HW5

Please type or photograph your solution and turn it into a pdf before submitting it to Canvas. The first two problems will be graded for correctness.

1. Show that the map $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined as $(t, x) \mapsto xe^t$ is a left $(\mathbb{R}, +)$ action on $\mathbb{R}$. Write down the corresponding permutation representation. Is the action effective? Is the action transitive? Is the action free?

2. Let G be a group, X and Y be two left G sets.

   (a) Show that $\cdot : G \times (X \times Y) \to X \times Y$ defined as $(g, (x, y)) = (gx, gy)$ is a left G action on $X \times Y$.

   (b) If $f : X \to Y$ is a map, show that f is G equivariant if and only if $\{(x, f(x)) : x \in X\}$ is a G invariant subset of the left G set $(X \times Y, \cdot)$.

3. Let G be a group with more than one elements, X be a non empty left G set. Show that $\cdot : G \times P(X) \to P(X)$ defined as $(g, A) \mapsto \{ga : a \in A\}$ is a left G action on $P(A)$. Can this action be free? Can this action be transitive?

4. Any non empty set X is a left S_X set via the action $\cdot : S_X \times X \to X$, $(\sigma, x) \mapsto \sigma(x)$. Show that if X has 3 or more elements then the only S_X equivariant map from X to X is the identity.

5. Write down a group G, two left G sets X and Y, such that there are no G equivariant map from X to Y.