1 Probability and random variables

- Probability: S sample space, $F \subset \mathcal{P}(S)$ a σ-algebra, $P : F \to \mathbb{R}$ a measure, such that $P(S) = 1$.
- Random variable: $X : S \to \mathbb{R}$, such that preimages of open sets are in F (i.e. has a well defined probability).
- (Cumulative) distribution function of random variable: $F_X(t) = P(X \leq t)$.
- Probability distribution of random variable: g such that $F_X(t) = \sum_{x \leq t, x \in C} g(x)$.
- Probability density function: f such that $F_X(t) = \int_{-\infty}^{t} f(s) ds$.

Example: uniform distribution.

1.1 Independence of random event, conditional probability

- $A, B \in F$ are independent iff $P(A \cap B) = P(A)P(B)$.
- If $P(B) \neq 0$, $P(A \cap B) = P(B)P(A|B)$. Here $P(A|B)$ is the conditional probability of A when B is known to happen.

1.2 Independence of random variables, conditional distribution

- X and Y are two random variables. The joint (cumulative) distribution function is $F(s, t) = P(X \leq s, Y \leq t)$
- If $F(s, t) = \sum_{(x,y) \in C, x \leq s, y \leq t} g(s, t)$, we call g the joint probability distribution.
- If $F(s, t) = \int_{(-\infty,s] \times (-\infty,t]} f(x, y) dx dy$ we call f the joint probability density.
- X and Y are called independent iff the joint cdf is $F(x, y) = F_X(x)F_Y(y)$.
- Knowing the joint distribution of X and Y, the distribution of X or Y are called the marginal distribution, their p.d. or p.d.f. the marginal p.d. or marginal p.d.f.
- If X and Y has a "good" joint probability sensity f, we can define conditional distribution of X at $Y = y$ as the one with desnity $\frac{f(x,y)}{h(y)}$ where h is the marginal p.d.f $h(y) = \int_{\mathbb{R}} f(x, y) dx$.

Example: X and Y are two independent random variable with uniform distribution on $[0, 1]$. What is the joint distribution function of X and Y? How about $max(X, Y)$ and $min(X, Y)$?

1.3 Expectation, moment generating function, characteristic function

Example

2 Special probability distributions, central limit theorem

3 Sample statistics

4 Point estimators and their properties

5 Method of moments, Maximum likelihood

6 Maximum a posteriori

7 Hypothesis testing

8 Examples of hypothesis testing

9 Confidence interval

10 Linear Regression

11 ANOVA

12 Example of non parametric methods