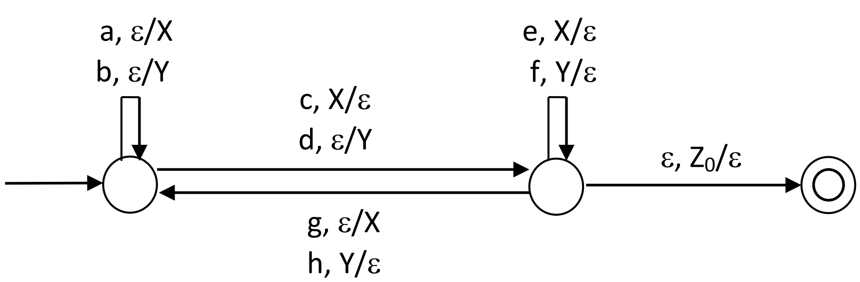
CS 475/575 Exam 2 **Fall 2012** **Name ___________________**

This exam has 120 possible points (includes 20 points extra credit).

1. Draw a parse tree for the string a [a . a] . a [a] using this context-free grammar: **[10 points]**

   $S \rightarrow a\ T$
   $T \rightarrow$ **[** S **]** T | **.** $a\ T$ | ε

2. Write all strings with length exactly 4 that are accepted by this pushdown automaton: **[10 points]**



3. Let language $L_1 = \{a^m b^n \mid m \leq 2n \text{ and } n \leq 2m\}$.

   a. Write a context-free grammar that generates language L_1. Ambiguity is permitted. **[10 points]**

   b. Draw a pushdown automaton that accepts language L_1 both by final state and by empty stack. Non-determinism is permitted. **[10 points]**

4. Let language $L_2 = \{a^m b^n c^p d^q \mid m\geq1, n\geq1, p\geq1, q\geq1, \text{ and } m+n=p+q\}$. Examples: $a^3b^2c^4d^1$ = aaabbccccd, $a^2b^5c^3d^4$ = aabbbbbcccdddd.

   a. Write an *unambiguous* context-free grammar that generates language L_2. **[10 points]** (Half credit for an ambiguous grammar.)

   b. Draw a *deterministic* pushdown automaton that accepts language L_2 both by final state and by empty stack. **[10 points]** (Half credit for a non-deterministic machine.)

5. Let language L_3 contain strings of balanced left and right parentheses '(' and ')' with the additional provision that a right bracket ']' may be used to match all (one or more) preceding unmatched left parentheses. Examples: (( ) ( )), (((( )], ((( ] ( ), ( ) (( ) ( ] (( )) (( ) (( ].

   a. Write an *unambiguous* context-free grammar that generates language L_3. **[10 points]** (Half credit for an ambiguous grammar.)

   b. Draw a *deterministic* pushdown automaton that accepts language L_3 by final state, with only the bottom-of-stack symbol Z_0 remaining on the stack if the string is accepted. **[10 points]** (Half credit for a non-deterministic machine.)

6. Eliminate all useless symbols, ε-productions, and unit productions from this context-free grammar. Your grammar should be equivalent to the original grammar. Bonus if you convert the grammar to Chomsky normal form. **[10 points + 4 points]**

   $S \rightarrow UX \mid TU \mid YT \mid RaR$
   $T \rightarrow b \mid cT \mid Sd$
   $U \rightarrow e \mid V$
   $V \rightarrow W \mid f$
   $W \rightarrow g \mid h$
   $X \rightarrow \varepsilon \mid TZ \mid XY$
   $Y \rightarrow XX \mid ZS \mid YY$
   $Z \rightarrow ZQ \mid QZ$
   $Q \rightarrow i \mid \varepsilon$
   $R \rightarrow jk \mid \varepsilon$

7. Let $L_4 = \{ a^m b^n c^q \mid q = \max(m,n) \}$. Examples: $a^2b^4c^4$ = aabbbbcccc, $a^4b^2c^4$ = aaaabbcccc, $a^3b^3c^3$ = aaabbbccc. Use the pumping theorem to show that L_4 is not context-free. **[16 points]**

   First complete this statement of the pumping theorem for context-free languages:
   For every context-free language L, there exists some constant p such that
   for every string s with _______________ and _______________ ,
   it is possible to write _______________ such that _______________ , _______________ ,
   and for every integer $i \geq 0$, ____________________________ .

   Next apply the pumping theorem to show that $L_4 = \{ a^m b^n c^q \mid q = \max(m,n) \}$ is not context-free.
   Choose string s = _________________ .
   Determine the possible cases and show a contradiction in each case:

8. Trace the CYK dynamic programming algorithm for input string "abcbab" using this Chomsky normal form grammar. Complete the table below. **[10 points]**

   $S \rightarrow XV \mid WU$
   $T \rightarrow WX \mid VT$
   $U \rightarrow VW \mid XS$
   $V \rightarrow a \mid UX \mid XW$
   $W \rightarrow b \mid TW \mid WV$
   $X \rightarrow c \mid SV \mid VX$

	1	2	3	4	5	6
1						
2	–					
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