# Honor's Assignment 2

December 6, 2020

1. (Exercise 7.13) Show that the composite Trapezium rule always give accurate answer to $\int_0^{2\pi} \sin(x)dx$.

2. (Exercise 10.7) Let $[a, b] = [-1, 1]$, let $p_{n-1}$ be the degree $n-1$ orthogonal polynomial of weight $1 - x^2$, and let $I_n$ be the quadrature rule where the quadrature points are roots of $(x^2 - 1)p_{n-1}(x)$.

- Show that if $q$ is a polynomial of degree no more than $2n - 1$, then $\int_{-1}^{1} q dx = I_n(q)$.
- Show that all quadrature weights are positive.
- Suppose $f$ is smooth, find a constant $C$ such that

$$|\int_{-1}^{1} f dx - I_n(f)| \leq C \max_{x \in [-1,1]} |f^{(2n)}(x)|$$

3. Consider the initial value problem $y' = \sin(y)$, $y(0) = 1$.

- Write down the formula for two step Adams-Bashforth.
- Show that the two step Adams-Bashforth has order of accuracy 2 for this problem.
- Suppose we use starting points $z(0) = 1$, $z(h) = 1 + h$ to carry out Adams-Bashforth till time $t = nh = 1$. Find number $C$ such that

$$|z(1) - y(1)| \leq Ch^2$$