

1 9/5 Matrices, vectors, and their applications

Algebra: study of objects and operations on them.

Linear algebra: object: matrices and vectors. operations: addition, multiplication etc.

Algorithms/Geometric intuition/sets and maps

$m \times n$ matrix: numbers forming a rectangular grid, m rows and n columns. Motivation: coefficients of a system of linear equations. Data tables in statistics.

(i, j) -th entry of a matrix.

Vectors: matrices with one row/column. Motivation: coordinates in plane and space.

Operations: (1) Addition. (2) Scalar multiplication. (3) Matrix-vector multiplication. (4) Transpose.

Example: $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Ax , $A(Ax)$.

Example: Averaging over columns. Covariance? Other statistical concepts?

Laws: The usual laws one may expect. e.g. $A(x + y) = Ax + Ay$, $(A + B)^T = A^T + B^T$, $(A^T)^T = A$.
Note: $A(Bx) \neq B(Ax)$!

Zero and one matrix. Standard vectors.

Example: Rotation by 60 degrees (or $\pi/3$).

Consequence: Matrix is completely determined by its action on the standard vectors! Matrix-matrix multiplication.

Example: 2×2 case.

The concept of linear combination. Relationship with matrix-vector multiplication.

Example: Rotation and Translation.

Example: Random walk on graphs.