Honor's Assignment 2

November 23, 2020

1. (Exercise 7.13) Show that the composite Trapezium rule always give accurate answer to $\int_0^{2\pi} \sin(x)dx$.

2. (Exercise 10.7) Let $[a,b] = [-1,1]$, let p_{n-1} be the degree $n-1$ Legendre polynomial, and let I_n be the quadrature rule where the quadrature points are roots of $(x^2-1)p_{n-1}(x)$.

- Show that if q is a polynomial of degree no more than $2n-1$, then $\int_{-1}^1 q dx = I_n(q)$.
- Show that all quadrature weights are positive.
- Suppose f is smooth, find a constant C such that

$$|\int_{-1}^{1} f dx - I_n(f)| \leq C \max_{x\in[-1,1]} |f^{(2n)}(x)|$$

3. Suppose f is smooth and periodic with period 1, $|f^{(4)}| \leq 1$. Let I_n be the result of composite trapezium rule for $\int_0^1 f dx$ using n subintervals. Find the largest integer d, and a number C, such that

$$|\int_0^1 f dx - I_n(f)| \leq \frac{C}{n^d}$$

Hint: You can do it using Hermit cubic spline. The integral of the linear spline is the result from composite trapezium rule, show that this integral is the same as the integral of the

Hermit cubic spline.

4. Consider the initial value problem $y' = \sin(y)$, $y(0) = 1$.

- Write down the formula for two step Adams-Bashforth.
- Show that the two step Adams-Bashforth has order of accuracy 2 for this problem.
- Suppose we use starting points $z(0) = 1$, $z(h) = 1 + h$ to carry out Adams-Bashforth till time $t = nh = 1$. Find number C such that

$$|z(1) - y(1)| \leq Ch^2$$