Levene's & F-test

Group 1

H24091265 葉詠馨

N86124127 游博凱

R26121057 林承寬*

RE6125015 劉仁忠

*: Group Leader

Contents

01 Assumption (Limitation)

Levene’s test - Introduction

1. A statistical tool used to check should the variances of different samples are equal, which is an important assumption in various statistical analyses.

2. This test evaluates the null hypothesis that the population variances are homogeneous, meaning they are **equal** across different groups or samples.

Levene's test - Introduction

If the p-value for the Levene test is greater than critical value **(typically 0.05)**, then the variances are not significantly different from each other (i.e., the homogeneity assumption of the variance is met).

H_0: Groups have **equal** variance
H_1: Groups have **different** variance

Levene’s test - Introduction

If the p-value from Levene's test is **less than critical value (typically 0.05)**, it suggests that the differences in sample variances are unlikely to have occurred based on random sampling.

Consequently, the null hypothesis of equal variances is rejected, indicating **differences** in population variances.

Levene’s test - Assumption

Levene’s test basically requires two assumptions:

1. **Independent observations:**
   Samples from the two samples are independent.

2. **The test variable is quantitative:**
   that is, not nominal or ordinal but cardinal.

F-test - Introduction

A statistical test in which the test statistic has an F-distribution under the null hypothesis.

It is most often used when comparing statistical models that have been fitted to a data set, in order to identify the model that best fits the population from which the data were sampled.

F-test - Assumption

Generally, there are two assumptions as follows:

1. **Data are normally distributed (Normal Distribution).**

2. **Samples are independent from one another.**

02 Purpose & Type of data

Levene’s Test

1. **Priori comparisons.**

2. **Test homogeneity of variances among different groups.**

F-test - purpose

1. **Variances of two independent normal data**

$$H_0: \sigma_i^2 = \sigma_j^2, H_0: \sigma_i^2 \geq \sigma_j^2, H_0: \sigma_i^2 \leq \sigma_j^2$$

2. **ANOVA or Regression analysis**

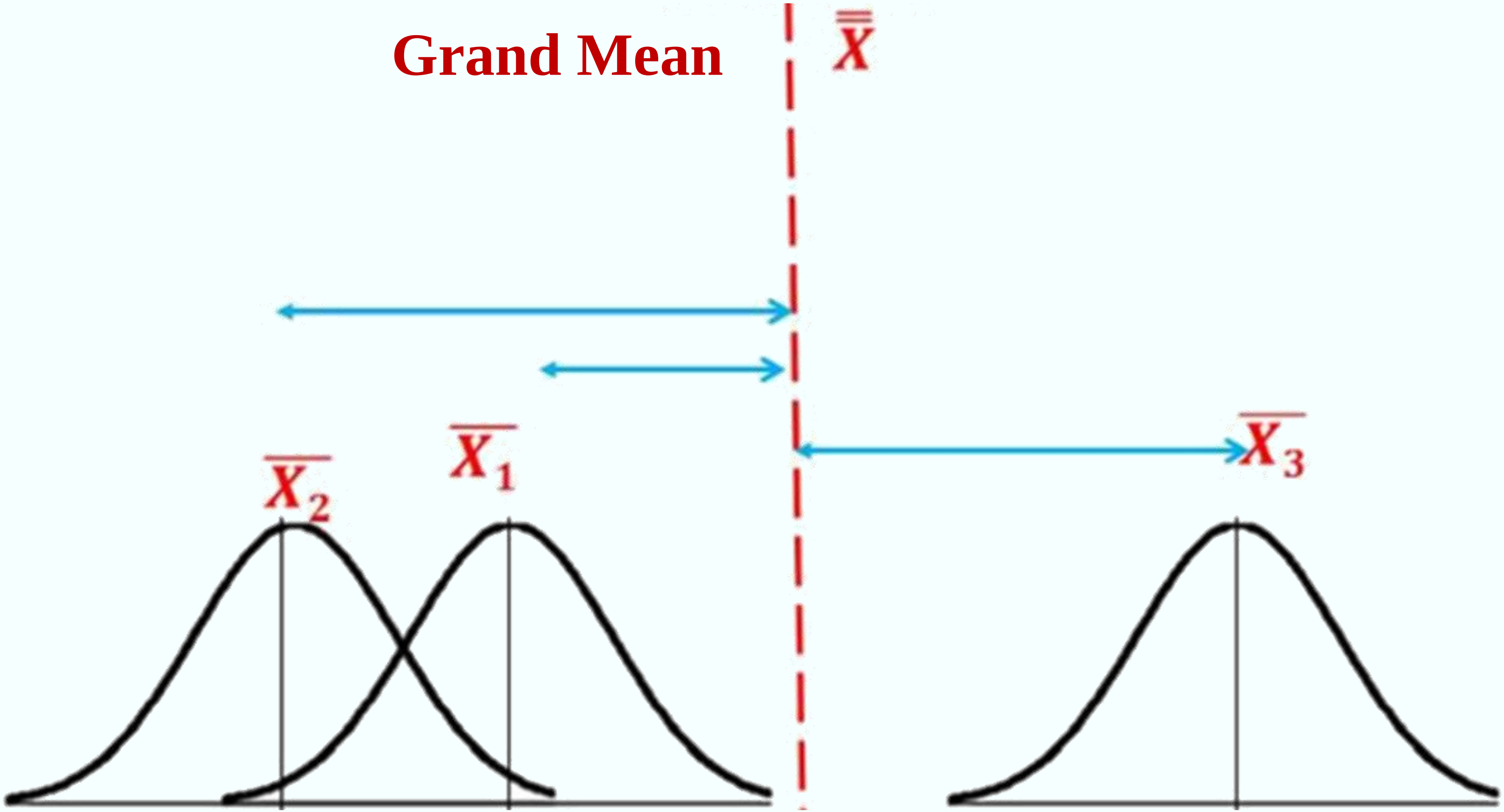
F-test: one-way ANOVA

1. **Divide the samples into m distinct groups**

$$Y_i : \mu_i + \varepsilon \, ; i = 1, 2, 3 \ldots n$$

$$Y_{ij} - \mu = (\mu_i - \mu) + \left(Y_{ij} - \mu_i\right) = \alpha_i + \varepsilon_{ij}$$

2. **Total variance = explained variance + unexplained variance**


Grand Mean
$\bar{\bar{X}}$
$\overline{X_2}$
$\overline{X_1}$
$\overline{X_3}$

1. **Use to investigate whether at least one group's explained variance has a significantly different.**

2. **m = 2, the F-test is equivalent to a two-tailed T test for the difference in population means**

3. **Comparing the means of multiple groups**

4. **If we use pairwise T-test, type 1 error will be**

$$1-(1-\alpha)^{\binom{m}{2}}$$

F-test In multiple linear regression model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_m X_{mi} + \mu_i$$
$$= \beta_0 + \sum_1^q \beta_i + \sum_{q+1}^m \beta_i + \mu_i$$

1. **Overall F test**

$$H_0: \beta_1 = \beta_2 = \cdots \beta_m = 0$$

2. **Partial F test**

$$H_0: \beta_1 = \beta_2 = \cdots \beta_q = 0$$

Restricted Model $Y_i = \beta_0 + \beta_{q+1}X_{q+1i} + \beta_m X_{mi} + \mu_i$

Unrestricted Model $Y_i = \beta_0 + \sum_1^q \beta_i + \sum_{\alpha+1}^m \beta_i + \mu_i$

<table>
<tr><th></th><th>Restricted Model</th><th>Unrestricted Model</th></tr>
<tr><td rowspan="3">SST</td><td>SSR_r</td><td>SSR_u</td></tr>
<tr><td rowspan="2">SSE_r</td><td>extra sums of squares</td></tr>
<tr><td>SSE_u</td></tr>
</table>

$= SSR_u - SSR_r$

$= SSE_r - SSE_u$

Type of data

1. **Continuous or numerical data**
2. **Usually in regression analysis or analysis of the variance**

03 Hypothesis Testing

Levene's Test

$H_0 = \sigma_1 = \sigma_2 = \cdots = \sigma_n$ (homogeneity of variances)
$H_a =$ at least one of the variance is not equal

Levene's Test

$$W = \frac{N-k}{K-1} \cdot \frac{\sum_{i=1}^{k} N_i (Z_{i.} - Z_{..})^2}{\sum_{i=1}^{k} \sum_{j=1}^{N_i} (Z_{ij} - Z_{i.})^2} \sim F_{k-1,\, N-k}$$

N = the number of all observations

K = the number of different groups

N_i = the number of observation in i^{th} group

$Z_{ij} = |Y_{ij} - \overline{Y}_i|$, $\overline{Y}_i$ is the mean of i^{th} group

We reject H_0 if W > $F_{\alpha,\, k-1,\, N-k}$ or p-value < α

F-test

$$H_0 = \sigma_1 = \sigma_2 \text{ vs. } H_a = \sigma_1 \neq \sigma_2$$

$$\mathrm{F} = \frac{\sum_{i=1}^{n_1} \frac{x_i - \bar{x}}{n_1 - 1}}{\sum_{i=1}^{n_2} \frac{y_i - \bar{y}}{n_2 - 1}} = \frac{S_1^2}{S_2^2} \sim \mathrm{F}_{n_1 - 1, n_2 - 1}$$

We reject H_0 if F > $\mathrm{F}_{\frac{\alpha}{2}, n_1 - 1, n_2 - 1}$ or p-value < α

04 Case Implementation

Data Explanation

```
A <- runif(30, 0, 3)
B <- runif(30, 0, 5)
C <- runif(30, 1, 7)

#create data frame
data <- data.frame(program = rep(c("A", "B", "C"), each = 30),
                   weight_loss = c(A, B, C))
```

Data	
data	90 obs. of 2 variables

	program	weight_loss
22	A	2.8041157
23	A	0.6364276
24	A	1.9550213
25	A	0.3766653
26	A	0.8016620
27	A	1.1583423
28	A	0.0401710
29	A	1.1471639
30	A	2.6090725
31	B	1.7017450
32	B	2.4104006
33	B	2.9978291
34	B	2.4677065
35	B	0.9310880
36	B	4.1368666
37	B	3.3423337

Levene's Test in R

```
data <- as.data.frame(unclass(data), stringsAsFactors = TRUE)

#load car package
library(car)

#conduct Levene's Test for equality of variances
leveneTest(weight_loss ~ program, data = data)
```

```
Warning message:
In leveneTest.default(y = y, group = group, ...) : group coerced to factor.
```

Usage

```
leveneTest(y, ...)
## S3 method for class 'formula'
leveneTest(y, data, ...)
## S3 method for class 'lm'
leveneTest(y, ...)
## Default S3 method:
leveneTest(y, group, center=median, ...)
```

Arguments

y	response variable for the default method, or a `lm` or right-hand-side of the model must all be factors and
group	factor defining groups.

Result

```
Levene's Test for Homogeneity of Variance (center = median)
      Df F value  Pr(>F)
group  2  4.1716 0.01862 *
      87
```

The p-value of the test is **0.01862**, which is less than our significance level of 0.05. Thus, we **reject** the null hypothesis and conclude that the variance among the three groups is ***not*** equal.

Levene's Test in Python

```
import scipy.stats as stats
stats.levene(A, B, C, center='median')
```

```
write.csv(data, "data.csv", row.names=FALSE)
```

```
data = pd.read_csv("data.csv")
data
```

```
A = data[data['program'] == 'A']
B = data[data['program'] == 'B']
C = data[data['program'] == 'C']
A = A['weight_loss']
B = B['weight_loss']
C = C['weight_loss']
```

Result

```
LeveneResult(statistic=4.171573799184559, pvalue=0.018620651676600257)
```

The result shows the same result with **statistic = F value = 4.1716** and **p-value = 0.01862**, which is less than our significance level of 0.05. Thus, we **reject** the null hypothesis and conclude that the variance among the three groups is ***not*** equal.

F-test in R

Note that F test are limited to compare two variances

```
var.test(A, B)
var.test(A, C)
var.test(B, C)
```

Result

```
> var.test(A, B, alternative="less")

        F test to compare two variances

data:  A and B
F = 0.53402, num df = 29, denom df = 29, p-value = 0.04833
```

```
> var.test(A, C, alternative="less")

        F test to compare two variances

data:  A and C
F = 0.33304, num df = 29, denom df = 29, p-value = 0.002076
```

```
> var.test(B, C, alternative="less")

        F test to compare two variances

data:  B and C
F = 0.62364, num df = 29, denom df = 29, p-value = 0.1048
```

F-test in Python

Note that F test are limited to compare two variances

```
# Converting the list to an array
x = np.array(A)
y = np.array(B)
z = np.array(C)

def f_test(group1, group2):
    f = np.var(group1, ddof=1)/np.var(group2, ddof=1)
    df1 = len(group1) - 1
    df2 = len(group2) - 1
    p_value = stats.f.cdf(f, df1, df2)
    return f, p_value

# perform F-test
f_test(x, y)
```

Lower tail of the distribution

F-value ← (0.5340183043017579, 0.04832536955222022) → **p-value**

```
f_test(x, z)
```

(0.3330366574991992, 0.002075733814973075)

```
f_test(y, z)
```

(0.6236427755686256, 0.10478760629445498)

From the result of F test tested in R and Python, we can conclude that the variance of A is not equal to both variance of B and C (reject Ho) as their p value shown are < 0.05, but the variance of B is equal to the variance of C as their p value is 0.104 (> 0.05, fail to reject H_0)

Thank you