COA 616 Geostatistics in Environmental Sciences

Lecture 3 Semivariogram

Wei Wu

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Data posting

- A map on which each data location is plotted along with its corresponding data value.

Spatial continuity – h-scatterplot

An h-scatterplot shows all possible pairs of data values whose locations are separated by a certain distance in a particular direction.

V(s) – V(s+**h**)

The x-coordinate of a point corresponds to the V value at a particular location and y-coordinate to the V value at a distance and direction **h** away.

The shape of the cloud of points on an h-scatterplot shows how continuous the data values are over a certain distance in a particular direction.

Spatial continuity cont.

h: straight line distance

Θ: direction (angle) from one point to another Θ=0 is north, clockwise increase

h or $\vec{h}$: vector (distance and direction) separating two locations

head: the end of the vector

tail: the beginning of the vector

H-scatterplot is a plot of the head value vs. tail value for all pairs of data points that are separated by $\vec{h}$.

$$h_{12} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\theta_{12} = \tan^{-1}\frac{y_2 - y_1}{x_2 - x_1} \qquad (x_2 > x_1)$$

$$\theta_{12} = \tan^{-1}\frac{y_2 - y_1}{x_2 - x_1} + 180 \qquad (x_2 < x_1)$$

Spatial continuity cont.

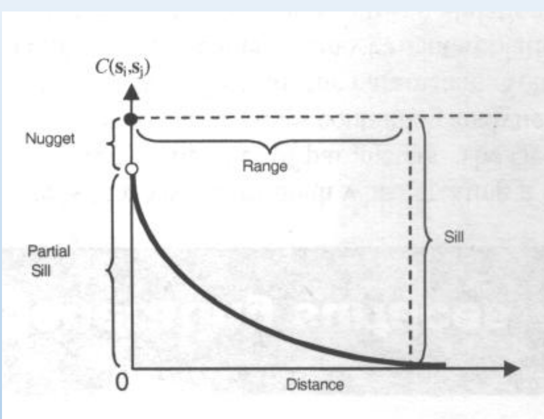
- Indicator for dispersion of cloud on an h-scatterplot

  Correlation coefficient: The relation between the coefficient correlation of an h-scatterplot and **h** correlation function or correlogram.

  Covariance: The relation between the covariance of an h-scatterpot and **h** is covariance function.

  Moment of inertia: $\frac{1}{2n}\sum_{i=1}^{n}(x_i - y_i)^2$

Spatial covariance



$$\mathrm{cov}(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})$$

$$\hat{c}(\vec{h}) = \frac{1}{m(\vec{h})}\sum_{i=1}^{m(\vec{h})}(z(\vec{x_i}) - \overline{z}_1)(z(\vec{x_i} + \vec{h}) - \overline{z}_2)$$

$\hat{c}(\vec{h})$: spatial covariance

$m(\vec{h})$: # of pairs of points separated by $\vec{\mathrm{h}}$

$z(\vec{x_i})$: data value at location $\vec{\mathrm{x}_\mathrm{i}}$ (tail value)

$z(\vec{x_i} + \vec{h})$: data value at location $\vec{x_i} + \vec{h}$ (head value)

$$\overline{z}_1 = \frac{1}{m(\vec{h})}\sum_{i=1}^{m(\vec{h})} z(\vec{x_i}) : \text{mean of tail value}$$

$$\overline{z}_2 = \frac{1}{m(\vec{h})}\sum_{i=1}^{m(\vec{h})} z(\vec{x_i} + \vec{h}) : \text{mean of head value}$$

Spatial Covariance cont.

$$\hat{c}(0) = \frac{1}{m(0)} \sum_{i=1}^{m(0)} (z(\overrightarrow{x_i}) - \bar{z}_1)(z(\overrightarrow{x_i} + 0) - \bar{z}_2)$$

$$m(0) = n$$

$$\bar{z}_1 = \bar{z}_2 = \bar{z}$$

$$\text{So } \hat{c}(0) = \frac{1}{n} \sum_{i=1}^{n} (z(\overrightarrow{x_i}) - \bar{z})(z(\overrightarrow{x_i}) - \bar{z}) \cong \text{var}(z) \frac{n-1}{n}$$

Assume stationarity if 1) the mean value does not vary significantly in space

2) the separation $\vec{h}$ is small relative to the size of the study area

then $\bar{z}_1 \cong \bar{z}_2 \cong \bar{z}$ thus

$$\hat{c}(\vec{h}) = \frac{1}{m(\vec{h})} \sum_{i=1}^{m(\vec{h})} (z(\overrightarrow{x_i}) - \bar{z})(z(\overrightarrow{x_i} + \vec{h}) - \bar{z}) = \frac{1}{m(\vec{h})} \sum_{i=1}^{m(\vec{h})} [(z(\overrightarrow{x_i}) z(\overrightarrow{x_i} + \vec{h})] - \bar{z}^2$$

Correlation function

Normalize the spatial covariance by dividing standard deviations

$$\hat{\rho}(\vec{h}) = \frac{\hat{c}(\vec{h})}{s_1 s_2}$$

s_1 : standard deviation of $z(\overrightarrow{x_i})$ values (tails)

s_2 : standard deviation of $z(\overrightarrow{x_i} + \vec{h})$ values (heads)

$$s_1 = \sqrt{\frac{1}{m(\vec{h}) - 1} \sum_{i=1}^{m(\vec{h})} (z(\overrightarrow{x_i}) - \bar{z}_1)^2}$$

Correlogram is a plot of $\hat{\rho}(\vec{h})$ vs. $\vec{h}$, should be similar in form to $\hat{c}(\vec{h})$ plot.

Simpler form if $\bar{z}_1 \cong \bar{z}_2 \cong \bar{z}$ and $s_1 = s_2 = s_z$ then

$$\hat{\rho}(\vec{h}) = \frac{\hat{c}(\vec{h})}{s_z^2}$$

(semi)Variogram: Half of the average squared difference between the paired data values – measures dissimilarity of values separated by a vector

$$\hat{\gamma}(\vec{h}) = \frac{1}{2m(\vec{h})} \sum_{i=1}^{m(\vec{h})} (z(\vec{x}_i) - z(\vec{x}_i + \vec{h}))^2$$

If $z(\vec{x}_i) = z(\vec{x}_i + \vec{h}) \Rightarrow \hat{\gamma}(\vec{h}) = 0$

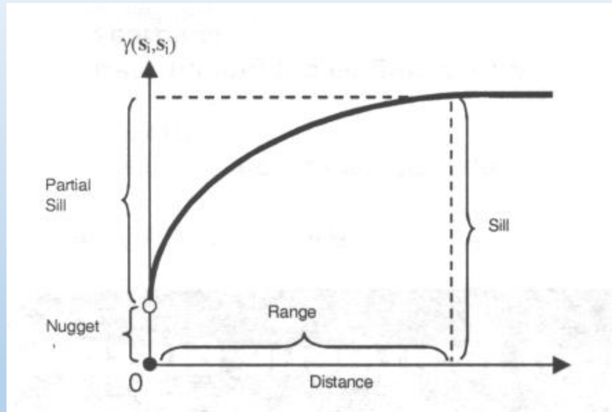
If $z(\vec{x}_i) - z(\vec{x}_i + \vec{h})$ is large $\Rightarrow \hat{\gamma}(\vec{h})$ is large

In principle $\hat{\gamma}(\vec{h}) = 0$ at $\vec{h} = 0$

In practice $\hat{\gamma}(\vec{h})$ rarely converges to 0 as $\vec{h} \to 0$

Nuggest effect: 1) Sampling error

2) Spatial variation at scales smaller than the samllest $\vec{h}$



Medogram; Rodegram

Semivariances are sensitive to extreme values,
medogram and rodegram are alternatives to semivariogram
Define a family of variogram

$$\hat{\gamma}_w(\vec{h}) = \frac{1}{2m(\vec{h})} \sum_{i=1}^{m(\vec{h})} | z(\vec{x}_i) - z(\vec{x}_i + \vec{h}) |^w$$

$w \in (0,2]$

$w = 2$: semivariogram

$w = 1$: medogram

$w = 1/2$: rodogram

Geometric interpretation of h-scatterplot in relation to semivariance

1. What if all data fall on the 1:1 line in h-scatterplot?

$$\hat{\gamma}(\vec{h}) = 0$$

2. How far is any point from the 1:1 line?

Semivariance = average of squared distance of points to the 1:1 line on an h-scatterplot

Variogram cloud/map

1. Isotropic

2. Anisotropic