Homework 3

1. Prove the shadowing lemma for circle doubling map. In other words, let $T : \mathbb{R}/\mathbb{Z}$ be $T(x) = 2x$, for any $\epsilon > 0$, there is some $\delta > 0$, such that any sequence $\{x_i\}$ that satisfies $|x_{i+1} - T(x_i)| \leq \delta$, there is some $\{y_i\}$ such that $|x_i - y_i| < \delta$ and $y_{i+1} = T(y_i)$.

2. Let f be a smooth map on interval $[0, 1]$. Suppose there is some positive number M, $k > 1$, such that the number of critical points of f^n is bounded by Mk^n, show that the topological entropy of f is no more than $\log(k)$.