

Example: N points arranged into a circle, adjacent points have different colors, each point has one of 4 colors. What is the number of possibilities of coloring these N points?

Answer: When $N = 2$ there are 12 possibilities. When $N = 3$ there are 24 possibilities. Let C_n be the number of possible colorings with n points. Pick one point on the circle, the two adjacent points can have the same color or different color. In the former case, remove the point and identify its neighbors, we get $n - 2$ points on a circle where adjacent ones have different colors, hence C_{n-2} possibilities; with the latter, remove the point and make its neighbors adjacent, we have C_{n-1} possibilities. Hence $C_n = 3C_{n-2} + 2C_{n-1}$. We also know that $C_2 = 12$ and $C_3 = 24$, hence we can calculate C_n for all n using this inductive relationship. A closed form for C_n can be obtained via linear difference equations.

Question:

- How about 4 kinds of cubes arranged into one layer of the pyramid (of length $4n$)?
- Knowing the configuration of one layer of the pyramid, can you tell how many configurations are for the next layer?
- Can you find a formula calculating the possible number of configurations for a “section” of the pyramid consisting of 2 adjacent layers? How about 3?