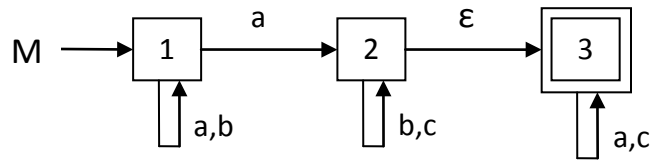


2. Let language L be as defined in problem 1, and let n be an arbitrary positive integer. Let X_n denote the number of strings in L that have length exactly n . Write a formula for X_n , and prove by mathematical induction that your formula is correct.

3. Consider the non-deterministic finite-state machine M shown below. Draw a deterministic finite-state machine M' that is equivalent to M , and label each state of M' with its corresponding set of states from M .



4. Let machines M and M' be same as in problem 3. For each of M and M' , write all the accepting computation sequences for the input string $abaca$. [If you omitted problem 3, note that you can still obtain a computation sequence for M' by determining all possible states for M after M processes each prefix of the input string.]

5. Let language L be the strings over alphabet $\{a,b,c\}$ such that the character in the third-rightmost position also appears somewhere earlier in the string. For example, in the string $S = \text{acab}\underline{\text{c}}\underline{\text{a}}\underline{\text{b}}\text{ac}$, the character b appears both in the third-rightmost position and also earlier in S . Draw a non-deterministic finite-state machine that accepts language L .
6. Draw a deterministic finite-state machine that models a memory that holds one value. The initial value is 0. The input alphabet $\{i,d,s\}$ corresponds to these unary operations: increment, double, square. Each operation is performed using mod 7 arithmetic. The machine should accept a string iff the corresponding sequence of operations yields a prime number (2, 3, 5). For example, the string “idsid” yields the value 3, so it is accepted. But the string “disidis” yields the value 4, so it is rejected.