HW1

Please type or photograph your solution and turn it into a pdf before submitting it to Canvas. The first two problems will be graded for correctness.

1. Let A be a set.

   (a) Let S be a non empty set of equivalence relations on A. Show that $\bigcap S$ is an equivalence relation on A.

   (b) Let R be a relation between A and A. Show that there is a unique equivalence relation on A, called $\sim_R$, such that any equivalence relation $\sim$ on A which contains R has $\sim_R$ as a subset.

2. Let A and B be two sets, $f : A \to B$ a function. Define function $F : P(B) \to P(A)$ as $F(C) = f^{-1}(C)$. Show that F is an injection iff f is a surjection, F is a surjection iff f is an injection.

3. Show that $\sim= \{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y \in \mathbb{Q}\}$ is an equivalence relation on $\mathbb{R}$.

4. Let A and B be two sets. $C = \{(x, i) \in (A \cup B) \times \{0, 1\} : x \in A \text{ if } i = 0, x \in B \text{ if } i = 1\}$. Show that there are injections $k : A \to C$, $j : B \to C$, such that $C = k(A) + j(B)$ and $k(A) \cap j(B) = \emptyset$.

5. Let $f : A \to B$ be a function. Show that there is a set C, an injection $g : A \to C$, and a surjection $h : C \to B$, such that $f = h \circ g$. (Hint: You may want to use the solution for the previous problem).