# HW 5

November 22, 2020

1. (Problem 7.6 in textbook) Consider the trapezium and Simpson's rule applied to $\int_0^1 (x^5 - Cx^4)dx$.

- Write down the error for trapezium and Simpson's rule, as functions of $C$.
- Find $C$ that makes the error under trapezium rule is 0.
- Find the range of $C$ where the trapezium rule is more accurate than Simpson's rule.

2. (Problem 7.11 in textbook) Suppose $f \in C^4([-1,1])$. Let $p$ be the Hermite interpolation polynomial of $f$ at $x_0 = -1$, $x_1 = 1$.

- Calculate $\int_{-1}^1 pdx$ and write it as a linear combination of $f(\pm 1)$, $f'(\pm 1)$.
- Prove that

$$|\int_{-1}^1 fdx - \int_{-1}^1 pdx| \leq \frac{2}{45} \max_{c \in [-1,1]} |f^{(4)}(c)|$$

3. (Problem 10.3 in textbook) Show that if $f \in C^2([0,1])$, then there is some point $c \in (0,1)$ such that

$$\int_0^1 xfdx = \frac{1}{2}f(2/3) + \frac{1}{72}f''(c)$$

Hint: use Gauss quadrature with weight $x$.

4.

- Suppose $f$ is continuous on $[0,1]$. Let $I_n$ be the estimate of $\int_0^1 f dx$ using composite trapezium rule with $n$ subintervals. Show that

$$\lim_{n\to\infty} \left| \int_0^1 f dx - I_n \right| = 0$$

Hint: There are many possible approaches. You can use the fact that any continuous function on a closed interval is uniformly continuous, or use the Weierstrass approximation theorem.

- (Optional) Find a continuous function $f$, such that there is $C > 0$ such that

$$\left| \int_0^1 f dx - I_n \right| \geq \frac{C}{n}$$

Hint: if $f$ has bounded second derivative then the error decays like $O(1/n^2)$, so you need to find some $f$ that doesn't have second order derivative or has unbounded second order derivative.