1. Let T be a map on X preserving a probability measure μ. Suppose μ measurable set A satisfies $\mu(A\backslash(T^{-1}(A))) = \mu(T^{-1}(A)\backslash A) = 0$, show that there is another measurable set B, such that $\mu(A\backslash B) = \mu(B\backslash A) = 0$ and $B = T^{-1}(B)$.

2. Let X be a compact separable metric space. Let $\{\phi_j\} \subset C(X)$ be a countable dense subset of $C(X)$, where topology is given by uniform convergence, M be the set of linear $\mathbb{R}$-valued functions on $C(X)$ such that 1 is sent to 1, and non negative functions are sent to non negative numbers. Prove that:

- The functions in M are all continuous. Hence $M \subset (C(X))^*$.
- For any $x \in M$, $x(\phi_j) \in [m_j, M_j]$, where m_j and M_j are the minimum and maximum of ϕ_j.
- Let $i : M \to \prod_j [m_j, M_j]$ be $x \mapsto (x(\phi_j))$. Here $\prod_j [m_j, M_j]$ has the product topology. Then M with weak-* topology is homeomorphic to $i(M)$ with subspace topology.
- $i(M)$ is closed in $\prod_j [m_j, M_j]$

3. Find a conjugate between circle doubling map $T : \mathbb{R}/\mathbb{Z} \to \mathbb{R}/\mathbb{Z}$, $T(x) = 2x$ and $U : \mathbb{R}/\mathbb{Z} \to \mathbb{R}/\mathbb{Z}$, $T(x) = 2x + 1/2$.

4. Classify the ergodic measures of a rational rotation $T : \mathbb{R}/\mathbb{Z}$, $T(x) = x + 1/3$.

5. Find an ergodic probability measure of the circle doubling map which is neither Lebesgue measure nor a linear combination of delta measures. (You may need to use ideas from symbolic dynamics, which we will discuss this week and next week).