Applied Bayesian Method HW4

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1 Assignment Description

- Problem 1: Use Bayesian large sample approximation for normal example in MCMC-init-conv.pdf.
- Problem 2: Obtain ML-II for μ and τ in MCMC3-Hier.pdf. That is, maximize $p(y|\mu, \tau)$ subject to $\tau > 0$.

2 Methodology

For the first problem, the **normal approximation** in Bayesian statistics is a technique used to approximate a posterior distribution $p(\theta|D)$ when the exact posterior is difficult to compute. The key idea is that for large datasets or certain types of posteriors, the posterior distribution can be approximated by a Gaussian distribution, which is given by

$$p(\theta|D) \approx N(\hat{\theta}, \Sigma_\theta) \tag{1}$$

where $\hat{\theta}$ is the mode of the posterior and Σ_θ is the covariance matrix derived from the Hessian of the log-posterior.

For the second problem, ML-II is a method used to estimate parameters by maximizing the marginal likelihood, which involves integrating over the uncertainty in parameters. Our goal function $p(y|\mu, \tau)$ is already given in the slide as follow:

$$p(y|\mu, \tau) = \prod_j \phi\big(\bar{y}_j; \mu, \sqrt{\sigma_j^2 + \tau^2}\big) \tag{2}$$

Thus, our objective is to find out μ and τ that maximize our goal function.

3 Result

Before showing the estimate results using normal approximation, I write down the model considered in this assignment. Consider the model:

$$\begin{aligned} & y_i \sim N(\mu, \tau^{-1}), i = 1, ..., n \\ & \mu \sim N(\mu_0, \tau_0^{-1}) \\ & \tau \sim Gamma(a, b) \end{aligned} \tag{3}$$

By using normal approximation, the estimated μ is 2.9639 and the estimated τ is 0.4897. Figure (1) shows the distribution of μ and τ by sampling 1000 times.

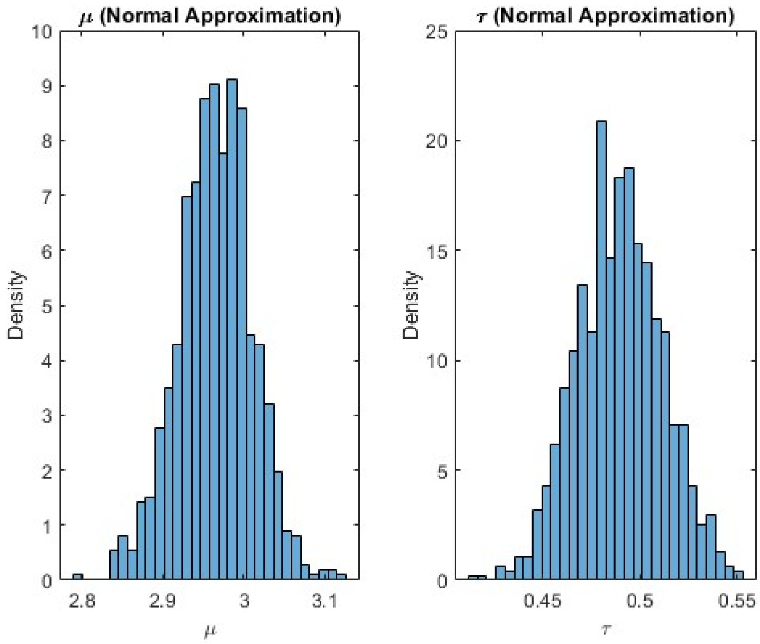

Figure 1: Distribution for normal approximation

For problem 2, we simply use *fmincon* in MATLAB to maximize the marginal likelihood and obtain the estimate results $\mu = 7.6856$ and $\tau = 0.0068$.