COA 616 Geostatistics in Environmental Sciences

Lecture 6 – More on Variogram and spatial interpolation

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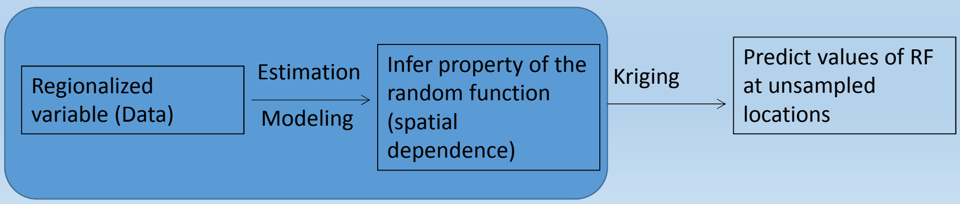


Talking about number of pairs, semivariogram map

- Number of pairs – 30
- Semivariogram map
- Send homework to geostats.hw@gmail.com, subject: homework#

Combining models

More complex functions to describe more complex variograms for large data set.

Combine two or more simple models. $\gamma(\vec{h}) = \sum_{i=1}^{n} |w_i| \gamma_i(\vec{h})$

Any combinations of CNSD function is CNSD.

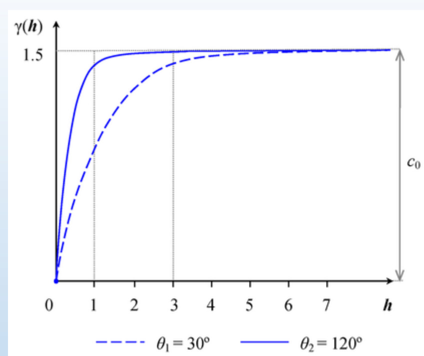
Spatial dependence may occur at two distinct scales – represented in the variogram as two spatial components – nested structure.

$$\gamma(\vec{h}) = \begin{cases} c_1\{\frac{3\vec{h}}{2a_1} - \frac{1}{2}(\frac{\vec{h}}{a_1})^3\} + c_2\{\frac{3\vec{h}}{2a_2} - \frac{1}{2}(\frac{\vec{h}}{a_2})^3\} & \text{for } 0 < \vec{h} < a_1 \\ c_1 + c_2\{\frac{3\vec{h}}{2a_2} - \frac{1}{2}(\frac{\vec{h}}{a_2})^3\} & \text{for } a_1 < \vec{h} < a_2 \\ c_1 + c_2 & \text{for } \vec{h} > a_2 \end{cases}$$

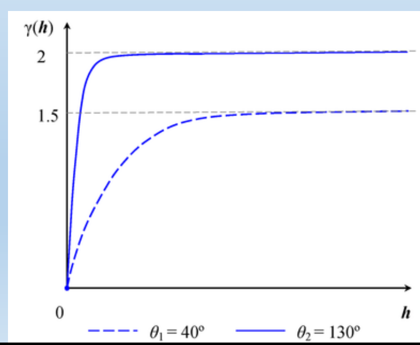
Anisotropy

Variation varies with direction

Geometric anisotropy: It occurs when the same sill (or scale) parameter is present in all directions but the range changes with direction.

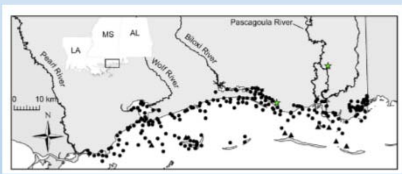


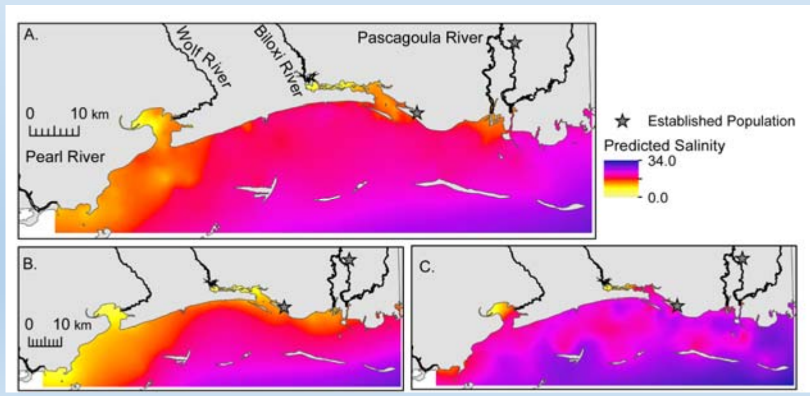
Zonal anisotropy: the sill (or scale) parameter is different for different directions. It is not possible to transform such a structure into an isotropic semivariogram.



Spatial interpolation

Examples
Predict rainfall from rain gauges
Predict salinity from limited measurements
Predict total nitrogen concentration in soil





Lowe et al. PLoS ONE 7(7) E41580.

Spatial interpolation

Nearly all the methods of prediction can be seen as weighted averages of data

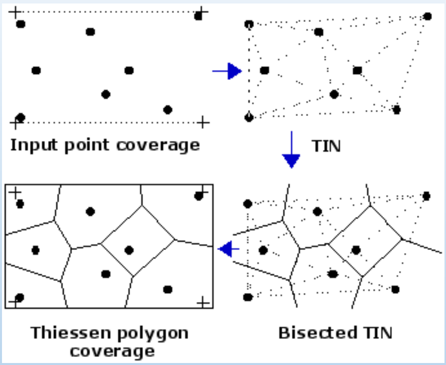
$$z^*(\overrightarrow{x_0}) = \sum_{i=1}^{N} \lambda_i z(\overrightarrow{x_i})$$

where $\overrightarrow{x_0}$ is a target point for which we want an estimate

$\overrightarrow{x_i}$s are locations where we have measurements

λ_i are the weights

Spatial interpolation – Thiessen polygons (nearest neighbor)

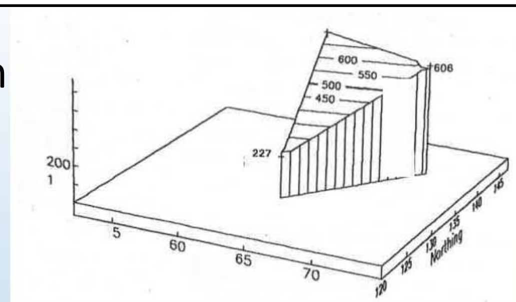


$$\lambda_i = \begin{cases} 1 & \text{if } \overrightarrow{\mathrm{x}_\mathrm{i}} \in V_i \\ 0 & \text{otherwise} \end{cases}$$

Each prediction is based on just one measurement, and information from neighboring points is ignored.

No estimate of the error.

Spatial interpolation – Triangulation



Fitting a plane through three samples that surround the point being estimated.

We can calculate the coefficients a, b, and c by solving the following system of equations:

$$\mathrm{a}x_1 + by_1 + c = z_1$$
$$\mathrm{a}x_2 + by_2 + c = z_2$$
$$\mathrm{a}x_3 + by_3 + c = z_3$$

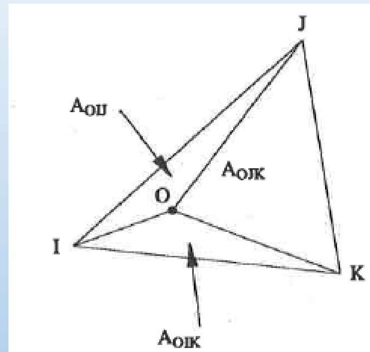
Then the estimate at (x_0, y_0) becomes: $\mathrm{a}x_0 + by_0 + c$

Spatial interpolation – Triangulation II

We could also express our estimates as a weighted linear combination of the three sample values.

$$\hat{v}_o = \frac{A_{OJK} v_I + A_{OIK} v_J + A_{OIJ} v_K}{A_{IJK}}$$

where As represent the areas of the triangles given in their subscripts.



Disadvantage
Prediction depends on only three data
No measure of error
No obvious triangulation that is better than any other (a **Delaunay triangulation** for a set **P** of points in a plane is a triangulation DT(**P**) such that no point in **P** is inside the circumcircle of any triangle in DT(**P**).

Spatial interpolation – Inverse functions of distance

$$\lambda_i = 1/|x_i - x_0|^{\beta} \quad \beta > 0$$
$$\sum \lambda_i = 1$$

The interpolation is exact, no discontinuity.

Disadvantage:
The choice of weighting function is arbitrary
No measured of error
Takes no account of configuration of the sampling

Spatial interpolation – Trend surfaces

A form of multiple regression in which the predictors are the spatial coordinates.

$$z(x_1, x_2) = f(x_1, x_2) + \varepsilon$$

Plausible functions, usually simple polynomials (1st, 2nd and 3rd order), are fitted by least squares to the spatial coordinates.

$$\mathrm{z} = \mathrm{b}_0 + \mathrm{b}_1\mathrm{x}_1 + \mathrm{b}_2 x_2$$

$$z = b_0 + b_1x_1 + b_2x_2 + b_3x_1^{\;2} + b_4x_2^{\;2} + b_5x_1x_2$$

Spatial variation is complex that a polynomial of very higher order is needed, and the results are unstable.
The residuals from the trend are autocorrelated.
It can represent long-range trend but not sufficiently local trend

Spatial interpolation – Splines

It consists of polynomials but each polynomial only describes pieces of a line or surface. They are fitted together so that they join smoothly.