n-bit blocks. The input is processed one block at a time in an iterative fashion to produce an n-bit hash function.

One of the simplest hash functions is the bit-by-bit exclusive-OR (XOR) of every block. This can be expressed as follows:

$$C_i = b_{i1} \oplus b_{i2} \oplus \ \ldots \ \oplus b_{im}$$

where

C_i = ith bit of the hash code, $1 \leq i \leq n$
m = number of n-bit blocks in the input
b_{ij} = ith bit in jth block
$\oplus$ = XOR operation

Figure 21.1 illustrates this operation; it produces a simple parity for each bit position and is known as a longitudinal redundancy check. It is reasonably effective for random data as a data integrity check. Each n-bit hash value is equally likely. Thus, the probability that a data error will result in an unchanged hash value is 2^{-n}. With more predictably formatted data, the function is less effective. For example, in most normal text files, the high-order bit of each octet is always zero. So if a 128-bit hash value is used, instead of an effectiveness of 2^{-128}, the hash function on this type of data has an effectiveness of 2^{-112}.

	Bit 1	Bit 2	• • •	Bit *n*
Block 1	b_{11}	b_{21}		b_{n1}
Block 2	b_{12}	b_{22}		b_{n2}
	• • •	• • •	• • •	• • •
Block *m*	b_{1m}	b_{2m}		b_{nm}
Hash code	C_1	C_2		C_n

Figure 21.1 Simple Hash Function Using Bitwise XOR

A simple way to improve matters is to perform a 1-bit circular shift, or rotation, on the hash value after each block is processed. The procedure can be summarized as follows:

1. Initially set the *n*-bit hash value to zero.
2. Process each successive *n*-bit block of data as follows:
   a. Rotate the current hash value to the left by 1 bit.
   b. XOR the block into the hash value.

This has the effect of "randomizing" the input more completely and overcoming any regularities that appear in the input.

Although the second procedure provides a good measure of data integrity, it is virtually useless for data security when an encrypted hash code is used with a plaintext message, as in Figures 2.6a and b. Given a message, it is an easy matter to produce a new message that yields that hash code: Simply prepare the desired alternate message and then append an *n*-bit block that forces the new message plus block to yield the desired hash code.

Although a simple XOR or rotated XOR (RXOR) is insufficient if only the hash code is encrypted, you may still feel that such a simple function could be useful when the message as well as the hash code is encrypted. But one must be careful. A technique originally proposed by the National Bureau of Standards used the simple XOR applied to 64-bit blocks of the message and then an encryption of the entire message that used the cipher block chaining (CBC) mode. We can define the scheme as follows: Given a message consisting of a sequence of 64-bit blocks $X_1, X_2, \ldots, X_N$, define the hash code C as the block-by-block XOR or all blocks and append the hash code as the final block:

$$C = X_{N+1} = X_1 \oplus X_2 \oplus \ \ldots \ \oplus X_N$$

Next, encrypt the entire message plus hash code, using CBC mode to produce the encrypted message $Y_1, Y_2, \ldots, X_{N+1}$. [JUEN85] points out several ways in which the ciphertext of this message can be manipulated in such a way that it is not detectable by the hash code. For example, by the definition of CBC (Figure 20.6), we have:

$$\begin{aligned} X_1 &= IV \oplus \mathrm{D}(K, Y_1) \\ X_i &= Y_{i-1} \oplus \mathrm{D}(K, Y_i) \\ X_{N+1} &= Y_N \oplus \mathrm{D}(K, Y_{N+1}) \end{aligned}$$

But X_{N+1} is the hash code:

$$X_{N+1} = X_1 \oplus X_2 \oplus \ldots \oplus X_N$$
$$= [IV \oplus \mathrm{D}(K, Y_1)] \oplus [Y_1 \oplus \mathrm{D}(K, Y_2)] \oplus \ldots \oplus [Y_{N-1} \oplus \mathrm{D}(K, Y_N)]$$

Because the terms in the preceding equation can be XORed in any order, it follows that the hash code would not change if the ciphertext blocks were permuted.

The SHA Secure Hash Function

The Secure Hash Algorithm (SHA) was developed by the National Institute of Standards and Technology (NIST) and published as a federal information processing standard (FIPS 180) in 1993; a revised version was issued as FIPS 180-1 in 1995 and is generally referred to as SHA-1. SHA-1 is also specified in RFC 3174, which essentially duplicates the material in FIPS 180-1 but adds a C code implementation.

SHA-1 produces a hash value of 160 bits. In 2002, NIST produced a revision of the standard, FIPS 180-2, that defined three new versions of SHA, with hash value lengths of 256, 384, and 512 bits, known as SHA-256, SHA-384, and SHA-512 (Table 21.1). Collectively, these hash algorithms are known as **SHA-2**. These new versions have the same underlying structure and use the same types of modular arithmetic and logical binary operations as SHA-1. In 2005, NIST announced the intention to phase out approval of SHA-1 and move to a reliance on the other SHA versions by 2010. Shortly thereafter, a research team described an attack in which two separate messages could be found that deliver the same SHA-1 hash using 2^{69} operations, far fewer than the 2^{80} operations previously thought needed to find a collision with an SHA-1 hash [WANG05]. This result has hastened the transition to the other versions of SHA.

In this section, we provide a description of SHA-512. The other versions are quite similar. The algorithm takes as input a message with a maximum length of less than 2^{128} bits and produces as output a 512-bit message digest. The input is processed in 1024-bit blocks. Figure 21.2 depicts the overall processing of a message to produce a digest. The processing consists of the following steps:

- **Step 1: Append padding bits.** The message is padded so that its length is congruent to 896 modulo 1024 [length $\equiv$ 896 (mod 1024)]. Padding is always

Table 21.1 Comparison of SHA Parameters

	SHA-1	SHA-256	SHA-384	SHA-512
Message digest size	160	256	384	512
Message size	$< 2^{64}$	$< 2^{64}$	$< 2^{128}$	$< 2^{128}$
Block size	512	512	1024	1024
Word size	32	32	64	64
Number of steps	80	64	80	80
Security	80	128	192	256

Notes: 1. All sizes are measured in bits.
2. Security refers to the fact that a birthday attack on a message digest of size n produces a collision with a work factor of approximately $2^{n/2}$.

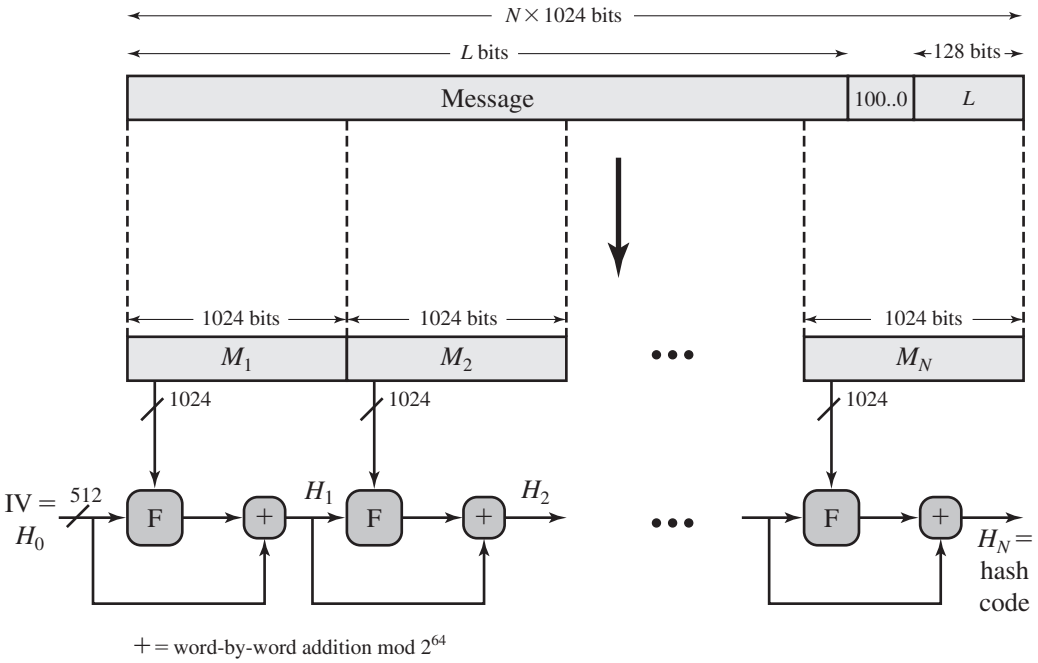


Figure 21.2 **Message Digest Generation Using SHA-512**

added, even if the message is already of the desired length. Thus, the number of padding bits is in the range of 1 to 1024. The padding consists of a single 1-bit followed by the necessary number of 0-bits.

- **Step 2: Append length.** A block of 128 bits is appended to the message. This block is treated as an unsigned 128-bit integer (most significant byte first) and contains the length of the original message (before the padding).

  The outcome of the first two steps yields a message that is an integer multiple of 1024 bits in length. In Figure 21.2, the expanded message is represented as the sequence of 1024-bit blocks $M_1, M_2, \ldots, M_N$, so that the total length of the expanded message is $N \times 1024$ bits.

- **Step 3: Initialize hash buffer.** A 512-bit buffer is used to hold intermediate and final results of the hash function. The buffer can be represented as eight 64-bit registers (a, b, c, d, e, f, g, h). These registers are initialized to the following 64-bit integers (hexadecimal values):

  a = 6A09E667F3BCC908 e = 510E527FADE682D1
  b = BB67AE8584CAA73B f = 9B05688C2B3E6C1F
  c = 3C6EF372FE94F82B g = 1F83D9ABFB41BD6B
  d = A54FF53A5F1D36F1 h = 5BE0CD19137E2179

  These values are stored in big-endian format, which is the most significant byte of a word in the low-address (leftmost) byte position. These words were obtained by taking the first 64 bits of the fractional parts of the square roots of the first eight prime numbers.

- **Step 4: Process message in 1024-bit (128-word) blocks.** The heart of the algorithm is a module that consists of 80 rounds; this module is labeled F in Figure 21.2. The logic is illustrated in Figure 21.3.

  Each round takes as input the 512-bit buffer value *abcdefgh* and updates the contents of the buffer. At input to the first round, the buffer has the value of the intermediate hash value, H_{i-1}. Each round t makes use of a 64-bit value W_t, derived from the current 1024-bit block being processed (M_i). Each round also makes use of an additive constant K_t, where $0 \leq t \leq 79$ indicates one of the 80 rounds. These words represent the first 64 bits of the fractional parts of the cube roots of the first 80 prime numbers. The constants provide a "randomized" set of 64-bit patterns, which should eliminate any regularities in the input data. The operations performed during a round consist of circular shifts, and primitive Boolean functions based on AND, OR, NOT, and XOR.

  The output of the eightieth round is added to the input to the first round (H_{i-1}) to produce H_i. The addition is done independently for each of the eight words in the buffer, with each of the corresponding words in H_{i-1}, using addition modulo 2^{64}.

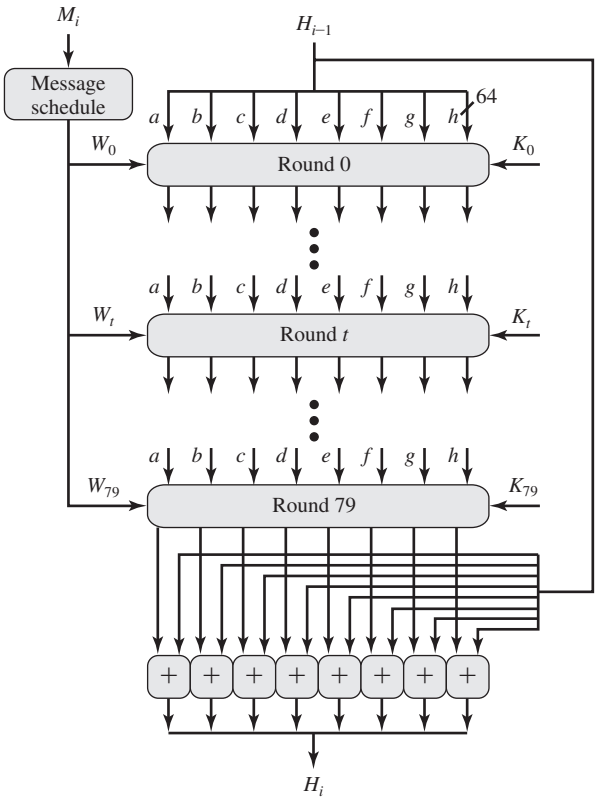


Figure 21.3 **SHA-512 Processing of a Single 1024-Bit Block**

- **Step 5: Output.** After all *N* 1024-bit blocks have been processed, the output from the *N*th stage is the 512-bit message digest.

The SHA-512 algorithm has the property that every bit of the hash code is a function of every bit of the input. The complex repetition of the basic function F produces results that are well mixed; that is, it is unlikely that two messages chosen at random, even if they exhibit similar regularities, will have the same hash code. Unless there is some hidden weakness in SHA-512, which has not so far been published, the difficulty of coming up with two messages having the same message digest is on the order of 2^{256} operations, while the difficulty of finding a message with a given digest is on the order of 2^{512} operations.

SHA–3

SHA-2, particularly the 512-bit version, would appear to provide unassailable security. However, SHA-2 shares the same structure and mathematical operations as its predecessors, and this is a cause for concern. Because it would take years to find a suitable replacement for SHA-2, should it become vulnerable, NIST announced in 2007 a competition to produce the next generation NIST hash function, to be called SHA-3. The basic requirements that must be satisfied by any candidate for SHA-3 are the following:

1. It must be possible to replace SHA-2 with SHA-3 in any application by a simple drop-in substitution. Therefore, SHA-3 must support hash value lengths of 224, 256, 384, and 512 bits.
2. SHA-3 must preserve the online nature of SHA-2. That is, the algorithm must process comparatively small blocks (512 or 1024 bits) at a time instead of requiring that the entire message be buffered in memory before processing it.

21.2 HMAC

In this section, we look at the hash code approach to message authentication. Appendix E looks at message authentication based on block ciphers. In recent years, there has been increased interest in developing a MAC derived from a cryptographic hash code, such as SHA-1. The motivations for this interest are as follows:

- Cryptographic hash functions generally execute faster in software than conventional encryption algorithms such as DES.
- Library code for cryptographic hash functions is widely available.

A hash function such as SHA-1 was not designed for use as a MAC and cannot be used directly for that purpose because it does not rely on a secret key. There have been a number of proposals for the incorporation of a secret key into an existing hash algorithm. The approach that has received the most support is HMAC [BELL96]. HMAC has been issued as RFC 2104, has been chosen as the mandatory-to-implement MAC for IP Security, and is used in other Internet

protocols, such as Transport Layer Security (TLS, soon to replace Secure Sockets Layer) and Secure Electronic Transaction (SET).

HMAC Design Objectives

RFC 2104 lists the following design objectives for HMAC:

- To use, without modifications, available hash functions—in particular, hash functions that perform well in software, and for which code is freely and widely available.
- To allow for easy replaceability of the embedded hash function in case faster or more secure hash functions are found or required.
- To preserve the original performance of the hash function without incurring a significant degradation.
- To use and handle keys in a simple way.
- To have a well-understood cryptographic analysis of the strength of the authentication mechanism based on reasonable assumptions on the embedded hash function.

The first two objectives are important to the acceptability of HMAC. HMAC treats the hash function as a "black box." This has two benefits. First, an existing implementation of a hash function can be used as a module in implementing HMAC. In this way, the bulk of the HMAC code is prepackaged and ready to use without modification. Second, if it is ever desired to replace a given hash function in an HMAC implementation, all that is required is to remove the existing hash function module and drop in the new module. This could be done if a faster hash function were desired. More important, if the security of the embedded hash function were compromised, the security of HMAC could be retained simply by replacing the embedded hash function with a more secure one.

The last design objective in the preceding list is, in fact, the main advantage of HMAC over other proposed hash-based schemes. HMAC can be proven secure provided that the embedded hash function has some reasonable cryptographic strengths. We return to this point later in this section, but first we examine the structure of HMAC.

HMAC Algorithm

Figure 21.4 illustrates the overall operation of HMAC. Define the following terms:

H = embedded hash function (e.g., SHA)
M = message input to HMAC (including the padding specified in the embedded hash function)
Y_i = ith block of $M, 0 \leq i \leq (L - 1)$
L = number of blocks in M
b = number of bits in a block
n = length of hash code produced by embedded hash function
K = secret key; if key length is greater than b, the key is input to the hash function to produce an n-bit key; recommended length is $\geq n$

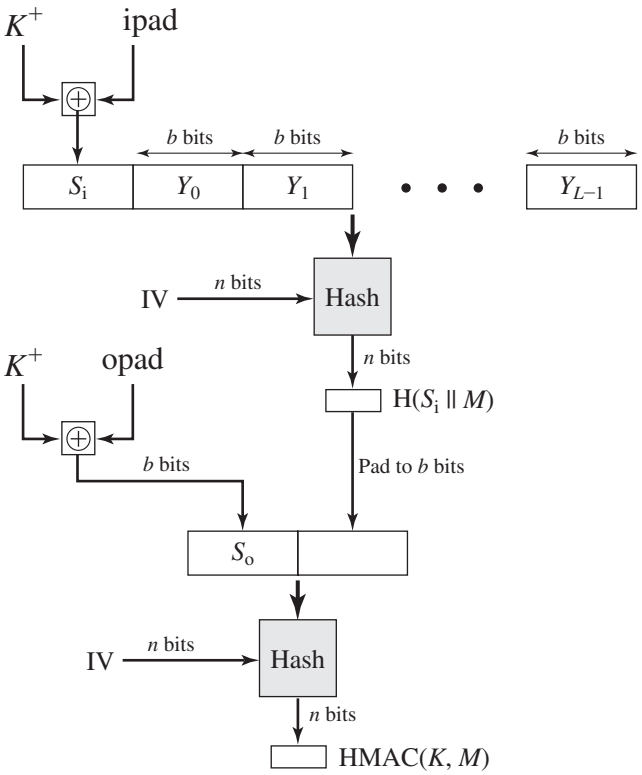


Figure 21.4 **HMAC Structure**

K^+ = K padded with zeros on the left so that the result is b bits in length
ipad = 00110110 (36 in hexadecimal) repeated $b/8$ times
opad = 01011100 (5C in hexadecimal) repeated $b/8$ times

Then HMAC can be expressed as follows:

$$\text{HMAC}(K, M) = \text{H}[(K^+ \oplus \text{opad}) \| \text{H}[K^+ \oplus \text{ipad}] \| M]]$$

In words,

1. Append zeros to the left end of K to create a b-bit string K^+ (e.g., if K is of length 160 bits and $b = 512$, then K will be appended with 44 zero bytes 0x00).
2. XOR (bitwise exclusive-OR) K^+ with ipad to produce the b-bit block S_i.
3. Append M to S_i.
4. Apply H to the stream generated in step 3.
5. XOR K^+ with opad to produce the b-bit block S_o.
6. Append the hash result from step 4 to S_o.
7. Apply H to the stream generated in step 6 and output the result.

Note that the XOR with ipad results in flipping one-half of the bits of K. Similarly, the XOR with opad results in flipping one-half of the bits of K, but a different set of bits. In effect, by passing S_i and S_o through the hash algorithm, we have pseudorandomly generated two keys from K.

HMAC should execute in approximately the same time as the embedded hash function for long messages. HMAC adds three executions of the basic hash function (for S_i, S_o, and the block produced from the inner hash).

Security of HMAC

The security of any MAC function based on an embedded hash function depends in some way on the cryptographic strength of the underlying hash function. The appeal of HMAC is that its designers have been able to prove an exact relationship between the strength of the embedded hash function and the strength of HMAC.

The security of a MAC function is generally expressed in terms of the probability of successful forgery with a given amount of time spent by the forger and a given number of message-MAC pairs created with the same key. In essence, it is proved in [BELL96] that for a given level of effort (time, message-MAC pairs) on messages generated by a legitimate user and seen by the attacker, the probability of successful attack on HMAC is equivalent to one of the following attacks on the embedded hash function:

1. The attacker is able to compute an output of the compression function even with an IV that is random, secret, and unknown to the attacker.
2. The attacker finds collisions in the hash function even when the IV is random and secret.

In the first attack, we can view the compression function as equivalent to the hash function applied to a message consisting of a single b-bit block. For this attack, the IV of the hash function is replaced by a secret, random value of n bits. An attack on this hash function requires either a brute-force attack on the key, which is a level of effort on the order of 2^n, or a birthday attack, which is a special case of the second attack, discussed next.

In the second attack, the attacker is looking for two messages M and M' that produce the same hash: $H(M) = H(M')$. This is the birthday attack mentioned previously. We have stated that this requires a level of effort of $2^{n/2}$ for a hash length of n. On this basis, the security of MD5 is called into question, because a level of effort of 2^{64} looks feasible with today's technology. Does this mean that a 128-bit hash function such as MD5 is unsuitable for HMAC? The answer is no, because of the following argument. To attack MD5, the attacker can choose any set of messages and work on these offline on a dedicated computing facility to find a collision. Because the attacker knows the hash algorithm and the default IV, the attacker can generate the hash code for each of the messages that the attacker generates. However, when attacking HMAC, the attacker cannot generate message/code pairs offline because the attacker does not know K. Therefore, the attacker must observe a sequence of messages generated by HMAC under the same key and perform the attack on these known messages. For a hash code length of 128 bits, this requires 2^{64} observed blocks (2^{72} bits) generated using the same

key. On a 1-Gbps link, one would need to observe a continuous stream of messages with no change in key for about 150,000 years in order to succeed. Thus, if speed is a concern, it is fully acceptable to use MD5 rather than SHA as the embedded hash function for HMAC.

21.3 THE RSA PUBLIC-KEY ENCRYPTION ALGORITHM

Perhaps the most widely used public-key algorithms are RSA and Diffie-Hellman. We examine RSA plus some security considerations in this section.[1] Diffie-Hellman is covered in Section 21.4.

Description of the Algorithm

One of the first public-key schemes was developed in 1977 by Ron Rivest, Adi Shamir, and Len Adleman at MIT and first published in 1978 [RIVE78]. The RSA scheme has since that time reigned supreme as the most widely accepted and implemented approach to public-key encryption. RSA is a block cipher in which the plaintext and ciphertext are integers between 0 and $n - 1$ for some n.

Encryption and decryption are of the following form, for some plaintext block M and ciphertext block C:

$$C = M^e \mod n$$
$$M = C^d \mod n = (M^e)^d \mod n = M^{ed} \mod n$$

Both sender and receiver must know the values of n and e, and only the receiver knows the value of d. This is a public-key encryption algorithm with a public key of $PU = \{e,n\}$ and a private key of $PR = \{d,n\}$. For this algorithm to be satisfactory for public-key encryption, the following requirements must be met:

1. It is possible to find values of e, d, n such that $M^{ed} \mod n = M$ for all $M < n$.
2. It is relatively easy to calculate M^e and C^d for all values of $M < n$.
3. It is infeasible to determine d given e and n.

The first two requirements are easily met. The third requirement can be met for large values of e and n.

More should be said about the first requirement. We need to find a relationship of the form

$$M^{ed} \mod n = M$$

The preceding relationship holds if e and d are multiplicative inverses modulo $\phi(n)$, where $\phi(n)$ is the Euler totient function. It is shown in Appendix B that for p, q prime, $\phi(pq) = (p - 1)(q - 1)$. $\phi(n)$, referred to as the Euler totient of n, is the number of positive integers less than n and relatively prime to n. The relationship between e and d can be expressed as

$$ed \mod \phi(n) = 1$$

[1]This section uses some elementary concepts from number theory. For a review, see Appendix B.

This is equivalent to saying

$$ed \bmod \phi(n) = 1$$
$$d \bmod \phi(n) = e^{-1}$$

That is, e and d are multiplicative inverses mod $\phi(n)$. According to the rules of modular arithmetic, this is true only if d (and therefore e) is relatively prime to $\phi(n)$. Equivalently, $\gcd(\phi(n),d) = 1$; that is, the greatest common divisor of $\phi(n)$ and d is 1.

Figure 21.5 summarizes the RSA algorithm. Begin by selecting two prime numbers, p and q, and calculating their product n, which is the modulus for encryption and decryption. Next, we need the quantity $\phi(n)$. Then select an integer e that is relatively prime to $\phi(n)$ [i.e., the greatest common divisor of e and $\phi(n)$ is 1]. Finally, calculate d as the multiplicative inverse of e, modulo $\phi(n)$. It can be shown that d and e have the desired properties.

Suppose that user A has published its public key and that user B wishes to send the message M to A. Then B calculates $C = M^e \pmod{n}$ and transmits C. On receipt of this ciphertext, user A decrypts by calculating $M = C^d \pmod{n}$.

An example, from [SING99], is shown in Figure 21.6. For this example, the keys were generated as follows:

1. Select two prime numbers, $p = 17$ and $q = 11$.
2. Calculate $n = pq = 17 \times 11 = 187$.
3. Calculate $\phi(n) = (p - 1)(q - 1) = 16 \times 10 = 160$.

Key Generation	
Select p, q	p and q both prime, $p \neq q$
Calculate $n = p \times q$	
Calculate $\phi(n) = (p - 1)(q - 1)$	
Select integer e	$\gcd(\phi(n), e) = 1;\ 1 < e < \phi(n)$
Calculate d	$de \bmod \phi(n) = 1$
Public key	$KU = \{e, n\}$
Private key	$KR = \{d, n\}$

Encryption	
Plaintext:	$M < n$
Ciphertext:	$C = M^e \pmod{n}$

Decryption	
Ciphertext:	C
Plaintext:	$M = C^d \pmod{n}$

Figure 21.5 The RSA Algorithm

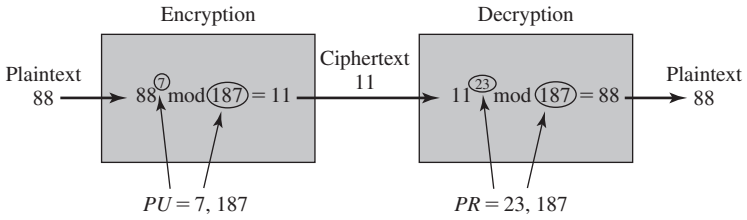


Figure 21.6 Example of RSA Algorithm

4. Select e such that e is relatively prime to $\phi(n) = 160$ and less than $\phi(n)$; we choose $e = 7$.
5. Determine d such that $de \bmod 160 = 1$ and $d < 160$. The correct value is $d = 23$, because $23 \times 7 = 161 = (1 \times 160) + 1$.

The resulting keys are public key $PU = \{7, 187\}$ and private key $PR = \{23, 187\}$. The example shows the use of these keys for a plaintext input of $M = 88$. For encryption, we need to calculate $C = 88^7 \bmod 187$. Exploiting the properties of modular arithmetic, we can do this as follows:

$$88^7 \bmod 187 = [(88^4 \bmod 187) \times (88^2 \bmod 187) \times (88^1 \bmod 187)] \bmod 187$$
$$88^1 \bmod 187 = 88$$
$$88^2 \bmod 187 = 7744 \bmod 187 = 77$$
$$88^4 \bmod 187 = 59{,}969{,}536 \bmod 187 = 132$$
$$88^7 \bmod 187 = (88 \times 77 \times 132) \bmod 187 = 894{,}432 \bmod 187 = 11$$

For decryption, we calculate $M = 11^{23} \bmod 187$:

$$11^{23} \bmod 187 = [(11^1 \bmod 187) \times (11^2 \bmod 187) \times (11^4 \bmod 187) \times (11^8 \bmod 187) \times (11^8 \bmod 187)] \bmod 187$$
$$11^1 \bmod 187 = 11$$
$$11^2 \bmod 187 = 121$$
$$11^4 \bmod 187 = 14{,}641 \bmod 187 = 55$$
$$11^8 \bmod 187 = 214{,}358{,}881 \bmod 187 = 33$$
$$11^{23} \bmod 187 = (11 \times 121 \times 55 \times 33 \times 33) \bmod 187 = 79{,}720{,}245 \bmod 187 = 88$$

The Security of RSA

Four possible approaches to attacking the RSA algorithm are as follows:

- **Brute force:** This involves trying all possible private keys.
- **Mathematical attacks:** There are several approaches, all equivalent in effort to factoring the product of two primes.
- **Timing attacks:** These depend on the running time of the decryption algorithm.

- **Chosen ciphertext attacks:** This type of attack exploits properties of the RSA algorithm. A discussion of this attack is beyond the scope of this book.

The defense against the brute force approach is the same for RSA as for other cryptosystems; namely, use a large key space. Thus, the larger the number of bits in d, the better. However, because the calculations involved, both in key generation and in encryption/decryption, are complex, the larger the size of the key, the slower the system will run.

In this subsection, we provide an overview of mathematical and timing attacks.

THE FACTORING PROBLEM We can identify three approaches to attacking RSA mathematically:

- Factor n into its two prime factors. This enables calculation of $\phi(n) = (p - 1) \times (q - 1)$, which, in turn, enables determination of $\mathrm{d} \equiv e^{-1} \pmod{\phi(n)}$.
- Determine $\phi(n)$ directly, without first determining p and q. Again, this enables determination of $\mathrm{d} \equiv e^{-1} \pmod{\phi(n)}$.
- Determine d directly, without first determining $\phi(n)$.

Most discussions of the cryptanalysis of RSA have focused on the task of factoring n into its two prime factors. Determining $\phi(n)$ given n is equivalent to factoring n [RIBE96]. With presently known algorithms, determining d given e and n appears to be at least as time consuming as the factoring problem. Hence, we can use factoring performance as a benchmark against which to evaluate the security of RSA.

For a large n with large prime factors, factoring is a hard problem, but not as hard as it used to be. Just as it had done for DES, RSA Laboratories issued challenges for the RSA cipher with key sizes of 100, 110, 120, and so on, digits. The latest challenge to be met is the RSA-200 challenge with a key length of 200 decimal digits, or about 663 bits. Table 21.2 shows the results to date. The level of effort is measured in MIPS-years: a million-instructions-per-second processor running for one year, which is about 3×10^{13} instructions executed (MIPS-year numbers not available for last 3 entries).

Table 21.2 Progress in Factorization

Number of Decimal Digits	Approximate Number of Bits	Date Achieved	MIPS-Years
100	332	April 1991	7
110	365	April 1992	75
120	398	June 1993	830
129	428	April 1994	5000
130	431	April 1996	1000
140	465	February 1999	2000
155	512	August 1999	8000
160	530	April 2003	—
174	576	December 2003	—
200	663	May 2005	—

A striking fact about Table 21.2 concerns the method used. Until the mid-1990s, factoring attacks were made using an approach known as the quadratic sieve. The attack on RSA-130 used a newer algorithm, the generalized number field sieve (GNFS), and was able to factor a larger number than RSA-129 at only 20% of the computing effort.

The threat to larger key sizes is twofold: the continuing increase in computing power, and the continuing refinement of factoring algorithms. We have seen that the move to a different algorithm resulted in a tremendous speedup. We can expect further refinements in the GNFS, and the use of an even better algorithm is also a possibility. In fact, a related algorithm, the special number field sieve (SNFS), can factor numbers with a specialized form considerably faster than the generalized number field sieve. It is reasonable to expect a breakthrough that would enable a general factoring performance in about the same time as SNFS, or even better. Thus, we need to be careful in choosing a key size for RSA. For the near future, a key size in the range of 1024 to 2048 bits seems secure.

In addition to specifying the size of n, a number of other constraints have been suggested by researchers. To avoid values of n that may be factored more easily, the algorithm's inventors suggest the following constraints on p and q:

1. p and q should differ in length by only a few digits. Thus, for a 1024-bit key (309 decimal digits), both p and q should be on the order of magnitude of 10^{75} to 10^{100}.
2. Both $(p - 1)$ and $(q - 1)$ should contain a large prime factor.
3. $\gcd(p - 1, q - 1)$ should be small.

In addition, it has been demonstrated that if $e < n$ and $d < n^{1/4}$, then d can be easily determined [WIEN90].

Timing Attacks If one needed yet another lesson about how difficult it is to assess the security of a cryptographic algorithm, the appearance of timing attacks provides a stunning one. Paul Kocher, a cryptographic consultant, demonstrated that a snooper can determine a private key by keeping track of how long a computer takes to decipher messages [KOCH96]. Timing attacks are applicable not just to RSA, but also to other public-key cryptography systems. This attack is alarming for two reasons: It comes from a completely unexpected direction and it is a ciphertext-only attack.

A timing attack is somewhat analogous to a burglar guessing the combination of a safe by observing how long it takes for someone to turn the dial from number to number. The attack exploits the common use of a modular exponentiation algorithm in RSA encryption and decryption, but the attack can be adapted to work with any implementation that does not run in fixed time. In the modular exponentiation algorithm, exponentiation is accomplished bit by bit, with one modular multiplication performed at each iteration and an additional modular multiplication performed for each 1 bit.

As Kocher points out in his paper, the attack is simplest to understand in an extreme case. Suppose the target system uses a modular multiplication function that is very fast in almost all cases but in a few cases takes much more time than an entire average modular exponentiation. The attack proceeds bit-by-bit starting with the leftmost bit, b_k. Suppose that the first j bits are known (to obtain the entire exponent,

start with $j = 0$ and repeat the attack until the entire exponent is known). For a given ciphertext, the attacker can complete the first j iterations. The operation of the subsequent step depends on the unknown exponent bit. If the bit is set, $d \leftarrow (d \times a) \bmod n$ will be executed. For a few values of a and d, the modular multiplication will be extremely slow, and the attacker knows which these are. Therefore, if the observed time to execute the decryption algorithm is always slow when this particular iteration is slow with a 1 bit, then this bit is assumed to be 1. If a number of observed execution times for the entire algorithm are fast, then this bit is assumed to be 0.

In practice, modular exponentiation implementations do not have such extreme timing variations, in which the execution time of a single iteration can exceed the mean execution time of the entire algorithm. Nevertheless, there is enough variation to make this attack practical. For details, see [KOCH96].

Although the timing attack is a serious threat, there are simple countermeasures that can be used, including the following:

- **Constant exponentiation time:** Ensure that all exponentiations take the same amount of time before returning a result. This is a simple fix but does degrade performance.
- **Random delay:** Better performance could be achieved by adding a random delay to the exponentiation algorithm to confuse the timing attack. Kocher points out that if defenders do not add enough noise, attackers could still succeed by collecting additional measurements to compensate for the random delays.
- **Blinding:** Multiply the ciphertext by a random number before performing exponentiation. This process prevents the attacker from knowing what ciphertext bits are being processed inside the computer and therefore prevents the bit-by-bit analysis essential to the timing attack.

RSA Data Security incorporates a blinding feature into some of its products. The private-key operation $M = C^d \bmod n$ is implemented as follows:

1. Generate a secret random number r between 0 and $n - 1$.
2. Compute $C' = C(r^e) \bmod n$, where e is the public exponent.
3. Compute $M' = (C')^d \bmod n$ with the ordinary RSA implementation.
4. Compute $M = M'r^{-1} \bmod n$. In this equation, r^{-1} is the multiplicative inverse of $r \bmod n$. It can be demonstrated that this is the correct result by observing that $r^{ed} \bmod n = r \bmod n$.

RSA Data Security reports a 2 to 10% performance penalty for blinding.

21.4 DIFFIE-HELLMAN AND OTHER ASYMMETRIC ALGORITHMS

Diffie-Hellman Key Exchange

The first published public-key algorithm appeared in the seminal paper by Diffie and Hellman that defined public-key cryptography [DIFF76] and is generally referred to as Diffie-Hellman key exchange. A number of commercial products employ this key exchange technique.

The purpose of the algorithm is to enable two users to exchange a secret key securely that can then be used for subsequent encryption of messages. The algorithm itself is limited to the exchange of the keys.

The Diffie-Hellman algorithm depends for its effectiveness on the difficulty of computing discrete logarithms. Briefly, we can define the discrete logarithm in the following way. First, we define a primitive root of a prime number p as one whose powers generate all the integers from 1 to $p - 1$. That is, if a is a primitive root of the prime number p, then the numbers

$$a \bmod p, a^2 \bmod p, \ldots, a^{p-1} \bmod p$$

are distinct and consist of the integers from 1 through $p - 1$ in some permutation.

For any integer b less than p and a primitive root a of prime number p, one can find a unique exponent i such that

$$b = a^i \bmod p \qquad \text{where } 0 \leq i \leq (p - 1)$$

The exponent i is referred to as the discrete logarithm, or index, of b for the base a, mod p. We denote this value as $\text{dlog}_{a,p}(b)$.[2]

The Algorithm With this background we can define the Diffie-Hellman key exchange, which is summarized in Figure 21.7. For this scheme, there are two publicly known numbers: a prime number q and an integer α that is a primitive root of q. Suppose the users A and B wish to exchange a key. User A selects a random integer $X_A < q$ and computes $Y_A = \alpha^{X_A} \bmod q$. Similarly, user B independently selects a random integer $X_B < q$ and computes $Y_B = \alpha^{X_B} \bmod q$. Each side keeps the X value private and makes the Y value available publicly to the other side. User A computes the key as $K = (Y_B)^{X_A} \bmod q$ and user B computes the key as $K = (Y_A)^{X_B} \bmod q$. These two calculations produce identical results:

$$\begin{aligned}
K &= (Y_B)^{X_A} \bmod q \\
&= (\alpha^{X_B} \bmod q)^{X_A} \bmod q \\
&= (\alpha^{X_B})^{X_B} \bmod q \\
&= \alpha^{X_B X_A} \bmod q \\
&= (\alpha^{X_A})^{X_B} \bmod q \\
&= (\alpha^{X_A} \bmod q)^{X_B} \bmod q \\
&= (Y_A)^{X_B} \bmod q
\end{aligned}$$

The result is that the two sides have exchanged a secret value. Furthermore, because X_A and X_B are private, an adversary only has the following ingredients to work with: q, α, Y_A, and Y_B. Thus, the adversary is forced to take a discrete logarithm to determine the key. For example, to determine the private key of user B, an adversary must compute

$$X_B = \text{dlog}_{\alpha,q}(Y_B)$$

The adversary can then calculate the key K in the same manner as user B calculates it.

---

[2]Many texts refer to the discrete logarithm as the *index*. There is no generally agreed notation for this concept, much less an agreed name.

Global Public Elements

q	Prime number
α	$\alpha < q$ and α a primitive root of q

User A Key Generation

Select private X_A	$X_A < q$
Calculate public Y_A	$Y_A = \alpha^{X_A} \bmod q$

User B Key Generation

Select private X_B	$X_B < q$
Calculate public Y_B	$Y_B = \alpha^{X_B} \bmod q$

Generation of Secret Key by User A

$K = (Y_B)^{X_A} \bmod q$

Generation of Secret Key by User B

$K = (Y_A)^{X_B} \bmod q$

Figure 21.7 The Diffie-Hellman Key Exchange Algorithm

The security of the Diffie-Hellman key exchange lies in the fact that, while it is relatively easy to calculate exponentials modulo a prime, it is very difficult to calculate discrete logarithms. For large primes, the latter task is considered infeasible.

Here is an example. Key exchange is based on the use of the prime number $q = 353$ and a primitive root of 353, in this case $\alpha = 3$. A and B select secret keys $X_A = 97$ and $X_B = 233$, respectively. Each computes its public key:

$$\text{A computes } Y_A = 3^{97} \bmod 353 = 40.$$
$$\text{B computes } Y_B = 3^{233} \bmod 353 = 248.$$

After they exchange public keys, each can compute the common secret key:

$$\text{A computes } K = (Y_B)^{X_A} \bmod 353 = 248^{97} \bmod 353 = 160.$$
$$\text{B computes } K = (Y_A)^{X_B} \bmod 353 = 40^{233} \bmod 353 = 160.$$

We assume an attacker would have available the following information:

$$q = 353;\ \alpha = 3;\ Y_A = 40;\ Y_B = 248$$

In this simple example, it would be possible by brute force to determine the secret key 160. In particular, an attacker E can determine the common key by discovering a solution to the equation $3^a \bmod 353 = 40$ or the equation $3^b \bmod 353 = 248$. The brute force approach is to calculate powers of 3 modulo 353, stopping when the result equals either 40 or 248. The desired answer is reached with the exponent value of 97, which provides $3^{97} \bmod 353 = 40$.

With larger numbers, the problem becomes impractical.

Key Exchange Protocols Figure 21.8 shows a simple protocol that makes use of the Diffie-Hellman calculation. Suppose that user A wishes to set up a connection with user B and use a secret key to encrypt messages on that connection. User A can generate a one-time private key X_A, calculate Y_A, and send that to user B. User B responds by generating a private value X_B, calculating Y_B, and sending Y_B to user A. Both users can now calculate the key. The necessary public values q and α would need to be known ahead of time. Alternatively, user A could pick values for q and α and include those in the first message.

As an example of another use of the Diffie-Hellman algorithm, suppose that in a group of users (e.g., all users on a LAN), each generates a long-lasting private key and calculates a public key. These public values, together with global public values for q and α, are stored in some central directory. At any time, user B can access user A's public value, calculate a secret key, and use that to send an encrypted message to user A. If the central directory is trusted, then this form of communication provides both confidentiality and a degree of authentication. Because only A and B can determine the key, no other user can read the message (confidentiality). User A knows that only user B could have created a message using this key (authentication). However, the technique does not protect against replay attacks.

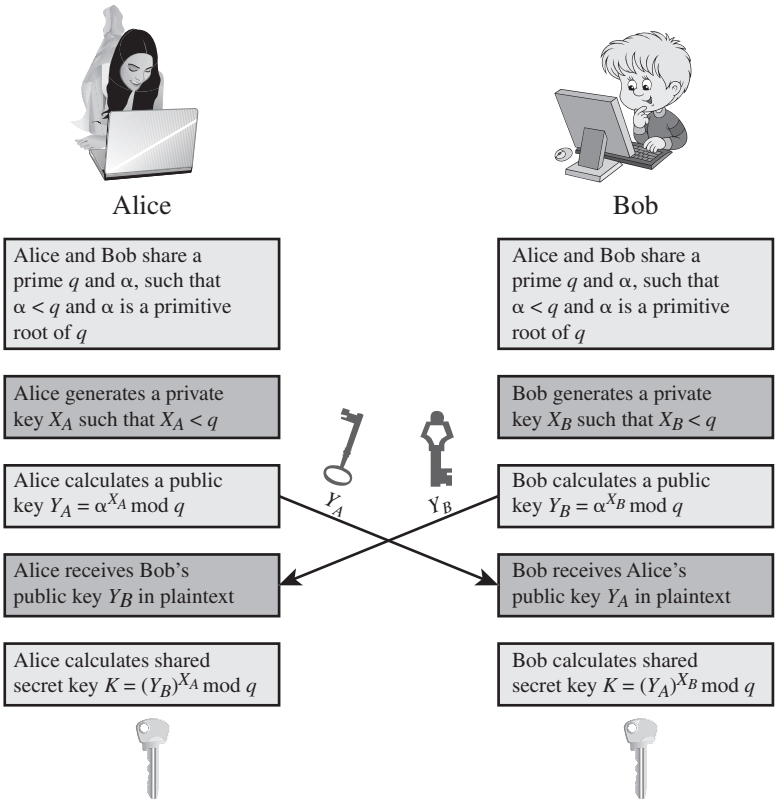


Figure 21.8 Diffie-Hellman Key Exchange

Man-in-the-Middle Attack The protocol depicted in Figure 21.8 is insecure against a man-in-the-middle attack. Suppose Alice and Bob wish to exchange keys, and Darth is the adversary. The attack proceeds as follows:

1. Darth prepares for the attack by generating two random private keys X_{D1} and X_{D2} and then computing the corresponding public keys Y_{D1} and Y_{D2}.
2. Alice transmits Y_A to Bob.
3. Darth intercepts Y_A and transmits Y_{D1} to Bob. Darth also calculates $K2 = (Y_A)^{X_{D2}} \bmod q$.
4. Bob receives Y_{D1} and calculates $K1 = (Y_{D1})^{X_B} \bmod q$.
5. Bob transmits Y_B to Alice.
6. Darth intercepts Y_B and transmits Y_{D2} to Alice. Darth calculates $K1 = (Y_B)^{X_{D1}} \bmod q$.
7. Alice receives Y_{D2} and calculates $K2 = (Y_{D2})^{X_A} \bmod q$.

At this point, Bob and Alice think that they share a secret key, but instead Bob and Darth share secret key $K1$ and Alice and Darth share secret key $K2$. All future communication between Bob and Alice is compromised in the following way:

1. Alice sends an encrypted message M: E($K2$, M).
2. Darth intercepts the encrypted message and decrypts it, to recover M.
3. Darth sends Bob E($K1$, M) or E($K1$, M'), where M' is any message. In the first case, Darth simply wants to eavesdrop on the communication without altering it. In the second case, Darth wants to modify the message going to Bob.

The key exchange protocol is vulnerable to such an attack because it does not authenticate the participants. This vulnerability can be overcome with the use of digital signatures and public-key certificates; these topics are explored later in this chapter and in Chapter 2.

Other Public-Key Cryptography Algorithms

Two other public-key algorithms have found commercial acceptance: DSS and elliptic-curve cryptography.

Digital Signature Standard The National Institute of Standards and Technology (NIST) has published Federal Information Processing Standard FIPS PUB 186, known as the Digital Signature Standard (DSS). The DSS makes use of the SHA-1 and presents a new digital signature technique, the Digital Signature Algorithm (DSA). The DSS was originally proposed in 1991 and revised in 1993 in response to public feedback concerning the security of the scheme. There was a further minor revision in 1996. The DSS uses an algorithm that is designed to provide only the digital signature function. Unlike RSA, it cannot be used for encryption or key exchange.

Elliptic-Curve Cryptography The vast majority of the products and standards that use public-key cryptography for encryption and digital signatures use RSA. The bit length for secure RSA use has increased over recent years, and this has put a heavier processing load on applications using RSA. This burden has ramifications, especially for electronic commerce sites that conduct large numbers of secure

transactions. Recently, a competing system has begun to challenge RSA: elliptic curve cryptography (ECC). Already, ECC is showing up in standardization efforts, including the IEEE P1363 Standard for Public-Key Cryptography.

The principal attraction of ECC compared to RSA is that it appears to offer equal security for a far smaller bit size, thereby reducing processing overhead. On the other hand, although the theory of ECC has been around for some time, it is only recently that products have begun to appear and that there has been sustained cryptanalytic interest in probing for weaknesses. Thus, the confidence level in ECC is not yet as high as that in RSA.

ECC is fundamentally more difficult to explain than either RSA or Diffie-Hellman, and a full mathematical description is beyond the scope of this book. The technique is based on the use of a mathematical construct known as the elliptic curve.

21.5 RECOMMENDED READING

Solid treatments of hash functions and message authentication codes are found in [STIN06] and [MENE97].

The recommended treatments of encryption provided in Chapter 2 cover public-key as well as symmetric encryption. [DIFF88] describes in detail the several attempts to devise secure two-key cryptoalgorithms and the gradual evolution of a variety of protocols based on them.

DIFF88 Diffie, W. "The First Ten Years of Public-Key Cryptography." *Proceedings of the IEEE*, May 1988.

MENE97 Menezes, A.; Oorshcot, P.; and Vanstone, S. *Handbook of Applied Cryptography.* Boca Raton, FL: CRC Press, 1997. Available free online at http://cacr.uwaterloo.ca/hac/

STIN06 Stinson, D. *Cryptography: Theory and Practice.* Boca Raton, FL: CRC Press, 2006.

21.6 KEY TERMS, REVIEW QUESTIONS, AND PROBLEMS

Key Terms

Diffie-Hellman key exchange	message authentication	RSA
digital signature	message authentication code (MAC)	secret key
Digital Signature Standard (DSS)	message digest	Secure Hash Algorithm (SHA)
elliptic-curve cryptography (ECC)	one-way hash function	secure hash function
HMAC	private key	SHA-1
key exchange	public key	SHA-2
MD5	public-key certificate	SHA-3
	public-key encryption	strong collision resistance
		weak collision resistance

Review Questions

21.1 In the context of a hash function, what is a compression function?

21.2 What basic arithmetical and logical functions are used in SHA?

21.3 What changes in HMAC are required in order to replace one underlying hash function with another?

21.4 What is a one-way function?

21.5 Briefly explain Diffie-Hellman key exchange.

Problems

21.1 Consider a 32-bit hash function defined as the concatenation of two 16-bit functions: XOR and RXOR, defined in Section 21.2 as "two simple hash functions."

a. Will this checksum detect all errors caused by an odd number of error bits? Explain.

b. Will this checksum detect all errors caused by an even number of error bits? If not, characterize the error patterns that will cause the checksum to fail.

c. Comment on the effectiveness of this function for use as a hash function for authentication.

21.2 **a.** Consider the following hash function. Messages are in the form of a sequence of decimal numbers, $M = (a_1, a_2, \ldots, a_t)$. The hash value h is calculated as $\left(\sum_{i=1}^{t} a_i\right) \bmod n$, for some predefined value n. Does this hash function satisfy the requirements for a hash function listed in Section 2.2? Explain your answer.

b. Repeat part (a) for the hash function $h = \left(\sum_{i=1}^{t} (a_i)^2\right) \bmod n$

c. Calculate the hash function of part (b) for $M = (189, 632, 900, 722, 349)$ and $n = 989$.

21.3 It is possible to use a hash function to construct a block cipher with a structure similar to DES. Because a hash function is one way and a block cipher must be reversible (to decrypt), how is it possible?

21.4 Now consider the opposite problem: using an encryption algorithm to construct a one-way hash function. Consider using RSA with a known key. Then process a message consisting of a sequence of blocks as follows: Encrypt the first block, XOR the result with the second block and encrypt again, and so on. Show that this scheme is not secure by solving the following problem. Given a two-block message B1, B2, and its hash

$$\text{RSAH}(\text{B1}, \text{B2}) = \text{RSA}(\text{RSA}(\text{B1}) \oplus \text{B2})$$

and given an arbitrary block C1, choose C2 so that RSAH(C1, C2) = RSAH(B1, B2). Thus, the hash function does not satisfy weak collision resistance.

21.5 Figure 21.9 shows an alternative means of implementing HMAC.

a. Describe the operation of this implementation.

b. What potential benefit does this implementation have over that shown in Figure 21.4?

21.6 Perform encryption and decryption using the RSA algorithm, as in Figure 21.6, for the following:

a. $p = 3; q = 11, e = 7; M = 5$

b. $p = 5; q = 11, e = 3; M = 9$

c. $p = 7; q = 11, e = 17; M = 8$

d. $p = 11; q = 13, e = 11; M = 7$

e. $p = 17; q = 31, e = 7; M = 2$

Hint: Decryption is not as hard as you think; use some finesse.

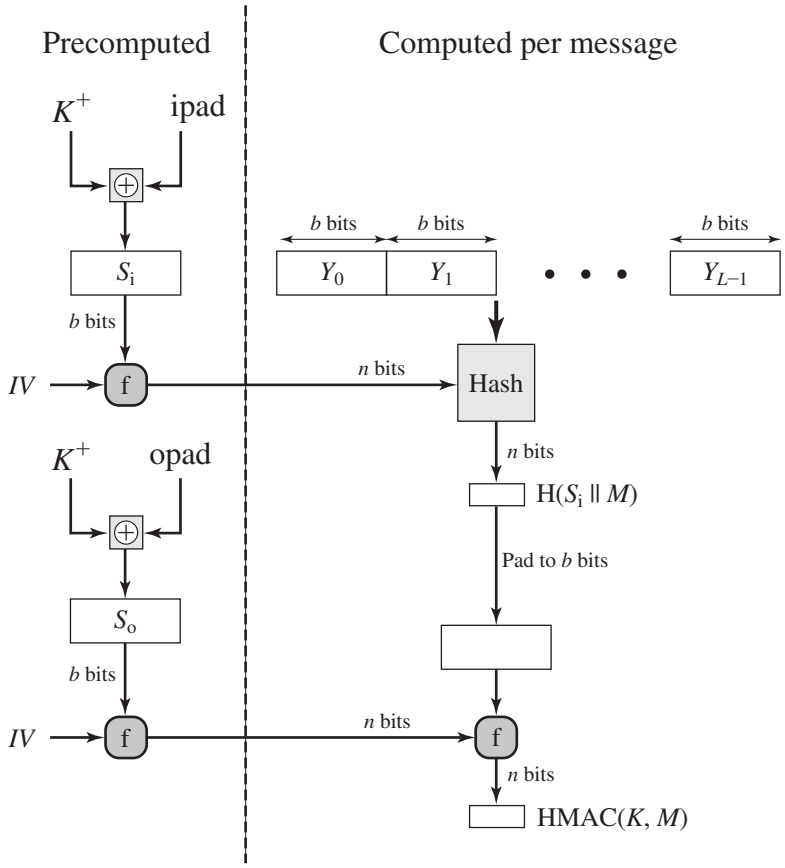


Figure 21.9 **Alternative Implementation of HMAC**

21.7 In a public-key system using RSA, you intercept the ciphertext $C = 10$ sent to a user whose public key is $e = 5, n = 35$. What is the plaintext M?

21.8 In an RSA system, the public key of a given user is $e = 31, n = 3599$. What is the private key of this user?

21.9 Suppose we have a set of blocks encoded with the RSA algorithm and we do not have the private key. Assume $n = pq$, e is the public key. Suppose also someone tells us they know one of the plaintext blocks has a common factor with n. Does this help us in any way?

21.10 Consider the following scheme:

1. Pick an odd number, E.
2. Pick two prime numbers, P and Q, where $(P - 1)(Q - 1) - 1$ is evenly divisible by E.
3. Multiply P and Q to get N.
4. Calculate $D = \dfrac{(P - 1)(Q - 1)(E - 1) + 1}{E}$.

Is this scheme equivalent to RSA? Show why or why not.

21.11 Suppose Bob uses the RSA cryptosystem with a very large modulus n for which the factorization cannot be found in a reasonable amount of time. Suppose Alice sends a message to Bob by representing each alphabetic character as an integer between 0 and 25 (A $\rightarrow$ 0, . . . , Z $\rightarrow$ 25), and then encrypting each number separately using RSA with large e and large n. Is this method secure? If not, describe the most efficient attack against this encryption method.

21.12 Consider a Diffie-Hellman scheme with a common prime $q = 11$ and a primitive root $\alpha = 2$.

a. If user A has public key $Y_A = 9$, what is A's private key X_A?

b. If user B has public key $Y_B = 3$, what is the shared secret key K?

PART FIVE: Network Security

CHAPTER 22

INTERNET SECURITY PROTOCOLS AND STANDARDS