1 1.1

1. $\begin{bmatrix} 8 & -4 & 20 \\ 12 & 16 & 4 \end{bmatrix}$.

3. $\begin{bmatrix} 6 & -4 & 24 \\ 8 & 10 & -4 \end{bmatrix}$.

5. $\begin{bmatrix} 2 & 4 \\ 0 & 6 \\ -4 & 8 \end{bmatrix}$.

9. $\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 5 & 1 \end{bmatrix}$.

17. $A - B$ is undefined.

19. $\begin{bmatrix} 7 & 1 \\ -3 & 0 \\ 3 & -3 \\ 4 & -4 \end{bmatrix}$.

23. $\begin{bmatrix} -7 & -1 \\ 3 & 0 \\ -3 & 3 \\ -4 & 4 \end{bmatrix}$.

25. -2.

37-56. (T=True, F=False) TTTFFTFFTFTTTFTTTTTT

71. For example, the zero and identity matrices of size 2×2 and 3×3 are both symmetric.

75. $(B + B^T)^T = B^T + (B^T)^T = B^T + B = B + B^T$.

79. The (i, i)-th entry of A^T is the same as the (i, i)-th entry of A. By skew-symmetry, it is also the negative of the (i, i)-th entry of A, hence it must be 0.

81. For any 3×3 matrix A, $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$.

82. (a) This is because the (i, i)-th entry of $A + B$ is the sum of the (i, i)-th entry of A and the (i, i)-th entry of B.
(b) This is because the (i, i)-th entry of cA is c times the (i, i)-th entry of A.
(b) This is because the (i, i)-th entry of A^T equals the (i, i)-th entry of A.

2 1.2

1. $\begin{bmatrix} 12 \\ 14 \end{bmatrix}$.

3. $\begin{bmatrix} 11 \\ 0 \\ 10 \end{bmatrix}$.

9. $\begin{bmatrix} as \\ bt \\ cu \end{bmatrix}$.

15. $\begin{bmatrix} 21 \\ 13 \end{bmatrix}$.

17. $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$.

19. $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{3\sqrt{3}-1}{2} \\ \frac{3+\sqrt{3}}{2} \end{bmatrix}$.

29. $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

31. u is not a linear combination of elements of $\mathcal{S}$.

35. $u = 3 \begin{bmatrix} 1 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

37. The answer is not unique, e.g. $u = 7 \begin{bmatrix} 1 \\ 2 \end{bmatrix} - 2 \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

39. u is not a linear combination of elements of $\mathcal{S}$.

45-63. TFTTTFFFTFTTFTFTFFT

67. $A_\theta(A_\beta v) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} (\begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix})$

$$= \begin{bmatrix} (\cos(\theta)\cos(\beta) - \sin(\theta)\sin(\beta))v_1 - (\sin(\theta)\cos(\beta) + \cos(\theta)\sin(\beta))v_2 \\ (\cos(\theta)\cos(\beta) - \sin(\theta)\sin(\beta))v_2 + (\sin(\theta)\cos(\beta) + \cos(\theta)\sin(\beta))v_1 \end{bmatrix} = A_{\theta+\beta}v.$$

68. $A_\theta^T = A_{-\theta}$, hence by 67. both are u.

75. $Au = \begin{bmatrix} a \\ 0 \end{bmatrix}$.

76. $A(Au) = A \begin{bmatrix} a \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix} = Au$.

77. Such a vector v must be of the form $\begin{bmatrix} a \\ 0 \end{bmatrix}$, hence $Av = \begin{bmatrix} a \\ 0 \end{bmatrix} = v$.

78. $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.