可靠度資料分析 Reliability Data Analysis
Spring 2023 Midterm Exam

Please write out thought process or derivations to receive full credit.

Question 1: (10 points)

Suppose that the life distribution of an item has the hazard rate function $\lambda(t) = t^3$, $t > 0$. What is the probability that

(a) the item survives to age 2? (4pts)

(b) the item's lifetime is between .4 and 1.4? (3pts)

(c) a 1-year-old item will survive to age 2? (3pts)

Solution:

(a) Let X denote the life distribution. The reliability function can be determined from the hazard function. First compute the probability function of the life distribution.

$$P(t) = 1 - \exp\left(-\int_0^t \lambda(u)du\right)$$
$$= 1 - \exp\left(-\frac{1}{4}t^4\right) \qquad \textbf{2pts}$$

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(2) = e^{-4} \approx 0.0183$$

2pts

(b) The probability is given by

$$P(0.4 \leq X \leq 1.4) = P(X \leq 1.4) - P(X \leq 0.4)$$
$$= P(1.4) - P(0.4)$$
$$= (1 - e^{-0.9604}) - (1 - e^{-0.0064})$$
$$= e^{-0.0064} - e^{-0.9604} \approx 0.6109$$

3pts

(c) The probability that a 1-year-old item will survive to age 2 is

$$P(X \geq 2|X \geq 1) = \frac{P(X \geq 2)}{P(X \geq 1)} = \frac{1 - P(2)}{1 - P(1)} = = \frac{e^{-4}}{e^{-1/4}}$$
$$= e^{-15/4} \approx 0.0235$$

3pts

Question 2: (5 points)

Thermocouples of a particular design have a failure rate of λ = 0.008/hr. How many thermocouples must be placed in parallel if the system is to run for 100 hrs with a system failure probability of no more than 0.05? Assume that all failures are independent.

Solution:

Component Reliability,

$$R(t) = \exp\left[-\int_0^t \lambda(t')dt'\right]$$

$\rightarrow R = e^{-\int_0^{100} \lambda dt} = e^{-\lambda t|_0^{100}} = e^{-0.008\times 100} = 0.4493$ **2pts**

$F \leq 0.05 \rightarrow (1 - \mathrm{R})^{\mathrm{N}} \leq 0.05$, **2pts**

$N \cdot ln(1 - R) \leq ln\, 0.05 \rightarrow N \geq 5.02$ ~6 units **1pt**

Question 3: (3pts)

A newly designed electrical component has undergone accelerated testing at 200℃. The electrical component is found to follow a Weibull distribution with a shape parameter of 2.5 and scale parameter of 120 hrs. The acceleration factor is 20.

(a) What is the characteristic lifetime at use conditions? (2pts)

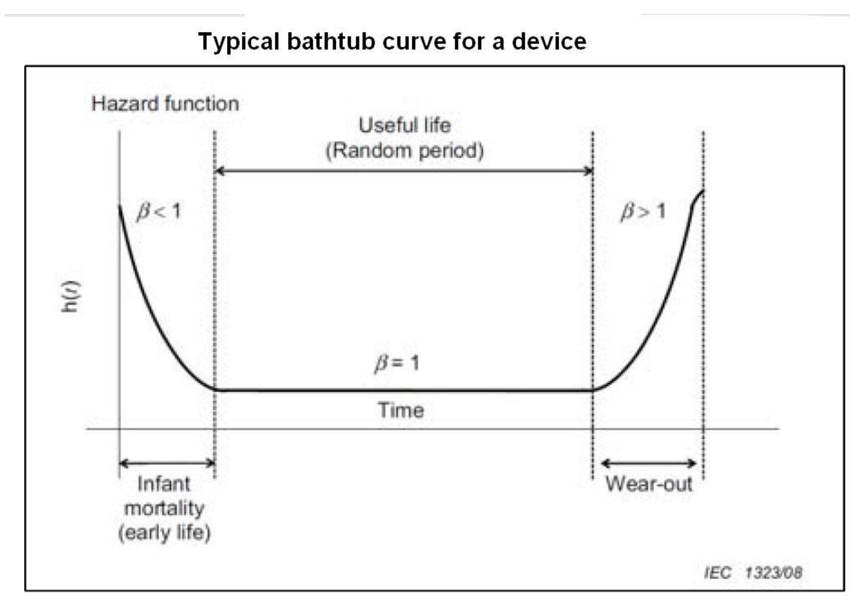
(b) If the shape parameter is changed from 2.5 to 1, how does this affect the instantaneous failure rate? (1pts)

Solution:

(a) $AF = \frac{\eta_{at\,use}}{\eta_{accelerated}} = \frac{\eta_{at\,use}}{120} = 20 \rightarrow \eta_{at\,use} = 120 \times 20 = 2400hrs$

2pts

(b) Instantaneous failure rate (hazard rate) goes from increasing failure rate to constant failure rate. **1pts**



Ref: https://www.windpowerengineering.com/weibull-analysis-applied-wind-projects/

<u>Question 4:</u> (14 points)

A motor is known to have an operating life (in hours) that fits the distribution $f(t) = \frac{a}{(t+b)^3}$, $t \geq 0$.

The mean life of the motor has been estimated to be 3000 hr.

(a) Find a and b. (6pts)

(b) What is the probability that the motor will fail in less than 2000 hr? (4pts)

(c) If the manufacturer wants no more than 5% of the motors returned for warranty service, how long should the warranty be? (4pts)

<u>Solution:</u>

(a)

$$\int_{-\infty}^{\infty} f(t)dt = \int_0^{\infty} \frac{a}{(t+b)^3} dt = 1 \rightarrow \left[-\frac{a}{2(t+b)^2}\right]_0^{\infty} = \left[0 - \left(-\frac{a}{2b^2}\right)\right] \rightarrow a = 2b^2$$

$$\mu = \int_0^{\infty} tf(t)dt = \int_0^{\infty} t \frac{a}{(t+b)^3} dt = 3{,}000$$

$$\rightarrow a \int_0^{\infty} \frac{t+b-b}{(t+b)^3} dt = a \int_0^{\infty} \left(\frac{1}{(t+b)^2} - \frac{b}{(t+b)^3}\right) dt =$$

$$a\left[-\frac{1}{(t+b)} + \frac{b}{2(t+b)^2}\right]_0^{\infty} = a\left[0 - \left(-\frac{1}{b} + \frac{1}{2b}\right)\right] = \frac{a}{2b} = 3{,}000 \rightarrow b = 3{,}000, a = 18 \times 10^6$$

6pts

Note: Integrals can also be found using the substitution method:

First Integral:

Problem:

$$\int \frac{a}{(t+b)^3} \mathrm{d}t$$

Substitute $u = t + b \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = 1$ (steps) $\longrightarrow \mathrm{d}t = \mathrm{d}u$:

$$= a \int \frac{1}{u^3} \mathrm{d}u$$

Now solving:

$$\int \frac{1}{u^3} \mathrm{d}u$$

Apply power rule:

$$\int u^{\mathrm{n}} \mathrm{d}u = \frac{u^{\mathrm{n}+1}}{\mathrm{n}+1} \text{ with } \mathrm{n} = -3:$$

$$= -\frac{1}{2u^2}$$

Plug in solved integrals:

$$a\int\frac{1}{u^3}\,\mathrm{d}u$$

$$=-\frac{a}{2u^2}$$

Undo substitution $u=t+b$:

$$=-\frac{a}{2(t+b)^2}$$

The problem is solved:

$$\int\frac{a}{(t+b)^3}\,\mathrm{d}t$$

$$=-\frac{a}{2(t+b)^2}+C$$

Second Integral:

Problem:

$$\int\frac{at}{(t+b)^3}\,\mathrm{d}t$$

Apply linearity:

$$=a\int\frac{t}{(t+b)^3}\,\mathrm{d}t$$

Now solving:

$$\int\frac{t}{(t+b)^3}\,\mathrm{d}t$$

Substitute $u=t+b \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}t}=1$ (steps) $\longrightarrow \mathrm{d}t=\mathrm{d}u$:

$$=\int\frac{u-b}{u^3}\,\mathrm{d}u$$

Expand:

$$=\int\left(\frac{1}{u^2}-\frac{b}{u^3}\right)\mathrm{d}u$$

Apply linearity:

$$=\int\frac{1}{u^2}\,\mathrm{d}u-b\int\frac{1}{u^3}\,\mathrm{d}u$$

Now solving:

$$\int\frac{1}{u^2}\,\mathrm{d}u$$

Apply power rule:

$\int u^{\mathrm{n}}\,\mathrm{d}u=\frac{u^{\mathrm{n}+1}}{\mathrm{n}+1}$ with $\mathrm{n}=-2$:

$$=-\frac{1}{u}$$

Now solving:

$$\int\frac{1}{u^3}\,\mathrm{d}u$$

Apply power rule with $\mathrm{n}=-3$:

$$=-\frac{1}{2u^2}$$

Plug in solved integrals:

$$\int \frac{1}{u^2}\,\mathrm{d}u - b\int \frac{1}{u^3}\,\mathrm{d}u$$

$$= \frac{b}{2u^2} - \frac{1}{u}$$

Undo substitution $u = t + b$:

$$= \frac{b}{2(t+b)^2} - \frac{1}{t+b}$$

Plug in solved integrals:

$$a\int \frac{t}{(t+b)^3}\,\mathrm{d}t$$

$$= \frac{ab}{2(t+b)^2} - \frac{a}{t+b}$$

The problem is solved:

$$\int \frac{at}{(t+b)^3}\,\mathrm{d}t$$

$$= \frac{ab}{2(t+b)^2} - \frac{a}{t+b} + C$$

Rewrite/simplify:

$$= -\frac{a\,(2t+b)}{2(t+b)^2} + C$$

(b)

$$F(t) = \int_0^t \frac{a}{(t+b)^3}\,dt = \left[-\frac{a}{2(t+b)^2}\right]_0^t$$

$$\rightarrow F(2{,}000) = \left[-\frac{18\times10^6}{2(2{,}000+3{,}000)^2} - \left(-\frac{18\times10^6}{2(0+3{,}000)^2}\right)\right]$$

$$= \left[-\frac{18\times10^6}{2\times25\times10^6} - \left(-\frac{18\times10^6}{2(0+3{,}000)^2}\right)\right] = \left[-\frac{18}{50}+1\right] = \frac{32}{50} = 0.64$$

4pts

(c)

$$F(t) = \int_0^t \frac{a}{(t+b)^3}\,dt = \left[-\frac{a}{2(t+b)^2}\right]_0^t$$

$$\rightarrow F(t) = \left[-\frac{18\times10^6}{2(t+3{,}000)^2} - \left(-\frac{18\times10^6}{2(0+3{,}000)^2}\right)\right] = 0.05 \rightarrow \frac{18\times10^6}{2(t+3{,}000)^2} = 0.95$$

$$(t+3{,}000)^2 = \frac{18\times10^6}{2\times0.95}$$

$\rightarrow t = 77.94$ hrs

Set the warranty to 77.94 hrs **4pts**