

5.11 Let the unknowns be  $\lambda, x_1, \dots, x_n$ , and the system of equations be  $F(\lambda, x_1, \dots, x_n) = ((A - \lambda I)x, x^T x - 1) = 0$ . Then by calculation the Jacobian matrix is

$$J = \begin{bmatrix} A - \lambda I & -x \\ 2x^T & 0 \end{bmatrix}$$

Hence Newton's method gives us

$$\begin{bmatrix} x^{(1)} \\ \lambda^{(1)} \end{bmatrix} = \begin{bmatrix} x^{(0)} \\ \lambda^{(0)} \end{bmatrix} + \begin{bmatrix} \delta x \\ \delta \lambda \end{bmatrix}$$

$$J \begin{bmatrix} \delta x \\ \delta \lambda \end{bmatrix} = - \begin{bmatrix} (A - \lambda^{(0)} I)x^{(0)} \\ (x^{(0)})^T x^{(0)} - 1 \end{bmatrix}$$

Hence

$$(A - \lambda^{(0)} I)\delta x - \delta \lambda x^{(0)} = -(A - \lambda^{(0)} I)x^{(0)}$$

$$-x^{(0)T} \delta x = \frac{1}{2}(x^{(0)T} x^{(0)} - 1)$$

It's easy to see that the direction of eigenvectors changes in the same way as inverse iteration, the difference being that at each step their norms are not strictly normalized into 1.