Partially coherent ptychography

Tong Wu

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Coherent model

$$f_j = \left|\mathcal{F}\left(\mathcal{S}_j u \circ \omega\right)\right|^2 \tag{1}$$

In a discrete setting, $u \in \mathbb{C}^n$ is a 2D image with $\sqrt{n} \times \sqrt{n}$ pixels, $\omega \in \mathbb{C}^{\bar{m}}$ is a localized 2D probe with $\sqrt{\bar{m}} \times \sqrt{\bar{m}}$ pixels. $f_j \in \mathbb{R}_+^{\bar{m}} (\forall 0 \leq j \leq J-1)$ is a stack of phaseless measurements. Here $|\cdot|$ represents the element-wise absolute value of a vector, o denotes the elementwise multiplication, and $\mathcal{F}$ denotes the normalized 2 dimensional discrete Fourier transform. Each $\mathcal{S}_j \in \mathbb{R}^{\bar{m} \times n}$ is a binary matrix that crops a region j of size $\bar{m}$ from the image u.

Partially coherent model: a general one

Thibault, Pierre, and Andreas Menzel. "Reconstructing state mixtures from diffraction measurements." Nature 494.7435 (2013): 68-71.

Blind ptychography model + quantum state tomography[1].

Phobe w is assumed to be in mixed state to represent partially coherent effect.

$$\text{Find } u, r \text{ othogonal } w_k \ s.t.$$
$$f_{pc,j} = \sum_{k=1}^{r} \left| \mathcal{F}\left(\mathcal{S}_j u \circ (\omega_k) \right) \right|^2 \tag{2}$$

[1] `https://homepage.univie.ac.at/reinhold.bertlmann/pdfs/T2_Skript_Ch_9corr.pdf` Many symbols in quantum mechanics are included here.

Connection to a particular model

Continuous setting:

$$f_{pc,j}(q) = \int \left| \mathcal{F}_{x \to q} \left(\mathcal{S}_j u(x) \omega(x-y) \right) \right|^2 \kappa(y) \mathrm{d}y \qquad (3)$$

Discrete setting:

$$f_{pc,j} = \sum_i \kappa_i \left| \mathcal{F} \left(\mathcal{S}_j u \circ (\mathcal{T}_i \omega) \right) \right|^2 \qquad (4)$$

We put the κ_i inside:

$$f_{pc,j} = \sum_i \left| \mathcal{F} \left(\mathcal{S}_j u \circ (\sqrt{\kappa_i} \mathcal{T}_i \omega) \right) \right|^2 = \sum_i \left| \mathcal{F} \left(\mathcal{S}_j u \circ (\hat{\omega}_i) \right) \right|^2 \qquad (5)$$

Multiple modes $\hat{w}_i$ are produced by shifted w. Then we can construct density matrix and use truncated SVD to get a low-rank approximation. Then it is exactly (2)

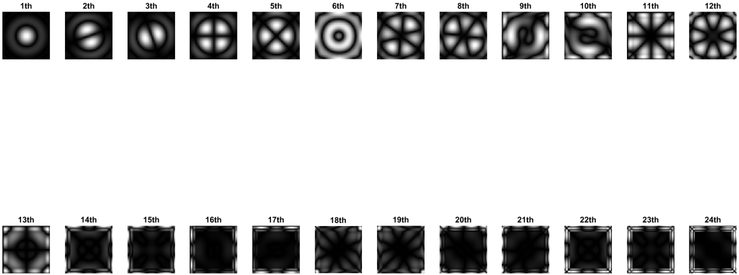
$$\rho = \sum_i \hat{w}_i \hat{w}_i^* \approx \sum_{k=1}^{r} w_k w_k^*$$

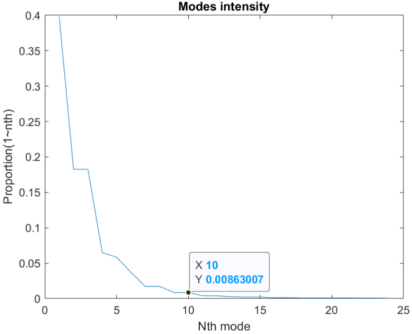
Algorithm design

Generally find a low-rank matrix ρ based on (2), which has special structures based on particular models like (4)
Two possible ways:

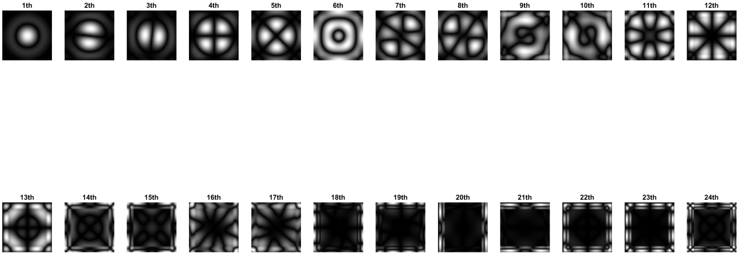
1. Transfer existing algorithms:
   related to Low-rank Matrix Recovery
2. Exploit new structure:
   Rank-1 Matrix + vibration kernel $\kappa \rightarrow$ matrix ρ
   The rank-1 matrix comes from the main mode (Bessel).
   Why approximated low-rank?
   Why special patterns for modes decomposed from ρ?

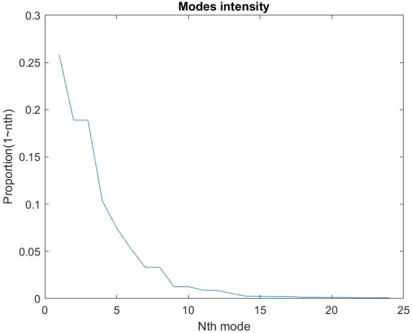
Example: Guassian κ (15 15)



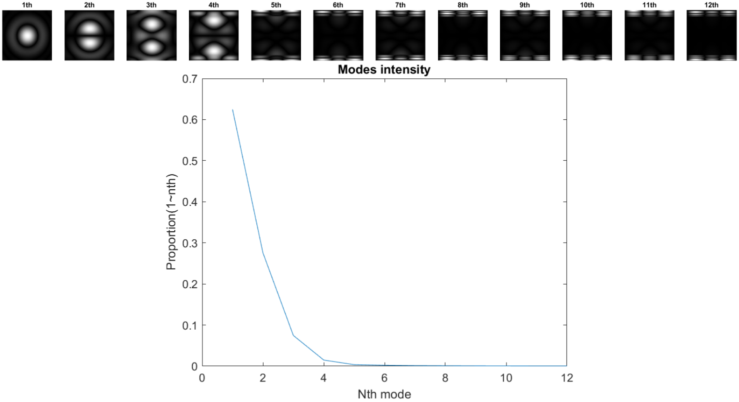


Example: Retangular κ (20 20)

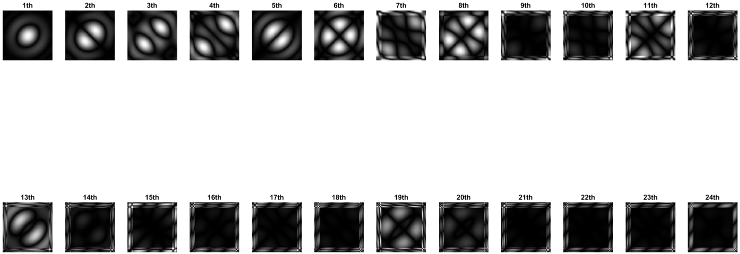


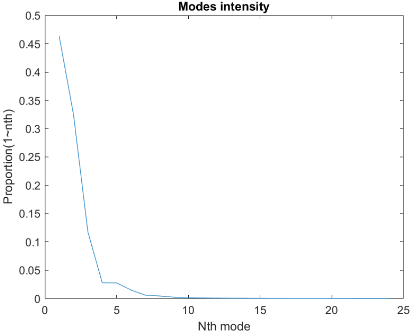


Example: Guassian κ (4 0)



Example: Motion κ (len=20,theta=45)





More about Bessel