RESEARCH STATEMENT

CHENXI WU

My research explores various topics in geometric topology and dynamics, including geometric group theory, low dimensional topology, symbolic dynamics, and the study of translation surfaces. The following are my current research projects:

1. Analogies of results for Riemann surface in the setting of metric graphs (Section 1)
2. Asymptotic translation length on curve complexes, free factor complexes and free splitting complexes. (Section 2)
3. Thurston's "master teapot" for interval maps of constant slope, and generalization to core entropies of quadratic maps. (Section 3)
4. Borel-Serre type compactification of the moduli space of translation surfaces into a real orbifold with corners. (Section 4)
5. Developing a Bowen-Series type coding for Kleinian groups. (Section 5)

A common theme in the first three projects is the analogy between Riemann surfaces and finite metric graphs, while a common idea in projects 3, 4 and 5 is to relate dynamics problems with combinatorics and formal languages.

1. Analogies of results for Riemann surface in the setting of metric graphs

A main motivation for this project is the analogies between closed Riemann surfaces and finite metric graphs. For example, since both graphs and closed hyperbolic surfaces are Eilenberg-MacLane spaces, the study of the outer automorphism groups of their fundamental groups ($Out(F_n)$ and mapping class groups, respectively), can often be reduced to understanding the dynamics of maps from graphs or surfaces to themselves, which makes it possible to study these groups by their action on the Culler-Vogtmann Outer space [CV] and the Teichmüller spaces respectively. The analogy can also be found in the context of tropical geometry, in the Berkovich spaces [B]. We found another such analogy by generalizing a classical theorem by Kazhdan to the context of graphs. The theorem by Kazhdan shows that one can obtain the hyperbolic metric on a closed Riemann surfaces via the canonical, or Arakelov metric, and we showed that an analogous procedure on finite metric graphs will also result in a metric.

1.1. Canonical metric on graphs.

Let G be a finite simplicial graph and l the length function on its edges. We have the following definitions:

Definition 1.1.

1. *By a* 1*-form on* G *we mean an element in the space of first cellular cochains* $C^1(G)$*. A norm on the space of* 1*-forms can be defined as* $\|w\| = \left(\sum_{e\in E(G)} \frac{w(e)^2}{l(e)}\right)^{1/2}$.
2. *We call a* 1*-form harmonic, if it is locally the coboundary of a graph harmonic function. More specifically, if for every vertex* v *in* G*, for all the outgoing edges* e_i *from* v*,* $\sum_i \frac{w(e_i)}{l(e_i)} = 0$.
3. *The canonical, or Arakelov, metric* [Z, BF] *on a metric graph is defined as follows: for every edge* e*, the length of* e *under the new metric is:*

$$l_{can}(e) = \sup_{\|w\|=1, w \text{ harmonic}} |w(e)| .$$

One of our motivation is the following classical result by Kazhdan:

Theorem 1. [K]*, also cf.* [M, Appendix] *Let* $S \leftarrow S_1 \leftarrow S_2 \ldots$ *be a tower of finite regular covers of a compact Riemann surface* S*,* $\cap_i \pi_1(S_i) = \{1\}$*, then the canonical metric on* S_i *are pullbacks of metric* d_i *defined on* S*, and* d_i *converges uniformly to a multiple of the hyperbolic metric. Here, the canonical metric on a compact Riemann surface* S *is defined as:*

$$\|v\| = \sup_{\|w\|=1, w \text{ holomorphic 1-form on } S} |w(v)|$$

Where v *is any tangent vector.*

Farbod Shokrieh and I proved the following analogy of Kazhdan's theorem in the case of finite metric graphs:

Theorem 2. [SW] *Let $G \leftarrow G_1 \leftarrow G_2 \dots$ be a tower of finite regular covers of a finite metric graph G, then the canonical metric on G_i are pullbacks of metrics d_i defined on G, and d_i converges uniformly to some limiting metric that depends only on G and $\cap_i \pi_1(G_i)$. More precisely, let $G \leftarrow G'$ be the regular cover defined by $\cap_i \pi_i(G_i)$, then the limiting metric pulls back to the canonical metric on G'.*

A restatement of the result, using the language of graph theory, is the following:

Theorem 3. [SW] *Let $G \leftarrow G_1 \leftarrow G_2 \dots$ be a tower of finite regular covers of a finite simplicial (undirected) graphs G. For each edge $e \in E(G)$, let e' be a preimage of e in G_i, $d_i(e)$ be the proportion of spanning trees in G_i which does not contain e', then $\lim_{i\to\infty} d_i(e)$ exists for all edges e.*

The main ingredients of the argument is Lück's approximation theorem [LS]. The argument can be easily generalized to other settings - e.g. Riemann surfaces, compact Riemannian manifolds or simplicial complexes with piecewise Euclidean structures, with suitably defined concept of canonical metric. One such generalization is outlined in [BSW].

For Riemann surfaces, the limit of the canonical metric for a tower of covers that converges to the universal cover is the hyperbolic metric. Hence, one may want to see the limit of the canonical metric of a tower of covers as the analogy of the hyperbolic metric for a metric graph. An observation by Huiping Pan is that if one uses the limiting metric as the new l, and goes through the process of calculating canonical metric on a tower of regular cover again and again, eventually the metric will converge. My collaborators and I are working on answering these remaining questions, as well as generalizing the result to the case of complex manifolds.

As a related question, there is a τ-invariant defined in [C, CR, Z], which is the capacity of a measure defined from the canonical metric mentioned above, and is conjectured to be bounded from below by $1/108$ of the the total edge lengths of the graph [C]. Many cases for this conjecture have been shown by Cinkir, and Farbod Shokrieh and I are currently working on proving or disproving this conjecture.

1.2. Spectral radius of the train track map on homology.

In [M], McMullen proved the following:

Theorem 4. [M] *Let ϕ be a pseudo-Anosov map on Riemann surface. Let λ be the stretch factor of ϕ on the invariant measured foliations, ρ be the size of the leading eigenvalue of the action of ϕ on the homology of the surface. Then, either $\lambda = \rho$ when passing to a double cover, or there is some $\epsilon > 0$ so that when passing to any finite cover of the surface, $\lambda - \rho > \epsilon$.*

A conjecture by Thomas Koberda asserts that there is an analogous result for graph homotopy equivalences that admits an irreducible train-track representation. More precisely, let ψ be a train track map on a finite graph, with irreducible incidence matrix. Then, either after passing to some finite cover the stretch factor is identical to the spectral radius of the induced map on homology, or there is some $\epsilon > 0$ such that when passing to arbitrary finite cover, the stretch factor is at least ϵ above the spectral radius of the induced map on homology. Farbod Shokrieh and I are working on proving or disproving this conjecture.

1.3. Generalization and alternative proofs for Riemann-Roch theorem on metric graphs.

There is a graph-theoretic Riemann-Roch theorem by Baker-Norine [BN], which is related to the question of chip-firing games on graphs, which is as follows:

Theorem 5. [BN] *Let G be a finite simplicial graph with no loops, all edges are assumed to have length 1. A* divisor *on G is an integer linear combination of vertices of G, i.e. an element in $C^0(G;\mathbb{Z})$, the degree is the sum of all the coefficients in this linear combination. A divisor D is called* effective *if all its coefficients are non negative, two divisors D and D' are* linearly equivalent *if $\delta d(D - D') = 0$, and $r(D)$ is the largest natural number r such that $D - E$ is linearly equivalent to an effective divisor for any effective divisor of degree r, or, if no such natural number exists, -1. Then*

$$r(D) - r(K - D) = deg(D) + 1 - g$$

Here g is the first betti number of G, K is the canonical divisor $K = \sum_v (\deg(v) - 2)(v)$.

The proof however is very different from the Riemann-Roch theorem on surfaces, as r can no longer be interpreted as the dimension of some vector space. The theorem can be generalized to arbitrary metric graphs as well, see Mikhalkin-Zharkov [MZ], Hladkỳ-Král-Norine [HKN], Gathmann-Kerber [GK] etc. Farbod Shokrieh and I managed to generalize it to the case when D is a more general signed measure on G, and we are working on finding a more topological or analytic proof of the theorem.

2. Asymptotic translation length on curve complexes, free factor complexes and free splitting complexes

2.1. Asymptotic translation length on curve complex and related questions. Let S be a surface of finite type. The *curve graph* of S, denoted as $\mathcal{C}(S)$, is the graph where the vertices are isotopy classes of simple closed curves on S and there is an edge between two vertices if they have disjoint representations. A metric can be assigned to the curve graph by setting all edge lengths to be 1. The mapping class group of S acts isometrically on S, and the *asymptotic translation length* of a mapping class g on $\mathcal{C} = \mathcal{C}(S)$ is

$$\ell_{\mathcal{C}}(g) = \lim_{n\to\infty} \frac{d_{\mathcal{C}}(g^n\gamma, \gamma)}{n}$$

where γ is any vertex in $\mathcal{C}$. $l_{\mathcal{C}}$ can be seen as a possible way to characterize the "combinatorial complexity" of the pseudo-Anosov map.

A motivating question of this project is to understand the relationship between the asymptotic translation lengths on curve graphs and group theoretic properties of elements in the mapping class group.

It is shown in [MM] that the curve graph is δ-hyperbolic, and that $\ell_{\mathcal{C}}$ is well defined and non-zero if g is pseudo-Anosov (not of finite order, and doesn't preserve any finite collection of disjoint simple closed curves up to isotopy). The technique in [MM] have been used by [GT, V, BSh] and others to provide asymptotics of the lower bound on $\ell_{\mathcal{C}}$ as the genus and number of punctures on S increases. Furthermore, in [BLM] it is shown that there is a relationship between $\ell_{\mathcal{C}}$ and the translation length $\ell_{\mathcal{T}}$ on the Teichmüller space under Teichmüller metric, which is that when $exp(\ell_{\mathcal{T}}) \leq g - 1/2$,

$$\ell_{\mathcal{C}} \leq \frac{4\ell_{\mathcal{T}}}{\log(g-1/2)}$$

In [KS], a sequence of pseudo-Anosov maps in different genus is constructed that realizes the asymptotic lower bound. The construction arises from an arithmetic sequence in a fibered cone of a hyperbolic 3-manifold. The concept of fibered cones is introduced in [T2]:

Definition 2.1. [T2] *Let M be a closed hyperbolic 3-manifold. A* fibered cone *is a rational cone (cone defined by linear inequalities with rational coefficients) of $H^1(M;\mathbb{Z})$, such that all integer cohomology class in it are pull backs of the generator of $H^1(S^1;\mathbb{Z})$ for some map $M \to S^1$ which is a surface bundle over the circle, which is maximal under containment.*

Generalizing the construction in [KS], Hyungryul Baik, Hyunshik Shin and I proved the following:

Theorem 6. [BShiW] *Suppose M is a closed hyperbolic 3-manifold and P a fibered cone* [T2] *in $H^1(M)$, L a rational subspace of $H^1(M)$ of dimendion r that intersects with P. For every primitive element $\alpha \in P \cap L \cap H^1(M;\mathbb{Z})$,*

$$l_{\mathcal{C}}(\phi_\alpha) \lesssim \|\alpha\|^{-(1+1/(r-1))} .$$

The main tool for this theorem is McMullen's Teichmüller polynomial [M2] which encodes the stretch factors of the monodromies of different fiberations of the same 3-manifold.

In a follow-up paper [BKSW], Hyungryul Baik, Eiko Kin, Hyunshik Shin, and I showed that this asymptotic upper bound is sharp when $r \leq 3$.

There is also the question of how constraints on the dimension of invariant cohomology may affect the minimal asymptotic translation length. For example, in [BSh], it is shown that

Theorem 7. [BSh] *The minimal asymptotic translation length on the curve graph of a pseudo-Anosov map in the Torelli group for closed surfaces of genus g grows at $O(1/g)$ as genus g increases.*

In a upcoming paper, together with my collaborators, we proved the following:

Theorem 8. *Let $L_c(k,g)$ be the minimal asymptotic translation length on curve graph of a pseudo-Anosov map on closed surface of genus at most g that preserved a subset of cohomology of dimension at least k. Then*

$$\frac{1}{g(2g-k+1)} \lesssim L_c(k,g) \lesssim \frac{k+1}{g^2} .$$

With my collaborators, I am working on strengthening the above-mentioned results in [BKSW] as well as Theorem 6, and possibly generalizing them to the setting of hierarchical hyperbolic spaces [BHS, MM2].

Furthermore, in [BKSW], we proved the following:

Theorem 9. [BKSW] *Suppose M is a closed hyperbolic 3-manifold and P a fibered cone in $H^1(M)$, L a rational subspace of $H^1(M)$ of dimension 2 that has non empty intersection with P. Then for all but finitely many primitive elements $\alpha \in P \cap L \cap H^1(M;\mathbb{Z})$, ϕ_α is a normal generator of the corresponding mapping class group.*

We are currently working to remove the condition that the dimension of L be 2.

2.2. **Asymptotic translation length on free factor and free splitting complexes.** The free factor and free splitting complexes are analogies of the curve complex in the study of $Out(F_n)$. The *free factor complex* [HV] is a simplicial complex whose vertices are proper free factors of F_n and a face correspond to a sequence of free factors ordered by inclusion, while the *free splitting complex* [HM] is a simplicial complex where the simplicies are splitting of F_n as graphs of groups with trivial edge groups, and gluing between simplicies are by edge collapsing. [BFe] and [HM, HM2] showed that they are both Gromov hyperbolic and characterized the $Out(F_n)$ elements which act on them loxidromically.

There is analogy of Thurston's fibered cone introduced in [DKL] which are called "McMullen cones" and "cone of sections", where each primitive integer vectors correspond to an element in some $Out(F_n)$. Together with Hyungryul Baik and Dongryul Kim, we proved the following analogy of Theorem 6, which will appear in an upcoming paper:

Theorem 10. *Let L be a rational slice of a proper subcone P' of a McMullen cone P, passing through origin. Then, for any primitive integer element $\beta \in L$, $l(\psi_\beta) \lesssim \|\beta\|^{-1-1/(d-1)}$, where $d = \dim(L)$, l is the asymptotic translation length on the 1-skeleton of the free factor or free splitting complexes, and $\psi_\beta \in Out(F_{\|\beta\|})$.*

We are working on understanding the lower bound in free factor/free splitting complex case as well as understanding their relationship with the curve complex case. In particular, we hope that by studying the $Out(F_n)$ action on the invariant train track we can get more insights on the asymptotic translation length on curve complexes.

3. Galois conjugate of entropies of interval maps and core entropy of complex quadratic maps

3.1. **Interval maps.** The dynamics of unimodal maps, as well as other continuous maps, on a finite interval is a classical subject that has been extensively studied. A main tool for studying interval maps is the kneading theory of Milnor-Thurston [MT], which says that the dynamic of such a map is semiconjugate to both a real quadratic map and a subshift. For any $\lambda \in (1, 2)$, let the tent map $f_\lambda : [0, 1] \to [0, 1]$ be defined as

$$f_\lambda(x) = \begin{cases} \lambda x & x \leq 1/\lambda \\ 2 - \lambda x & x > 1/\lambda \end{cases}$$

In [T], to study the number-theoretic properties of the entropies of 1-dimensional dynamical systems, Thurston proposed the "master teapot" which is a closed subset of $\mathbb{R}^3 = \mathbb{C} \times \mathbb{R}$, defined as follows:

$$T = \overline{\{(z, \lambda) : z \text{ is a Galois conjugate of } \lambda, \text{ for which there exists } n \in \mathbb{N},\ f_\lambda^{\circ n}(1) = 1\}}$$

Using kneading theory and the idea of "dominant words" in [Ti], Harrison Bray, Diana Davis, Kathryn Lindsey and I showed the following:

Theorem 11. [BDLW] *If $|z| = 1$ then $z \times [1, 2] \subset T$. If $(z, \lambda) \in T$, $|z| \leq 1$, then $z \times [\lambda, 2] \subset T$.*

In order to state our characterization of the master teapot, we introduce the following definitions:

Definition 3.1.

(i) Let $\lambda \in (1, 2)$. The itinerary of $x \in [0, 1]$ under f_λ, denoted as $it_\lambda(x)$, is the infinite string of 0 and 1, where the i-th entry is 1 iff the $i-1$-th iteration of x under f_λ is in $(1/\lambda, 1]$.
(ii) If s is an infinite string, $\sigma(s)$ is s with the first letter removed.
(iii) If $w = w_1 w_2 \ldots w_n$ is a finite word of length n consisting of 0 and 1, $F(z, w) = F_z^{w_1} \cdot F_z^{w_2} \ldots F_z^{w_n}(1)$, here $F_z^0(x) = x/z$, $F_z^1(x) = (2 - x)/z$.
(iv) We call a finite word w to be λ-suitable, if
  (a) For any $k \in \mathbb{N}$, $\sigma^k(w^\infty)$ is the itinerary of some $x \in [0, 1]$ under f_λ.
  (b) If a suffix of w is the same as a prefix of $it_\lambda(1)$, there are odd number of 1s in this suffix.

In a recent preprint [LW], Kathryn Lindsey and I showed this full characterization of Thurston's teapot:

Theorem 12. *$(z, \lambda) \in T$ iff one of the following is true:*
(i) $|z| = 1$, $\lambda \in [1, 2]$

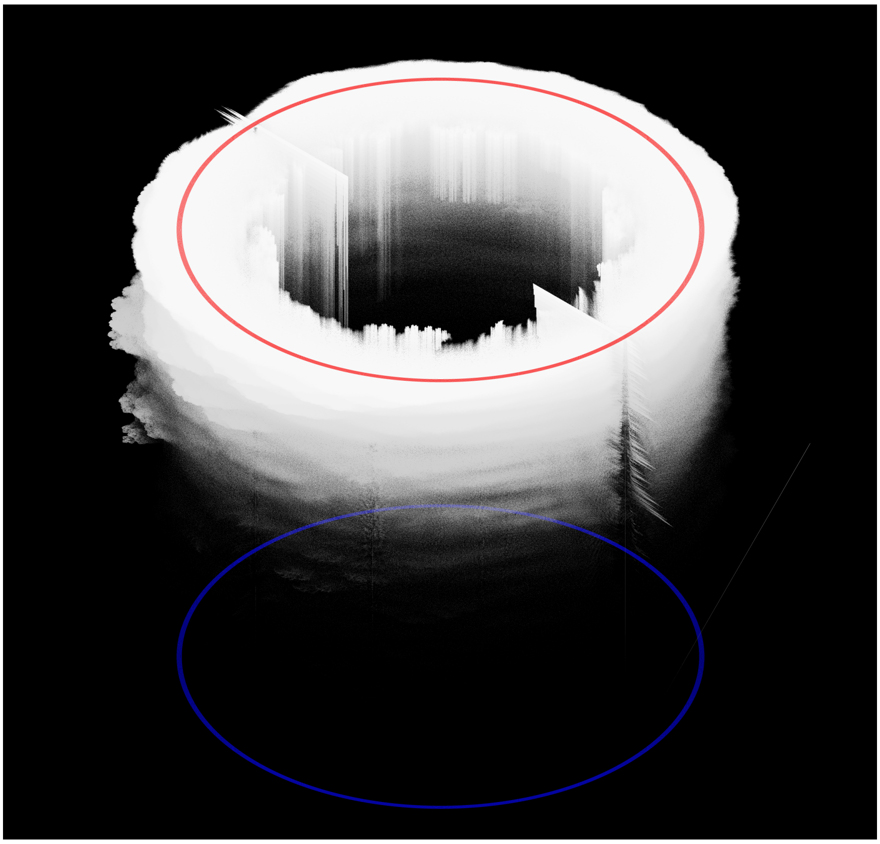

FIGURE 1. A picture of the Thurston's Teapot

(ii) $|z| > 1$, *and* $1/z$ *is a root of the kneading power series corresponding to* λ.
(iii) $|z| < 1$, $\lambda \in [\sqrt{2}, 2]$, *and there is some* $R > 0$, *such that for any* $\lambda' > \lambda$ *there are infinitely many* λ'*-suitable words* w *such that* $|F(z, w)| < R$
(iv) *There is some* k *such that* (z^{2^k}, λ^{2^k}) *is in one of the three previous cases.*

Kathryn Lindsey and I are currently working on to generalize this description to the case of more complicated interval maps. In particular, we will be considering this kind of interval maps (here $\lambda > 1$):

$$h_\lambda = \begin{cases} \lambda x - 2n & x \in [2n/\lambda, (2n+1)/\lambda] \\ 2n + 2 - \lambda x & x \in [(2n+1)/\lambda, (2n+2)/\lambda] \end{cases}$$

Based on numerical evidence, we conjectured that the analogy of Theorems 8 and 9 should be true for this new set

$$U = \overline{\{(z, \lambda) : z \text{ is a Galois conjugate of } \lambda, \text{ for which there exists } n \in \mathbb{N},\ h_\lambda^{\circ n}(1) = 1\}}$$

Kathryn Lindsey and I are currently work to prove the analogy of Julia-Mandelbrot correspondance (which asserts that locally, the Mandelbrot set resembles the corresponding Julia set) in the setting of Thurston teapots give a better explanation of the shape of the part of T and U inside the unit cylinder $\mathbb{D}_1 \times \mathbb{R}$. Based on numerical experiments we made the following conjecture:

Conjecture 3.2. Let $\lambda \in (1, 2)$, M_λ be the symbolic dynamics of all possible itinearies of the tent map of slope λ, z be a point inside the unit circle, Ω_λ the slice of Thurston's teapot at height λ, then the intersection of a neighborhood of z with Ω_λ will, up to scaling, be close to the intersection of limits of trajectories of the iterated function system $f_0 : x \mapsto zx$, $f_1 : x \mapsto 2 - zx$ labeled with sequences in M_λ intersecting with a neighborhood of 1.

3.2. **Maps on quadratic Hubbard tree.** The dynamics of unimodal interval maps can be seen as a special case of the dynamics on Hubbard trees of quadratic maps. Consider complex quadratic maps $f_c : z \mapsto z^2 + c$, we call it postcritically finite if the critical point 0 has finite forward orbit. A way to completely encode the dynamics of postcritically finite quadratic maps is through the concept of the

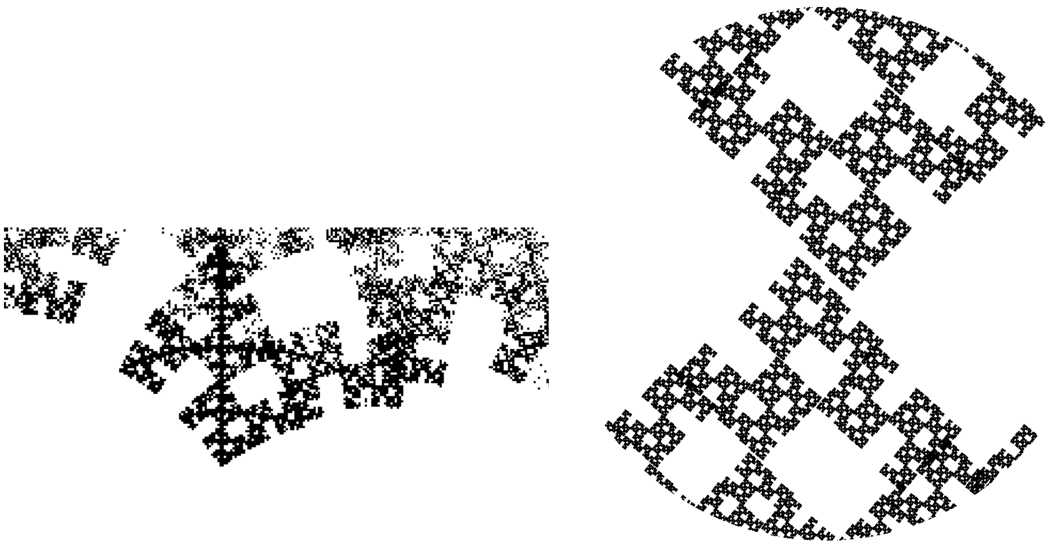

FIGURE 2. The neighborhood of a point on a horizontal slice of the teapot at $\lambda = 1.8$ (left), and the corresponding neighborhood in the limit set. (right)

Hubbard tree [DH], which is a tree connecting the forward orbit of 0 that lies within the filled Julia set. The quadratic map induces a continuous map on the Hubbard tree. The topological entropy of the map on the Hubbard tree is called by Thurston the *core entropy*. Thurston proposed a combinatorial algorithm [G] to calculate the core entropy from the direction of the corresponding external rays landing on the Mandelbrot set. Furthermore, Tiozzo showed in [Ti2] that the core entropy is related to the Hausdorff dimension of the set of external rays landing on the corresponding *principal vein* in the Mandelbrot set (which is an arc connecting origin to a point $c_{p/q}$, where the quadratic map sends the critical point to a fixed point where the external ray of the filled Julia set has angle 0, in exactly q steps), and in [Ti2] that the core entropy is continuous as a function of the angle of the external ray associated with it, resolving a conjecture by Thurston.

A critically periodic unimodal map can be seen as a superattracring parameter on the real slice of the Mandelbrot sets, so the study of core entropy is a generalization of the study of the topological entropy of these interval maps.

Together with Kathryn Lindsey and Giulio Tiozzo, by using the "combinatorial surgery" technique in [Ti2], I showed that there is an analogy of Theorem 12 along other principal veins of the Mandelbrot set. We are working on extending this result to the whole Mandelbrot set.

4. Borel-Serre type compactification of the moduli space of translation surfaces

A finite translation surface is a holomorphic differential on a closed Riemann surface. The set of translation surfaces where the orders of zeros are fixed and matched by the entries of a vector k, form an affine manifold under the "period coordinates", denoted as $\mathcal{H}(k)$ cf. [Wr]. There is a $GL(2,\mathbb{R})$ action on $\mathcal{H}(k)$ defined by

$$\begin{pmatrix} Re(Aw) \\ Im(Aw) \end{pmatrix} = A \begin{pmatrix} Re(w) \\ Im(w) \end{pmatrix}$$

This $GL(2,\mathbb{R})$ action is called the *affine action*. The action of the subgroup $\begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix}$ is called the *horocycle action*. The study of these two actions are connected to many questions in dynamics, such as the study of polygonal billiards and interval exchange maps. It was established in the seminal paper [EMM] that $GL(2,\mathbb{R})$ orbit closures of $\mathcal{H}(k)$ are affine submanifolds. In contrast, the recent work by Chaika-Smillie-Weiss shows that the horocycle closures are generally not submanifolds at all.

Many approaches have been proposed so far to give a compactification for the moduli space of finite translation surfaces $\mathcal{H}(k)$. For example, the "WYSIWYG" bordification in [MW] has been used to study the dynamics of affine invariant subspaces in such moduli spaces, and there is a compactification proposed in [BCGGM] that is known to have good properties and is particularly useful in the study of the geometry of such moduli spaces. Motivated by the Borel-Serre compactification [BS] and the Bestvina-Horbez compactification of outer space [BH], John Smillie and I are working on a bordification of the moduli space of translation surfaces as a (real) orbifold with corners (a space where around every point, there is a local coordinate chart which is a product of open and half open intervals). A points on this bordifications can be described with the following data:

- A *component diagram*, which is a finite graph G with some half edges which is divided into levels, such that every edge between levels is assigned a non-zero integer, every edge within a level is assigned 0, and the half edges are assigned numbers which are the entries of the vector k plus 1.
- A set of meromorphic differentials on possibly disjoint Riemann surfaces assigned to each level of G, such that each connected component of the Riemann surface corresponds to a vertex in G, each zero, pole or marked point correspond to an edge or a half edge, with turning angles determined by the assigned numbers, and the residue of the poles satisfy certain linear conditions.
- For each pair of matching poles or pole and zero, a gluing map along the boundary created via real oriented blowups.

Together with John Smillie, I am investigating the properties of this bordification and its application to study the fundamental groups of moduli spaces of translation surfaces, the geometry and dynamics of horocycle flows and the structure of the sup norm balls in the strata. Here the sup norm on the strata is defined via a norm on the cotangent space $T_M = H^1(M, \Sigma)$ (here M is a translation surface and σ its cone points) as the supremum of its pairing with homology classes defined by saddle connections divided by the length of said saddle connection. In particular, we are able to show that, the sup norm admits a product structure compatible with the Borel-Serre compactification.

5. Developing a Bowen-Series type coding for Kleinian groups

Bufetov-Klemenko-Series [BKS] described a coding of elements of a Fuchsian group by admissible sequences of a geometric Markov chain, which is motivated by Bowen-Series [BowS] and Wroten [Wro]. Here, a Geometric Markov chain is a generating set S together with a finite directed graph with two vertices sets $\mathcal{S}$ and $\mathcal{E}$, each vertex v labeled with g_v, such that the map $(v_1 \to v_2 \to \cdots \to v_n) \mapsto g_{v_n} g_{v_{n-1}} \dots g_{v_1}$ sends the set of paths of length n on the graph from $\mathcal{S}$ to $\mathcal{E}$ surjectively to the set of group elements of word length $n-1$.

This coding is bijective, and the finite directed graph used in the coding admits an involution that exchanges the sets $\mathcal{S}$ and $\mathcal{E}$ and is compatible with the map $g \mapsto g^{-1}$. Bufetov, Klemenko, and Series used this coding map to show the a.s. and L^2 convergence of spherical averages of the Fuchsian group action on functions on Lebesgue probability spaces. Together with Bufetov and Klemenko, we are able to generalize this Wroten-style coding to the case of cocompact Kleinian groups and prove the following:

Theorem 13. *If G is a Kleinian group with a compact, polygonal fundamental domain D, such that the 2-skeleton of the dual complex of the tiling of $\mathbb{H}^3$ by the G-orbit of D is a $C(4)-T(4)-P$ small cancellation complex* [GS]*, then elements of G admits a Wroten-style coding with an involution as in* [BKS]*. Let G_D be the graph whose vertices are the faces of D, where two are connected iff the corresponding face are not adjacent. Let F_D be the graph whose vertices are the faces of D, where two are connected iff the corresponding face are adjacent. If G_D is connected, and the 1-neighborhood of every subset of F_D of diameter 1 does not cover all vertices, then the graph representing the geometric Markov chain used in this coding is strongly connected.*

We are working on relaxing the assumptions on G in the theorem above, as well as showing further properties of this coding in order to obtain results on ergodicity as in [BKS].

6. Publications and preprints

The followings are my current publications and preprints:

- C. Wu. Deloné property of the holonomy vectors of translation surfaces. *Israel Journal of Mathematics* 214(2):733-740, 2016.
- C. Wu. Lattice surfaces and smallest triangle. *Geom Dedicata*, 2016.
- C. Wu. The relative cohomology of abelian covers of the flat pillowcase. *Journal of Modern Dynamics* 9:123-140, 2015.
- L. Clavier, A. Randecker, and C. Wu. Rotational component spaces for infinite-type translation surfaces. *Geometriae Dedicata* 201(1), 57-80, 2019.
- F. Shokrieh and C. Wu. Canonical measures on metric graphs and a Kazhdan's theorem *Invent. Math.* 215(3), 819-862, 2019.

- H. Baik, A. Rafiqi, and C. Wu. Constructing pseudo-Anosov maps with given dilatations. *Geometriae Dedicata* 180(1):39-48, 2016
- H. Baik, A. Rafiqi, and C. Wu. Is a typical bi-Perron number a pseudo-Anosov dilatation? *Ergodic Theory and Dynamical Systems* 39(7), 1745-1750, 2019.
- H. Baik, F. Shokrieh, and C. Wu. Limits of canonical forms on towers of Riemann surfaces *J. Reine Angew. Math.* 2019.
- Hyungryul Baik, Hyunshik Shin and Chenxi Wu. An upper bound on the asymptotic translation length on the curve graph and fibered faces *arXiv:* 1801.06638, accepted in *Indiana University Math Journal*, 2020.
- Harrison Bray, Diana Davis, Kathryn Lindsey and Chenxi Wu. The shape of Thurston's Master Teapot *arXiv:*1902.10805, accepted in *Advances in Mathematics*, 2020.
- Hyungryul Baik, Eiko Kin, Hyunshik Shin and Chenxi Wu. Asymptotic translation length and normal generation for the fibered cone *arXiv:*1909.00974
- K. Lindsey and C. Wu. A characterization of Thurston's Master Teapot. *arXiv*: 1909.10675

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