

Let $F(X_1, X_2) = P(Y = 1|X_1, X_2)$, and suppose the machine learning algorithm can capture F perfectly, then the ROC of the permutation of the second type is

$$\left(\sum_{F(a,b) > p} P(X_1 = a, X_2 = b|Y = 0), 1 - \sum_{F(a,b) < p} P(X_1 = a, X_2 = b|Y = 1) \right)$$

And the ROC of the permutation of the first type is:

$$\left(\sum_{F(a,b) > p} P(X_1 = a|Y = 0)P(X_2 = b|Y = 0), 1 - \sum_{F(a,b) < p} P(X_1 = a|Y = 1)P(X_2 = b|Y = 1) \right)$$

Hence, the AUC of the first permutation is no worse than the second iff

$$\int_0^1 \left(1 - \sum_{F(a,b) < p} P(X_1 = a|Y = 1)P(X_2 = b|Y = 1) \right) d \left(\sum_{F(a,b) > p} P(X_1 = a|Y = 0)P(X_2 = b|Y = 0) \right) \geq \int_0^1 \left(1 - \sum_{F(a,b) < p} P(X_1 = a, X_2 = b|Y = 1) \right) d \left(\sum_{F(a,b) > p} P(X_1 = a, X_2 = b|Y = 0) \right)$$

In particular, the AUCs are the same if X_1 and X_2 are conditionally independent with regards to Y .