可靠度資料分析
Reliability Data Analysis

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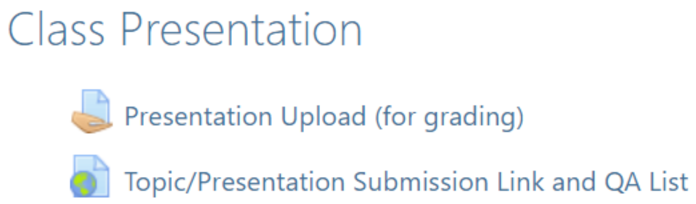
Lecture 6 – Review

Class Presentation Slides

- Need to submit slides **twice**!
  - One on Moodle: Presentation Upload (for grading)
  - One on Topic/Presentation Submission Link and QA List



- Please submit slides **the week before presentation** by 12:00 midnight
  - Ex. If your time slot is 4/8/2023, submit your presentation slides to Moodle by 4/1/2023 12:00 midnight

During Midterm Exam

- Can bring **one** A4 paper filled with notes to exam
- Bring calculator to exam
- Calculus integral table will be given in exam

- **Tip:** Make sure you practice doing all the example questions in the lecture notes!

Quick Calculus Review

u Substitution : The substitution $u = g(x)$ will convert $\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$ using $du = g'(x)\,dx$. For indefinite integrals drop the limits of integration.

Ex. $\int_1^2 5x^2 \cos(x^3)\,dx$

$u = x^3 \Rightarrow du = 3x^2 dx \Rightarrow x^2 dx = \frac{1}{3} du$

$x = 1 \Rightarrow u = 1^3 = 1 \;::\; x = 2 \Rightarrow u = 2^3 = 8$

$$\int_1^2 5x^2 \cos(x^3)\,dx = \int_1^8 \tfrac{5}{3}\cos(u)\,du$$

$$= \tfrac{5}{3}\sin(u)\Big|_1^8 = \tfrac{5}{3}\left(\sin(8) - \sin(1)\right)$$

Integration by Parts : $\int u\,dv = uv - \int v\,du$ and $\int_a^b u\,dv = uv\big|_a^b - \int_a^b v\,du$. Choose u and dv from integral and compute du by differentiating u and compute v using $v = \int dv$.

Ex. $\int x\mathbf{e}^{-x}\,dx$

$u = x \quad dv = \mathbf{e}^{-x} \quad \Rightarrow \quad du = dx \quad v = -\mathbf{e}^{-x}$

$$\int x\mathbf{e}^{-x}\,dx = -x\mathbf{e}^{-x} + \int \mathbf{e}^{-x}\,dx = -x\mathbf{e}^{-x} - \mathbf{e}^{-x} + c$$

Ex. $\int_3^5 \ln x\,dx$

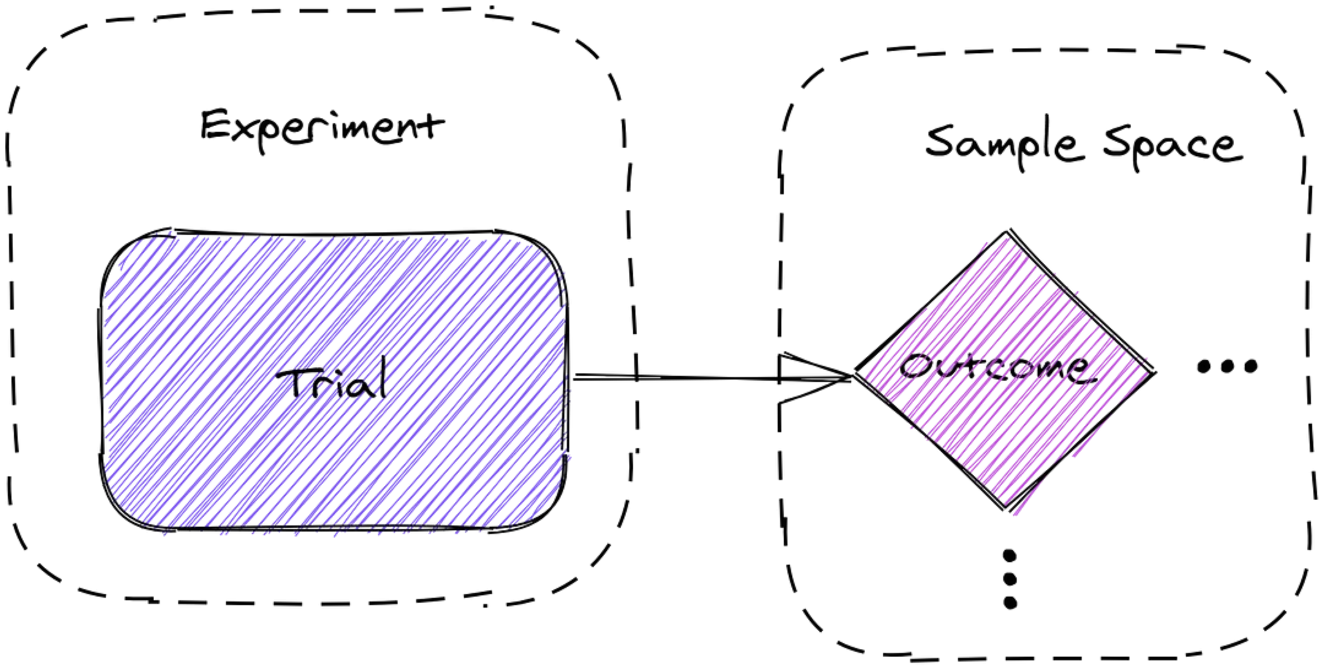
$u = \ln x \quad dv = dx \quad \Rightarrow \quad du = \frac{1}{x}dx \quad v = x$

$$\int_3^5 \ln x\,dx = x\ln x\Big|_3^5 - \int_3^5 dx = \left(x\ln(x) - x\right)\Big|_3^5$$

$$= 5\ln(5) - 3\ln(3) - 2$$

https://math.colorado.edu/math2300/resources/calc1/lamar/Calculus_Cheat_Sheet_All.pdf

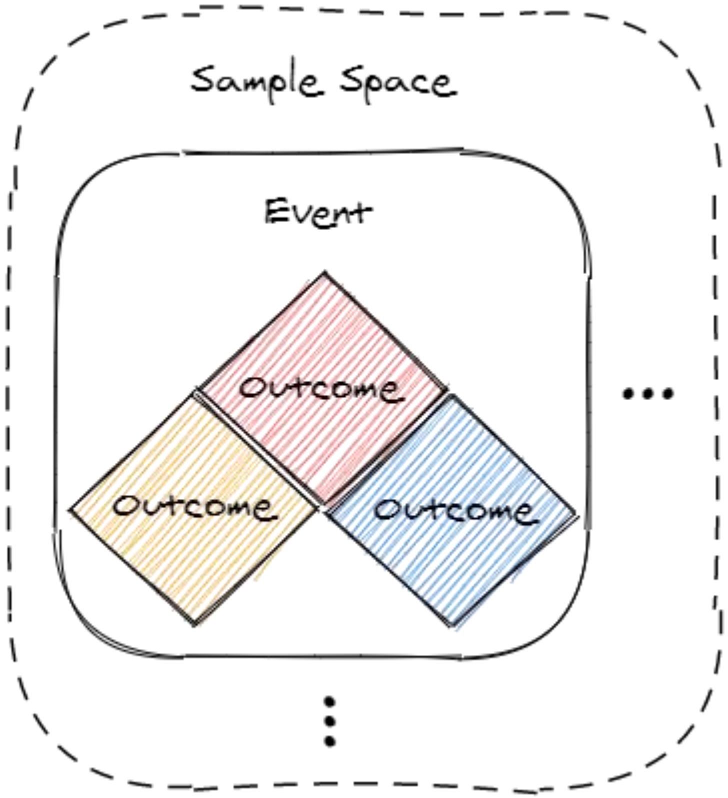
Terminology



Experiment: process of observation where the output cannot be predicted with certainty due to random effects
Trial: single occurrence of an experiment
Outcome: observed output of a trial
Sample space Ω: set of all possible outcomes of an experiment

https://towardsdatascience.com/probability-axioms-in-pictures-2de96880c868

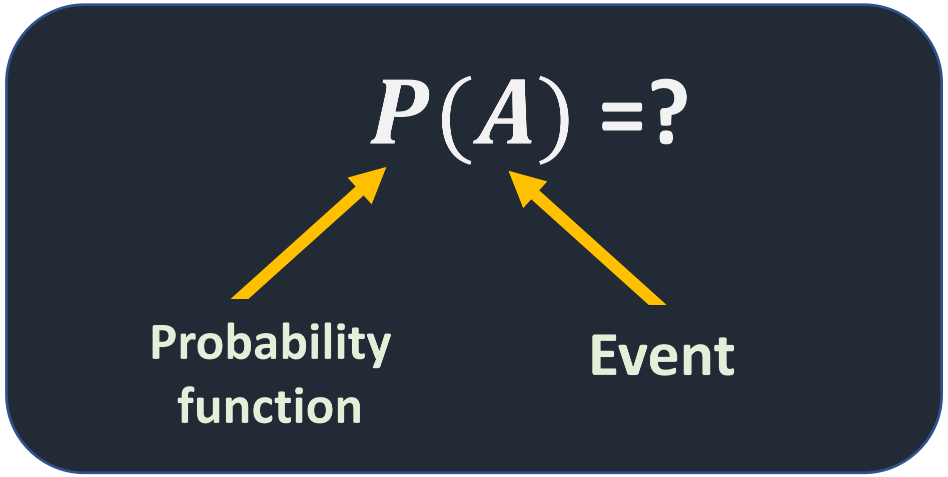
Terminology



Event A: subset of outcomes in the sample space Ω

https://towardsdatascience.com/probability-axioms-in-pictures-2de96880c868

What conditions must a probability function satisfy?



Axioms of Probability

Axiom 1: (non-negativity)	$P(A) \geq 0$
Axiom 2: (normalization)	$P(S) = 1$
Axiom 3: (finite additivity)	empty set If $A \cap B = \emptyset$, Then $P(A \cup B) = P(A) + P(B)$

Bayes' Theorem Physical Understanding

Likelihood:
How probable is the evidence given hypothesis is true?

Prior:
How probable was the hypothesis before observing the evidence?

$$\boldsymbol{P(H|e) = \frac{P(e|H)P(H)}{P(e)}}$$

Posterior:
How probable is the hypothesis given the observed evidence?

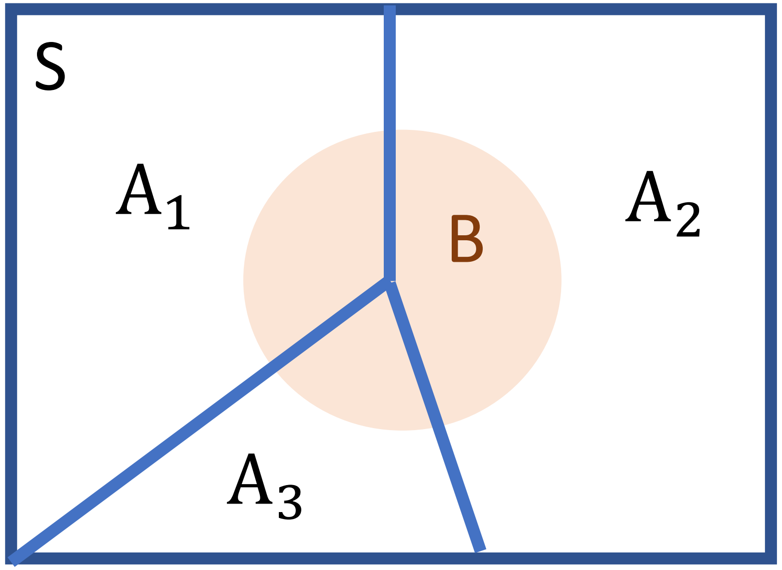
Marginal:
How probable is the new evidence under all possible hypothesis?

Law of Total Probabilities

- $P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \cdots + P(B|A_n)P(A_n)$

  where $A_1, \ldots, A_n$ is a partition of the sample space

  $$S(A_i \cap A_j = \emptyset \; for \; any \; i \neq j \; and \; A_1 \cup A_2 \cup \cdots A_n = S)$$



Bayes' Theorem

- Applying Law of Total Probabilities to Bayes' Theorem

$$P(H|e) = \frac{P(e|H)P(H)}{P(e)}$$

$$= \frac{P(e|H)P(H)}{P(H)P(e|H) + P(\neg H)P(e|\neg H)}$$

$\neg$ *means "not"*

The Reliability Function

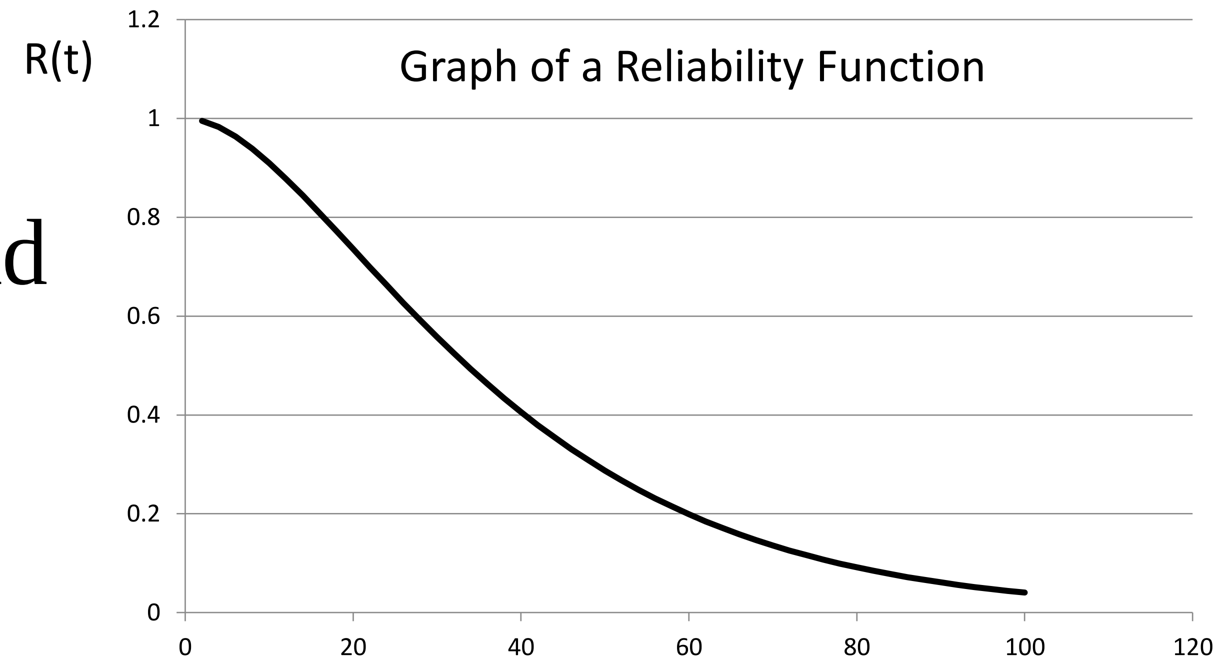
Let T = a random variable, the time to failure of a component

$$R(t) = Pr\{T \geq t\}$$

where $R(t) \geq 0$, $R(0) = 1$, and

$$\lim_{t \to \infty} R(t) = 0$$

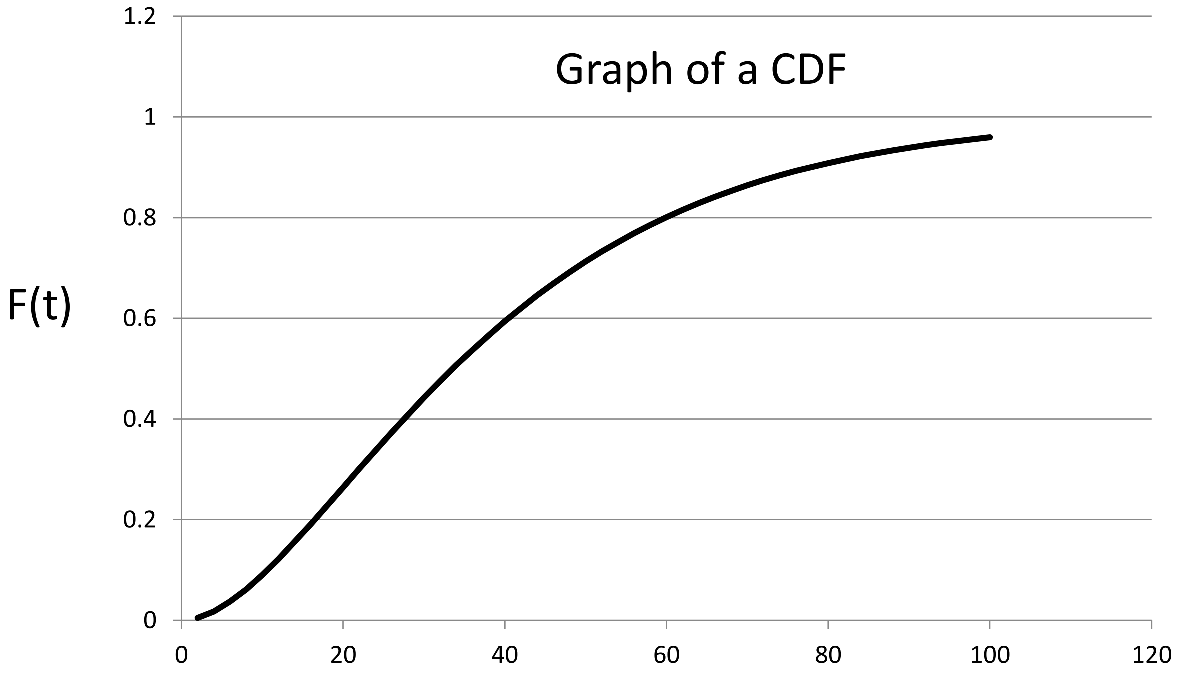
Often called the SURVIVAL FUNCTION



The Cumulative Distribution Function (CDF)

$$F(t) = 1 - R(t) = Pr\{T < t\}$$ ➡ The probability of a failure

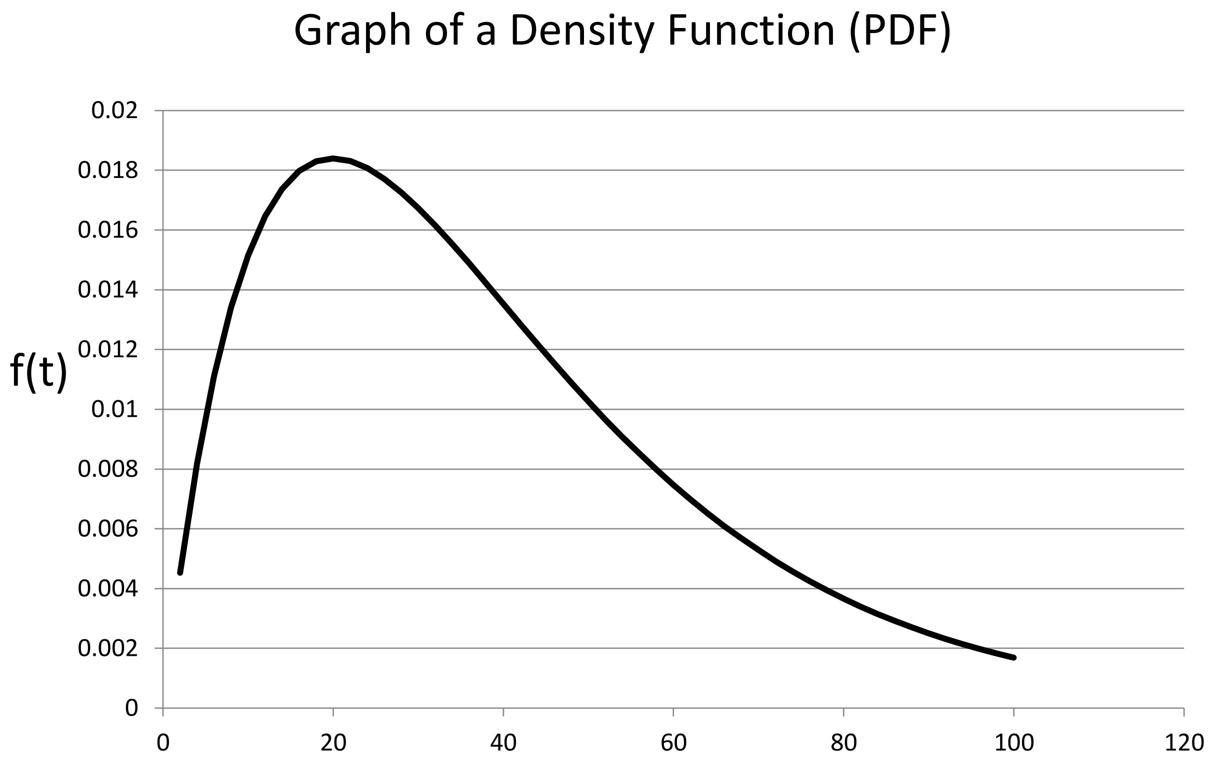
where $F(0) = 0$ and $\lim_{t \to \infty} F(t) = 1$



The Density Function (PDF)

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

$$f(t) \geq 0 \quad and \quad \int_0^\infty f(t)dt = 1$$

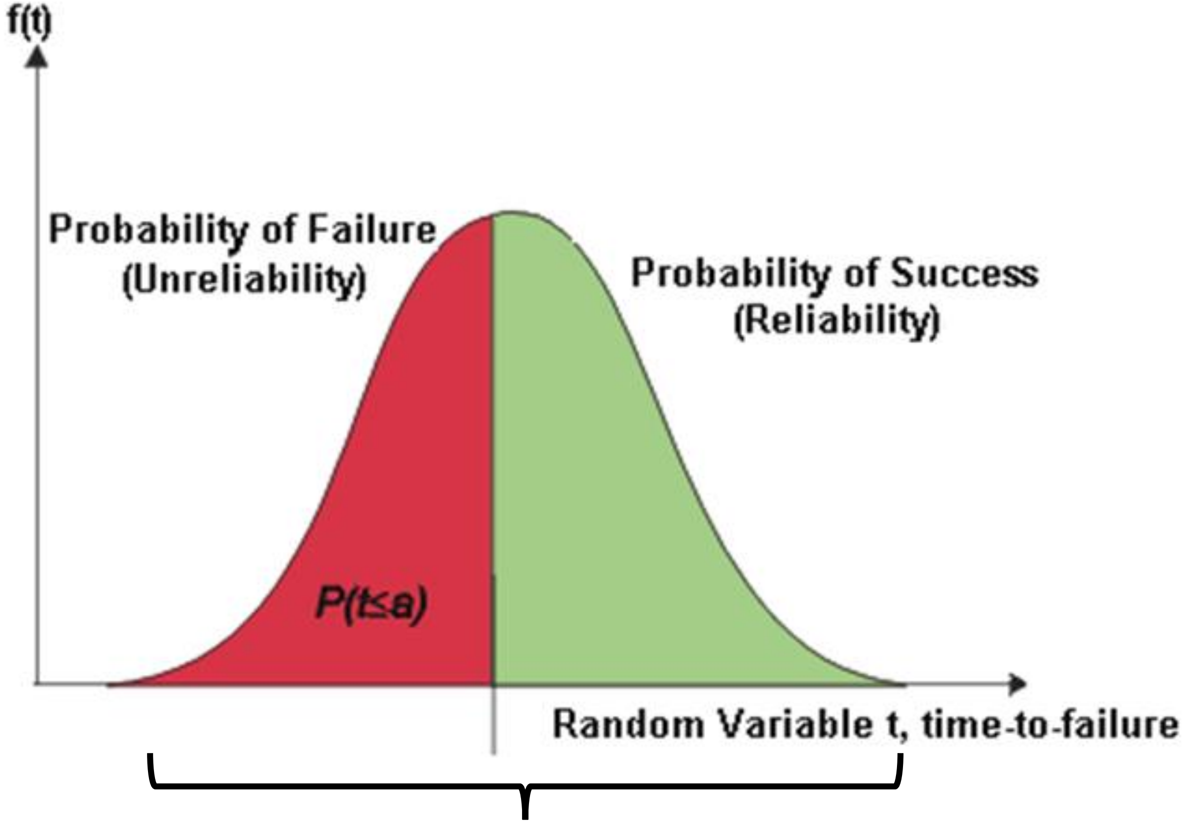


Relationship between PDF and CDF/Reliability Function

$$F(t) = \int_0^t f(t')\,dt'$$

$$R(t) = \int_t^\infty f(t')\,dt'$$

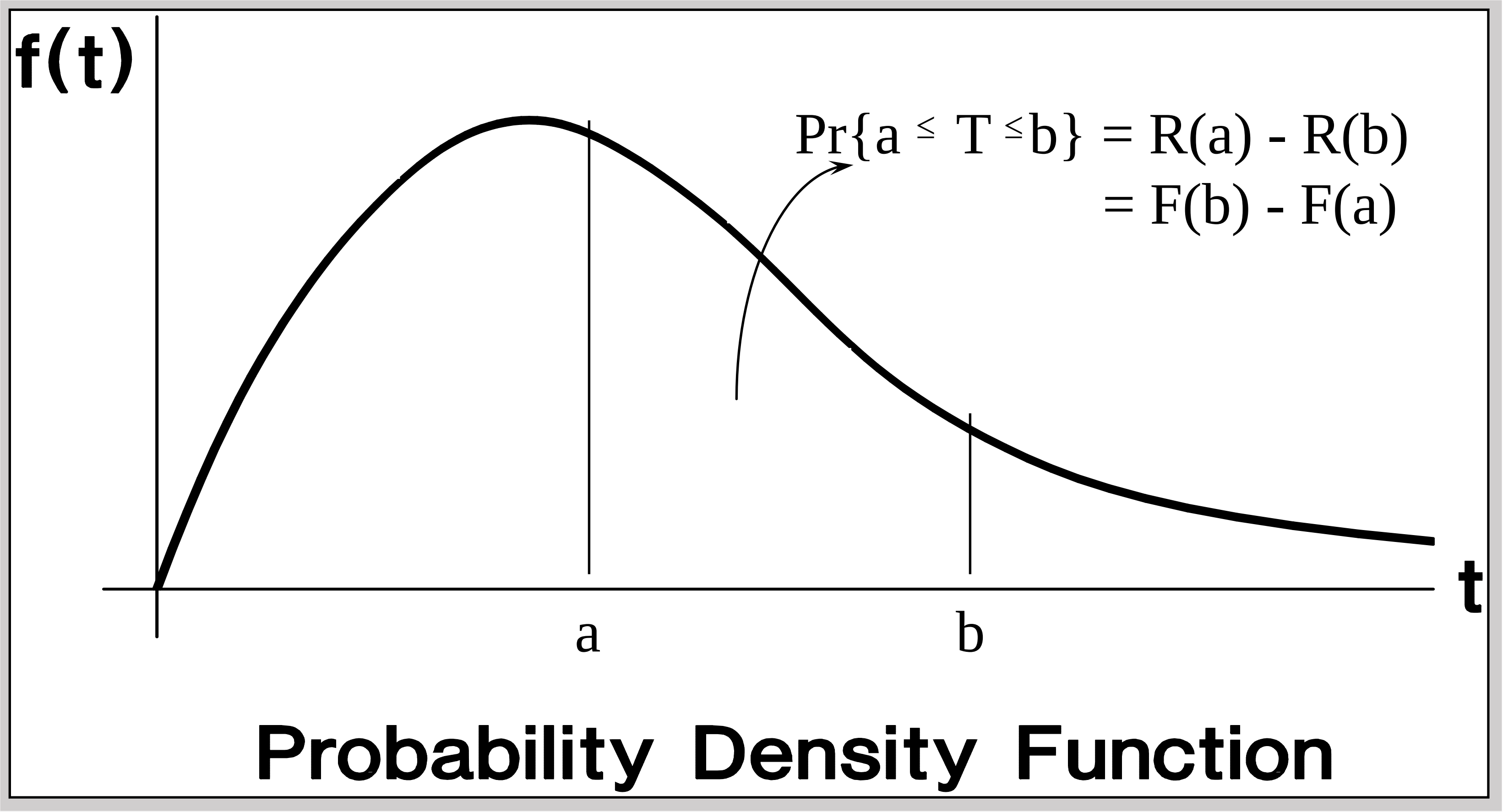
Understand PDF vs. CDF vs. Reliability Function



Integration of entire area under PDF from -∞ to +∞ equals 1

Therefore, $R(t) + F(t) = 1$ $\Rightarrow$ $\frac{dR(t)}{dt} = -f(t)$

Finding Failure Probabilities

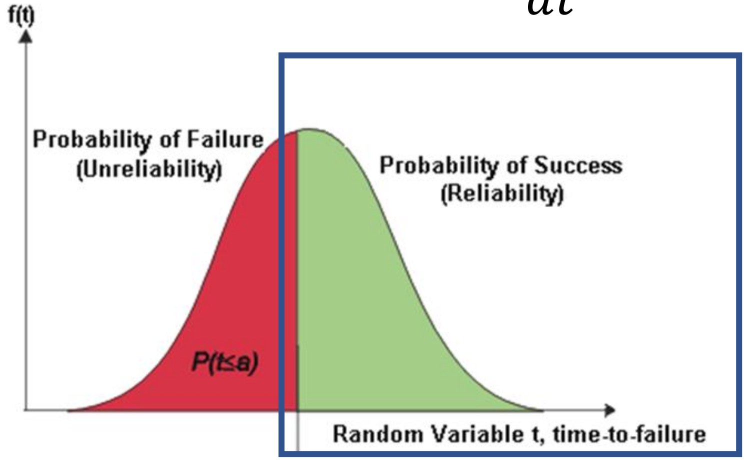


Mean Time to Failure (MTTF)

Note alternate notation:
MTTF = E[T]

$$MTTF = \int_0^\infty t \cdot f(t)dt = \int_0^\infty -t\boxed{\frac{dR(t)}{dt}}dt = \int_0^\infty R(t)dt$$

Remember: $\mathrm{f(t)} = -\frac{dR(t)}{dt}$

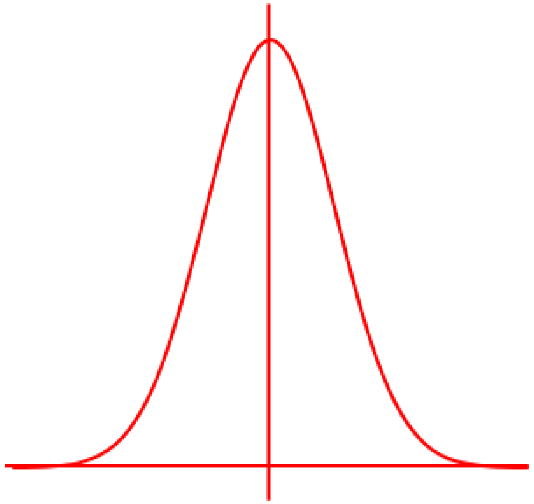


Variance

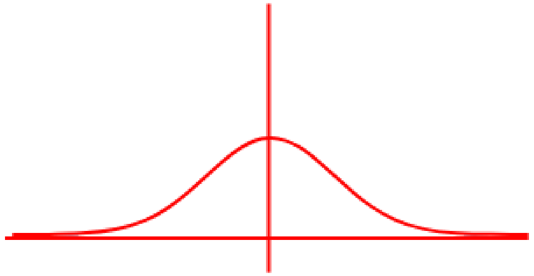
- For a continuous rv X with a state space R and pdf $f(x)$, we define its variance to be:

$$V(X) = \int \left(x - E(X)\right)^2 f(x)dx = \int x^2 f(x)dx - \left(\int xf(x)dx\right)^2$$

- The variance is a measure for the variability or <u>spread</u> about the mean in the state space. The standard deviation of X is $\sqrt{V(X)}$.



low variance



high variance

https://nowke.github.io/stats/normal-distributions/

Variance & Standard Deviation

definitional form:

$$\sigma^2 = \int_0^\infty (t - MTTF)^2 f(t)dt$$

computational form:

$$\sigma^2 = \int_0^\infty t^2 f(t)dt - (MTTF)^2$$

Failure Rate

- Failure Rate in the interval [t, t+ Δt]:
  - probability that a failure per unit time occurs in the interval given that no failure occurred prior to t

This is conditional probability given $R(t)$

$$\text{Failure rate} = \frac{R(t) - R(t+\Delta t)}{\Delta t R(t)}$$

Hazard Rate Function

- The Hazard Function is defined as the limit of the failure rate as Δt approaches zero, i.e., **instantaneous Failure Rate**.

$$\boxed{\lambda(t)} = \lim_{\Delta t \to 0} \frac{-[R(t+\Delta t)-R(t)]}{\Delta t}\,\frac{1}{R(t)} = \boxed{\frac{-dR(t)}{dt}\,\frac{1}{R(t)} = \frac{f(t)}{R(t)}}$$

Derivation of R(t) from the Hazard Rate Function

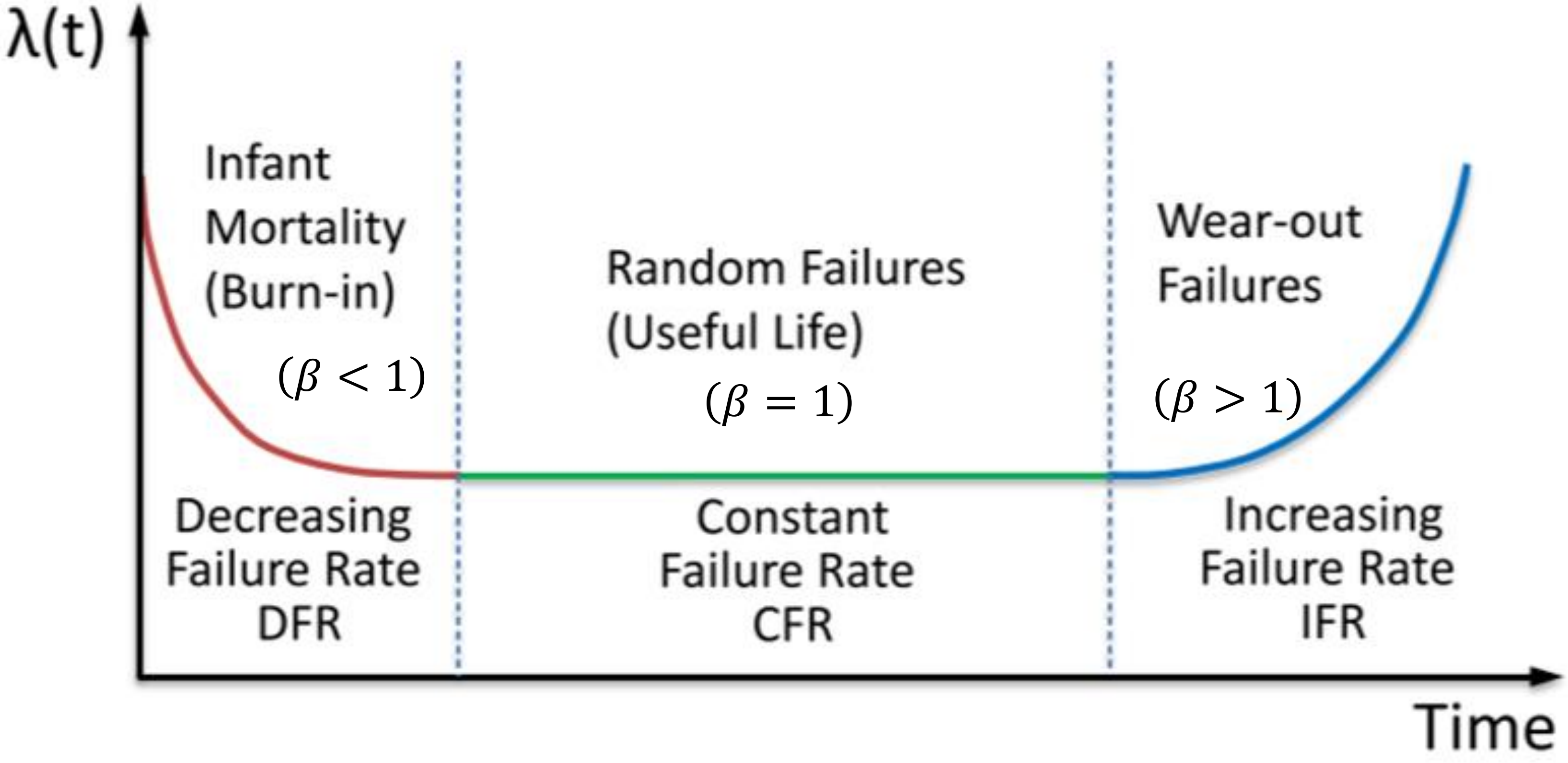
$$\lambda(t) = \frac{-d\,R(t)}{dt} \cdot \frac{1}{R(t)}$$

$$\longrightarrow \quad \lambda(t)\,dt = \frac{-d\,R(t)}{R(t)} \longrightarrow \int_0^t \lambda(t')dt' = -\int_1^{R(t)} \frac{d\,R(t')}{R(t')}$$

$$\longrightarrow \quad -\int_0^t \lambda(t')dt' = \ln R(t)$$

$$\longrightarrow \quad \boxed{R(t) = \exp\left[-\int_0^t \lambda(t')dt'\right]}$$

Bathtub Curve



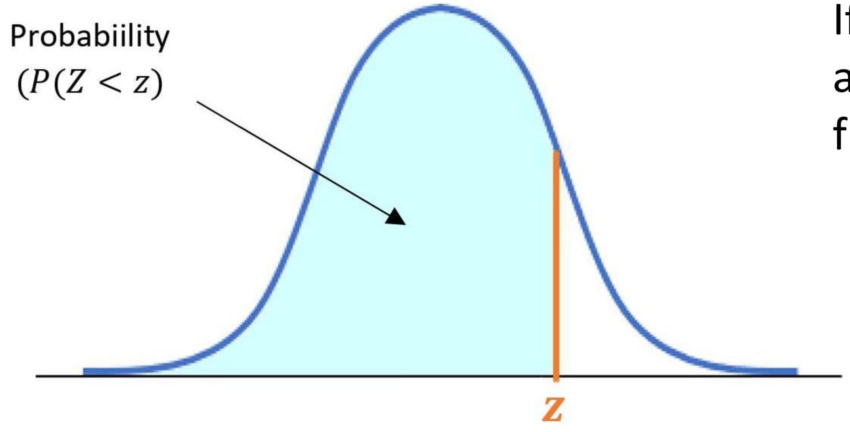
Conditional Reliability

$$R(t|T_0) = P\{\underbrace{T > T_0 + t}_{\text{Event A}} \mid \underbrace{T > T_0}_{\text{Event B}}\}$$

$$= \frac{P\{T > T_0 + t\}}{P\{T > T_0\}} = \frac{R(T_0 + t)}{R(T_0)}$$

Normal distribution

- Any normal distribution can be converted to a standard normal distribution to find the CDF



If mean and standard deviation are known, use the Z-Table to find the CDF

Z-Table

$$z = \frac{x - \mu}{\sigma}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879

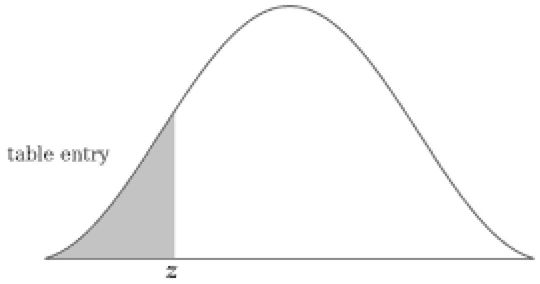
⋮

https://www.wizeprep.com/online-courses/16841/chapter/10/core/5/1

Note: Z-Score Tables come in different formats!

- **<u>Second type (use this):</u>**
  - A.k.a. Need two tables, one positive and one negative.

<u>Negative Z score table</u>



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476

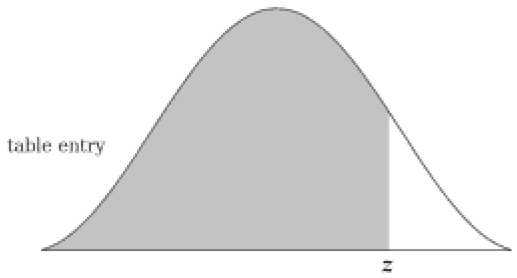
- Use the negative Z score table to find values on the left of the mean
- Corresponding values which are less than the mean are marked with a negative score in the z-table and represent the area under the bell curve to the left of z.

https://mathbitsnotebook.com/Algebra2/Statistics/STzScores.html
https://www.ztable.net/

Note: Z-Score Tables come in different formats!

- **<u>Second type (use this):</u>**
  - A.k.a. Need two tables, one positive and one negative.

<u>Positive Z score table</u>



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
+0.1	.53983	.54380	.54776	.55172	.55567	.55966	.56360	.56749	.57142	.57535
+0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
+0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
+0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
+0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
+0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
+0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
+0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
+0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
+1	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214

https://mathbitsnotebook.com/Algebra2/Statistics/STzScores.html
https://www.ztable.net/

- Use the positive Z score table to find values on the right of the mean.
- Corresponding values which are greater than the mean are marked with a positive score in the z-table and represent the area under the bell curve to the left of z.

Comparison of Distribution Functions

- Times of failures should lead your selection of appropriate probability distribution

Model	Description	Characteristic	Advantages	Disadvantages
Exponential	The failure rate is constant over time	Memoryless (the age of the item has no effect on its future failure rate)	Simple and easy to understand (only one parameter to estimate)	May not reflect real-life scenarios where the failure rate changes over time
Log-normal	The failure rate changes over time	Model is appropriate when underlying process has a large number of causes of failure and the failure times are spread out over a large range	Can reflect real-life scenarios where the failure rate changes over time	Requires more parameters to estimate
Weibull	The shape parameter determines the shape of the failure rate curve, which can be constant, increasing or decreasing over time	Flexibility in modeling	Can model a wide range of failure rate patterns	Estimation process can be more complex

Comparison of Distribution Functions

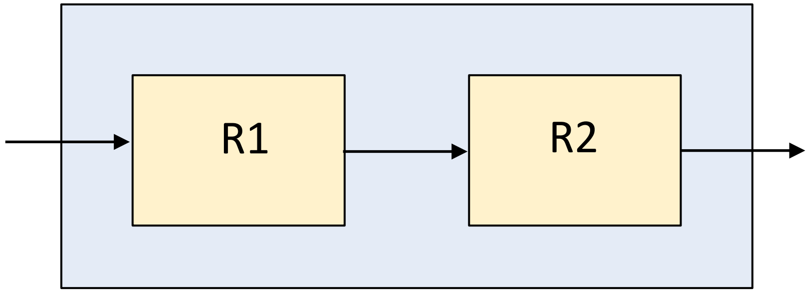
	Exponential	**Log-normal**	**Weibull**
PDF	$f(t) = \lambda e^{-\lambda t}$	$f(t) = \dfrac{1}{\sigma t\sqrt{2\pi}} exp\left\{-\left[\dfrac{\ln(t) - \ln(t_{50})}{\sigma\sqrt{2}}\right]^2\right\}$	$f(t) = \left(\dfrac{\beta}{\alpha}\right)\left(\dfrac{t}{\alpha}\right)^{\beta-1} exp^{\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right]}$
CDF	$F(t) = 1 - e^{-\lambda t}$	$F(t) = \dfrac{1}{2} erfc\left(\dfrac{\ln(t_{50}) - \ln(t)}{\sigma\sqrt{2}}\right)$	$F(t) = 1 - exp\left[-\left(\dfrac{t}{\alpha}\right)^{\beta}\right]$
Parameter	λ	$\mu = \ln(t_{50}), \sigma$ (*note: these refer to the parameters in normal form, i.e. $\mu = \mu_y, \sigma = \sigma_y$ on slide "Normal vs. Lognormal")	$\beta = shape\ parameter$ (affects shape of distribution) $\eta = scale\ parameter$ $or\ characteristic\ lifetime$ (affects spread of distribution)

System Reliability: Series

System fails if either R1 or R2 fail

The system reliability block diagram places these elements in series

The diagram below does not reflect functionality!



$$R_{system} = R_1 \cdot R_2$$

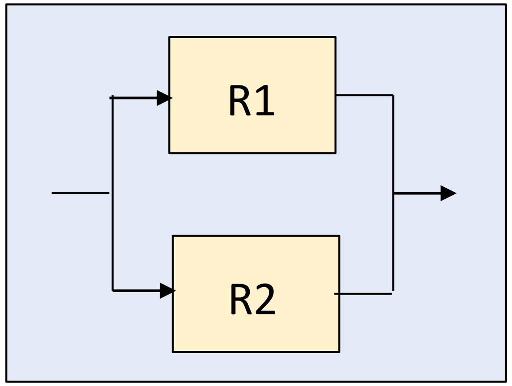
$$R_{system} = \mathbf{0.9} \cdot \mathbf{0.9} = \mathbf{0.81}$$

System Architecture: Parallel

System contains redundancy

Both R1 and R2 have to fail for the system to fail

Reliability block diagram places these elements in parallel

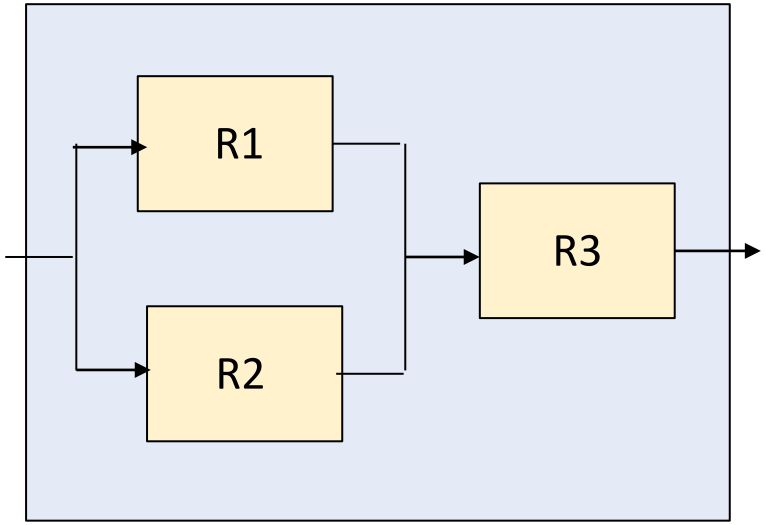


$$R_{system} = [1 - (1 - R_1) \cdot (1 - R_2)]$$

$$R_{system} = [1 - (1 - \mathbf{0.9}) \cdot (1 - \mathbf{0.9})] = 0.99$$

System Architecture: Parallel / Series

System fails when either condition happens:
both R1 and R2 fail OR R3 fails



$$R_{system} = [1 - (1 - R_1) \cdot (1 - R_2)] \cdot [R_3]$$

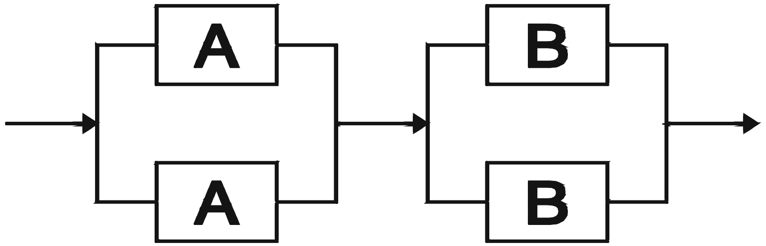
$$R_{system} = [1 - (1 - \mathbf{0.9}) \cdot (1 - \mathbf{0.9})] \cdot [\mathbf{0.9}] = 0.891$$

High vs Low Level Redundancy

Reliability of low-level redundancy is <u>greater</u> than or equal to high-level redundancy!

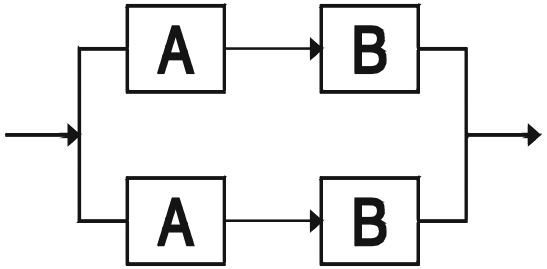
Low-level redundancy

Duplication at subsystem or component level



High-level redundancy

Entire system is duplicated



- Both systems will fail if either both components A fail or both components B fail
- High-level has one extra failure path → Easier to fail compared to low-level
  - Failure path: One A fails, and one B fails on separate paths

k-out-of-n Redundancy

Let n = the number of redundant, identical and independent components each having a reliability of R.

Let X = a random variable, the number of components (out of n components) operating. Then

$$\boxed{P(x) = \binom{n}{x} R^x (1-R)^{n-x}}$$

where $\binom{n}{k} = \frac{n!}{(n-k)!} \cdot \frac{1}{k!}$

If k ≤ n components must operate for the system to operate:

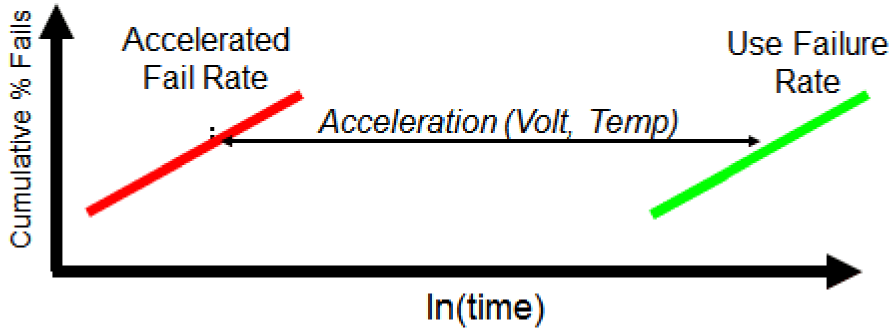
$$\boxed{R_s = \sum_{x=k}^{n} P(x)}$$

which is the probability of k or more successes from among the n components

$$\mathrm{P}\left(\text{at least } k \text{ successes out of } n\right) = \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}.$$

Accelerated Testing

- Failure time low stress (at use conditions) = AF x failure time high stress,
  - where AF is called an "acceleration factor."



- The acceleration factor is the ratio of the time to a given CDF value under low stress conditions to the time to that same CDF value under high stress conditions, that is:

$$AF = \left(\frac{t_{S_{low}}}{t_{S_{high}}} \right)$$

Reference: David C. Trindade, Exploring Reliability with JMP: Accelerated Testing
https://www.ti.com/support-quality/reliability/reliability-testing.html

Some practice problems…

Example 1

- The lifetime T of the cellphone product, defined as the amount of time (in years) the product works properly until it breaks down, satisfies

$$P(T \geq t) = 1 - e^{\frac{-t}{10}}, \text{ for all t≥0.}$$

- If the cellphone is purchased and used for one year without any problems, what is the probability that it breaks down in the next year?

Example 1

- The lifetime T of the cellphone product, defined as the amount of time (in years) the product works properly until it breaks down, satisfies

$$P(T \geq t) = 1 - e^{\frac{-t}{10}}, \text{ for all } t \geq 0.$$

- If the cellphone is purchased and used for one year without any problems, what is the probability that it breaks down in the next year?
- **<u>Solution:</u>**
- Let A be the event that the cellphone breaks down in the second year. Also, let B be the event that the cellphone does not break down in the first year. We are interested in P(A|B). We have

$$P(B) = P(T \geq 1) = 1 - e^{\frac{-1}{10}}$$

- We also have $P(A) = P(1 \leq T \leq 2) = P(T \geq 2) - P(T \geq 1) = (1 - e^{\frac{-2}{10}}) - \left(1 - e^{\frac{-1}{10}}\right) = e^{\frac{-1}{10}} - e^{\frac{-2}{10}}$
- Finally, since A⊂B, we have A∩B=A. Therefore,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{e^{\frac{-1}{10}} - e^{\frac{-2}{10}}}{1 - e^{\frac{-1}{10}}} = 0.9048$$

Example 2

- In a factory there are two machines manufacturing bolts. The first machine manufactures 75% of the bolts and the second machine manufactures the remaining 25%. From the first machine 5% of the bolts are defective and from the second machine 8% of the bolts are defective. A bolt is selected at random, what is the probability the bolt came from the first machine, given that it is defective?

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- Ans:

Let A be the event that a bolt is defective and let B be the event that a bolt came from Machine 1.

Check that you can see where these probabilites come from!

$P(B) = 0.75 \quad P(B') = 0.25 \quad P(A|B) = 0.05 \quad P(A|B') = 0.08$

Now, use Bayes' Theorem to find the required probability:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B')P(B')}$$

$$= \frac{0.05 \times 0.75}{0.05 \times 0.75 + 0.08 \times 0.25}$$

$$= 0.3846$$

Example 3

- The failure time for the wind panel has the random variable X, and follows the pdf:

$$f_X(x) = \begin{cases} 2x^{-2} & \text{for } 1 < x < 2, \\ 0 & \text{otherwise.} \end{cases}$$

- Find the MTTF and Var(x).

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Ans:

MTTF= $E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx = \int_1^2 x \times 2x^{-2}\, dx \;=\; \int_1^2 2x^{-1}\, dx$

$$\begin{aligned} &= \Big[2\log(x)\Big]_1^2 \\ &= 2\log(2) - 2\log(1) \\ &= 2\log(2). \end{aligned}$$

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Ans:

For Var(X), *we use*

$$\text{Var}(X) = \mathbb{E}(X^2) - \{\mathbb{E}(X)\}^2 .$$

Now

$$\begin{aligned} E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_1^2 x^2 \times 2x^{-2}\,dx &= \int_1^2 2\,dx \\ &= \Big[2x\Big]_1^2 \\ &= 2 \times 2 - 2 \times 1 \\ &= 2. \end{aligned}$$

Thus

$$\begin{aligned} \text{Var}(X) &= \mathbb{E}(X^2) - \{\mathbb{E}(X)\}^2 \\ &= 2 - \{2\log(2)\}^2 \end{aligned}$$

Note:

$$\text{var}(X) = E[X^2] - \underbrace{(E[X])}_{\text{mean}}{}^2$$

$$\sum_K K^2 P_X(K)$$

Example 4

- The time to failure for a battery is the random variable X, and it known that the pdf has the following form:

$$f(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- If the MTTF =3/5, find a and b.

Example 4

- The time to failure for a battery is the random variable X, and it known that the pdf has the following form:

$$f(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- If the MTTF =3/5, find a and b.
- Ans:
- First, f(x) is a probability density function, so the integration of a pdf over the entire sample space equals 1.

$$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = \int_0^1 a + bx^2\,\mathrm{d}x = ax + \frac{b}{3}x^3\Big|_0^1 = a + \frac{b}{3} = 1$$

MTTF= $E[X] = \int_{-\infty}^{\infty} xf(x)\,\mathrm{d}x = \int_0^1 ax + bx^3\,\mathrm{d}x = \frac{a}{2}x^2 + \frac{b}{4}x^4\Big|_0^1 = \frac{a}{2} + \frac{b}{4} = \frac{3}{5}$

$$\left\{ \begin{array}{rcl} 3a + b & = & 3 \\ 10a + 5b & = & 12 \end{array} \right. \Rightarrow$$ a = 3/5 and b = 6/5

Example 5

The light bulb has a reliability function of

$$R(t) = 0.01e^{-0.01t}, t \geq 0$$

What is the MTTF?

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Ans:

- $MTTF = \int_0^\infty R(t)dt \quad = \int_0^\infty 0.01e^{-0.01t}dt = 1$

Example 6

- A certain capacitor product has the following pdf.

$$f(t) = \frac{1}{5000} t, \; 0 \le t \le 100$$

- Calculate the hazard function.

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- Calculate the hazard function.
- Ans:

$$F(t) = \int_0^t f(t)dt = \int_0^t \frac{1}{5000} t \, dt = \frac{t^2}{10000}$$

$$R(t) = 1 - F(t) = 1 - \frac{t^2}{10000}$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{\left(\frac{1}{5000}\right)t}{1 - \frac{t^2}{10000}}$$

Example 7

- A simple computer consists of a processor, a bus and a memory. The computer will work only if all three are functioning correctly. The probability that the processor is functioning is 0.99, that the bus is functioning 0.95, and that the memory is functioning is 0.99. What is the probability that the computer will work?

Example 7

- A simple computer consists of a processor, a bus and a memory. The computer will work only if all three are functioning correctly. The probability that the processor is functioning is 0.99, that the bus is functioning 0.95, and that the memory is functioning is 0.99.
- Ans:

$$\text{input} \longrightarrow \underset{0.99}{\boxed{\textit{Processor}}} \longrightarrow \underset{0.95}{\boxed{\textit{Bus}}} \longrightarrow \underset{0.99}{\boxed{\textit{Memory}}} \longrightarrow \text{output}$$

The probability that the computer will work is:

$$Rel = .99 \times .95 \times .99 = 0.893475$$

So even though all the components have above 95% or more reliability, the overall Reliability of the computer is less that 90%.

Example 8

- A system has three components, which are connected in parallel configuration from a reliability point of view. The reliabilities of these components for a mission of 300 days are 0.60, 0.55 and 0.70, respectively. Evaluate the reliability of the system for a mission of 300 days. Assume that the components are independent.

Example 8

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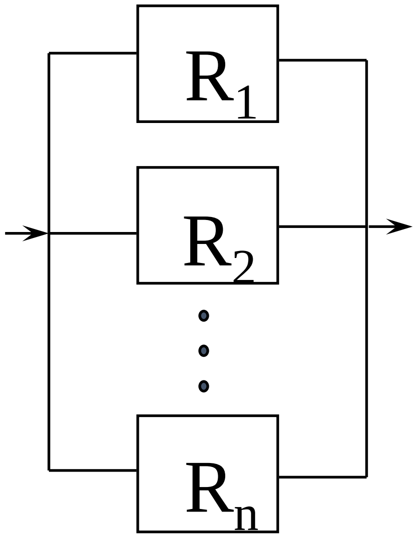
- Ans:
- We know that the reliability (R) of a parallel system having n components with reliabilities R_i, i = 1, 2, 3, …, n is given by the following equation:

$$R = 1 - (1 - R_1)\,(1 - R_2)(1 - R_3) \ldots (1 - R_n)$$



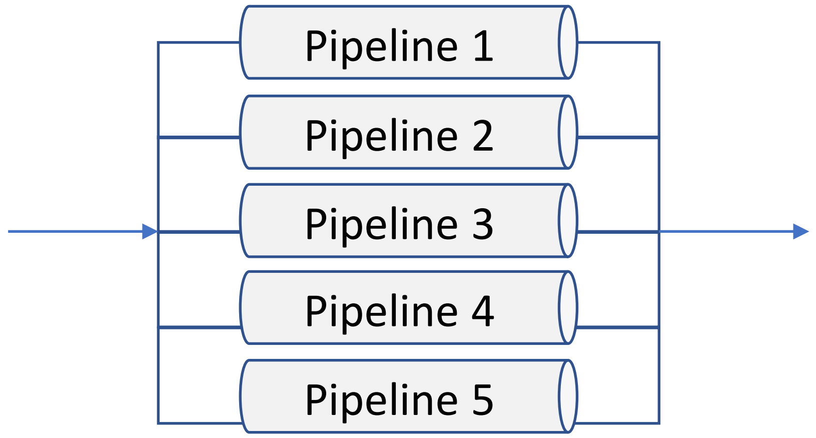
In this case, R_1 = 0.60, R_2 = 0.55, R_3 = 0.70 and n = 3.

$$R = 1 - (1 - 0.60)\,(1 - 0.55)(1 - 0.70) = 1 - 0.40 \times 0.45 \times 0.30$$

= 1 – 0.054 = 0.946 for a mission of 300 days

Example 9

- Consider a piping system having 5 pipes connected in parallel as shown in figure. Assume that all pipes are identical and independent. If the reliability of flow from each pipeline is 0.80 for a mission of 1 year, evaluate the reliability of the system working successfully. The system is said to work successfully if at least 3 pipelines perform their intended function successfully.



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- Ans:
- Since it is given that for successful operation of the system, at least 3 pipelines should work successfully, it is a k-out-of-n system, where k = 3 and n = 5. If R denotes the reliability of each pipeline and R_s that of the 3-out-of-5 system, then

$$R_s = \sum_{k=3}^{5} {}^5C_k R^k (1-R)^{5-k} = {}^5C_3 R^3 (1-R)^2 + {}^5C_4 R^4 (1-R)^1 + {}^5C_5 R^5 (1-R)^0$$
$$= 10(0.8)^3(1-0.8)^2 + 5(0.8)^4(1-0.8) + (0.8)^5$$
$$= 10 \times 0.512 \times 0.04 + 5 \times 0.4096 \times 0.2 + 0.32768$$
$$= 0.2048 + 0.4096 + 0.32768 = 0.94208$$