COA 616 Geostatistics in Environmental Sciences

Lecture 9 – Block & Indicator Kriging

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Regionalized variable (Data) → (Estimation / Modeling) → Infer property of the random function (spatial dependence) → (Kriging) → Predict values of RF at unsampled locations

Block Kriging

Predictions over an area (change of support)

$$\hat{z}(B) = \frac{1}{|B|}\int \hat{z}(x_B)dx$$

where B = block, | B |= area of block, $\hat{z}(x_B)$ = estimated value of z at point x_B in the block.

or

$$\hat{z}(B) = \sum_{i=1}^{n} {}^{B}\lambda_i z(x_i)$$

where ${}^{B}\lambda_i$ = block kriging weight for point x_i

$${}^{B}\lambda_i = A^{-1}\,{}^{B}b$$

$$A = \begin{bmatrix} \gamma(x_1,x_1) & \gamma(x_1,x_2) & \gamma(x_1,x_3)\ldots & \gamma(x_1,x_n) & 1 \\ \gamma(x_2,x_1) & \gamma(x_2,x_2) & \gamma(x_2,x_3)\ldots & \gamma(x_1,x_n) & 1 \\ \gamma(x_3,x_1) & \gamma(x_3,x_2) & \gamma(x_3,x_3)\ldots & \gamma(x_3,x_n) & 1 \\ \ldots & & & & \\ \gamma(x_n,x_1) & \gamma(x_n,x_2) & \gamma(x_n,x_3)\ldots & \gamma(x_n,x_n) & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$${}^{B}b = \begin{bmatrix} \bar{\gamma}(x_1,B) \\ \bar{\gamma}(x_2,B) \\ \bar{\gamma}(x_3,B) \\ \ldots \\ \bar{\gamma}(x_n,B) \\ 1 \end{bmatrix}$$

The $\bar{\gamma}$ values are the average semivariances from each sampling points to the block

$$\bar{\gamma}(x_i, B) = \frac{1}{|B|}\int_B \gamma(x_i, x_B)dx$$

where $\mathrm{x_B}$: a point in the block

$\bar{\gamma}(x_i, B)$: average of all the semivariances between sampling points to all the points in the block

The point to block covariance that are requested for block kriging can be devleoped as follows:

$$\begin{aligned}
\mathrm{C_{iB}} &= \mathrm{cov}(z_i, z_B) \\
&= \mathrm{E}\{z_i z_B\} - E\{z_i\}E\{z_B\} \\
&= \mathrm{E}\{\frac{1}{|\mathrm{B}|}\sum_{j\in B} z_i z_j\} - E\{z_i\}E\{\frac{1}{|\mathrm{B}|}\sum_{j\in B} z_j\} \\
&= \frac{1}{|\mathrm{B}|}\sum_{j\in B} E\{z_i z_j\} - \frac{1}{|\mathrm{B}|}\sum_{j\in B} E\{z_i\}E\{z_j\} \\
&= \frac{1}{|\mathrm{B}|}\sum_{j\in B}[E\{z_i z_j\} - E\{z_i\}E\{z_j\}] \\
&= \frac{1}{|\mathrm{B}|}\sum_{j\in B}\mathrm{cov}(z_i, z_j)
\end{aligned}$$

The covariance between RV at the ith sample and the RV at the $\mathrm{z_B}$ representing the average value of the phenomena over the area B is the same as the average point to point covariance between $\mathrm{z_i}$ and RVs at all the points within B.

Choose a suitable N_B

Try N_B = 4 (1 D)
16 (2D)
64 (3D)
Trade off between # of discretizing points (accuracy) and computation time

$$\bar{\gamma}(x_i, B) = \frac{1}{N_B}\sum_{j=1}^{N_B}\gamma(x_i, x_{B_i})$$

where x_{B_i} : Points in the block used for estimation for $\bar{\gamma}$

$$\sigma^2(B) = {}^B b^{T\,B}\lambda - \bar{\gamma}(B,B)$$

$\bar{\gamma}(B,B) =$ *average* of $\gamma(x_i, x_j)$ for all x_i, x_j pairs in block B

Indicator Kriging

Indicator kriging converts a variable that has been measured on a continuous scale to several indicator variable, each taking the values of 0 and 1 at the sample sites (loss of information).

It is linear kriging of non-linear transforms of data.

It estimates the probability that the true values exceed specified thresholds at unknown points or blocks, therefore enables us to assess the risk we take by accepting the estimates at their face values.

Repeat the process for several thresholds.