# 1 1.1

1. $\begin{bmatrix} 8 & -4 & 20 \\ 12 & 16 & 4 \end{bmatrix}$.

3. $\begin{bmatrix} 6 & -4 & 24 \\ 8 & 10 & -4 \end{bmatrix}$.

5. $\begin{bmatrix} 2 & 4 \\ 0 & 6 \\ -4 & 8 \end{bmatrix}$.

9. $\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 5 & 1 \end{bmatrix}$.

17. $A - B$ is undefined.

19. $\begin{bmatrix} 7 & 1 \\ -3 & 0 \\ 3 & -3 \\ 4 & -4 \end{bmatrix}$.

23. $\begin{bmatrix} -7 & -1 \\ 3 & 0 \\ -3 & 3 \\ -4 & 4 \end{bmatrix}$.

25. $-2$.

37-56. (T=True, F=False) TTTFFTFFTFTTTFTTTTTT

71. For example, the zero and identity matrices of size $2 \times 2$ and $3 \times 3$ are both symmetric.

75. $(B + B^T)^T = B^T + (B^T)^T = B^T + B = B + B^T$.

79. The $(i, i)$-th entry of $A^T$ is the same as the $(i, i)$-th entry of $A$. By skew-symmetry, it is also the negative of the $(i, i)$-th entry of $A$, hence it must be 0.

81. For any $3 \times 3$ matrix $A$, $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$.

82. (a) This is because the $(i, i)$-th entry of $A + B$ is the sum of the $(i, i)$-th entry of $A$ and the $(i, i)$-th entry of $B$.
(b) This is because the $(i, i)$-th entry of $cA$ is $c$ times the $(i, i)$-th entry of $A$.
(b) This is because the $(i, i)$-th entry of $A^T$ equals the $(i, i)$-th entry of $A$.

# 2 1.2

1. $\begin{bmatrix} 12 \\ 14 \end{bmatrix}$.

3. $\begin{bmatrix} 11 \\ 0 \\ 10 \end{bmatrix}$.

9. $\begin{bmatrix} as \\ bt \\ cu \end{bmatrix}$.

15. $\begin{bmatrix} 21 \\ 13 \end{bmatrix}$.

17. $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$.

19. $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{3\sqrt{3}-1}{2} \\ \frac{3+\sqrt{3}}{2} \end{bmatrix}$.

29. $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

31. $u$ is not a linear combination of elements of $\mathcal{S}$.

35. $u = 3 \begin{bmatrix} 1 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

37. The answer is not unique, e.g. $u = 7 \begin{bmatrix} 1 \\ 2 \end{bmatrix} - 2 \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

39. $u$ is not a linear combination of elements of $\mathcal{S}$.

45-63. TFTTTFFFTFTTFTFTFFT

67. $A_\theta(A_\beta v) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} ( \begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} )$

$$= \begin{bmatrix} (\cos(\theta)\cos(\beta) - \sin(\theta)\sin(\beta))v_1 - (\sin(\theta)\cos(\beta) + \cos(\theta)\sin(\beta))v_2 \\ (\cos(\theta)\cos(\beta) - \sin(\theta)\sin(\beta))v_2 + (\sin(\theta)\cos(\beta) + \cos(\theta)\sin(\beta))v_1 \end{bmatrix} = A_{\theta+\beta}v.$$

68. $A_\theta^T = A_{-\theta}$, hence by 67. both are $u$.

75. $Au = \begin{bmatrix} a \\ 0 \end{bmatrix}$.

76. $A(Au) = A \begin{bmatrix} a \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix} = Au$.

77. Such a vector $v$ must be of the form $\begin{bmatrix} a \\ 0 \end{bmatrix}$, hence $Av = \begin{bmatrix} a \\ 0 \end{bmatrix} = v$.

78. $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

# 3 1.3

1. $\begin{bmatrix} 0 & -1 & 2 \\ 1 & 3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 & 2 & 0 \\ 1 & 3 & 0 & -1 \end{bmatrix}$.

3. $\begin{bmatrix} 1 & 2 \\ -1 & 3 \\ -3 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 2 \\ -3 & 4 & 1 \end{bmatrix}$.

7. $\begin{bmatrix} 0 & 2 & -4 & 4 & 2 \\ -2 & 6 & 3 & -1 & 1 \\ 1 & -1 & 0 & 2 & -3 \end{bmatrix}$.

9. $\begin{bmatrix} 1 & -1 & 0 & 2 & -3 \\ 0 & 4 & 3 & 3 & -5 \\ 0 & 2 & -4 & 4 & 2 \end{bmatrix}$.

11. $\begin{bmatrix} 1 & -1 & 0 & 2 & -3 \\ -2 & 6 & 3 & -1 & 1 \\ 0 & 1 & -2 & 2 & 1 \end{bmatrix}$.

23. Yes.

25. No.

39. $x_1 = 2 + x_2$, $x_2$ free.

41. $x_1 = 2x_2 + 6$, $x_2$ free.

43. Inconsistent.

45. $x_1 = 4 + 2x_2$, $x_3 = 1/3$, $x_2$ free.

47. $x_4 \begin{bmatrix} 3 \\ 4 \\ -5 \\ 1 \end{bmatrix}$.

49. $\begin{bmatrix} -3 \\ -4 \\ 5 \\ 0 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

51 $\begin{bmatrix} 6 \\ 0 \\ 7 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ -4 \\ 1 \end{bmatrix}$

53 Inconsistent.

55. $n - k$, because a variable is either free or basic.

57-76. FFTFTTFTTFTTFTTTFTFT

81. There are 3 cases when the last row is non-zero, 3 when the last row is 0 and the first row isn't, and 1 when the matrix is zero, so 7 in total.

# 4 1.4

1 $x_1 = -2 - 3x_2$, $x_2$ free.

3 $x_2 = -5$, $x_1 = 4$.

5 Inconsistent.

7 $x_3 = 2$, $x_1 = 2x_2 - 1$, $x_2$ free.

11 $x_1 = -3x_2 + x_4 - 4$, $x_3 = 3 - 2x_4$, $x_2$, $x_4$ free.

13 Inconsistent.
17 $-12$.

19 Anything non-zero.

23 By row reduction one gets $\begin{bmatrix} -1 & r & 2 \\ 0 & r^2-9 & 6+2r \end{bmatrix}$. Hence 3.

27 When $r$ is not 2 it has exactly one solution, when $r$ is 2 and $s$ is 15 it has infinitely many solutions, when $r$ is 2 and $s$ is not 15 it has no solution.

35 Rank 3, nullity 1.

37 Rank 2, nullity 3.

43 (a) Mine 1: 10 days, Mine 2: 20 days, Mine 3: 25 days. (b) The system of equations has a unique solution which is not non-negative, hence no.

53-72. TFTTTTFFTTTFFTTFTTFT

74. 0. 0 matrix has rank 0.

75. 4. There can be at most one pivot per row.

76. 4. There can be at most one pivot per column.

77. 3. Because of problem 75.

78. 0. Because of problem 76.

81. No. Do row reduction of $A$, the last row must be 0. Do the reverse of the row reduction to the vector $e_4$, then it is a $b$ for which $Ax = b$ has no solution.

82. The rank of $A$ must be $n$ so that there aren't any free variable.

83. It can never have just one solution.

84. (a) $x_1 = 1$, $x_1 = 2$. (b) $x_1 = 1$, $2x_1 = 2$. (c) $x_1 + x_2 = 0$, $2x_1 + 2x_2 = 0$, $3x_1 + 3x_2 = 0$.

87. Yes. Because $A(cu) = c(Au) = c0 = 0$.

88. Yes. Because $A(u + v) = Au + Av = 0 + 0 = 0$.

89. $A(u - v) = Au - Av = b - b = 0$.

90. $A(u + v) = Au + Av = 0 + b = b$.

91. If there is some $v$ so that $Av = b$, then $A(cv) = cb$ hence $Ax = cb$ is consistent.

# 5 1.6

45-64. TTTFTTTFFFTTTTTTTTTT