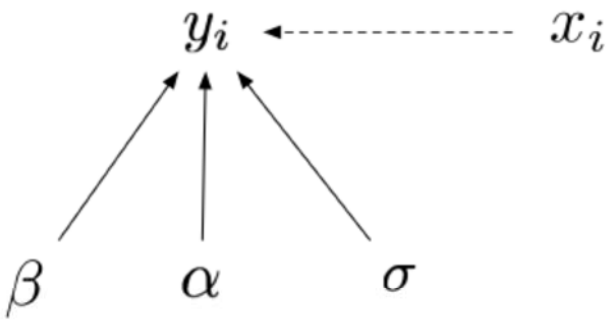
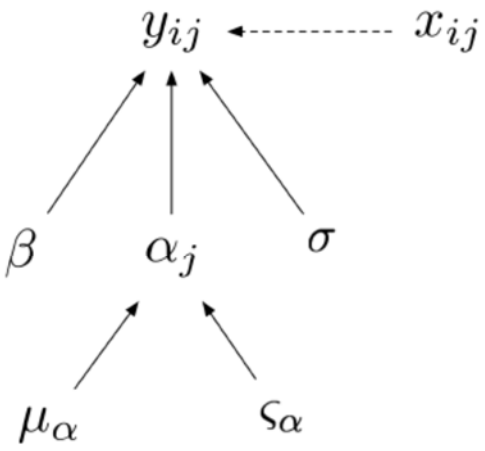
Pooled model



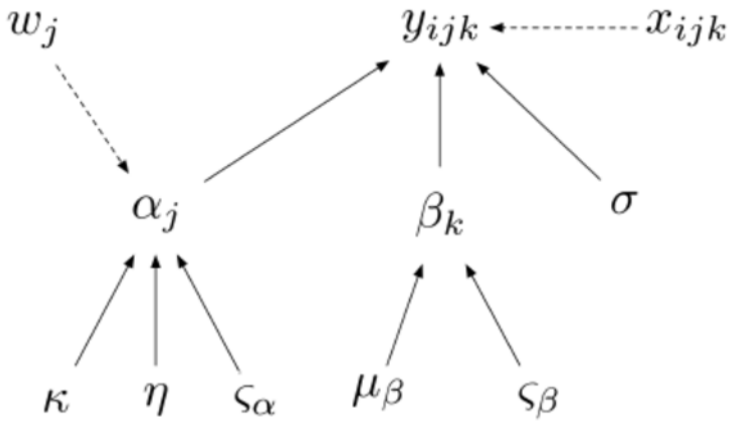
$$
\begin{aligned}
\mu_i &= \gamma x^{\beta} \\
\alpha &= \log(\gamma) \\
\log(\mu_i) &= \alpha + \beta \log(x_i) \\
g(\alpha, \beta, x_i) &= \alpha + \beta \log(x_i) \\
[\alpha, \beta, \sigma \mid y_i] &\propto [\log(y_i) \mid g(\alpha, \beta, x_i), \sigma^2][\alpha][\beta][\sigma]
\end{aligned}
$$

Intercepts for each site



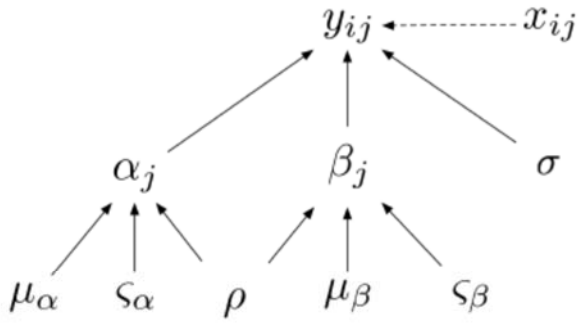
$$
\begin{aligned}
g(\alpha_j, \beta, x_{ij}) &= \alpha_j + \beta \log(x_{ij}) \\
[\alpha_j, \beta, \mu_\alpha, \sigma, \varsigma_\alpha \mid y_{ij}] &\propto [\log(y_{ij}) \mid g(\alpha_j, \beta, x_{ij}), \sigma^2] [\alpha_j \mid \mu_\alpha, \varsigma_\alpha^2] [\beta] [\sigma] [\mu_\alpha] [\varsigma_\alpha]
\end{aligned}
$$

Intercepts vary with carbon level in site soils and slopes vary with fertilizer type



$$
\begin{aligned}
g_1\left(\alpha_j, \beta_k, x_{ijk}\right) &= \alpha_j + \beta_k \log(x_{ijk}) \\
g_2\left(\kappa, \eta, w_j\right) &= \kappa + \eta \text{logit}(w_j) \\
\left[\boldsymbol{\alpha}, \boldsymbol{\beta}, \sigma, \varsigma_\alpha, \kappa, \eta, \mu_\beta, \varsigma_\beta \mid \mathbf{y}\right] &\propto \prod_{j=1}^{J} \prod_{k=1}^{K_j} \prod_{i=1}^{n_j} \text{normal}\left(\log\left(y_{ijk}\right) \mid g_1\left(\alpha_j, \beta_k, x_{ijk}\right), \sigma^2\right) \\
&\times \text{normal}\left(\alpha_j \mid g_2\left(\kappa, \eta, w_j\right), \varsigma_\alpha^2\right) \\
&\times \text{normal}\left(\beta_k \mid \mu_\beta, \varsigma_\beta^2\right) \\
&\times \text{normal}\left(\eta \mid 0, 1000\right) \\
&\times \text{normal}\left(\kappa \mid 0, 1000\right) \\
&\times \text{uniform}\left(\sigma \mid 0, 100\right) \\
&\times \text{uniform}\left(\varsigma_\alpha \mid 0, 200\right) \\
&\times \text{normal}\left(\mu_\beta \mid 0, 1000\right) \\
&\times \text{uniform}\left(\varsigma_\beta \mid 0, 200\right)
\end{aligned}
$$

Slope *and* intercepts vary by site



$$g(\alpha_j, \beta_j, x_{ij}) = \alpha_j + \beta_j \log(x_{ij})$$

$$\begin{aligned}
[\boldsymbol{\alpha}, \boldsymbol{\beta}, \mu_\alpha, \mu_\beta, \sigma, \varsigma_\alpha, \varsigma_\beta, \rho \mid \mathbf{y}] \propto & \prod_{j=1}^{J} \prod_{i=1}^{n_j} \text{normal}\big(\log(y_{ij}) \mid g(\alpha_j, \beta_j, x_{ij}), \sigma^2\big) \\
& \times \text{multivariate normal}\left(\begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix} \middle| \begin{pmatrix} \mu_\alpha \\ \mu_\beta \end{pmatrix}, \begin{pmatrix} \varsigma_\alpha^2 & \rho\varsigma_\alpha\varsigma_\beta \\ \rho\varsigma_\alpha\varsigma_\beta & \varsigma_\beta^2 \end{pmatrix}\right) \\
& \times \text{normal}(\mu_\alpha \mid 0, 1000) \\
& \times \text{normal}(\mu_\beta \mid 0, 1000) \\
& \times \text{uniform}(\sigma \mid 0, 100) \\
& \times \text{uniform}(\varsigma_\alpha \mid 0, 200) \\
& \times \text{uniform}(\varsigma_\beta \mid 0, 200) \\
& \times \text{uniform}(\rho \mid 0, 1)
\end{aligned}$$