1 1.1

2. $u_{xx} + u_{yy} = (2x \cdot \frac{1}{2}(x^2+y^2)^{-1})_x + (2y \cdot \frac{1}{2}(x^2+y^2)^{-1})_y = 2(x^2+y^2)^{-1} - (2x^2+2y^2)\cdot(x^2+y^2)^{-2} = 0.$

4. The statement of this problem is somewhat unclear in whether they mean $(u_{xx})^2 + (u_{yy})^2 = 0$ (the more likely one) or $(u^2)_{xx} + (u^2)_{yy} = 0$, so either interpretation would be considered correct. With the first interpretation it is obvious that all function in the stated form satisfy that $u_{xx} = u_{yy} = 0$. With the second there would need to be additional constraints on a, b, c, d for it to work.

5. The general solution is $u = xF(t) + G(t)$, hence one can let $u = t^2 + x(1-t^2)$.

6. $u_{tt} = (g(x+ct) + g(x-ct))_t = c(g'(x+ct) - g'(x-ct))$, $u_{xx} = c^{-1}(g(x+ct) - g(x-ct))_x = c^{-1}(g'(x+ct) - g'(x-ct))$.

7. $(e^{at}\sin bx)_t = ae^{at}\sin bx$, $(e^{at}\sin bx)_{xx} = -b^2e^{at}\sin bx$, hence $a = -kb^2$.

8. $(u_x)_t = 1 - 3u_x$, hence $u_x = \frac{1}{3} + e^{-3t}f(x)$ for some arbitrary function f, hence $u(x,t) = \frac{x}{3} + e^{-3t}F(x) + G(t)$ for arbitrary function F (which is the anti-derivative of f) and G.

12. To sketch wave profile, pick some k, A, D or c, sketch $u(x,t)$ for different values of t, and if u is complex-valued you can sketch either the real or imaginary part.

Dispersion relations: a) $\omega = -iDk^2$. b) $\omega = \pm ck$. c) $\omega = -k^3$. d) $\omega = k^2$. e) $\omega = ck$.

14. Dispersion relation is $\omega = (-1 + \delta k^2 - k^4)i$ hence this is diffusive. When $\delta = k^2 + 1/k^2$ the solution has growth rate 0. When $k^2 + 1/k^2 > \delta$ the solution decays.

2 1.2

1. From equation (1.7) in the text we have $\frac{d}{dt}\int_a^b uAdx = A\phi|_a - A\phi|_b$. Differentiate with respect to b (or use some other argument, for example as in the textbook), we have $Au_t = -A_x\phi - A\phi_x$, hence $u_t + \phi_x = -A'\phi/A$.

3. By chain rule, $u_x = u_\xi$, $u_t = -cu_\xi + u_\tau$, hence the equation (1.12) becomes $u_\tau = -\lambda u$, hence the general solution is $u = e^{-\lambda\tau}F(\xi) = e^{-\lambda t}F(x - ct)$.