Machine/Statistical Learning

Recurrent Neural Networks (RNN), Long Short Time Memory Networks (LSTM)

Florent Chatelain, Olivier.JJ.Michel
Univ. Grenoble-Alpes; Grenoble-INP; GIPSA-Lab

2-5. février 2021, ENSTA

Additionnal course material, and document sources for this lecture

Youtube MIT online courses :

- on NN : https ://www.youtube.com/watch ?v=njKP3FqW3Sk, by Alexander Amini
- on RNNs and LSTM : https ://www.youtube.com/watch ?v=_h66BW-xNgk by Ava Soleimani
- From Perceptrons to LSTM : https ://www.youtube.com/watch ?v=yKGm4yLuTkU by Caio Benatti Moretti

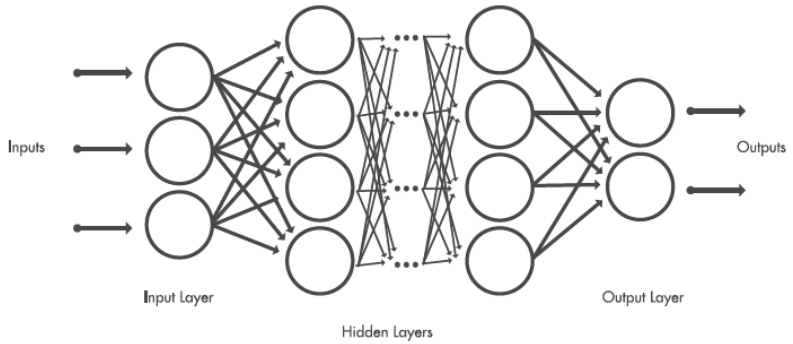
Most material and figures in the slides below are from (excellent) Colah's blog : https ://colah.github.io/posts/2015-08-Understanding-LSTMs/

☞The reference book on Deep learning :
I.J.Goodfellow, Y.Bengio, A.Courville : "Deep Learning", MIT Press 2016 (http ://www.deeplearningbook.org)

Motivation

Remind the main features of a feed forward Neural Network

- Inputs are assumed iid
- Learning is performed via the Back propagation of gradient, to minimize a Loss function



But

- The meaning of a word in a sentence depends on both
  1. its own meaning, signification
  2. previous words, from a recent past in the sentence
  3. context, possibly defined way back in the past
- The value of a time series
  1. rarely fully uncorrelated from its predecessor (except for white noise...)
  2. may depend on an underlying process with many characteristic time of dependencies

RNN :

Recurrent Neural network precisely address this problem :

☞ How can a system gets a memory about previous encountered events ?

Solutions proposed in the 1990's (Jeffrey Elmann (90)), as being "Simple Recurrent Neural Network" :

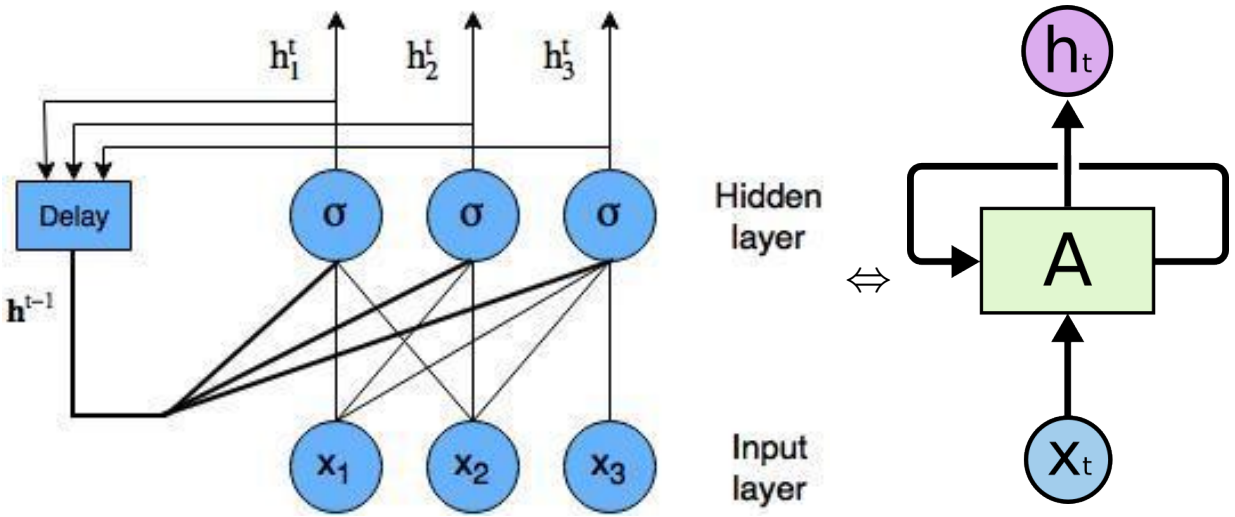
- Introduce some Neuron Internal State (NIS) h_t, depending on the current input, and past NIS :

$$\begin{cases} h_t = & \sigma(W_{hx}x_t + W_{hh}h_{t-1} + b_h) \\ y_t = & f_o(W_{yh}h_t + b_y) \end{cases}$$

- This introduces a loop (like in many system from control theory) in the learning neuron, allowing the information to persist

Notations : σ=sigmoid function ; h_t=NIS ; x_t=input vector at t, $x_t \in \mathbb{R}^d$; y_t=output vector, $y_t \in \mathbb{R}^{d'}$; W_{kl} is a weight matrix describing the connecting edges between $'l'$; $'k'$, $b_{(.)}$ stand for the biases, and f_o is the output function relating y_t to h_t.

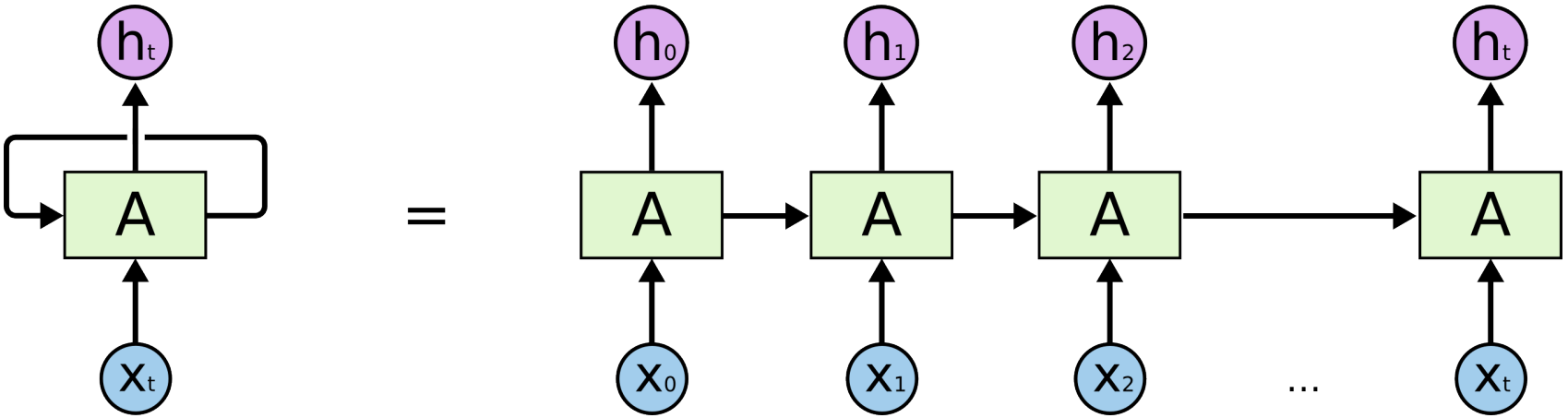
RNN cell :



Nota Bene :

- the output y_t is not represented here. $y_t = f_o(W_{yh}h_t + b_y)$ may take different forms, depending on the learning task (linear operator for regression/regression or e.g. Softmax for classification)
- On this graph ; at each time t, $x_t \in \mathbb{R}^3$, $h_t \in \mathbb{R}^3$
- Each neuron (or cell) is a MLP with a single hidden layer
- The loop is applied on the hidden layer

An alternate representation of Elman's equations, accounting for time evolution is :



The RNN can be thought of as a Deep Neural Network, with all layers having identical weight matrices and bias, or as multiple copies of the same network, each passing a message (h_{n-1}) to its successor.

- The unrolled structure highlights the relation of RNNs with lists or sequences
- RNNs where recently extremely successfull for speech recognition, translation, time series forecasting, ... **BUT with a shlightly different functionnal bloc 'A'**...

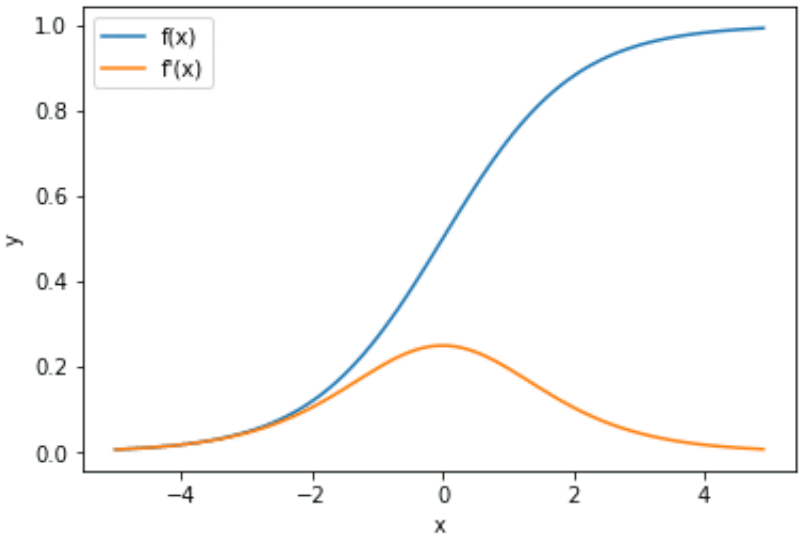
Vanishing gradient problem

Consider the unrolled structure for e.g. 3 time iterations. Learning the parameters of 'A' using GBP algorithm leads to equation that look (roughly) like this

$$\frac{\partial L}{\partial U} = \frac{\partial L_3}{\partial out_3}\frac{\partial out_3}{\partial h_3}\frac{\partial h_3}{\partial h_2}\frac{\partial h_2}{\partial h_1}\frac{\partial h_1}{\partial h_0}\frac{\partial h_0}{\partial U}$$

where
- U is any coefficient from the weight matrices or bias coeff.
- $out_{(.)}$ is e.g. the desired output at third iteration
- L stand for the loss function.

☞In this product of gradients, each involves to evaluate the gradient of a sigmoid function : if the input values are such that a term in the product is very weak (wich occurs if the output is close to either 0 or 1), the product becomes very small.
(Sigmoid gradient as a function of input value →).



Vanishing gradient problem, cont'd

- The gradient will become basically 0 when dealing with (too) many time steps, as all terms are between 0 and 1 (the same occurs by replacing the sigmoid by the tanh function).
- The weights will not adjust to take into account values that occured too early in the sequence (remind the structure of the GBP algorithm)
- The network cannot learn relationships separated by significant period of time
- One may thin about using RELU function instead. This des not solve the problem, as the gradient may as well explode....

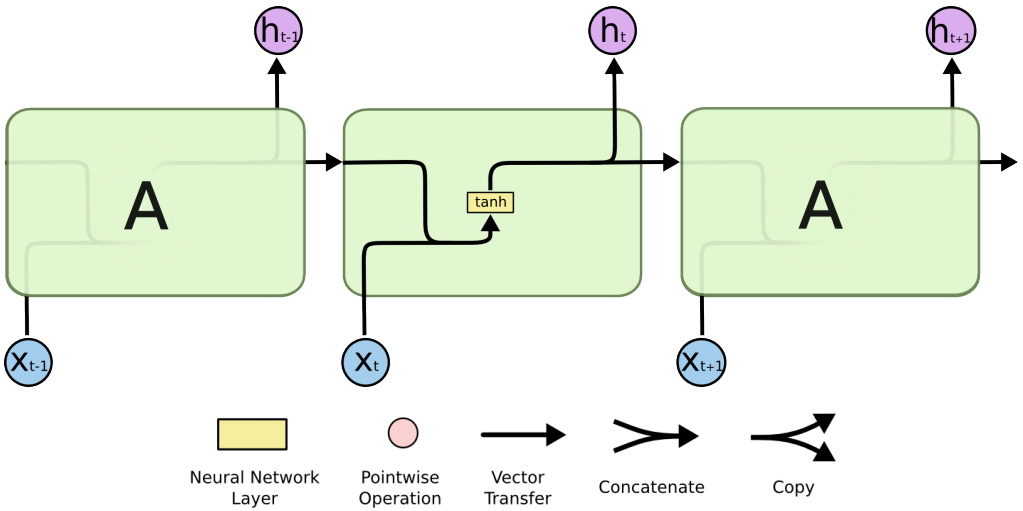
A solution was found by Hochreiter and Schmidhuber in 1997, now very widely used :

Long Short Time Memory RNN : LSTM networks

LSTM network... or simply 'LSTM'

LSTM

- Keeps the form of a repeating chain of similar modules, as in simple RNN
- Keeps the principle of creating a cell (neuron) internal state, that is transferred to the next module
- The unrolled RNN structure will be represented as follows, where 'A' is simply containing concatenation, sigmoid (or tanh) and copy functions :

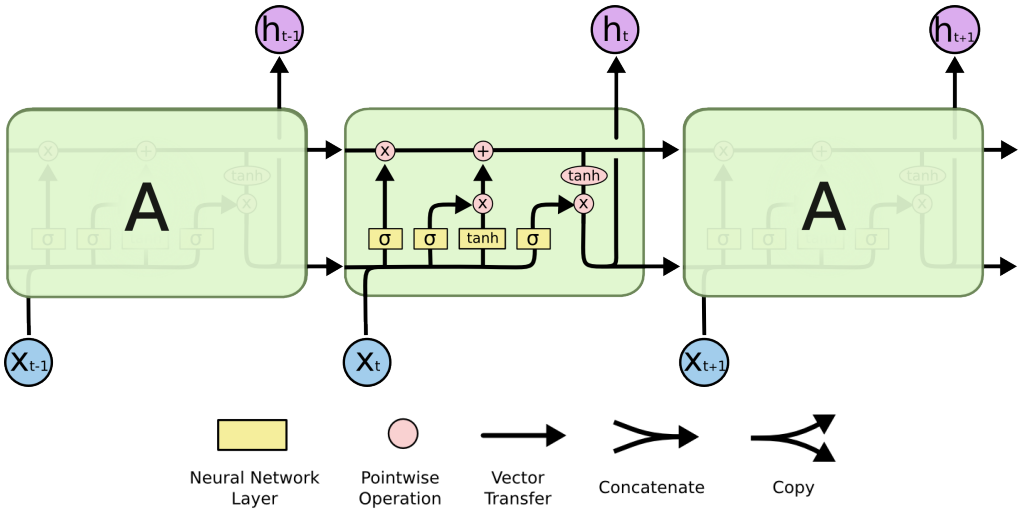


- Note that a repeating cell in a standard RNN is basically a MLP with a single hidden layer

LSTM cont'd

LSTM

- Instead of a single hidden layer MLP, a LSTM will count four (4) layer interacting in a special way. This increase in complexity has the unique purpose of avoiding vanishing gradient problems !

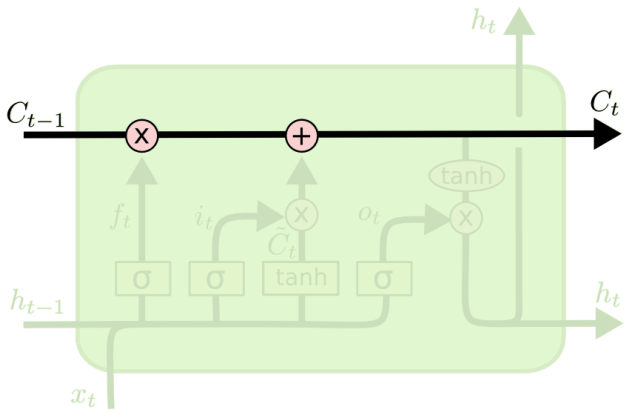


Note that each line in the diagram above carries an entire vector.

- Each of the (yellow) boxes represents a learned neural network (as a multiple output perceptron)

LSTM as an information conveyor belt

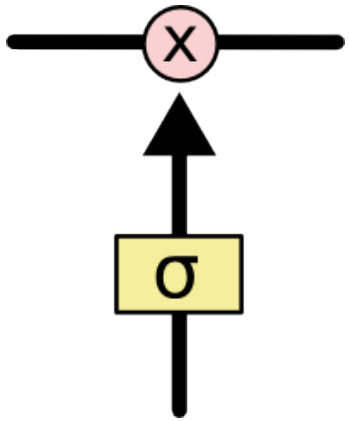
The key information transfer between modules (remind that in the unrolled representation, a module is associated to a time step) is represented by the upper horizontal line running through the cells :



- The first operator on the information conveyor, is a point wise multiplication : this will allow to cancel (remove via a multiplication y a very small value) irrelevant information.
- The second operator is a poinwise addition : this allows to add information to the cell state
- This operations are regulated by structures called **GATES**

Gates

- Gates regulate how information is either cancelled or let through. Gates composed out of sigmoid neural net layer (whose outputs component values are between 0 and 1).

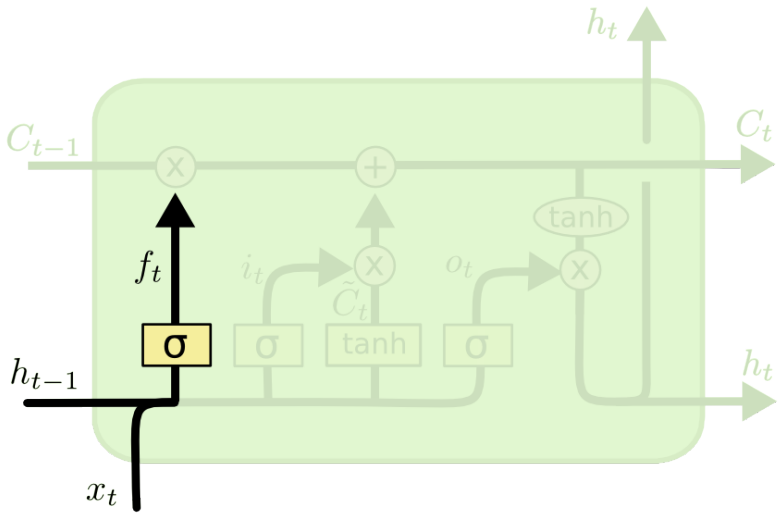


- A gate output component value of 0 means "stop this information"
- A gate output component value of 1 means "let this information through"

Step by step LSTM

Forget gate

The purpose of the forget gate is to decide what information must be thrown away from the cell state, by considering both the new input x_t and the previous time internal state h_{t-1} :



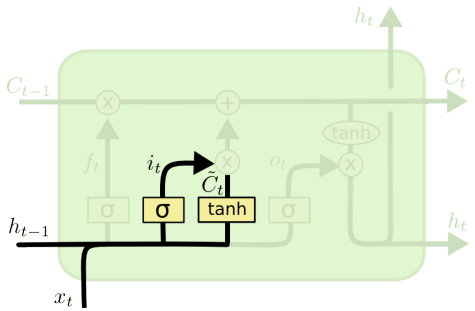
$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \; + \; b_f\right)$$

Step by step LSTM

Input gate

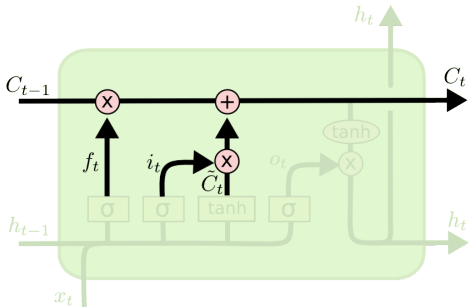
The input gate decides which information must be stored in the cell state. This is a three stages operation :

- First decide which values should be updated
- Next, a 'tanh' layer creates a new candidate $\tilde{C}_t$ that could be added to the state



$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \; + \; b_i\right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \; + \; b_C)$$

- Third, combine the two previous results to create the state update :

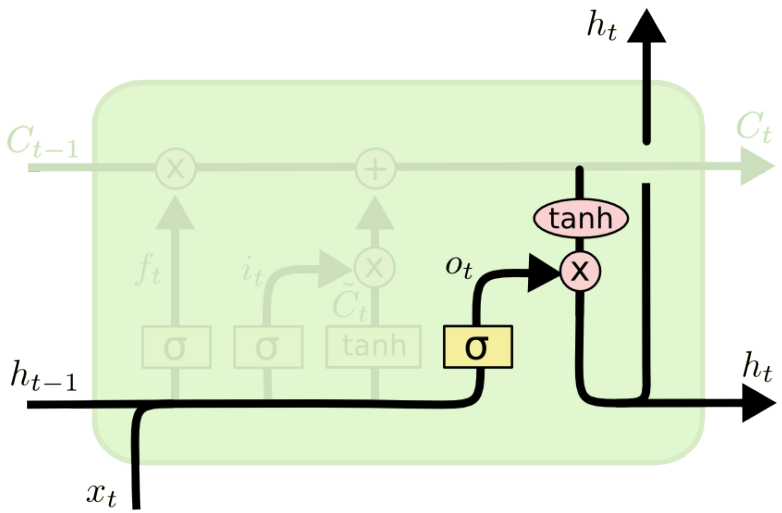


$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Step by step LSTM

Output gate

The output's gate role is to output a value h_t that combines both information from the current input x_t, the information state C_t and the previous hidden state h_{t-1} :



$$o_t = \sigma\left(W_o\left[h_{t-1}, x_t\right] + b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

LSTM : Summary

- A LSTM network is a recurrent network or equivalently a chain of identical modules or cells
- In comparison to simple RNN, LSTM contains not only a hidden state but also a cell state
- The cell state is transferred between modules with a constant unit gain, thus avoiding vanishing gradient problems
- The cell state allows long term memory storing (again, no vanishing gradient problem arises for C_t
- The hidden states cares for short term memory
- The structure of each cell is based on three (3) gates :
  1. A forget gate, deciding which internal information to cancel
  2. An input gate, deciding which information to store in the internal cell state
  3. An output gate, combining last input, previous hidden state and updated cell state to provide the new hidden state

LSTM variants

Some variants of the preceding structure may be found in the literature, among which some popular are just cited here :

- Adding "Peephole connections" : all three or only a subset of the NNs (σ) also take C_{t-1} as inputs.
- Some authors proposed to couple forget and input gates : all forget info is also replaced at the input stage.
- Gated Recurrent Units (GRU) -2014- : merge input and ouput to make an 'update gate', merge C_t and h_t

For LSTM application example, see Notebook LSTM_RNN.jpynb.