Statistical methods
Homework 3

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Due data: 23:59, October 16, 2023

Given the following distributions, use the method of moments (MME) to estimate the model parameters for the two datasets in Homework 2.

- Normal distribution with mean μ and variance σ^2.
- Exponential distribution with rate parameter λ.
- Laplace distribution with location parameter μ and scale parameter b.
- Gamma distribution with shape parameter α and rate parameter λ.

Please answer the following questions:

(a) Use the method of moments (MME) to estimate the model parameters.

Distributions	**Dataset 1**	**Dataset 2**
Normal	$(\hat{\mu} = 0.02471087, \hat{\sigma}^2 = 0.7138611)$	$(\hat{\mu} = 19.24671, \hat{\sigma}^2 = 5.177468)$
Exponential	$\hat{\lambda} = 40.46803$	$\hat{\lambda} = 0.05195693$
Laplace	$(\hat{\mu} = 0.02471087, \hat{b} = 0.5974366)$	$(\hat{\mu} = 19.24671, \hat{b} = 3.661023)$
Gamma	$(\hat{\alpha} = 0.01764013, \hat{\lambda} = 0.03461579)$	$(\hat{\alpha} = 13.81905, \hat{\lambda} = 0.7179953)$

Table 1: MMEs

Normal Distribution:
Let $x_1, x_2, \ldots, x_n \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mu, \sigma^2)$
$\bar{X} \triangleq E(X) = \mu$
$M_r \triangleq \mu_r f(\theta) = \sigma^2$
$\hat{\mu} = \bar{X}$
$\hat{\sigma^2} = M_2$

Exponential Distribution:
Let $x_1, x_2, \ldots, x_n \overset{\text{i.i.d.}}{\sim} \exp(\lambda)$
$\bar{X} \triangleq E(X) = \frac{1}{\lambda}$
$\hat{\lambda} = \frac{1}{\bar{X}}$

Laplace Distribution:
Let $x_1, x_2, \ldots, x_n \overset{\text{i.i.d.}}{\sim} \text{Laplace}(\mu, b)$
$\bar{X} \triangleq E(X) = \mu$
$M_r \triangleq \mu_r f(\theta) = 2b^2$
$\hat{\mu} = \bar{X}$
$\hat{b} = \sqrt{\frac{\bar{X}}{2}}$

Gamma Distribution:
Let $x_1, x_2, \ldots, x_n \overset{\text{i.i.d.}}{\sim} \text{Gamma}(\alpha, \beta)$
$\bar{X} \triangleq E(X) = \frac{\alpha}{\beta}$
$M_r \triangleq \mu_r f(\theta) = \frac{\alpha}{\beta^2}$
$\hat{\alpha} = \bar{X}\frac{\bar{X}}{M_2}$
$\hat{\beta} = \frac{\bar{X}}{M_2}$

(b) For each dataset, add the "fitted probability density functions" of the given distributions to the histograms of the two datasets. Trying to select more suitable distributions based on your opinion.

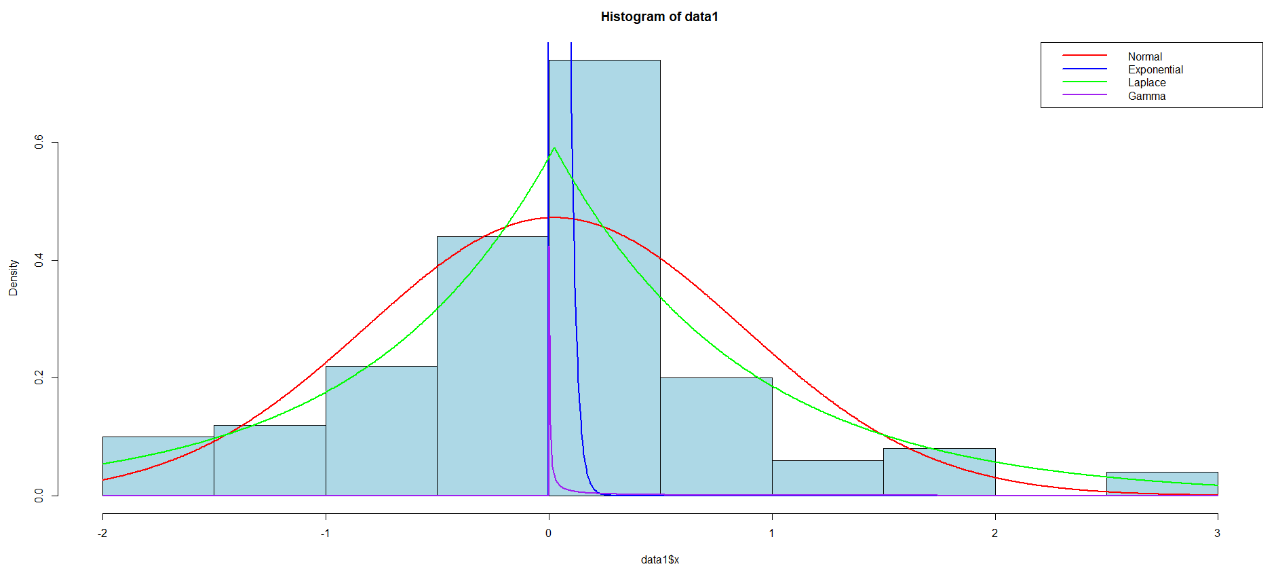

Figure 1: Histogram of set1

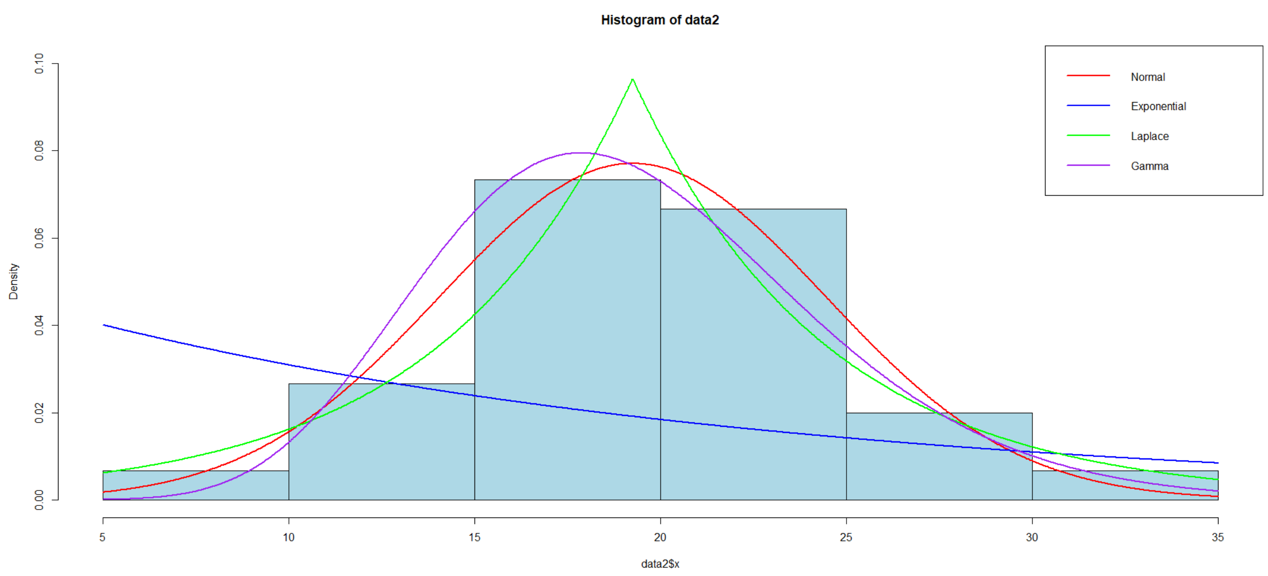

Figure 2: Histogram of set1

(c) Plot the "fitted cumulative distribution functions (cdf)" with the empirical cdf of the two datasets.

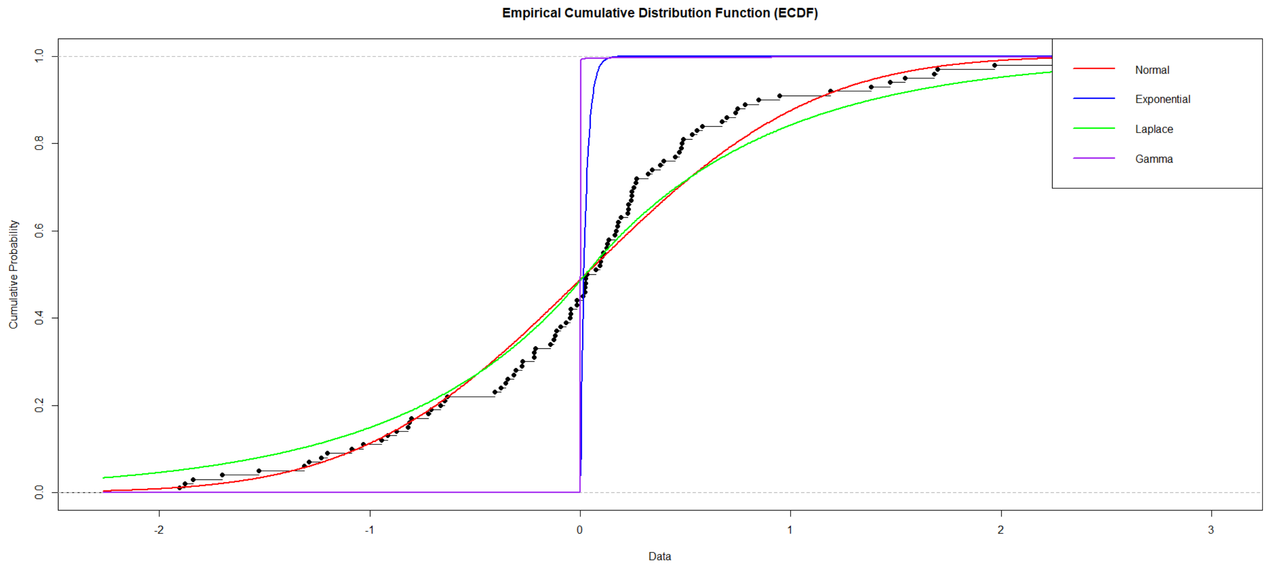

Figure 3: Histogram of set1

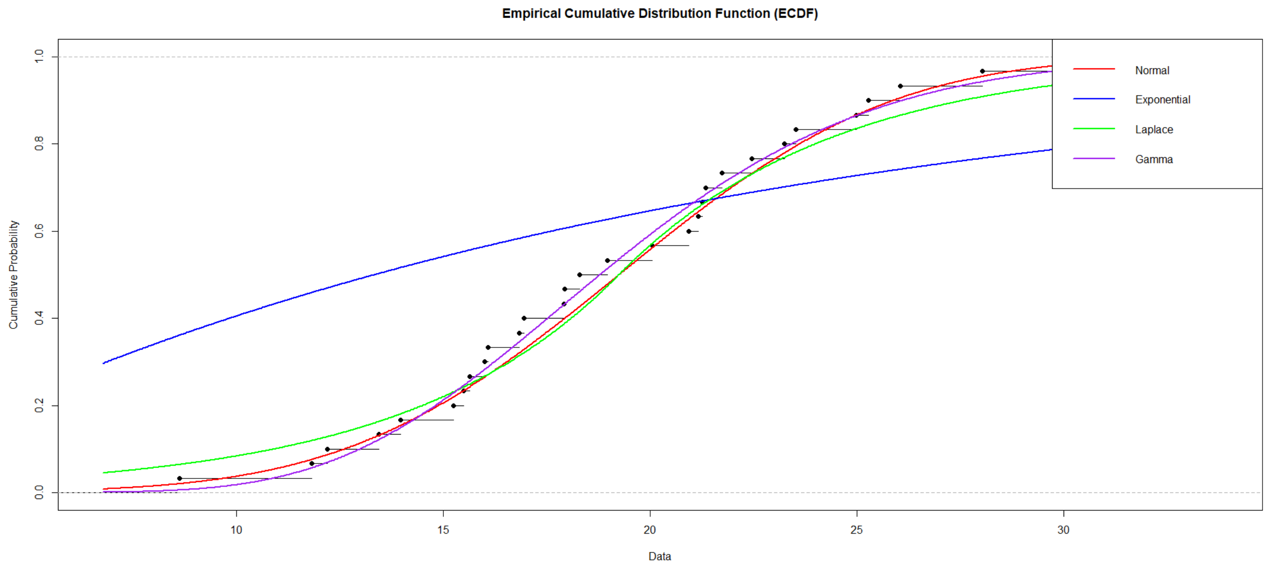

Figure 4: Histogram of set1

(d) Provide the necessary evidence for selecting suitable models for the two datasets via suitable hypothesis testings. (Could be choose one or more.
H0 : the samples are from the distribution F0
H1 : the samples are not from the distribution F0

Table 1: Test Table of set1

	Normal	Exponential	Laplace	Gamma
Kolmogorov–Smirnov Test	0.201	0.000	0.939	0.000
Anderson–Darling Test	0.157	0.000	0.822	0.000
Cramer-Von Mises Test	0.134	0.000	0.772	0.000

Table 2: Test Table of set2

	Normal	Exponential	Laplace	Gamma
Kolmogorov–Smirnov Test	0.994	0.000	0.620	0.945
Anderson–Darling Test	0.999	0.000	0.667	0.999
Cramer-Von Mises Test	0.991	0.000	0.540	0.996

In Set1:

Normal Distribution: The p-value is greater than 0.05, do not reject the null hypothesis H_0. The data does not significantly deviate from a normal distribution.

Exponential Distribution: The p-value is less than 0.05, suggests rejecting the null hypothesisH_0. The data significantly deviates from an exponential distribution.

Laplace Distribution: The p-value is greater than 0.05, do not reject the null hypothesis H_0. The data does not significantly deviate from a normal distribution.

Gamma Distribution: The p-value is less than 0.05, suggests rejecting the null hypothesisH_0. The data significantly deviates from an exponential distribution.

In Set2:

Normal Distribution: The p-value is greater than 0.05, do not reject the null hypothesis H_0. The data does not significantly deviate from a normal distribution.

Exponential Distribution: The p-value is less than 0.05, suggests rejecting the null hypothesisH_0. The data significantly deviates from an exponential distribution.

Laplace Distribution: The p-value is greater than 0.05, do not reject the null hypothesis H_0. The data does not significantly deviate from a normal distribution.

Gamma Distribution: The p-value is greater than 0.05, do not reject the null hypothesis H_0. The data does not significantly deviate from a normal distribution.