A Briedf Intro of Catalan Numbers

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1. Introduction

Catalan Numbers were first discovered by Swiss mathematician *Leonhard Euler* and Hungarian mathematician *Johann Andreas von Segner* by studying the problem of triangulation of the convex polygon. Although recursive relations of these numbers are introduced from Segner, many of the properties and identities of these numbers are discovered by the side of French-Belgian mathematician Eugene Charles Catalan in 1838 through the study of well-formed sequences of parentheses.

Catalan numbers have a significant place and major importance in combinatorics and computer science. They form a sequence of natural numbers that occur in studying astonishingly many combinatorial problems. In mathematics, Catalan numbers describe the number of ways a polygon with +2 sides can be cut into triangles, the number of rooted, trivalent trees with n+1 nodes, the number of paths of length 2n through an n×n grid that do not rise above the main diagonal.

2. Recurrence Relation

The definition of Catalan Numbers is given by the following formula

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}. \quad n \geq 0.$$

or is more commonly defined by the following recurrence relation:

$$C_0 = 1,$$
$$C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}, \quad n \geq 0.$$

These two definitions are proved equivalent by using the method of generating functions. Suppose there is a function $g(x)$ such that

$$g(x) = 1 + g(x)^2.$$

Then we have

$$g(x) = \frac{1 - \sqrt{1 - 4x}}{2x} = \sum_{n=0}^{\infty} C_n x^n.$$

3. Conclusion

From medicine to applied mathematics, to analytics, and then to logic, Curry adjusted his research direction three times in his life. As an double-major student, I'm deeply inspired by Curry's perseverance and decisiveness. I now major in theoretical computer science and mathematics (especially logic), and hope that I could make a difference in these fields, like Dr. Curry did.

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