Notes for Peano Arithmetic

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(i) $x + 0 \equiv 0 + x$

$$(m + 1) + 0 \equiv m + (0 + 1) \equiv (m + 0) + 1 \equiv (0 + m) + 1 \equiv 0 + (m + 1)$$

(ii) $x + (y + 1) \equiv (x + 1) + y$

$$x + (0 + 1) \equiv (x + 0) + 1 \equiv x + 1 \equiv (x + 1) + 0$$

$$x + ((m + 1) + 1) \equiv (x + (m + 1)) + 1 \equiv ((x + 1) + m) + 1 \equiv (x + 1) + (m + 1)$$

(iii) $x + y \equiv y + x$

$$x + (m + 1) \equiv (x + m) + 1 \equiv (m + x) + 1 \equiv m + (x + 1) \equiv (m + 1) + x$$

(iv) $(x + y) + z \equiv x + (y + z)$

$$(x + y) + (m + 1) \equiv ((x + y) + m) + 1 \equiv (x + (y + m)) + 1 \equiv x + ((y + m) + 1) \equiv x + (y + (m + 1))$$

(v) $0 \cdot x \equiv 0$

$$0 \cdot (m + 1) \equiv 0 \cdot m + 0 \equiv 0 + 0 \equiv 0$$

(vi) $(x + 1) \cdot y \equiv x \cdot y + y$

$$(x + 1) \cdot (m + 1) \equiv (x + 1) \cdot m + (x + 1) \equiv (x \cdot m + m) + (x + 1) \equiv \cdots \equiv (x \cdot m + x) \equiv (m + 1) \equiv x \cdot (m + 1) + (m + 1)$$

(vii) $x \cdot y \equiv y \cdot x$

$$x \cdot (m + 1) \equiv x \cdot m + x \equiv m \cdot x + x \equiv (m + 1) \cdot x$$

(viii) $(x + y) \cdot z \equiv x \cdot z + y \cdot z$

$$(x + y) \cdot (m + 1) \equiv (x + y) \cdot m + (x + y) = (x \cdot m + y \cdot m) + (x + y) \equiv \cdots \equiv (x \cdot m + x) + (y \cdot m + y) \equiv x \cdot (m + 1) + y \cdot (m + 1)$$

(ix) $x \cdot (y + z) \equiv x \cdot y + x \cdot z$

$$x \cdot (y + (m + 1)) \equiv x \cdot ((y + m) + 1) \equiv x \cdot (y + m) + x \equiv (x \cdot y + x \cdot m) + x \equiv x \cdot y + (x \cdot m + x) \equiv x \cdot y + x \cdot (m + 1)$$

(x) $(x \cdot y) \cdot z \equiv x \cdot (y \cdot z)$

$$(x \cdot y) \cdot (m + 1) \equiv (x \cdot y) \cdot m + x \cdot y \equiv x \cdot (y \cdot m) + x \cdot y \equiv x \cdot (y \cdot m + y) \equiv x \cdot (y \cdot (m + 1))$$