HW 6

November 23, 2020

1. (Problem 12.7 in textbook) Consider solving $y' = f(t,y), y(0) = y_0$ using the trapezium method

$$z((n+1)h) = z(nh) + \frac{h}{2}(f((n+1)h, z((n+1)h)) + f(nh, z(nh)))$$

Suppose further that $|y'''|$ is uniformly bounded by M.

(i) Prove that

$$|\frac{y((n+1)h) - y(nh)}{h} - \frac{1}{2}(f((n+1)h, y((n+1)h)) + f(nh, y(nh)))| \leq \frac{Mh^2}{12}$$

Hint: You can prove it by applying integration by parts to $\int_{nh}^{(n+1)h}(x - hn)(x - nh - h)f'''(x)dx$.

(ii) Let $e_n = z(nh) - y(nh)$, and assume that f is L-Lipschitz with respect to the send parmeter, then

$$|e_{n+1}| \leq |e_n| + \frac{1}{2}hL(|e_{n+1}| + |e_n|) + \frac{h^3 M}{12}$$

2. (Problem 12.12 in textbook) Consider solving the initial value problem $y' = f(t,y), y(0) = y_0$ via linear multistep method:

$$z((n+3)h) + bz((n+1)h) + az(nh) = hf((n+2)h, z((n+2)h))$$

(i) Find a, b such that the method is consistent.

(ii) Verify that for such a, b, the order of accuracy is 1.

(iii) Show that for such a, b, the method is not zero stable.