HW7, Due: Friday, March 8

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Question 1 *Use the Calculus method to find*

$$\sum_{k=1}^{n} \frac{1}{k^2}\binom{n}{k}.$$

Question 2 *Let m, n be positive integers. Show that*

$$\sum_{\substack{0\le k_1,k_2\le n\\ l_1+l_2=m}} (-1)^{l_2}\binom{n}{k_1}\binom{2k_1}{l_1}\binom{n}{k_2}\binom{2k_2}{l_2} = \begin{cases} \binom{n}{m/4}4^{n-m/4} & 4 \mid m. \\ 0 & \text{otherwise.} \end{cases}$$

Here are some steps.

1. *Show that*
$$(x^4+4)^n = ((x+1)^2+1)^n((x-1)^2+1)^n. \tag{1}$$

2. *Show that*
$$((x{+}1)^2{+}1)^n((x{-}1)^2{+}1)^n = \left(\sum_{k_1,l_l}\binom{n}{k_1}\binom{2k_1}{l_1}x^{l_1}\right)\left(\sum_{k_2,l_2}\binom{n}{k_2}\binom{2k_2}{l_2}(-x)^{l_2}\right).$$

3. *Compare the coefficients of x^m on both sides of* (1).

Question 3 *Problems 1,4,7 in textbook Chapter 6.*