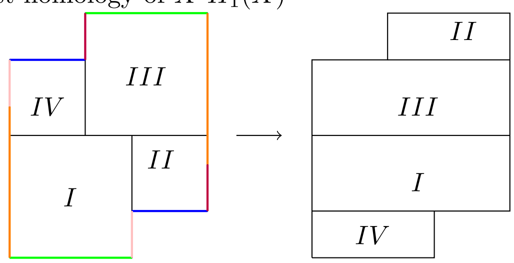
1. Let X be the polygon formed by two squares of size $\lambda = \frac{\sqrt{5}+1}{2}$ and two squares of size 1 in the picture below, where opposite sides with the same color are glued by translation. Show that the map shown below, where each square is flipped upside down and then scaled horizontally by $\frac{\sqrt{5}+1}{2}$ and vertically by $\frac{2}{\sqrt{5}+1}$, gives a diffeomorphism from X to itself. Find the induced map on the first homology of X $H_1(X)$



2. Show that if G is a finite directed graph with vertices labeled as $\{1, \dots, m\}$, it is strongly connected (given any pair of vertices there is a path from one to another), and the greatest common divisor of the length of loops in G is 1. Let M be a $m \times m$ matrix where the (i, j)-th entry of M is 1 if there is an edge from i to j and 0 if otherwise. Show that there is some N such that all entries of M^N are positive.

3. If $S_G \subset \{1, \dots, m\}^{\mathbb{Z}}$ is a topological markov chain defined by a directed graph as in the previous problem. Let P_n be the set of points $x \in S_G$ such that $\sigma^n(x) = x$. Let μ_n be defined as $\mu_n(A)$ equals the number of elements in $A \cap P_n$ divided by the number of elements of P_n. Show that as n increases μ_n weak-* converges to the measure defined using Perron-Frobenious theorem, i.e. the measure of maximal entropy.

4. Let S_G be the same as above, show that there is some $N > 0$, some σ^N-invariant set in S_G, such that σ^N action on it is conjugate to the shift map on $\{0, 1\}^{\mathbb{Z}}$ via an homeomorphism.