Galois conjugate of entropies of interval maps

Chenxi Wu

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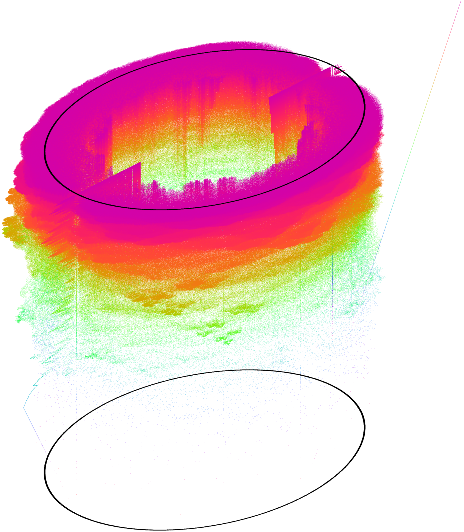
In collaboration with Kathryn Lindsey, Diana Davis, Harrison Bray and Giulio Tiozzo

Motivting question

If f is an unimodal map on a finite interval I, the unique critical point is sent to itself after some iteration, then the forward orbit of the critical point provides a Markov decomposition of I into subintervals. The leading eigenvalue of the incidence matrix is the exponent of the topological entropy of f, which must be an algebraic integer.
Question: which algebraic integers are the exp of such entropies?

Thurston's Teapot

Plot all $(z, \lambda) \in \mathbb{C} \times \mathbb{R}$, such that λ is the exp of some entropy and z a Galois conjugate. T is the closure of all such pairs.



Summary of results

1. If $(z, \lambda) \in T$, $|z| < 1$, then $(z, y) \in T$ for all $\lambda < y < 2$. ("persistence")
2. The part of the horizontal slices of T outside the unit circle changes continuously. (Known by Thurston, Tiozzo etc.)
3. There is an algorithm that can certify $(z, \lambda) \notin T$.

Main Tool: Admissibility criteria in Milnor-Thurston kneading theory

- If f is a unimodal interval map, which has positive derivative on the first interval and negative derivative on the second. Then, we define the "itinerary" of a point x to be $i \in \{0, 1\}^{\mathbb{N}}$, such that $i_j = 0$ or 1 if $f^{\circ j}(x)$ is in the first or second interval.
- Suppose i' is i with any prefix deleted. Then i' and i satisfies the following condition: if k is the first index where $i'_k \neq i_k$, then $(-1)^{\sum_{j<k} i_j}(i_k - i'_k) > 0$.
- Milnor-Thurston showed that this is actually "iff".
- If the ciritcal value b has periodic itinerary with period n, let $f_0(x) = zx$, $f_1(x) = 2 - zx$,

$$P(z) = f_{i_{n-1}} \circ \cdots \circ f_{i_0}(1) - 1$$

has the exp of entropy λ as a root. It is the "parry polynomial" P_λ.

Outline of the proof

1. Define T' to be the closure of $\{(z, \lambda) : P_\lambda(z) = 0\}$.
2. Use a long combinatorics argument to show results 1 and 2 when T is replaced with T'.
3. Use Eisenstein's criteria to show that for a dense set of z, P_λ is irreducible except some cyclotomic factors.
4. Use above two to show results 1 and 2
5. Use 1 and 2 plus the observation 3 above to find the algorithm.

Generalization to core entropy

- Question: Find properties of the core entropy of $z \mapsto z^2 + c$, where c is in the Mandelbrot set and is a postciritcally finite parameter.
- When c is on the real axis and has periodic critical orbit then it reduces to the study of unimodal interval maps.
- Tiozzo described a integer coefficient polynomial P_c such that the exp of core entropy is a root.

Some new results

- Along some veins, if λ is the exp of core entropy of a critically periodic parameter c, z is a Galois conjugate of λ, then for any c' which is closer to the tip of the vein, there is some critically periodic parameter c'' arbitrarily close to c' such that a root of $P_{c''}$ is arbitrarily close to z. (i.e. persistence along a vein).
- The roots of P_c outside the unit circle changes continuously with the turning angle of c. The continuity of core entropy itself was shown by Tiozzo.

References

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- Giulio Tiozzo. Topological entropy of quadratic polynomials and dimension of sections of the Mandelbrot set. Adv. Math. 273, pp. 651-715, 2015