Friedman Test

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Use cases

1. Test whether public relations make a difference in the prestige of doctors, lawyers, police officers, and teachers by asking 10 people to rank the reputation of these four professions.
2. Test if the price of 14 different types of soft drinks, sold in 5 different outlets, on a specific date, when neither promotion nor special discount was applied are different.
3. Test whether three kinds of drug have different effects on patients.

Purpose

- Test whether there are difference between three or more repeated measures over subjects
- Commonly used in two situations:
  - Compare the measures of subjects among three or more time points
  - Compare the measures of subjects under three or more different conditions
- Can be used to when data are not normally distributed

Assumption

- The group is a random sample from the population
- Groups (rows) are independent, treatments (columns) are independent
- Homogeneity of variance

Treatment

Group/Block

Patient	Drug 1	Drug 2	Drug 3
Patient 1	4	5	2
Patient 2	6	6	4
Patient 3	3	8	4
Patient 4	4	7	3
Patient 5	3	7	2
Patient 6	2	8	2
Patient 7	2	4	1
Patient 8	7	6	4
Patient 9	6	4	3
Patient 10	5	5	2

Data Type:

- Ordinal
- Numeric

Hypothesis: all treatment are the same?

Treatment

Tx 1	Tx 2	Tx 3	…	Tx k
# 1	# 1	# 1	…	# 1
# 2	.	# 2		# 2
# 3	.	# 3		.
.	.	.	…	.
.	.	.		.
.	.			.
.	.	# N3		.
# N1	.			# Nk
	# N2			

In repeated (related) measures

- Whether 3 painkillers have similar effect?
- Are the 4 tests of mathematical statistics equally easy?

Repeated measures for ordinal data

Treatment

Group Block

Case	Tx 1	Tx 2	Tx 3	…	Tx k
# 1	X_{11}	X_{12}		…	X_{1k}
# 2	X_{21}	X_{22}		…	X_{2k}
# 3	X_{31}	X_{32}		…	X_{3k}
# 4					.
.					.
.					.
.					.
# N	X_{N1}				X_{Nk}

Repeated measures for ordinal data

Treatment

Group Block

Case	Tx 1	Tx 2	Tx 3	Tx 4	Tx k = 5
# 1	1.6	6.0	3.3	8.1	9.3
# 2	7.0	4.0	5.5	4.0	2.5
# 3			.		
# 4			.		.
			.		.
.			.		.
.			.		.
.			.		
# N	3.5	3.5	5.0	3.5	8.5

Repeated measures for ordinal data

Treatment

Group

Case	Tx 1	Tx 2	Tx 3	Tx 4	Tx k = 5
# 1	1.6 (1)	6.0 (3)	3.3 (2)	8.1 (4)	9.3 (5)
# 2	7.0 (5)	4.0 (2.5)	5.5 (4)	4.0 (2.5)	2.5 (1)
# 3			.		
# 4			.		.
.			.		.
.			.		.
.			.		.
			.		
# N	3.5 (2)	3.5 (2)	5.0 (4)	3.5 (2)	8.5 (5)

Hypothesis: all treatment effect are the same

- H_0: $\mathcal{R}_1 = \mathcal{R}_2 = \cdots = \mathcal{R}_k$
- H_1: $at\ least\ one\ \mathcal{R}_i \neq \mathcal{R}_j\ for\ some\ i,j$

- Test Statistic Q =

$$\frac{12}{Nk(k+1)}\sum_{j=1}^{k}\left(R_j - \frac{N(k+1)}{2}\right)^2$$

$\sim\chi^2$ $with\ degree\ of\ freedon\ k-1$

Case	Tx 1	Tx 2	Tx 3	Tx 4	Tx = 5
# 1	1.6 (1)	6.0 (3)	3.3 (2)	8.1 (4)	9.3 (5)
# 2	7.0 (5)	4.0 (2.5)	5.5 (4)	4.0 (2.5)	2.5 (1)
# 3	2.5 (1)	6.5 (5)	3.0 (2)	4.0 (3)	5.1 (4)
			⋮		
# N	3.5 (2)	3.5 (2)	5.0 (4)	3.5 (2)	8.5 (5)
Total	R_1	R_2	R_3	R_4	R_5

Example (Using in R)

The following table shows the reaction time of five patients on four different drugs.

Since each patient is measured on each of the four drugs, we will use the Fridman test to determine

if the mean reaction time differs between drugs.

Score	Drug			
Person	1	2	3	4
1	30	28	16	34
2	14	18	10	22
3	24	20	18	30
4	38	34	20	44
5	26	28	14	30

Example (Using in R)

Score	Drug			
Person	1	2	3	4
1	30	28	16	34
2	14	18	10	22
3	24	20	18	30
4	38	34	20	44
5	26	28	14	30

R code:

```
install.packages(“stats”)

library(stats)

data <- data.frame(person = rep(1:5,each=4),

    drug = rep(c(1:4),times=5),

    score = c(30,28,16,34,14,18,10,22,24,20,18,30,38,34,20,44,26,28,14,30))
```

	person	drug	score
1	1	1	30
2	1	2	28
3	1	3	16
4	1	4	34
5	2	1	14
6	2	2	18
7	2	3	10
8	2	4	22
9	3	1	24
10	3	2	20
11	3	3	18
12	3	4	30
13	4	1	38
14	4	2	34
15	4	3	20
16	4	4	44
17	5	1	26
18	5	2	28
19	5	3	14
20	5	4	30

Example (Using in R)

R code:

```
install.packages("nonpar")

library(nonpar)

cochrans.q(matrix(data$score,5,4))
```

Result:

```
Cochran's Q Test

H0: There is no difference in the effectiveness of treatments.
HA: There is a difference in the effectiveness of treatments.

Q = -0.37034878881212

Degrees of Freedom = 3

Significance Level = 0.05
The p-value is  1
```

Since p-value = 1 > 0.05, do not reject H0.

That is, we don't have significant evidence to support that the variance of different drugs are different.

Hence, we can assume the homogeneity of variance.

Example (Using in R)

R code:

```
friedman.test(y=data$score,groups=data$drug,blocks=data$person)
#y: a "vector" of a response values
#groups: a "vector" of values indicating the "group" an observations belongs in.
#blocks: a "vector" of values indicating the "blocking" variable.
```

Result:

```
        Friedman rank sum test

data:  data$score, data$drug and data$person
Friedman chi-squared = 13.56, df = 3, p-value = 0.00357
```

Since p-value = 0.00357 < 0.05, reject H0, that is, we have significant evidence to support that at least one drug's reaction time is different from the others.

Reference

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2. 沈明來. 實用無母數統計學. 第二版. 2007. 九州
3. Milton Friedman (1937) The Use of Ranks to Avoid the Assumption of Normality Implicit in the Analysis of Variance, Journal of the American Statistical Association, 32:200, 675-701, DOI: 10.1080/01621459.1937.10503522