

HW 6

December 2, 2020

1. (Problem 12.7 in textbook) Consider solving $y' = f(t, y)$, $y(0) = y_0$ using the trapezium method

$$z((n+1)h) = z(nh) + \frac{h}{2}(f((n+1)h, z((n+1)h)) + f(nh, z(nh)))$$

Suppose further that $|y'''|$ is uniformly bounded by M .

(i) Prove that

$$\left| \frac{y((n+1)h) - y(nh)}{h} - \frac{1}{2}(f((n+1)h, y((n+1)h)) + f(nh, y(nh))) \right| \leq \frac{Mh^2}{12}$$

Hint: You can prove it by applying integration by parts to $\int_{nh}^{(n+1)h} (x - hn)(x - nh - h)y'''(x)dx$.

(ii) Let $e_n = z(nh) - y(nh)$, and assume that f is L -Lipschitz with respect to the second parameter, then

$$|e_{n+1}| \leq |e_n| + \frac{1}{2}hL(|e_{n+1}| + |e_n|) + \frac{h^3M}{12}$$

2. (Problem 12.12 in textbook) Consider solving the initial value problem $y' = f(t, y)$, $y(0) = y_0$ via linear multistep method:

$$z((n+3)h) + bz((n+1)h) + az(nh) = hf((n+2)h, z((n+2)h))$$

- (i) Find a, b such that the method is consistent.
- (ii) Show that for such a, b , the method is not zero stable.