Statistics Method Report

Wilcoxon Signed Rank Test

Group 8

數據所 黃亮臻 RE6124035 統計所 張伊萱 R26121023

統計所 施文千 R26124055 企管所 陳冠瑜 R46121213

Table of content

1.

Introduction

What is Wilcoxon Signed Rank Test

- A non-parametric test
- The type of variable could be continuous or ordinal (rank)
- Situations:
  1. Population medium (η) vs. Constant (η_0)
  2. Difference between Paired samples (η_A, η_B)
- Non-parametric alternative to One sample t-test and Paired t-test

Properties of Nonparametric Test

- Distribution-Free
- Allow smaller sample size
- Less sensitive to outliers

Why don't we always use non-parametric tests?

- Low statistical power

2.

Theory

Hypothesis Testing

- One – sample test

The one-sample Wilcoxon signed-rank test can be used to test whether data comes from a population with a specified median (η_0).

$$\begin{cases} H_0 : \eta = \eta_0 \\ H_1 : \eta \neq \eta_0 \end{cases}$$

Hypothesis Testing

- Paired data test

The paired data Wilcoxon signed-rank test can be used to test that is difference between two sample median(η_A and η_B).

$$\begin{cases} H_0 : \eta_A = \eta_B \\ H_1 : \eta_A \neq \eta_B \end{cases}$$

Test Statistic

- Calculate the differences :

If we have one sample$x_1, x_2, \dots, x_n$,

we define $d_i = x_i - \eta_0$, for i = 1,2,...,n

If we have paired data(x_i, y_i) , for i = 1, 2,...,n

we define $d_i = x_i - y_i$, for i = 1,2,...,n

Test Statistic

- Take the absolute value of each difference ($|d_i|$) and rank them from smallest to largest, define it as R_i
- Let W$^+ = \sum_{i=1}^{n} R_i \times I_{\{d_i>0\}}$, W$^- = \sum_{i=1}^{n} R_i \times I_{\{d_i<0\}}$

Note: I is indicator function

$$\begin{cases} I_{\{d_i>0\}} = 0,\ if\ d_i < 0 \\ I_{\{d_i>0\}} = 1,\ if\ d_i > 0 \end{cases} \qquad \begin{cases} I_{\{d_i<0\}} = 1,\ if\ d_i < 0 \\ I_{\{d_i<0\}} = 0,\ if\ d_i > 0 \end{cases}$$

Test Statistic

- The test statistic is

$$W = \text{Min}\{\, W^{+}, W^{-} \}$$

- If the sample size is large enough, the test statistic approximates a normal distribution with

$$\mu = \frac{n(n+1)}{4} \text{ and } \sigma = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

$$Z = \frac{W-E(W)}{\sqrt{Var(w)}} = \frac{W-\frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \sim N(0,1)$$

Level Of Significance & Rejection Region

- Level of significance(α)

  We usually use $\alpha = 0.05$

- Rejection region(RR)

  RR=$\left\{W < W_{\frac{\alpha}{2}}(n)\right\}$, find Wilcoxon signed-rank table

- If the sample size is large, then RR=$\left\{Z < -Z_{\frac{\alpha}{2}}\right\}$, find Standard Normal Distribution Table

Wilcoxon Signed-Rank Table

	alpha values						
n	0.001	0.005	0.01	0.025	0.05	0.10	0.20
5	--	--	--	--	--	0	2
6	--	--	--	--	0	2	3
7	--	--	--	0	2	3	5
8	--	--	0	2	3	5	8
9	--	0	1	3	5	8	10
10	--	1	3	5	8	10	14
11	0	3	5	8	10	13	17
12	1	5	7	10	13	17	21
13	2	7	9	13	17	21	26
14	4	9	12	17	21	25	31
15	6	12	15	20	25	30	36
16	8	15	19	25	29	35	42
17	11	19	23	29	34	41	48
18	14	23	27	34	40	47	55
19	18	27	32	39	46	53	62
20	21	32	37	45	52	60	69
21	25	37	42	51	58	67	77
22	30	42	48	57	65	75	86
23	35	48	54	64	73	83	94
24	40	54	61	72	81	91	104
25	45	60	68	79	89	100	113
26	51	67	75	87	98	110	124
27	57	74	83	96	107	119	134

	alpha values						
n	0.001	0.005	0.01	0.025	0.05	0.10	0.20
28	64	82	91	105	116	130	145
29	71	90	100	114	126	140	157
30	78	98	109	124	137	151	169
31	86	107	118	134	147	163	181
32	94	116	128	144	159	175	194
33	102	126	138	155	170	187	207
34	111	136	148	167	182	200	221
35	120	146	159	178	195	213	235
36	130	157	171	191	208	227	250
37	140	168	182	203	221	241	265
38	150	180	194	216	235	256	281
39	161	192	207	230	249	271	297
40	172	204	220	244	264	286	313
41	183	217	233	258	279	302	330
42	195	230	247	273	294	319	348
43	207	244	261	288	310	336	365
44	220	258	276	303	327	353	384
45	233	272	291	319	343	371	402
46	246	287	307	336	361	389	422
47	260	302	322	353	378	407	441
48	274	318	339	370	396	426	462
49	289	334	355	388	415	446	482
50	304	350	373	406	434	466	503

3.

Example

Example

Example 5.1.1. In computer memory manufacturing, chips that come off the assembly line can actually have widely varying properties, even from the same piece of silicon. To counter for this, manufacturer will measure the properties of the chips and bin them (as in product binning) to choose the product line best suits the measured value of some specification. In some application, for the example of running RAM chips in dual- or quad-channel architecture for better performance, it is important to have RAM's running at matching speeds. From one of the production line, consecutive two DDR4 RAM's are sampled to measure their memory clock speed (MHz). Tests made on 13 randomly selected pairs of RAM's are reported in the table below, and the production manager are wondering if the overall process can produce consecutive RAM sticks in pairs of two with identical memory clock speed before further analysis. Denote by X_i the clock speed of the first component, Y_i of the second of pair i, and D_i the absolute difference of $Y_i - X_i$. Is the production making pairs of RAM's with matching clock speeds?

Table 5.1: Memory clock speed measured in MHz

Pair i	1	2	3	4	5	6	7	8	9
X_i	300.02	299.99	300.00	300.00	300.01	300.01	299.99	299.98	299.99
Y_i	299.97	299.99	299.99	300.04	299.98	299.95	300.03	300.02	299.99
$\|Y_i - X_i\|$	0.05	0.00	0.01	0.04	0.03	0.06	0.04	0.04	0.00

Pair i	10	11	12	13
X_i	299.99	299.98	299.97	300.01
Y_i	300.01	300.00	299.98	300.00
$\|Y_i - X_i\|$	0.02	0.02	0.01	0.01

Example from applied nonparamatric statistics wayne W.Daniel

Example

Pair i	Xi	Yi	di = Xi – Yi	Sign	Rank	
1	300.02	299.97	0.05	+	10	
2	299.99	299.99	0			
3	300.00	299.99	0.01	+	2	
4	300.00	300.04	-0.04	–		8
5	300.01	299.98	0.03	+	6	
6	300.01	299.95	0.06	+	11	
7	299.99	300.03	-0.04	–		8
8	299.98	300.02	-0.04	–		8
9	299.99	299.99	0			
10	299.99	300.01	-0.02	–		4.5
11	299.98	300.00	-0.02	–		4.5
12	299.97	299.98	-0.01	–		2
13	300.01	300.00	0.01	+	2	
			Sum		31	35

$$\begin{cases} H_0 : \eta_X = \eta_Y \\ H_1 : \eta_X \neq \eta_Y \end{cases}$$

By Wilcoxon table

W = min{ |31| , |35| }

By Z table

$$E(W) = \frac{13*14}{4} = 45.5$$

$$Var(W) = \frac{13*14*27}{24} = 204.75$$

$$Z = \frac{(31-14)}{\sqrt{204.75}} = 1.188$$

P-value = 0.119

Example

Use library **MASS**

alpha values							
n	0.001	0.005	0.01	0.025	0.05	0.10	0.20
5	--	--	--	--	--	0	2
6	--	--	--	--	0	2	3
7	--	--	--	0	2	3	5
8	--	--	0	2	3	5	8
9	--	0	1	3	5	8	10
10	--	1	3	5	8	10	14
11	0	3	5	8	10	13	17
12	1	5	7	10	13	17	21
13	2	7	9	13	17	21	26
14	4	9	12	17	21	25	31
15	6	12	15	20	25	30	36
16	8	15	19	25	29	35	42
17	11	19	23	29	34	41	48

```
library(MASS)
df <- data.frame(X = c(300.02, 299.99 ,300.00, 300.00, 300.01, 300.01, 299.99,
                 299.98, 299.99, 299.99, 299.98, 299.97, 300.01),
             Y = c(299.97, 299.99, 299.99, 300.04, 299.98, 299.95, 300.03,
                 300.02, 299.99, 300.01, 300.00, 299.98, 300.00))
wilcox.test(df$X, df$Y, paired=TRUE)
```

```
        Wilcoxon signed rank test with continuity correction

data:  df$X and df$Y
V = 31, p-value = 0.8936
alternative hypothesis: true location shift is not equal to 0
```

$W = \min\{ |31| , |-35| \} = 31$

From table, 31 > 8 , so we don't reject H_0

Thanks!

CREDITS: This presentation template was created by **Slidesgo**, and includes icons by **Flaticon**, and infographics & images by **Freepik**

Introduction : 黃亮臻
Theory : 張伊萱
Example : 施文千
PPT 彙整 : 陳冠瑜、施文千