# HW5

Please type or photograph your solution and turn it into a pdf before submitting it to Canvas. The first two problems will be graded for correctness.

1. Show that the map $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined as $(t, x) \mapsto xe^t$ is a left $(\mathbb{R}, +)$ action on $\mathbb{R}$. Write down the corresponding permutation representation. Is the action effective? Is the action transitive? Is the action free?

2. Let $G$ be a group, $X$ and $Y$ be two left $G$ sets.

   (a) Show that $\cdot : G \times (X \times Y) \to X \times Y$ defined as $(g, (x, y)) = (gx, gy)$ is a left $G$ action on $X \times Y$.

   (b) If $f : X \to Y$ is a map, show that $f$ is $G$ equivariant if and only if $\{(x, f(x)) : x \in X\}$ is a $G$ invariant subset of the left $G$ set $(X \times Y, \cdot)$.

3. Let $G$ be a group with more than one elements, $X$ be a non empty left $G$ set. Show that $\cdot : G \times P(X) \to P(X)$ defined as $(g, A) \mapsto \{ga : a \in A\}$ is a left $G$ action on $P(A)$. Can this action be free? Can this action be transitive?

4. Any non empty set $X$ is a left $S_X$ set via the action $\cdot : S_X \times X \to X$, $(\sigma, x) \mapsto \sigma(x)$. Show that the only $S_X$ equivariant map from $X$ to $X$ is the identity.

5. Write down a group $G$, two left $G$ sets $X$ and $Y$, such that there are no $G$ equivariant map from $X$ to $Y$.