1 9/5 PDE models

ODE: equation for a univariate function that involves its derivatives.

PDE: equation for a multivariate function that involves its partial derivatives. Order of a pde. Linear PDE. Homogeneous PDE.

Some PDEs we will study later:
Heat: $u_t = u_{xx}$: (heat transmission, diffusion)
Laplace: $u_{xx} + u_{yy} = 0$: (static electric field, Newton's gravity, random walk)
Wave: $u_{tt} = u_{xx}$: (sound wave, other waves in physics)
Dispersive wave equations: $u_{tt} = u_{xx} - ku_{xxxx}$ (stiff string)
Cauchy-Riemann equation: $u_x = v_y$, $u_y = -v_x$

PDE you may see in later classes: Navier-Stokes, Nonlinear Schrodinger $iu_t = -\Delta u + k|u|^2 u$, KdV $u_t + u_{xxx} + 6uu_x = 0$, etc.

Example: growth of the zebra fish embryo. Baseline: GMCF (geodesic mean curvature flow) $u_t = A\frac{\nabla u}{|\nabla u|} \cdot \nabla u + B|\nabla u|\nabla \cdot \frac{\nabla u}{|\nabla u|}$.

General solution. Example: $u = u(x, t), u_t = x$.

Evolution model (with time): Boundary condition. Initial condition. Initial value problem. Initial-boundary value problem.

Steady state model (no time): boundary value problem.

Typical questions in the theory of PDE: Existence, uniqueness, regularity, continuous dependency on boundary, etc.

Typical strategy: integral transform: $(Tu)(y) = \int u(x)K(x, y)dx$, then $T(u_x) = \int u_x(x)K(x, y)dx = -\int u(x)K_x(x, y)dx$, assume some decay conditions on the boundary (or infinity). Connection with harmonic analysis. Use of symmetry (method of mirror images, spherical symmetry etc.)