HW 6

November 23, 2020

1. (Problem 12.7 in textbook) Consider solving $y' = f(t, y), y(0) = y_0$ using the trapezium method

$$z((n+1)h) = z(nh) + \frac{h}{2}(f((n+1)h, z((n+1)h)) + f(nh, z(nh)))$$

Suppose further that $|y'''|$ is uniformly bounded by M.

(i) Prove that

$$|\frac{y((n+1)h) - y(nh)}{h} - \frac{1}{2}(f((n+1)h, y((n+1)h)) + f(nh, y(nh)))| \leq \frac{Mh^2}{12}$$

Hint: You can prove it by applying integration by parts to $\int_{nh}^{(n+1)h}(x - hn)(x - nh - h)f'''(x)dx$.

(ii) Let $e_n = z(nh) - y(nh)$, and assume that f is L-Lipschitz with respect to the send parmeter, then

$$|e_{n+1}| \leq |e_n| + \frac{1}{2}hL(|e_{n+1}| + |e_n|) + \frac{h^3 M}{12}$$

2. (Problem 12.12 in textbook) Consider solving the initial value problem $y' = f(t, y), y(0) = y_0$ via linear multistep method:

$$z((n+3)h) + bz((n+1)h) + az(nh) = hf((n+2)h, z((n+2)h))$$

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(i) Find a, b such that the method is consistent.

(ii) Show that for such a, b, the method is not zero stable.