A PROOF OF ERDŐS-RADO THEOREM

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Abstract. We discuss some of the key ideas of Perelman's proof of Poincaré's conjecture via the Hamilton program of using the Ricci flow, from the perspective of the modern theory of nonlinear partial differential equations.

Contents

1. Introduction

2. Preliminary Set Theory

The notion of regular uncountable cardinal is a generalization of the pigeonhole principle for uncountable sets.

Definition 2.1 (ordinal)**.** An *ordinal* is a set α such that $\bigcup \alpha \subset \alpha$ and well-ordered by $\in$.

Definition 2.2 (cardinal)**.**

Definition 2.3 (regular cardinal)**.** An uncountable cardinal number λ is said to be *regular* if for every $\kappa < \lambda$, every set S of cardinal λ, and every function $f : S \to \kappa$, there exists $H \subset S$ of cardinal λ such that the function f is constant on H.

Definition 2.4 (successor cardinal)**.** Let μ be a cardinal, define the *successor cardinal* of μ to be the least cardinal strictly greater than μ, denoted by μ^+.

Proposition 2.5. For every infinite cardinal μ, the cardinal μ^+ is regular.

Proof. □

Notation 1. Let n be a positive integer, and let S be a linearly ordered set. Denote the set $\{(j_1, j_2, \ldots, j_n) \mid j_1 < j_2 < \ldots < j_n\}$ by $[S]^n$. For a subset $H \subset S$, let

$$[H]^n = \{(j_1, j_2, \ldots, j_n) \in H^n \mid j_1 < j_2 < \ldots < j_n\}.$$

Example. We can view $[\omega]^2$ as the complete undirected graph on ω, and any pair of distinct elements in ω is an edge in the graph.

Definition 2.6. Let $n < \omega$ and suppose λ, κ and μ are cardinal numbers (not necessarily infinite). We denote by $\lambda \to (\mu)^n_\kappa$ the following statement:
For every set S of cardinal λ and every function $c : [S]^n \to \kappa$, there exists $H \subset S$ of cardinal μ and there exists $i_0 < \kappa$ such thatc for every $\boldsymbol{j} \in [H]^n$, $c(\boldsymbol{j}) = i_0$.
We denote by $\lambda \not\to (\mu)^n_\kappa$ if the above statement is false.

In the example above, we call the function c *coloring with κ colors*, and the set H *monochromatic set*.

Notation 2. $[S]^{<\omega} = \bigcup_{n<\omega} [S]^n$.

Theorem 2.7 (infinitary Ramsey)**.** $\aleph_0 \to (\aleph_0)^2_2$

Theorem 2.8.

Definition 2.9 (weakly compactness)**.** An uncountable χ is called *weakly compact* if $\chi \to (\chi)^2_2$ holds.

3. Ramsey's Theorem

Theorem 3.1 (Ramsey's Theorem)**.** *Let ...*

4. Erdős-Rado Theorem

5. References