Honor's Assignment 2

November 23, 2020

1. (Exercise 7.13) Use relations $2\sin(a)\sin(b) = \cos(b-a) - \cos(b+a)$ to show that the composite Trapezium rule always give accurate answer to $\int_0^{2\pi} \sin(x)dx$.

2. (Exercise 10.7) Let $[a, b] = [-1, 1]$, let p_{n-1} be the degree $n-1$ Legendre polynomial, and let I_n be the quadrature rule where the quadrature points are roots of $(x^2-1)p_{n-1}(x)$.

- Show that if q is a polynomial of degree no more than $2n-1$, then $\int_{-1}^{1} q dx = I_n(q)$.
- Show that all quadrature weights are positive.
- Suppose f is smooth, find a constant C such that $|\int_{-1}^{1} f dx - I_n(f)| \leq C \max_{x\in[-1,1]} |f^{2n}(x)|$.

3. Suppose f is smooth and periodic with period 1, $|f''| \leq 1$. Let I_n be the result of composite quadrature rule for $\int_0^1 f dx$ using n subintervals. Find a number C such that

$$|\int_0^1 f dx - I_n(f)| \leq \frac{C}{n^2}$$

4. Consider the initial value problem $y' = \sin(y)$, $y(0) = 1$.

- Write down the formula for two step Adams-Bashforth.
- Show that the two step Adams-Bashforth has order of accuracy 2 for this problem.

- Suppose we use starting points $z(0) = 1$, $z(h) = 1 + h$ to carry out Adams-Bashforth till time $t = nh = 1$. Find number C such that

$$|z(1) - y(1)| \leq Ch^2$$