CS 475/575 Exam 2 **Spring 2011** **Solution**

1. Determine whether each identity below is true or false, and provide a convincing justification. Here R and S denote regular languages. **[10 points]**

   a. $R\,(R \cup S)^* \cup (R \cup S)^*\,S = (R \cup S)^*\,R \cup S\,(R \cup S)^*$

   False, because counterexamples can be shown when R = a and S = b:
   The left side generates string ab, but the right side cannot generate ab.
   Or, the right side generates string ba, but the left side cannot generate ba.

   b. $R\,(R \cup S)^* \cup S\,(R \cup S)^* = (R \cup S)^*\,R \cup (R \cup S)^*\,S$

   True, because both sides simplify to the same expression:
   Left side = $(R \cup S)\,(R \cup S)^* = (R \cup S)^+$.
   Right side = $(R \cup S)^*\,(R \cup S) = (R \cup S)^+$.

2. Write a regular expression that accepts the set of strings over alphabet {a,b} that contain at least one of the following: the substring aaa, or the substring bbb, or two aa substrings, or two bb substrings. **[8 points]**

   $(a \cup b)^*\,\big(aaa \cup bbb \cup aa\,(a \cup b)^*\,aa \cup bb\,(a \cup b)^*\,bb\big)\,(a \cup b)^*$

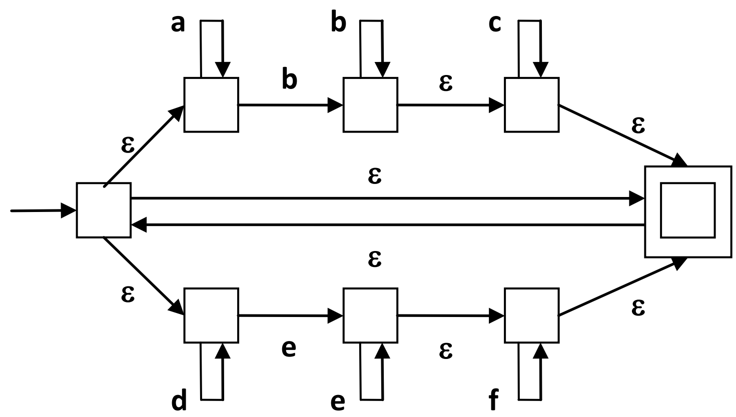
3. Write a regular expression that accepts the set of valid identifiers in a programming language in which each identifier can be any non-empty string of lowercase letters, other than these specified reserved words: {if, iff, of, off, on, one}. **[8 points]**

   $i \cup o \cup \big([\text{^}io] \cup i\,[\text{^}f] \cup o\,[\text{^}fn] \cup if\,[\text{^}f] \cup of\,[\text{^}f] \cup on\,[\text{^}e]\big)\,[a\text{–}z]^* \cup \big(iff \cup off \cup one\big)\,[a\text{–}z]^+$

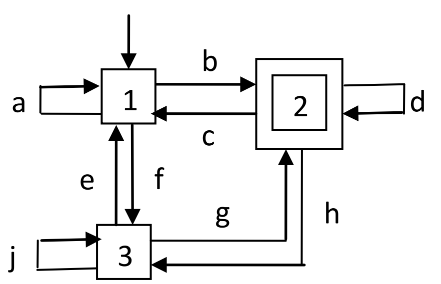
4. Let input alphabet {L, U, D, P} represent these four categories: lowercase, uppercase, digit, punctuation. Define a *strong* password as any string that contains at least two symbols from each category. Also define a *weak* password as any string that is not a strong password. Write a regular expression that accepts *weak* passwords. **[8 points]**

   $(U \cup D \cup P)^*\,L?\,(U \cup D \cup P)^* \;\cup\; (L \cup D \cup P)^*\,U?\,(L \cup D \cup P)^*$
   $\cup\; (L \cup U \cup P)^*\,D?\,(L \cup U \cup P)^* \;\cup\; (L \cup U \cup D)^*\,P?\,(L \cup U \cup D)^*$

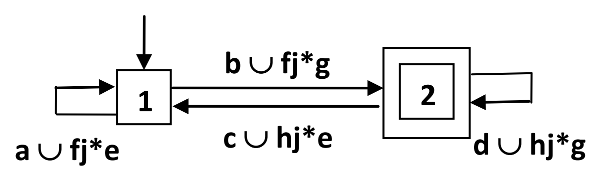
5. Draw a non-deterministic finite-state machine that is equivalent to this regular expression: $(a^* \, b^+ \, c^* \cup d^* \, e^+ \, f^*)^*$ **[8 points]**



6. Write a regular expression that is equivalent to this finite-state machine: **[10 points]**



First eliminate state 3:



$$(a \cup fj^*e)^* \, (b \cup fj^*g) \, (d \cup hj^*g)^* \, \big((c \cup hj^*e) \, (a \cup fj^*e)^* \, (b \cup fj^*g) \, (d \cup hj^*g)^*\big)^*$$

7. Let $L = \{ a^m b^n \mid m \leq n \}$. Prove that L is not a regular language. **[8 points]**

Suppose L is regular.
Let p = the pumping theorem constant, and choose string $s = a^p b^p \in L$.
Write $s = uvw$ such that $|uv| \leq p$, $|v| \geq 1$, and $u v^k w \in L$ for all $k \geq 0$.
Therefore $u = a^q$, $v = a^r$, and $w = a^{p-q-r} b^p$, where $q+r \leq p$ and $r \geq 1$.
But $u v^2 w = a^q a^{2r} a^{p-q-r} b^p = a^{p+r} b^p \notin L$, because $r \geq 1$ implies $p+r > p$.
Contradiction, so L is not regular.