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Research Article

A Super-Efficiency Approach to Rank Units of a Hospital

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Abstract

Efficiency measurement of units and their performance-based ranking are important matters in hospital management. Data envelopment analysis is one of the frequently used methods for these purposes, however, basic data envelopment analysis models are sometimes incapable of performance-based ranking of units. In this study, at first, as the classic ones, the models of Charnes, Cooper & Rhodes and Banker, Charnes & Cooper are summarized. Then the super-efficiency approach is briefly described. In the results section, the explained methods are compared to rank ten surgical departments of a hospital. Based on the obtained results, we show that the super-efficiency model is able to rank the units, though the others cannot do this completely and then we discuss the reason for this.

Keywords: Hospital management, Data envelopment analysis, Efficiency analysis, Super-efficiency, Gray relational analysis, Ranking of hospital units

Abbreviations

DEA: Data Envelopment Analysis; DMU: Decision Making Unit; CCR model: Charnes, Cooper and Rhodes Model; BCC model: Banker, Charnes and Cooper Model; GRA: Gray Relational Analysis; SE: Super Efficiency.

Introduction

Data envelopment analysis (DEA) is a commonly used method to assess the performance of healthcare organizations, which are generally composed of multiple units. Generally, the units whose efficiency is measured are named decision-making units (DMUs). Based on traditional DEA approaches, DMUs are divided into one of the efficient or inefficient groups and efficiency scores cannot exceed one. Thus, inefficient units can be sorted over their efficiency scores, however, this is not valid for the efficient units because all of them yield a score equal to one. In some studies, after determining efficient units with DEA methods they are sorted by other methods. The gray relational analysis (GRA) is one of the methods used for this purpose [5]. Also, the super efficiency (SE) models are applied to rank DMUs, in which, the efficiency score can be more than one [6].

The aim of this study is to make a comparison between some DEA methods to rank units. For this purpose, the efficiency of ten surgical services of a hospital is analyzed, whose dataset is taken from the literature. Girginer et al., at first have determined the efficient units with DEA for this dataset and then they have ranked them with GRA [5]. We use the DEA models too, however, we applied them with the mathematical models written in GAMS software. We present our results, which are somewhat different from the results in the literature. Then, we rank the units through a SE method that is also written in GAMS software. The acquired results are interpreted and a general conclusion is presented in the last section.

Methods

In this section, at first, the models of Charnes, Cooper & Rhodes (CCR) and Banker, Charnes & Cooper (BCC) are described. The used notations for the model are summarized in Table 1.

Notation	Description
y_{rj}	Output r for DMU j
u_r	Weight of output r
x_{ij}	Input i for DMU j
v_i	Weight of input i
z_j	Efficiency od DMU j

Table 1: Used notation.

CCR model

Suppose that in an organization there are n DMUs and each has m inputs and s outputs. To be maximized, the efficiency of the j-th DMU is defined as the ratio of the weighted sum of outputs to the weighted sum of inputs, as is in Equation 1 [4, 11].

$$Max\ z_o = \frac{\sum_{r=1}^{s} u_r\, y_{ro}}{\sum_{i=1}^{m} v_i\, x_{io}} \quad ....(1)$$

As in Inequality 2, it is clear that the efficiency of each DMU is at most equal to 1 [11].

$$s.t. \frac{\sum_{r=1}^{s} u_r\, Y_{rj}}{\sum_{i=1}^{m} v_i\, X_{ij}} \leq 1, j=1,..., n \quad ....(2)$$

Furthermore, the weights of the inputs and outputs are positive, which is expressed as in Inequalities 3 and 4 [11].

$$v_i \geq 0, i=1,..., m \quad ....(3)$$

$$u_r \geq 0, r=1,..., s \quad ....(4)$$

The non-linear model defined as in Equation 1 and Constraints 2- 4 is the fractional CCR model and is solved separately for each DMU [11].

Multiplier input-oriented CCR model

Generally, the linearized type of this problem is more easily solved. The linearization can be done by maximizing the nominator. In this case, it is an input-oriented model. The multiplier input-oriented CCR model is defined as in Equations 5-6 and Constraints 7-9 [11].

$$Max\ z_o = \sum_{r=1}^{s} u_r y_{ro} \quad ......(5)$$

$$s.t.\ \sum_{i=1}^{m} v_i x_{io} = 1 \quad ......(6)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0,\ j = 1, \dots, n \quad ......(7)$$

$$v_i \geq 0,\ i = 1, \dots, m \quad ......(8)$$

$$u_r \geq 0,\ r = 1, \dots, s \quad ......(9)$$

Multiplier output-oriented CCR model

The linearization can also be done by minimizing the denominator. In this case, it is an output-oriented model. The multiplier output-oriented CCR model is defined as in Equations 10-11 and Constraints 12-14 [11].

$$Min\ z_o = \sum_{i=1}^{m} v_i x_{io} \quad ......(10)$$

$$s.t.\ \sum_{r=1}^{s} u_r y_{ro} = 1 \quad ......(11)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0,\ j = 1, \dots, n \quad ......(12)$$

$$v_i \geq 0,\ i = 1, \dots, m \quad ......(13)$$

$$u_r \geq 0,\ r = 1, \dots, s \quad ......(14)$$

BCC model

The BCC model was designed based on the variable return to scale. The difference between the CCR and BCC models is related to variable or constant returns to scale. In variable returns to scale, it is supposed that the outputs do not change in proportion to the input. The BCC model can also be used as the input-oriented or output-oriented approaches [1-4, 8, 12].

Fractional input-oriented BCC model

The fractional input-oriented BCC model is defined as in Equation 15 and Constraints 16-18, which is a non-linear model [11].

$$Max\ z_o = \frac{\sum_{r=1}^{s} u_r y_{ro} - u_o}{\sum_{i=1}^{m} v_i x_{io}} \quad ....(15)$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj} - u_o}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \quad j = 1, \dots, n \quad ......(16)$$

$$v_i \geq 0, \quad i = 1, \dots, m \quad ......(17)$$

$$u_r \geq 0, \quad r = 1, \dots, s \quad ......(18)$$

$$u_o\ free.$$

Multiplier input-oriented BCC model

Linearized type of the fractional input-oriented BCC model is defined as in Equation 19 and Constraints 20-23, which is named as the Multiplier input-oriented BCC model [11].

$$Max\ z_o = \sum_{r=1}^{s} u_r y_{ro} - u_o \quad ......(19)$$

$$s.t. \quad \sum_{i=1}^{m} v_i x_{io} = 1 \quad ......(20)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_o \leq 0, \quad j = 1, \dots, n \quad ......(21)$$

$$v_i \geq 0, \quad i = 1, \dots, m \quad ......(22)$$

$$u_r \geq 0, \quad r = 1, \dots, s \quad ......(23)$$

$$u_o\ free.$$

Multiplier output-oriented BCC model

The multiplier output-oriented BCC model is defined as in Equation 24 and Constraints 25-28 [11].

$$Min\, z_o = \sum_{i=1}^{m} v_i x_{io} - v_o \quad ......(24)$$

$$s.t. \quad \sum_{r=1}^{s} u_r y_{ro} = 1 \quad ......(25)$$

$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - v_o \geq 0, \quad j = 1, \dots, n \quad ......(26)$$

$$v_i \geq 0, \quad i = 1, \dots, m \quad ......(27)$$

$$u_r \geq 0, \quad r = 1, \dots, s \quad ......(28)$$

$$u_o\ free.$$

In all of the models defined in the previous subsections, the efficiency score is at most equal to one which refer to an efficient unit. In some cases, all units can be defined as efficient by these models, especially, in the case that the number of the evaluated units is considerably lower than the total number of inputs and outputs [11].

SE model

The conclusion that all units are efficient is not helpful for decision-makers and cannot be used for ranking of units. In this case, one of the approaches used for ranking is the super-efficiency in which the evaluated units are removed from the set of DMUs and the evaluation is made over the updated frontier. So in this approach, the efficiency scores of inefficient units remain unchanged, however, the efficiency scores of efficient units are higher than one. Hence, this approach also ranks among efficient units. The used SE model is defined as in Equation 29 and Constraints 30-32 [6, 7, 10, and 11].

$$Min\, z_o = \theta \quad ......(29)$$

$$s.t. \quad \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1, \dots, m \quad ......(30)$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \dots, s \quad ......(31)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n \quad ......(32)$$

$$\theta\ free.$$

Experimental Results and Discussion

A dataset from the literature is selected for a comparison between the methods mentioned in previous sections. As summarized in Table 2, DMUs are the 10 surgical sections of a hospital, whose inputs and outputs are as in Table 3.

DMU_1	Neurosurgery
DMU_2	General surgery
DMU_3	Ophthalmology
DMU_4	Cardiology
DMU_5	Urology
DMU_6	Pediatric surgery
DMU_7	Orthopedics and Traumatology
DMU_8	Otorhinolaryngology (ENT)
DMU_9	Cardiovascular surgery
DMU_{10}	Plastic surgery

Table 2: DMUs of the dataset [5].

	Notation	Description
Inputs	X_1	Bed turnover rate
	X_2	Total number of physicians
	X_3	Number of other healthcare personnel
	X_4	Bed occupancy rate (Capacity Utilization Rate)
Outputs	Y_1	Number of hospitalized-discharged patients in the clinic
	Y_2	Total number of operations

Table 3: Inputs and outputs of DMUs [5].

X_1 and X_4 are defined as in Equations 5 and 6 [5].

$$X_1 = \frac{\text{Total number of discharged patients (including deaths) from the clinic in a given period}}{\text{Number of beds in that period}} \quad ....(33)$$

$$X_4 = \frac{\frac{Number\ of\ inpatient\ days}{Number\ of\ total\ beds} \times 180}{100} \quad ......(34)$$

The values of inputs and outputs are summarized in Table 4 [5].

Girginer *et al.*, find the efficiencies of units 9 and 10 less than one, while the efficiencies of the other ones are found to be equal to one using the BCC model. They note that since hospital services focus on output maximization, the BCC model directed at output maximization with variable inputs is used. Then, efficient units are ranked with the GRA. The results obtained with these two methods are summarized in Table 5 [5].

We apply CCR, BCC and SE methods in the dataset using GAMS software and BARON solver [10]. The obtained results are summarized in Table 6. We note that for this data set, we obtain the same results using the fractional, multiplier input-oriented and output-oriented types of the models. Hence, each of the CCR and BCC models is presented only in one column of Table 6.

		DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}
Inputs	X_1	35.92	50.75	238.5	69.29	47.6	49.33	18.37	114.29	1325	2830
	X_2	3	14	10	10	8	1	8	11	9.13	51.2
	X_3	14	30	4	35	14	2	22	7	7	3
	X_4	63	90.92	662.5	135.73	102.66	113.33	39.83	206.76	17	8
Outputs	Y_1	464	1609	1431	1661	1029	116	800	1096	18.94	74.66
	Y_2	681	3038	2737	2452	1812	207	4214	13172	301	104

Table 4: The values of variables [5].

Services	Rank
DMU_1	5
DMU_2	6
DMU_3	1
DMU_4	4
DMU_5	7
DMU_6	3
DMU_7	8
DMU_8	2
DMU_9	10
DMU_{10}	9

Table 5: Rank of the DMUs obtained by combined use of DEA and GRA [5].

	CCR model		BCC model		SE model	
Services	**Efficiency**	**State**	**Efficiency**	**State**	**Efficiency**	**Rank**
DMU_1	0.93	In efficient	1.00	Efficient	0.93	7
DMU_2	1.00	Efficient	1.00	Efficient	1.20	5
DMU_3	1.00	Efficient	1.00	Efficient	2.28	3
DMU_4	1.00	Efficient	1.00	Efficient	1.26	4
DMU_5	1.00	Efficient	1.00	Efficient	1.14	6
DMU_6	0.75	In efficient	1.00	Efficient	0.75	8
DMU_7	1.00	Efficient	1.00	Efficient	2.43	2
DMU_8	1.00	Efficient	1.00	Efficient	6.84	1
DMU_9	0.19	In efficient	1.00	Efficient	0.19	10
DMU_{10}	0.51	In efficient	1.00	Efficient	0.51	9

Table 6: Efficiency and rank of the DMUs obtained by SE. Since the same results are obtained using the fractional, multiplier input-oriented and output-oriented types of the models, each of the CCR and BCC models is presented only in one column.

As seen in Table 6, according to the BCC model, all DMUs are found as efficient, however, based on the results obtained with the CCR model, only six units are efficient. According to the results obtained with the SE model, the efficiency of six DMUs is more than one and the others appear to be inefficient as in the CCR model. Also, the ranking obtained with the SE model is given in the last column of Table 6, which is different from the results in Table 5.

Conclusion

The main purpose of this study is to compare the ability of some DEA models to rank units of organizations. The used models are CCR, BCC, and SE, whose mathematical models are written in GAMS software. In the section of experimental results, a dataset from the literature is used, which contains the value of the defined inputs and outputs for ten surgical units of a hospital. In the literature, DMUs of this hospital are ranked based on an approach combined with DEA and GRA methods. Such that, at first, efficient units are found using the BCC model and then they are ranked with the GRA method. The reason to use the GRA is that in the BCC model the efficiency values of the units that are found as efficient are equal to one and therefore it is not possible to rank among them. But the efficient units can be ranked using the SE model because in this approach the efficiency values can be more than one. We rank the DMUs using the SE model, whose result is not the same as the order found with the approach that combines DEA and GRA methods. In addition, in this study, it is shown that the results obtained with BCC and CCR models can be different. In general, the BCC model provides more tolerant results than the CCR model, and hence the number of efficient units can be higher in the BCC model than in the CCR model. Moreover, although in the results from the literature, which are obtained with the BCC model, eight units are efficient, all units are found to be efficient with the same model in our results, which is maybe due to the software and also the solver used for optimization.

DEA has also interesting applications in obesity researches [9]. In future studies, we plan to use the models of this paper for larger problems of obesity researches.

Conflict of Interest

The authors declare that they have no conflict of interest.

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