

## 1 Linked Lists

There are many different implementations of sequences in Python. Today, we'll explore the linked list implementation.

A linked list is either an empty linked list, or a `Link` object containing a `first` value and the `rest` of the linked list.

To check if a linked list is an empty linked list, compare it against the class attribute `Link.empty`:

```
if link is Link.empty:
    print('This linked list is empty!')
else:
    print('This linked list is not empty!')
```

## Implementation

```
class Link:
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest:
            rest_str = ', ' + repr(self.rest)
        else:
            rest_str = ''
        return 'Link({0}{1})'.format(repr(self.first), rest_str)

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ' '
            self = self.rest
        return string + str(self.first) + '>'
```

## A Brief Note on Str and Repr

There are two main ways to produce the "string" of an object in Python: `str()` and `repr()`. While the two are similar, they are used for different purposes. `str()` is used to describe the object to the end user while `repr()` is mainly used for debugging and development.

When we define a class in Python, `str()` and `repr()` are both built-in functions for the class. We can call an object's `str()` and `repr()` by using their respective functions. In addition, the `print()` function calls the `str()` function of the object, while simply calling the object in interactive mode calls the `repr()` function. Here's an example:

```
class Test:
    def __str__(self):
        return "str"
    def __repr__(self):
        return "repr"
```

```
>>> a = Test()
>>> str(a)
'str'
>>> repr(a)
'repr'
>>> print(a)
str
>>> a
repr
```

## Questions

- 1.1 Write a function that takes in a linked list and returns the sum of all its elements. You may assume all elements in `lnk` are integers.

```
def sum_nums(lnk):
    """
    >>> a = Link(1, Link(6, Link(7)))
    >>> sum_nums(a)
    14
    """

    if lnk == Link.empty:
        return 0
    return lnk.first + sum_nums(lnk.rest)
```

- 1.2 Write a function that takes in a Python list of linked lists and multiplies them element-wise. It should return a new linked list.

If not all of the `Link` objects are of equal length, return a linked list whose length is that of the shortest linked list given. You may assume the `Link` objects are shallow linked lists, and that `lst_of_lns` contains at least one linked list.

```
def multiply_lns(lst_of_lns):
    """
    >>> a = Link(2, Link(3, Link(5)))
    >>> b = Link(6, Link(4, Link(2)))
    >>> c = Link(4, Link(1, Link(0, Link(2))))
    >>> p = multiply_lns([a, b, c])
    >>> p.first
    48
    >>> p.rest.first
    12
    >>> p.rest.rest.rest is Link.empty
    True
    """
```

Recursive solution:

```
product = 1
for lnk in lst_of_lns:
    if lnk is Link.empty:
        return Link.empty
    product *= lnk.first
lst_of_lns_rests = [lnk.rest for lnk in lst_of_lns]
return Link(product, multiply_lns(lst_of_lns_rests))
```

For our base case, if we detect that any of the lists in the list of `Links` is empty, we can return the empty linked list as we're not going to multiply anything.

Otherwise, we compute the product of all the firsts in our list of `Links`. Then, the subproblem we use here is the rest of all the linked lists in our list of `Links`. Remember that the result of calling `multiply_lns` will be a linked list! We'll use the product we've built so far as the first item in the returned `Link`, and then the result of the recursive call as the rest of that `Link`.

Iterative solution:

```
import operator
from functools import reduce
def prod(factors):
    return reduce(operator.mul, factors, 1)

head = Link.empty
tail = head
while Link.empty not in lst_of_lns:
    all_prod = prod([l.first for l in lst_of_lns])
    if head is Link.empty:
        head = Link(all_prod)
        tail = head
    else:
        tail.rest = Link(all_prod)
        tail = tail.rest
    lst_of_lns = [l.rest for l in lst_of_lns]
return head
```

The iterative solution is a bit more involved than the recursive solution. Instead of building the list “backwards” as in the recursive solution (because of the order that the recursive calls result in, the last item in our list will be finished first), we'll build the resulting linked list as we go along.

We use `head` and `tail` to track the front and end of the new linked list we're creating. Our stopping condition for the loop is if any of the `Links` in our list of `Links` runs out of items.

Finally, there's some special handling for the first item. We need to update both `head` and `tail` in that case. Otherwise, we just append to the end of our list using `tail`, and update `tail`.

- 1.3 Implement `filter_link`, which takes in a linked list `link` and a function `f` and returns a generator which yields the values of `link` for which `f` returns `True`.

Try to implement this both using a while loop and without using any form of iteration.

```
def filter_link(link, f):
    """
    >>> link = Link(1, Link(2, Link(3)))
    >>> g = filter_link(link, lambda x: x % 2 == 0)
    >>> next(g)
    2
    >>> next(g)
    StopIteration
    >>> list(filter_link(link, lambda x: x % 2 != 0))
    [1, 3]
    """
```

```
while _____:
```

```
    if _____:
```

```
        _____
```

```
    _____
```

```
def filter_link(link, f):
    while link is not Link.empty:
        if f(link.first):
            yield link.first
        link = link.rest
```

```
def filter_no_iter(link, f):
    """
    >>> link = Link(1, Link(2, Link(3)))
    >>> list(filter_no_iter(link, lambda x: x % 2 != 0))
    [1, 3]
    """
```

```
if _____:
```

```
    return
```

```
elif _____:
```

```
    _____
```

```
    _____
```

```
def filter_no_iter(link, f):  
    if link is Link.empty:  
        return  
    elif f(link.first):  
        yield link.first  
    yield from filter_no_iter(link.rest, f)
```

## 2 Trees

Recall the tree abstract data type: a tree is defined as having a label and some branches. Previously, we implemented the tree abstraction using Python lists. Let's look at another implementation using objects instead:

```
class Tree:
    def __init__(self, label, branches=[]):
        for b in branches:
            assert isinstance(b, Tree)
        self.label = label
        self.branches = branches

    def is_leaf(self):
        return not self.branches
```

Notice that with this implementation we can mutate a tree using attribute assignment, which wasn't possible in the previous implementation using lists.

```
>>> t = Tree(3, [Tree(4), Tree(5)])
>>> t.label = 5
>>> t.label
5
```

## Questions

- 2.1 Define a function `make_even` which takes in a tree `t` whose values are integers, and mutates the tree such that all the odd integers are increased by 1 and all the even integers remain the same.

```
def make_even(t):
    """
    >>> t = Tree(1, [Tree(2, [Tree(3)]), Tree(4), Tree(5)])
    >>> make_even(t)
    >>> t.label
    2
    >>> t.branches[0].branches[0].label
    4
    """

    if t.label % 2 != 0:
        t.label += 1
    for branch in t.branches:
        make_even(branch)
```

- 2.2 Define a function `square_tree(t)` that squares every value in the non-empty tree `t`. You can assume that every value is a number.

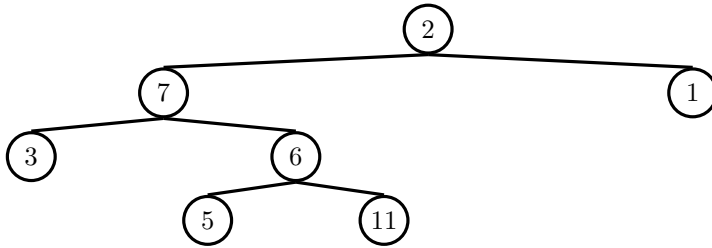
```
def square_tree(t):
    """Mutates a Tree t by squaring all its elements."""
```

```
    t.label = t.label ** 2
    for branch in t.branches:
        square_tree(branch)
```

- 2.3 Define the procedure `find_path` that, given a Tree `t` and an `entry`, returns a list containing the nodes along the path required to get from the root of `t` to `entry`. If `entry` is not present in `t`, return `False`.

Assume that the elements in `t` are unique. Find the path to an element.

For instance, for the following tree `tree_ex`, `find_path` should return:



```
>>> tree_ex = Tree(2, [Tree(7, [Tree(3), Tree(6, [Tree(5), Tree(11)])]), Tree(1)])
>>> find_path(tree_ex, 5)
[2, 7, 6, 5]
```

```
def find_path(t, entry):
```

```
    if t.label == entry:
        return [entry]
    for b in t.branches:
        path = find_path(b, entry)
        if path:
            return [t.label] + path
    return False
```



- 2.4 Assuming that every value in `t` is a number, define `average(t)`, which returns the average of all the values in `t`. You may not need to use all the provided lines.

```
def average(t):
    """
    Returns the average value of all the nodes in t.
    >>> t0 = Tree(0, [Tree(1), Tree(2, [Tree(3)])])
    >>> average(t0)
    1.5
    >>> t1 = Tree(8, [t0, Tree(4)])
    >>> average(t1)
    3.0
    """
    def sum_helper(t):

        total, count = _____

        for _____:

            _____

            _____

            _____

        return total, count

    total, count = _____

    return total / count

def sum_helper(t):
    total, count = t.label, 1
    for b in t.branches:
        b_total, b_count = sum_helper(b)
        total += b_total
        count += b_count
    return total, count
total, count = sum_helper(t)
return total / count
```

[Video walkthrough](#)

- 2.5 Write a function that combines the values of two trees `t1` and `t2` together with the `combiner` function. Assume that `t1` and `t2` have identical structure. This function should return a new tree.

Hint: consider using the `zip()` function, which returns an iterator of tuples where the first items of each iterable object passed in form the first tuple, the second items are paired together and form the second tuple, and so on and so forth.

```
def combine_tree(t1, t2, combiner):
    """
    >>> a = Tree(1, [Tree(2, [Tree(3)])])
    >>> b = Tree(4, [Tree(5, [Tree(6)])])
    >>> combined = combine_tree(a, b, mul)
    >>> combined.label
    4
    >>> combined.branches[0].label
    10
    """

    combined = [combine_tree(b1, b2, combiner) for b1, b2
                 in zip(t1.branches, t2.branches)]
    return Tree(combiner(t1.label, t2.label), combined)
```

- 2.6 Implement the `alt_tree_map` function that, given a function and a `Tree`, applies the function to all of the data at every other level of the tree, starting at the root.

```
def alt_tree_map(t, map_fn):
    """
    >>> t = Tree(1, [Tree(2, [Tree(3)]), Tree(4)])
    >>> negate = lambda x: -x
    >>> alt_tree_map(t, negate)
    Tree(-1, [Tree(2, [Tree(-3)]), Tree(4)])
    """

    def helper(t, depth):
        if depth % 2 == 0:
            label = map_fn(t.label)
        else:
            label = t.label
        branches = [helper(b, depth + 1) for b in t.branches]
        return Tree(label, branches)
    return helper(t, 0)
```

Alternate solution without a helper function:

```
def alt_tree_map(t, map_fn):
    label = map_fn(t.label)
    branches = []
    for b in t.branches:
```

```
    next_branches = [alt_tree_map(bb, map_fn) for bb in b.branches]
    branches.append(Tree(b.label, next_branches))
return Tree(label, branches)
```