1. Determine whether each identity below is true or false, and provide a convincing justification. Here R and S denote regular languages.

a.
$$R(R \cup S)^* \cup (R \cup S)^* S = (R \cup S)^* R \cup S(R \cup S)^*$$

b.
$$R(R \cup S)^* \cup S(R \cup S)^* = (R \cup S)^* R \cup (R \cup S)^* S$$

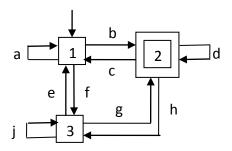
2. Write a regular expression that accepts the set of strings over alphabet {a,b} that contain at least one of the following: the substring aaa, or the substring bbb, or two aa substrings, or two bb substrings.

3. Write a regular expression that accepts the set of valid identifiers in a programming language in which each identifier can be any non-empty string of lowercase letters, other than these specified reserved words: {if, iff, of, off, on, one}.

4. Let input alphabet {L, U, D, P} represent these four categories: lowercase, uppercase, digit, punctuation. Define a *strong* password as any string that contains at least two symbols from each category. Also define a *weak* password as any string that is not a strong password. Write a regular expression that accepts *weak* passwords.

5. Draw a non-deterministic finite-state machine that is equivalent to this regular expression: $(a^* b^+ c^* \cup d^* e^+ f^*)^*$

6. Write a regular expression that is equivalent to this finite-state machine:



7. Let $L = \{a^m b^n \mid m \le n \}$. Prove that L is not a regular language.