Reconciling two models of public debt and interest rates

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Abstract: This paper provides a careful replication and evaluation of the differences between two models of the relationship between public debt and interest rates. The American Economic Association presidential address of Olivier Blanchard (2019) provided evidence that increasing government debt in times of low interest rates could be welfare improving. Evans (2020) recreates the Blanchard approach and finds an opposite result. Evans mentions some possibilities of what might be different in his study from Blanchards, but he provides no direct mapping between the two studies. This paper seeks to identify the differences with the goal of providing further evidence for the relationship between public debt and interest rates.

1. Methodology

This paper compares the differences of Blanchard (2019) and Evans (2020) in both derivation of the economic model and the simulation code. Although Blanchard (2019)'s code is written in MATLAB, Evans (2020) code is written in Python. To eliminate any possible discrepancy caused by programming languages, I used Julien Acalin's Replication of Blanchard (2019) code that is written in Python. It fully replicates the analysis and derivation of the stochastic overlapping generations(OLG) model developed in Blanchard(2019), and yields exactly the same results, so it is sufficient to become the substitute in my comparison between Blanchard(2019) and Evans (2020). Therefore, for the comparisons of both theoretical formula and programming code, I use mostly Acalin (2019) as my major reference. First, I will compare the differences in their theoretical formulas, and second, I will specify how these discrepancies lead to different coding structure and finally different simulation results.

2. Results

2.1 Basic set-up of the economic models

The basic setup of the overlapping generations model is the same: the economy is consisted of people who only live for two periods. In the first period, the young work and consume, and then in the second period, the old consume all savings. The utility maximization function is an Epstein-Zin utility function:

$$\mathbb{U}_t = (1-\beta)u\left(C_t^y\right) + \frac{\beta}{1-\gamma}u(\mathbb{E}_t\left[(C_{t+1}^o)^{1-\gamma}\right])$$

where C is consumption in each period of life respectively.

Production function is specified as a constant retuned production function:

$$Y_t = A_t F(K_{t-1}, L_t)$$

where N=1. And in calibration and simulation models, production function is defined more strictly as:

$$Y_t = A_t\left(bK_{t-1}^\rho + (1-b)N^\rho\right)^{1/\rho} = A_t(bK_{t-1}^\rho + (1-b))^{1/\rho}$$

Blanchard used two special cases of the production function in his simulations, where $\rho = 1$ or tends to zero. In the first case, $\rho = 1$, the production function is linear, and the linear production function is defined as:

$$Y_t = A_t(\alpha K_{t-1} + (1-\alpha)L_t)$$

In the second case, ρ tends to zero, and the production function will be the Cobb-Douglas production function defined as:

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$$

Evans (2020) focused mostly on the replication of Figure 7 in Blanchard (2019), the welfare effect of a transfer of 5% of saving using linear production function, and Figure 9, the welfare effect of a transfer of 5% of saving using Cobb-Douglas production function, and I will focus my comparison in these two cases specifically.

2.2 Difference in derivation of economic models

On the theorical side, Evans (2020) follows exactly the general theory of OLG model proposed in Blanchard (2019). Although Evans (2020) recreates Blanchard (2019)'s model following the same general structure and logic, there are still slight discrepancies in the final resulting formulas. The first major difference is in the treatment of endowment variable X. In both papers, endowment is calibrated to the value of wage. In Acalin's derivation, the variable was omitted for simplicity because it does not change the results, and in many of his equations, the wage variable W represents (W+X). For instance, the expression for capital is expressed as:

$$I_t = \beta W_t$$

$$I_t = \beta(W_t + X)$$

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Code: I = beta*(1+l)*WSS0list[r]
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Therefore, in the code, the presence of endowment is always demonstrated by "1+l", where "l" is a dummy variable of whether endowment exists.

However, in Evans (2020), the endowment variable is never omitted from theoretical derivation. In the linear model case, all of the formulas are the same because the linear model is relatively simple, but in the Cobb-Douglas case, the expressions for $\beta, X,$ and capital are all different. The discrepancy could be a result of the inclusion of an additional endowment factor in the derivation.

In the Evans' paper, $\beta, X,$ capital k_2 are denoted as:

$$\beta = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{2E[R_{t+1}]}$$

$$x_1 = \left[(1-\alpha)e^{\mu+\frac{\sigma^2}{2}}(2\beta)^{\alpha}\right]^{\frac{1}{1-\alpha}}$$

$$\bar{k}_2 = 2\beta\left[(1-\alpha)e^{\mu+\frac{\sigma^2}{2}}(2\beta)^{\alpha}\right]^{\frac{1}{1-\alpha}}$$

While in Acalin's paper, β and capital K_t are denoted as:

$$\mathbb{E}[k_t] = \frac{\log[\beta(1-\alpha)]+\mu}{1-\alpha}$$

$$\mathbb{E}[K_t] = \bar{K} = e^{\mathbb{E}k}e^{\frac{\mathbb{V}k}{2}} = e^{(\frac{\log[\beta(1-\alpha)]+\mu}{1-\alpha}+\frac{\sigma^2/2}{1-\alpha^2})}$$

The value of capital as well as endowment is very different between both models, and as wage W is calculated based on capital K, the value of wage is different between two models as well. As the resulting parameters of β, capital, wage, and endowment are all different, it makes sense to have a very different result from the simulation models.

Another major difference is the set-up of Total Factor Productivity, A_t. In Blanchard (2019), Total Factor Productivity is defined simply as a random variable with a log normal distribution, while in Evans (2020), the Total Factor Productivity also includes a z_t variable that follows a normally distributed AR(1) process. However, when setting the persistence to be zero and use the same A_t, the model still gives a different result than Blanchard (2019), so the set-up of Total Factor Productivity is probably not the reason for discrepancy in final results.

2.2 Difference in choice of parameters

The choice of parameters in most cases are identical in both papers, but there are slight differences in the choice of parameters for β and μ. In the linear case, as most formulas do not include β, the parameter β is set randomly at 0.325 for all simulations, and after a series of test simulations with different values of β ranging from 0.2 to 0.65, it seems that the value of β do not change the final result. The same goes for the value of μ in the Cobb-Douglas case. As in the second case, the parameter more relevant is the value of β instead of μ, μ is set randomly at value 3 for all simulations, and after I run the simulation on different values of μ, for example, $\mu = 2$, the result didn't change at all.

While in Evans (2020), in both cases, the model uses formulas to derive different β and μ, so both the value of β and μ changes according to different risky rate and safe rate assumptions. As the formula in both papers of μ is the same, this discrepancy in choice of μ should not be the cause of the completely opposite result. However, the difference in parameter β is more distinct, and it can be seen from Table 1.

Table 1. Value of β in different cases

	Evans	Blanchard
Linear Model	[0.25, 0.152, 0.094]	[0.325, 0.325, 0.325]
Cobb-Douglas	[0.25, 0.152, 0.094]	[0.332, 0.202, 0.125]

From the previous section, I mentioned that the theoretical formula for β is different in the Cobb-Douglas case. Plugging in the numerical values of α, I find that the β value in Blanchard (2019) model is 1.328 times the parameter values in Evans, and this discrepancy is caused by:

$$e^{\frac{\sigma^2}{1+\alpha}} = 1.328$$

Although the differences of parameter β can influence the result of the Cobb-Douglas case, it could not explain the different results in linear model, so this factor is still not the key factor for causing the differences.

Another difference of the choice of parameter is the choice of σ. In Blanchard, the model is greatly simplified by only using an annual $\sigma = 0.2$ across all different periods without changing it. While in Evans (2020), σ is treated as a variable that also gets compounded over the 25-year period, and the overall 25-year σ= 0.6. However, the different choice of σ still cannot explain the discrepancy of final results. By setting $\sigma = 0.2$ in Evans (2020) simulation and $\sigma = 0.6$ in Blanchard Python code, both results didn't change at all from their original output. Therefore, the difference in setting σ cannot explain the final opposition.

Overall, my preliminary examination and comparison of Evans (2020) and Blanchard (2019) Python code does not find the key factor that causes the discrepancy in the final results, but I've identify some differences in both the theoretical economic model and the code simulations.