HW4, Due: Friday, Feb. 16

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Question 1 *Find the smallest integer n, so that for any n people, there are three of them either know each other or don't know each other.*

Question 2 *Prove that for any $n+1$ integers $a_1, a_2, \ldots, a_{n+1}$, there exists two of the integers a_i, a_j with $i \neq j$ such that n divides $a_i - a_j$.*

Question 3 *Suppose there are n points $p_1, p_2, \ldots, p_n$ in $[0,1]^2$ (no three of them lie on the same line). Show that there exists three different points p_i, p_j, p_k, so that the triangle with these three points as vertices has area $\leq \frac{1}{n-2}$.*

Question 4 *Suppose there is a sequence of integers define in the following way.*

$$a_1 = 0, a_2 = 2024, a_{n+1} = a_n^{2024} + 2024 a_{n-1}.$$

Show that for any prime number p, there are infinitely many n such that p divides a_n. (Hint: use Recurrence Theorem discussed in the class.)

The next question is to test whether you know the definition of permutation group.

Question 5 *Let $(G_5, *)$ be the permutation group of $\{1,2,3,4,5\}$.*

1. *Calculate $(14253) * (24513)$.*
2. *Calculate $(34512)^{-1}$.*
3. *Recall $\varepsilon(g)$ is the sign of the permutation g. Calculate $\varepsilon(43251)$.*