Welch’s t test

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Purpose

Welch’s t test is a statistical hypothesis test, which is used to **compare the means** of two independent samples to determine if they are significantly different.

It is an extension of the Student's t-test, designed to handle **unequal population variances** or **unequal sample sizes**.

Assumption/Limitation

- Two samples are **independent**
- The sample means being compared for two populations are **normally distributed**
- It suggests that the sample sizes are both larger than five.

Data types

Welch's t test can only apply to **numerical data**. This can include numerical data such as measurement results, temperature, time, distance, and so on.

Null hypothesis H_0 vs. alternative hypothesis H_1

H_0 : the means of the two samples are equal.
H_1 : the means of the two samples are not equal

Advantages

- Handles Unequal Variances
- Accurate for Unequal Sample Sizes
- More robust
- Complex degrees of freedom
- Widely Supported by statistical software

$$t = \frac{\Delta\overline{X}}{s_{\Delta\bar{X}}} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{s^2_{\bar{X}_1} + s^2_{\bar{X}_2}}}$$

$$s_{\bar{X}_i} = \frac{s_i}{\sqrt{N_i}}$$

$$\nu \approx \frac{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}\right)^2}{\frac{s_1^4}{N_1^2\nu_1} + \frac{s_2^4}{N_2^2\nu_2}}$$

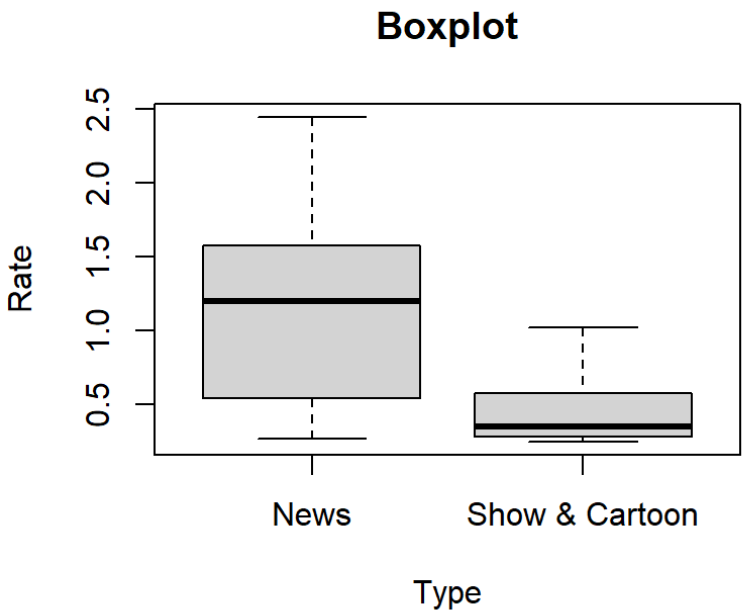
Real data

The data is about the ratings from Taiwanese channel. We want to compare that whether the ratings of two types of programs, such as news and show & cartoon , are different.

```
> data
    Channel       Program            Type Rate
1      中視  中視新聞全球              news 2.45
2      台視  台視晚間新聞              news 2.18
3   東森新  1800東森晚間              news 1.86
4      TVBN     九點熱話題              news 1.58
5   TVBS台     少康戰情室              news 1.56
6   三立新     台灣大頭條              news 1.43
7   東森新       黃金８點              news 1.23
8   ＥＲＡ  年代1900晚報              news 1.17
9   三立２     型男大主廚 show & cartoon 1.02
10     華視  華視晚間新聞              news 0.95
11  民視新     台灣最前線              news 0.90
12     台視  1800蠟筆小新 show & cartoon 0.75
13  東森綜     全民星攻略 show & cartoon 0.58
14  東森財     這不是新聞              news 0.54
15  衛電影  1930名偵探柯 show & cartoon 0.46
16  東森財     ５７爆新聞              news 0.44
17  非凡新     財經８點檔              news 0.41
18   GTV-C  1900蠟筆小新 show & cartoon 0.35
19   JET綜       大尋寶家 show & cartoon 0.30
20  MOMO台  2000新麵包超 show & cartoon 0.28
21  非凡新  錢線快報1800              news 0.27
22  緯來綜     鬼太郎之妻 show & cartoon 0.25
23  TVBS歡  1800食尚玩家 show & cartoon 0.25
```

Box plot

```
> data <- read.csv("C:/Users/user/Desktop/tv_rate.csv")
>
> news <- data[data$Type == "news",]$Rate
> show_cartoon <- data[data$Type == "show & cartoon",]$Rate
> str(news)
 num [1:14] 2.45 2.18 1.86 1.58 1.56 1.43 1.23 1.17 0.95 0.9 ..
> summary(news)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  0.270   0.630   1.200   1.212   1.575   2.450
> str(show_cartoon)
 num [1:9] 1.02 0.75 0.58 0.46 0.35 0.3 0.28 0.25 0.25
> summary(show_cartoon)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.2500  0.2800  0.3500  0.4711  0.5800  1.0200
> boxplot(news,show_cartoon, names=c("News", "Show & Cartoon"),
+         ylab = "Rate", xlab = "Type", main = "Boxplot" )
```



Check assumption

- Normal assumption:

$$H_0 : data\ \ follows\ \ normal\ \ distribution$$
$$H_a : data\ \ doesn't\ \ follow\ \ normal\ \ distribution$$

```
> ad.test(news)

        Anderson-Darling normality test

data:  news
A = 0.20678, p-value = 0.8347

> ad.test(show_cartoon)

        Anderson-Darling normality test

data:  show_cartoon
A = 0.61933, p-value = 0.07236
```

Check assumption

- F test

$$H_0 : \sigma_1^2 = \sigma_2^2 \ v.s. \ H_a : \sigma_1^2 \neq \sigma_2^2$$

$$\sigma_1^2 : The\ variance\ of\ News$$

$$\sigma_2^2 : The\ variance\ of\ Shows\ \&\ Cartoon$$

```
> var.test(news,show_cartoon,alternative = "two.sided")

        F test to compare two variances

data:  news and show_cartoon
F = 6.4208, num df = 13, denom df = 8, p-value = 0.01292
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
  1.542654 21.753550
sample estimates:
ratio of variances
          6.420788
```

Welch's t test

$$H_0 : \mu_1 = \mu_2 \ v.s. H_a \ \mu_1 \neq \mu_2$$

$$\mu_1 : The\ mean\ of\ News$$

$$\mu_2 : The\ mean\ of\ Shows\ \&\ Cartoon$$

```
> t.test(news, show_cartoon , alternative ="two.sided")

        Welch Two Sample t-test

data:  news and show_cartoon
t = 3.6803, df = 18.315, p-value = 0.001672
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.3185248 1.1635387
sample estimates:
mean of x mean of y
1.2121429 0.4711111
```

Reference

R統計 | 變異數分析 (neocities.org)

Welch's t-test - Wikipedia

Thank you