Statistical methods
Homework 2

RE6124019 Matthew

Due data: 23:59, September 25, 2023

1. Give the sample space (domain), probability density function, mean, and variance of the following distributions:

   (a) Poisson distribution with rate $\lambda = 3$. Sample space: $S = \{0, 1, 2, \ldots\}$
   Probability density function: $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$
   Mean: λ
   Variance: λ
   Example: Counting the number of emails received in an hour.

   (b) Geometric distribution with probability $p = \frac{1}{3}$.
   Sample space: $S = \{1, 2, 3, \ldots\}$
   Probability density function: $P(X = k) = (1 - p)^{k-1}p$
   Mean: $\frac{1}{p}$
   Variance: $\frac{1-p}{p^2}$
   Example: The number of coin flips until the first head.

   (c) Normal distribution with $\mu = 0$ and $\sigma = 1$.(Standard)
   Sample space: $S = (-\infty, \infty)$
   Probability density function: $\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$
   Mean: μ
   Variance: σ^2
   Example: Heights of individuals in a population.

   (d) Exponential distribution with rate $\lambda = 2$.
   Sample space: $S = [0, \infty)$
   Probability density function: $f(x) = \lambda e^{-\lambda x}$
   Mean: $\frac{1}{\lambda}$
   Variance: $\frac{1}{\lambda^2}$
   Example: Modeling the time between arrivals of customers at a store.

   (e) Laplace distribution with location $\mu = 0$ and scale $b = \frac{1}{2}$.
   Sample space: $S = (-\infty, \infty)$
   Probability density function: $\frac{1}{2b}e^{-\frac{|x-\mu|}{b}}$
   Mean: μ
   Variance: $2b^2$
   Example: Modeling the difference in stock prices between consecutive days.

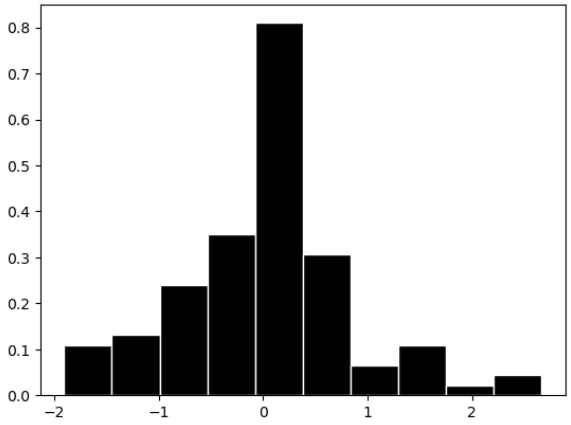
   (f) Gamma distribution with shape $\alpha = 10$ and rate $\lambda = \frac{1}{2}$.
   Sample space: $S = [0, \infty)$
   Probability density function: $\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}$
   Mean: $\frac{\alpha}{\lambda}$

Variance: $\frac{\alpha}{\lambda^2}$
Example: Modeling the time until a light bulb fails.

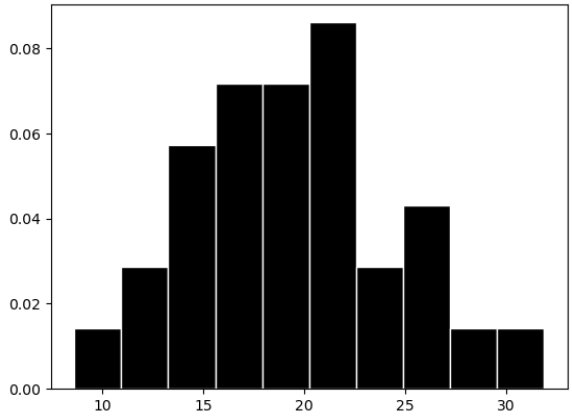
(g) Chi-square distribution with degrees of freedom 20. Sample space: $S = [0, \infty)$
Probability density function: $\frac{1}{2^{k/2}\Gamma(k/2)} x^{(k/2)-1} e^{-x/2}$
Mean: k
Variance: $2k$
Example: Testing the goodness of fit for observed and expected data in a contingency table.

2. Given two datasets,

(a) Please provide the histograms of two datasets.



(a) Histogram of set1



(b) Histogram of set2

Figure 1: Histogram

(b) For each dataset, add the probability density functions of the given distributions (c)-(g) in Question 1 to the figures in Question 2(a). Trying to select more suitable distributions to the data based on your opinion.

Set1 : Laplace
The probability density function of the Laplace distribution best matches the actual data distribution in the "set1" dataset(Figure 2).

Set2 : Gamma and Chi-square
The probability density functions of the Gamma and Chi-square distributions best match the actual data distribution in the "set2" dataset(Figure 3).

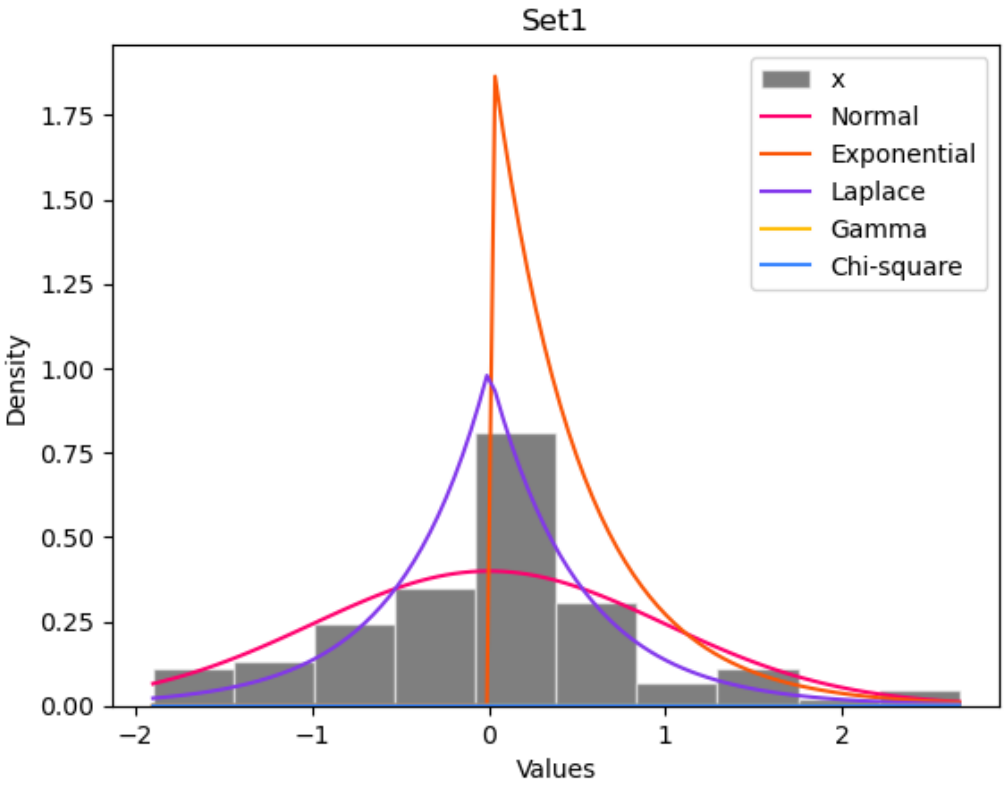


Figure 2: Histogram with distribution of set1

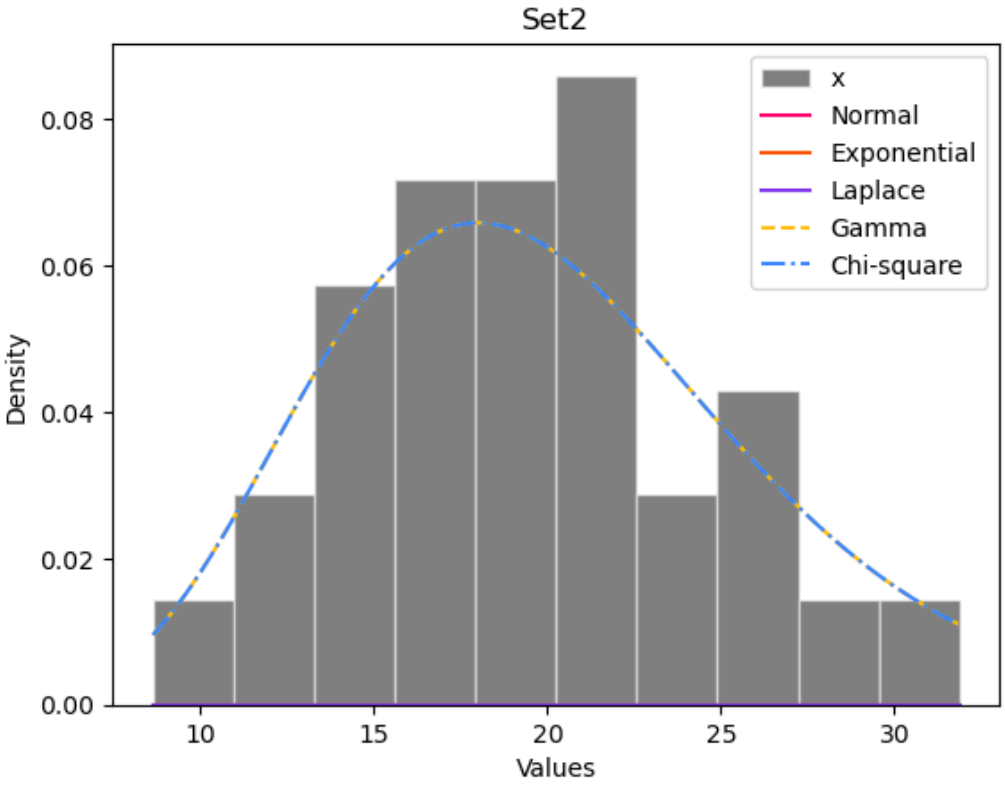


Figure 3: Histogram with distribution of set2