Data Mining

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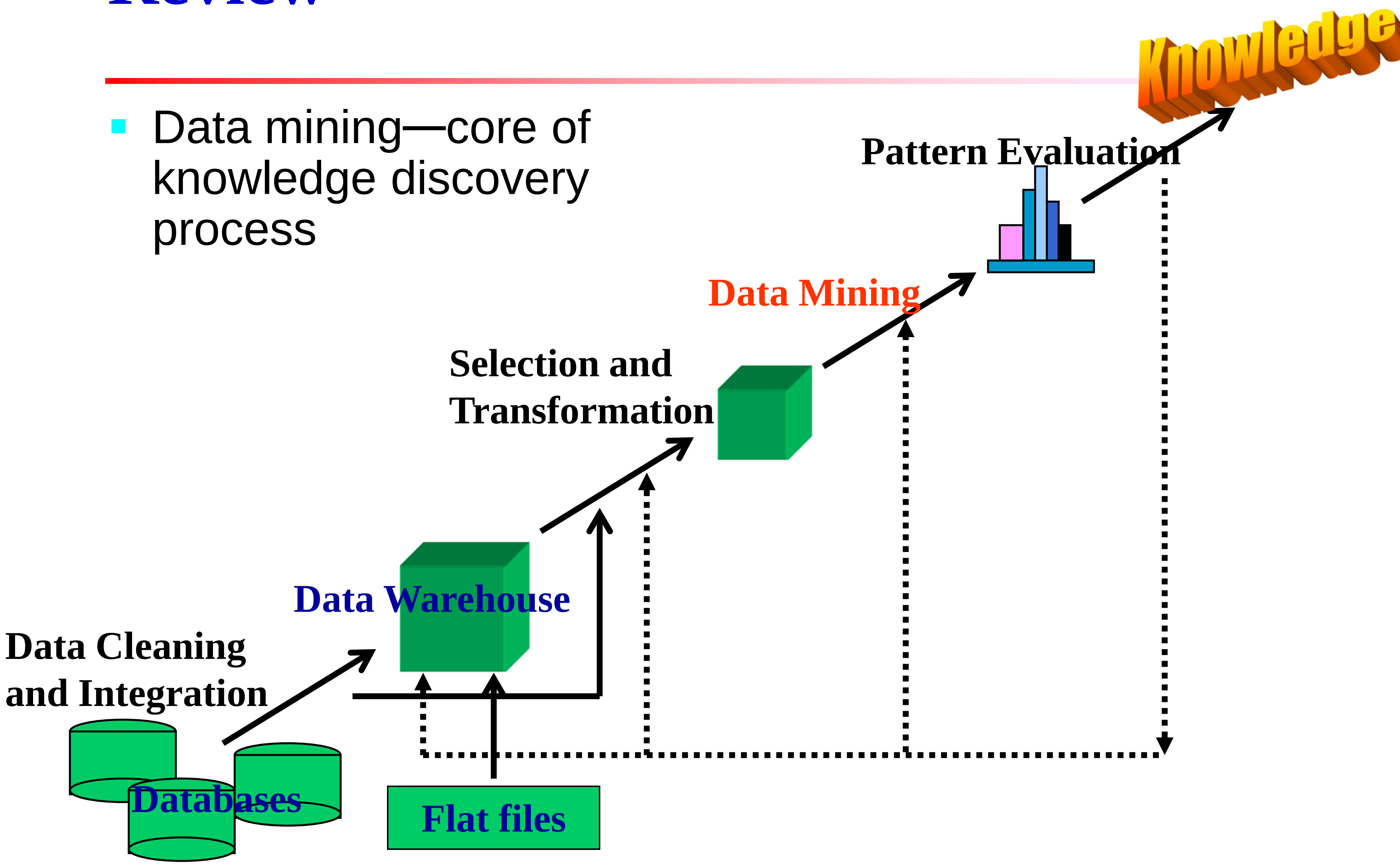
Review

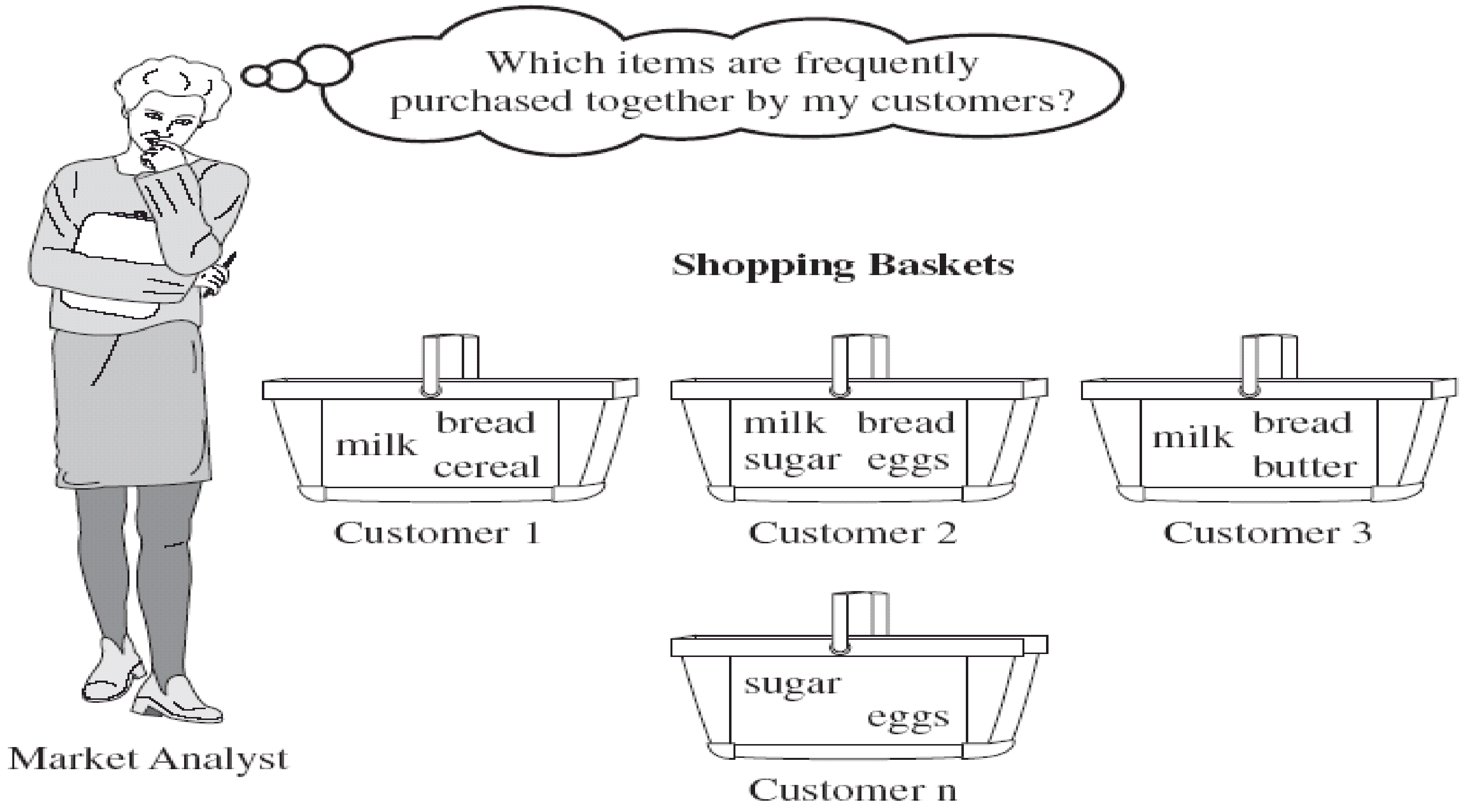
- Data mining—core of knowledge discovery process



Mining Association Rules in Large Databases

- Basic concepts and a road map
- Mining single-dimensional Boolean association rules
- Mining multilevel association rules
- Mining multidimensional association rules
- Summary

Market Basket Analysis



What Is Association Rules Mining?

- Association rules mining
  - Finding frequent patterns, associations among sets of items or objects in transaction databases, relational databases, and other information repositories.
- Examples
  - What products were often purchased together? — Beer and diapers?!
  - What DNA segments often occur together in DNA sequences?

What Is Association Rules Mining?

- Where does the data come from?
  - supermarket transactions, e-commerce orders/shopping lists, membership cards
- Applications
  - Basket data analysis
  - Cross-marketing
  - Catalog design
  - Sale campaign analysis
  - Web log (click stream) analysis
  - DNA sequence analysis

Basic Concepts

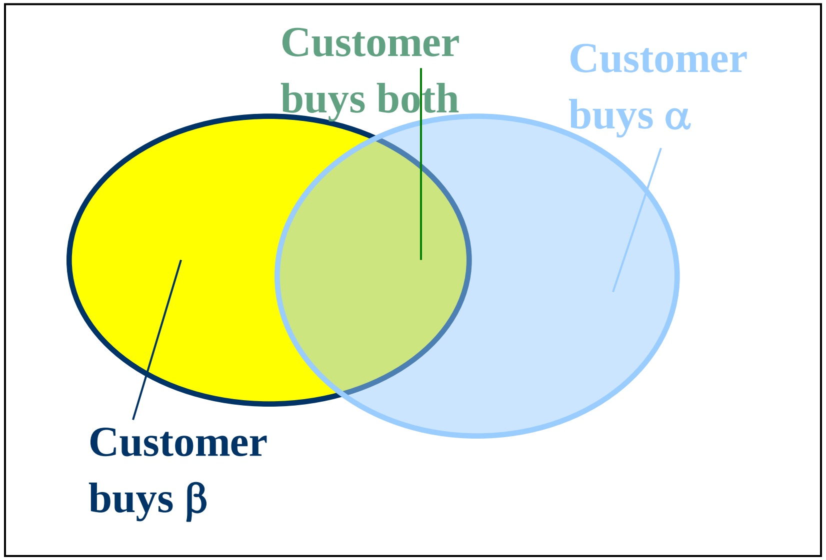
Transaction-id	Items bought
10	A, B, D
20	A, C, D
30	A, D, E
40	B, E, F
50	B, C, D, E, F

- Item collection $X = \{x_1, \ldots, x_m\}$
- Itemset: a set of items, *k*-itemset
- Transaction $T \subseteq X$, each T associates a unique Tid and items bought by a customer
- Rule form $\alpha \Rightarrow \beta$, $\alpha \subset X$, $\beta \subset X$, $\alpha \cap \beta = \emptyset$

Basic Concepts

- support, *s*, probability that a transaction contains α and β
  - support ($\alpha \Rightarrow \beta$) = $P(\alpha \cap \beta)$
- Frequent itemset, occurrence greater than a min_support
- Frequent itemset mining, find all the rules $\alpha \Rightarrow \beta$ satisfying min_support
- Let sup_{min} = 50%,
  frequent Itemsets {A:3, B:3, D:4, E:3, AD:3}

  support (A) = 3/5 = 60%, support (AD) = 3/5 = 60%



Basic Concepts

- confidence, *c,* conditional probability that a transaction having α also contains β

$$\text{Confidence}(\alpha \Rightarrow \beta) = P(\beta \mid \alpha) = \frac{P(\alpha \cap \beta)}{P(\alpha)} = \frac{\text{count}(\alpha \cap \beta)}{\text{count}(\alpha)}$$

- Measure of rule interestingness
- Rules satisfy min_support and min_confidence are strong
- Let $sup_{min} = 50\%$, $conf_{min} = 50\%$,
  frequent itemsets {A:3, B:3, D:4, E:3, AD:3}
  Association rules:
  A => D (60%, 100%)
  D => A (60%, 75%)

Interestingness Measure: Correlations (Lift)

- *play basketball* $\Rightarrow$ *eat cereal* [40%, 66.7%] is misleading
  - The overall % of students eating cereal is 75% > 66.7%.
- Measure of dependent/correlated events: lift

$$lift = \frac{P(A \cap B)}{P(A)P(B)}$$

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

$$lift(B,C) = \frac{2000/5000}{3000/5000 * 3750/5000} = 0.89 \qquad lift(B,\neg C) = \frac{1000/5000}{3000/5000 * 1250/5000} = 1.33$$

Association Rule Mining: A Road Map

- Boolean vs. quantitative associations (based on the types of values handled)
  - Boolean association rules, only concern presence or absence of items, buys(x, “SQLServer”) ^ buys(x, “DMBook”) => buys(x, “DBMiner”) [0.2%, 60%]
  - Quantitative association rules, concern quantitative attributes, age(x, “30…39”) ^ income(x, “42…48K”) => buys(x, “high resolution TV”) [1%, 75%]
- Single level vs. multiple-level analysis (based on the levels of abstraction involved)
  - age(x, “30…39”) => buys(x, “laptop computer”)
  - age(x, “30…39”) => buys(x, “computer”)
- Single dimension vs. multiple dimensional associations (based on dimensions involved)

Mining Association Rules in Large Databases

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- Mining multidimensional association rules
- Summary

Handling Exponential Complexity

- Given *n* transactions and *m* different items:
  - Number of possible association rules: $O(2^m)$
  - Computation complexity: $O(nm2^m)$

- Apriori Principle
  - Collect single item counts, find large items
  - Find candidate pairs, count them => large pairs of items
  - Find candidate triplets, count them => large triplets of items, And so on...
  - Guiding Principle: Every subset of a frequent itemset has to be frequent
    - Used for pruning many candidates

Apriori: A Candidate Generation-and-Test Approach

- Apriori uses prior knowledge of frequent itemsets
- Iterative approach, level-wise search
- The Apriori property (downward closure property，anti-monotone) of frequent patterns
  - Any subset of a frequent itemset must be frequent
  - If any itemset is infrequent, its superset should not be generated/tested
  - If {beer, diaper, nuts} is frequent, so is {beer, diaper}, every transaction having {beer, diaper, nuts} also contains {beer, diaper}
  - If {beer, diaper} is infrequent, {beer, diaper, nut} cannot be frequent at all

Apriori: A Candidate Generation-and-Test Approach

- Method:
  - Initially, scan DB once to get frequent 1-itemset
  - Generate length (k+1) candidate itemsets from length k frequent itemsets
  - Test the candidates against DB
  - Terminate when no frequent or candidate set can be generated

Apriori Algorithm — An Example

$Sup_{min} = 2$

Database D

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan

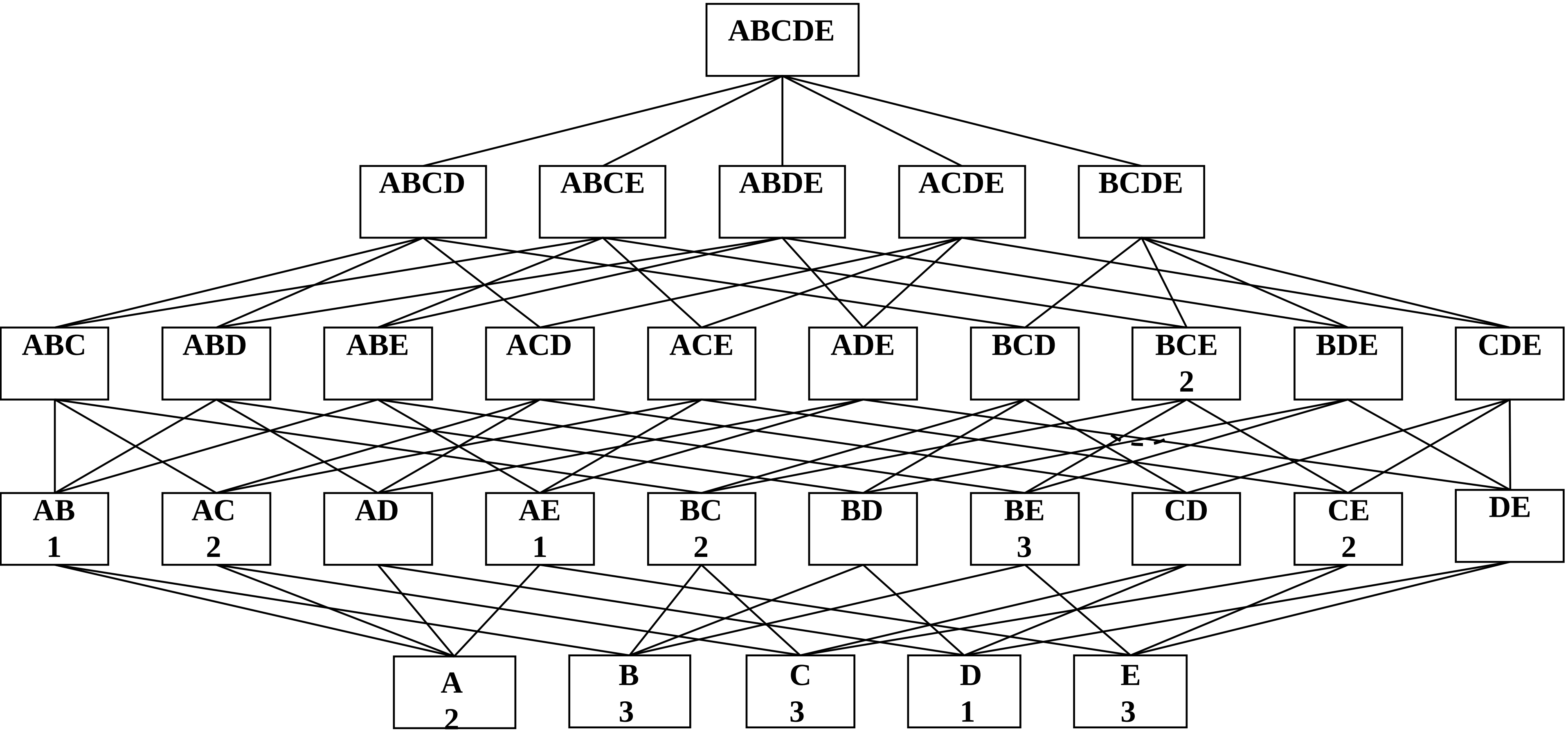
C_3

Itemset	sup
{B, C, E}	2

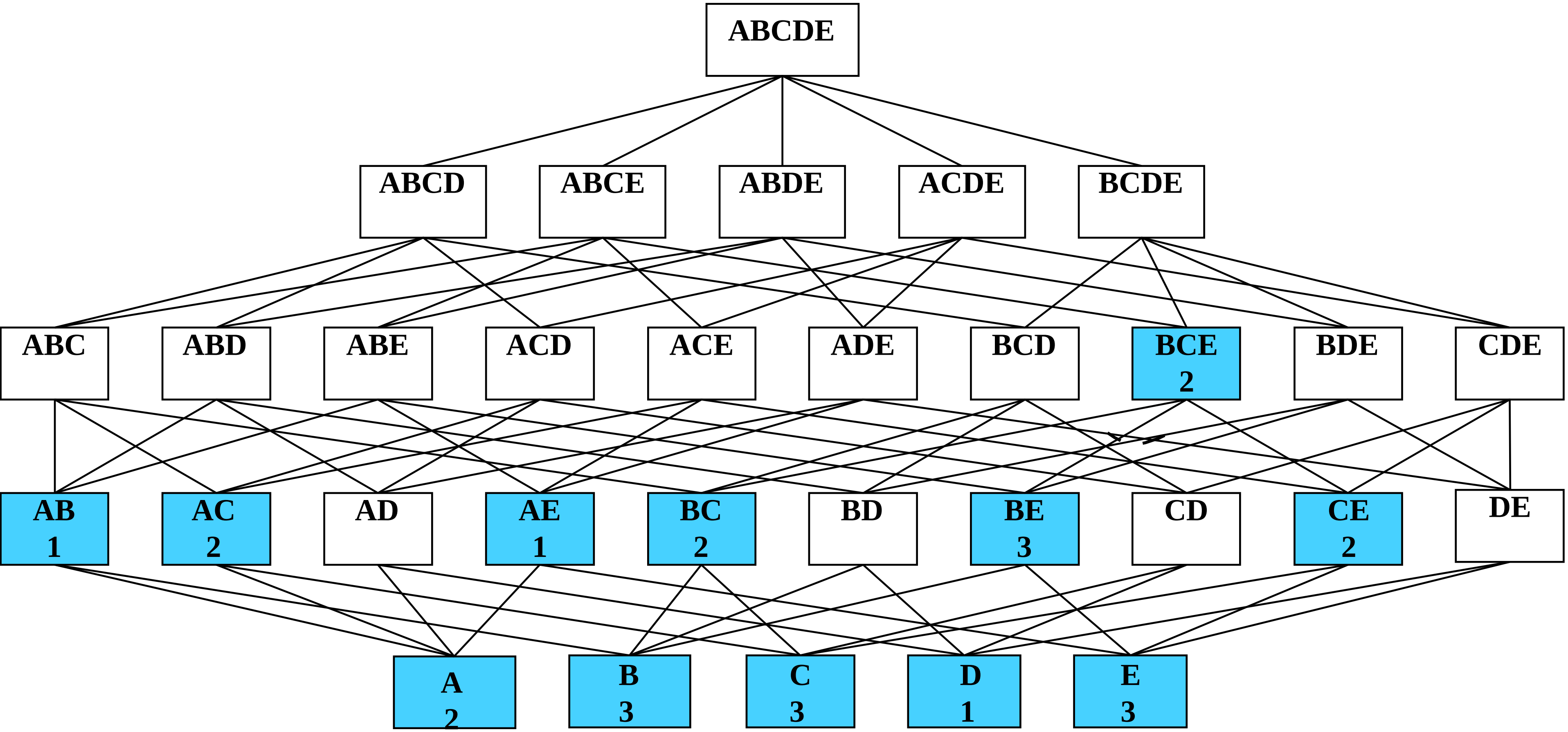
L_3

Itemset	sup
{B, C, E}	2

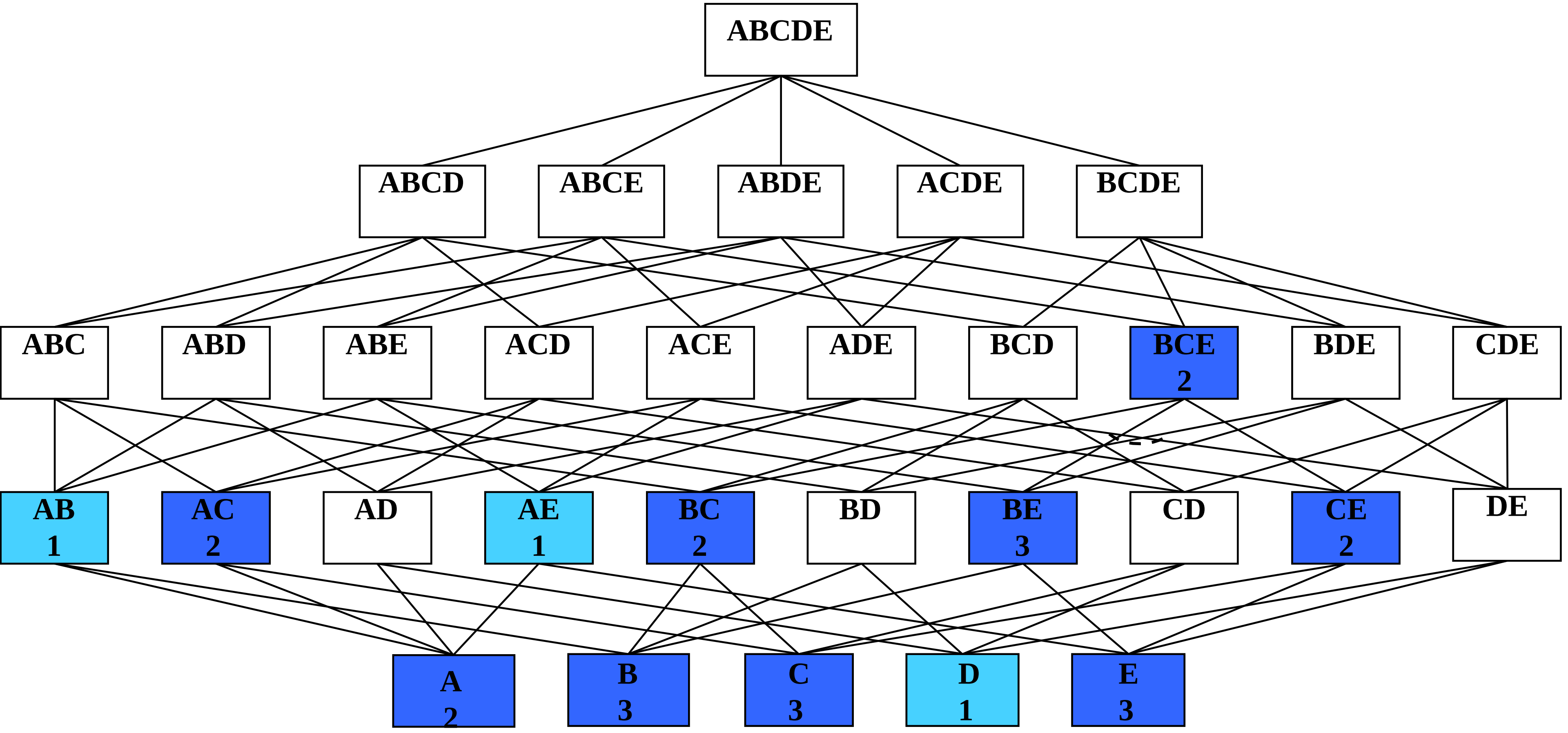
Apriori Algorithm — An Example



Apriori Algorithm — An Example



Apriori Algorithm — An Example



Apriori Algorithm

- Pseudo-code

  C_k: Candidate itemset of size k

  L_k : frequent itemset of size k

 Input: Database D, min_sup

 Output: frequent itemsets L

L_1 = {frequent single items from D};

for (k = 2; L_{k-1} !=$\emptyset$; k++) **do begin**

  C_k = candidates generated from L_{k-1};

  for each transaction $t \in D$ do

    increment the count of all candidates in C_k which are contained in t

  end

  L_k = candidates in C_k with min_support

end

return $L = \cup_k L_k$;

How to Generate Candidates?

- How to generate candidates?
  - Step 1: self-joining L_k
  - Step 2: pruning
- Example
  - L_3={*abc, abd, acd, ace, bcd*}
  - Self-joining: $L_3 * L_3$
    - *abc* and *abd* -> *abcd, acd* and *ace* -> *acde*
  - Pruning:
    - *acde* is pruned because *ade* is not in L_3
  - C_4={*abcd*}

How to Generate Candidates?

- Suppose the items in L_{k-1} are listed in order
- Step 1: self-joining L_{k-1}

for each itemset $l_1 \in L_{k-1}$

for each itemset $l_2 \in L_{k-1}$

if $(l_1[1]=l_2[1])$^$(l_1[2]=l_2[2])$^…^$(l_1[k-2]=l_2[k-2])$ then

$c = l_1$ *join* l_2

pruning (c)

end

end

- Step 2: pruning

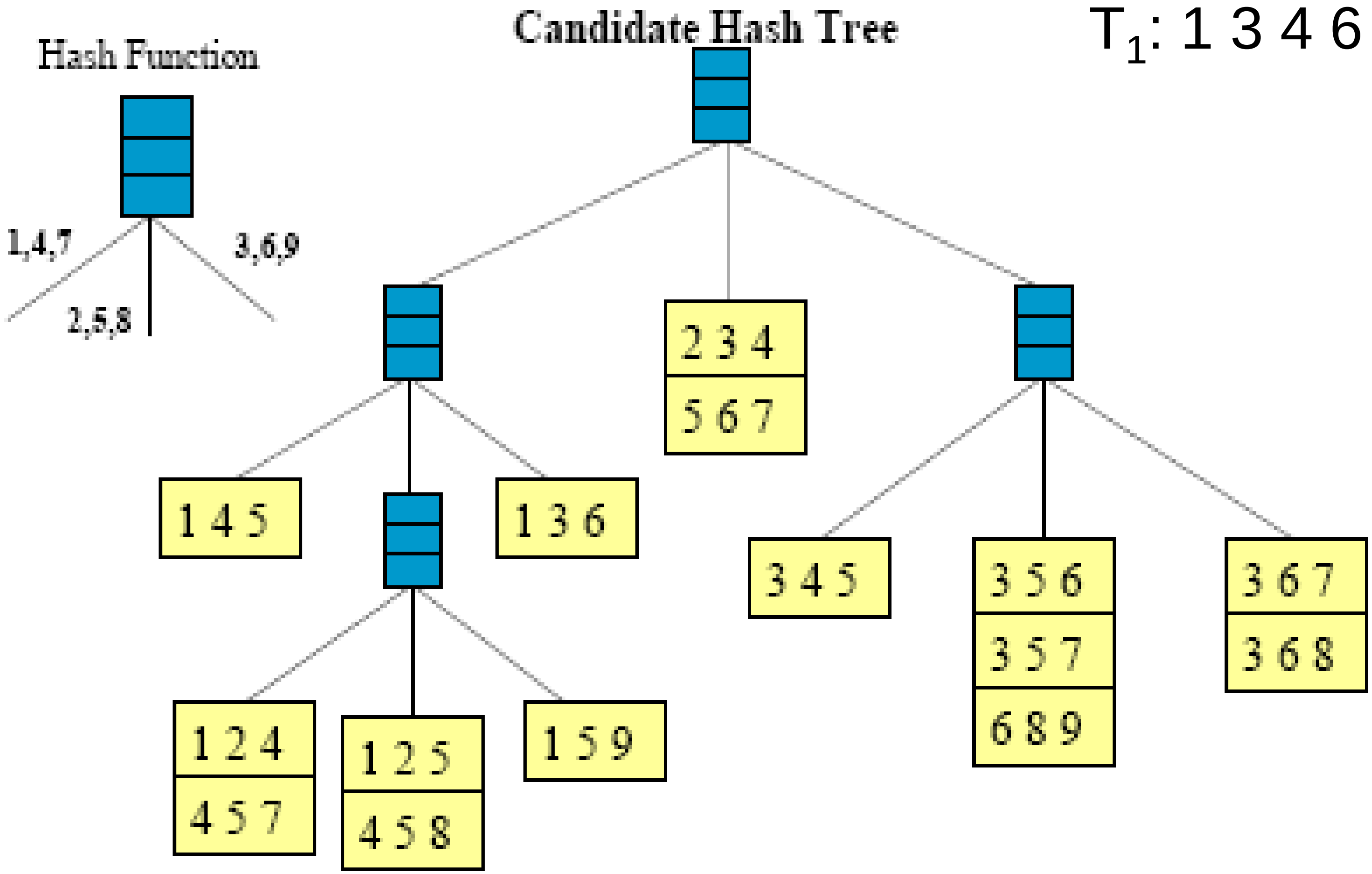
forall *(k-1)-subsets s of c* do

if *(s is not in* L_{k-1}*)* then delete c

How to Count Supports of Candidates?

- Why counting supports of candidates a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method:
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table

Example: Counting Supports of Candidates



T_1: 1 3 4 6 7 9

Exercise

1. A database has 9 transactions. Let *min_sup* = 20%. Please present all the candidates and frequent itemsets at each iteration.

TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

Challenges of Frequent Pattern Mining

- Challenges
  - Multiple scans of transaction database
  - Huge number of candidates
  - Tedious workload of support counting for candidates
- Improving Apriori
  - Reduce passes of transaction database scans
  - Shrink number of candidates
  - Facilitate support counting of candidates

Partition: Scan Database Only Twice

- A. Savasere, E. Omiecinski, and S. Navathe. An efficient algorithm for mining association in large databases. In *VLDB'95.*
- Partitioning technique
  - Partition the data into *N* small partitions
  - Phase 1: find local frequent itemsets on each data partition. Record all local frequent itemsets.
  - Phase 2: Integrate all local frequent itemsets, scan database, find global frequent itemsets.
- Correctness: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions

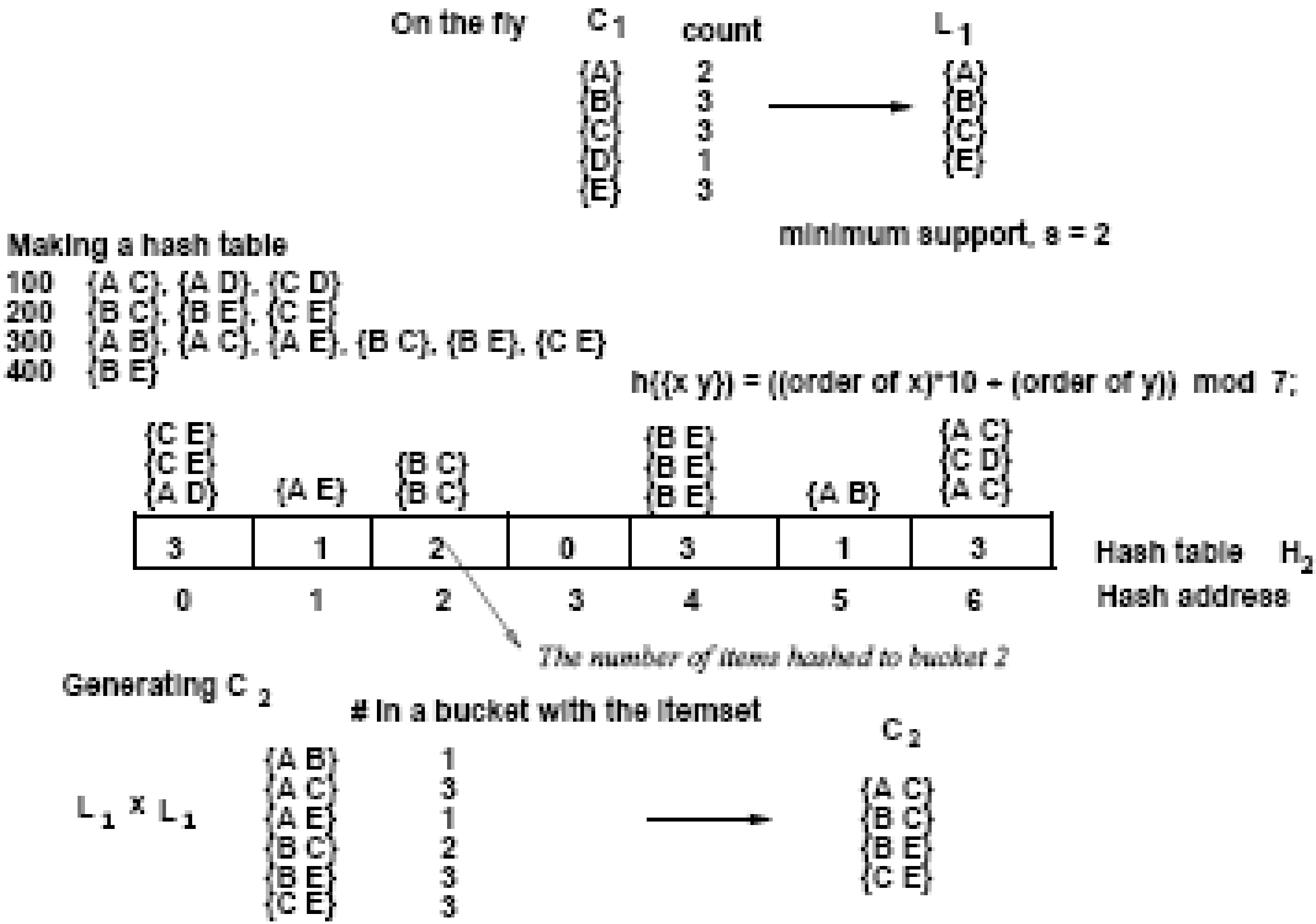
Partition: Scan Database Only Twice

- Each partition can be fit into memory
- Scan database only twice! Reduce I/O cost!
- Execution time scales linearly
- Good for very large-scale database
- Applicable to parallel/distributed computing systems
  - Each processor performs FIM on its local data
  - Central server aggregates local frequent itemsets, broadcast potential global itemsets
  - Each processor scans local data to count the frequency
  - Central server aggregates the counts, find the global itemsets

DHP: Reduce the Number of Candidates

- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. In *SIGMOD'95*
- Hash-based technique
  - When scanning transactions to generate frequent *k*–itemsets, L_k, generate all (*k*+1)-itemsets for each transaction
  - Hash all (*k*+1)-itemsets into buckets, increase bucket count
  - If a (*k*+1)-itemset bucket count is below *min_sup*, it must be removed from (*k*+1) candidate itemsets, C_{k+1}
- Correctness：A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent

DHP: Reduce the Number of Candidates



DHP: Reduce the Number of Candidates

- Pros
  - Reduce the number of candidates, C_k, especially for $C_{2.}$ Size of C_2 is usually huge, reduce C_2 is crucial
  - Execution time scales linearly when varying the size of data

Comparison of time (T15.I4.D100)

	Apriori number	DHP number
L_1	820	820
C_2	335,790	338
L_2	207	207
C_3	618	618
L_3	201	201
C_4	184	184
L_4	98	98
C_5	30	30
L_5	23	23
C_6	1	1
L_6	1	1
total time	39.39	13.91

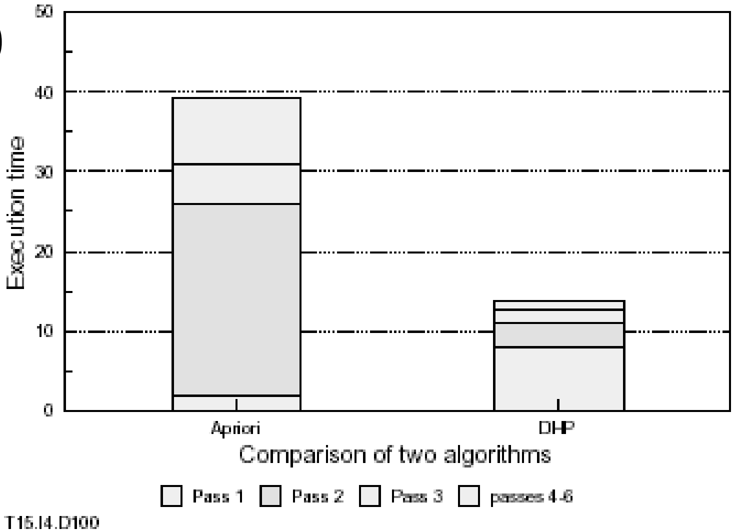
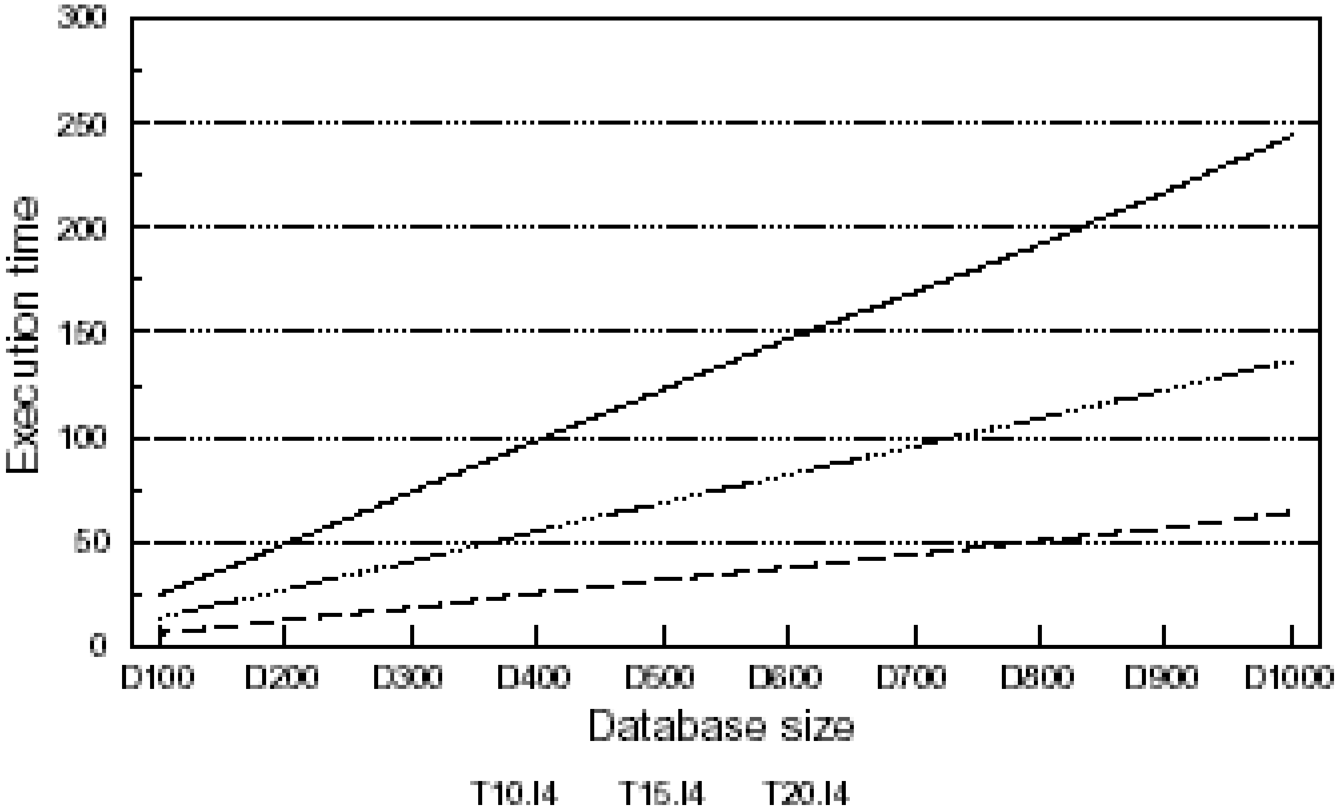


Figure 8: Execution time of Apriori and DHP

Comparison of time (T15.I4.D100)

DHP: Reduce the Number of Candidates



Performance of DHP when increasing the size of database

DHP: Reduce the Number of Candidates

- Cons
  - Consume more memory, for hash table
  - The larger the hash table, the smaller C_k *and* L_k

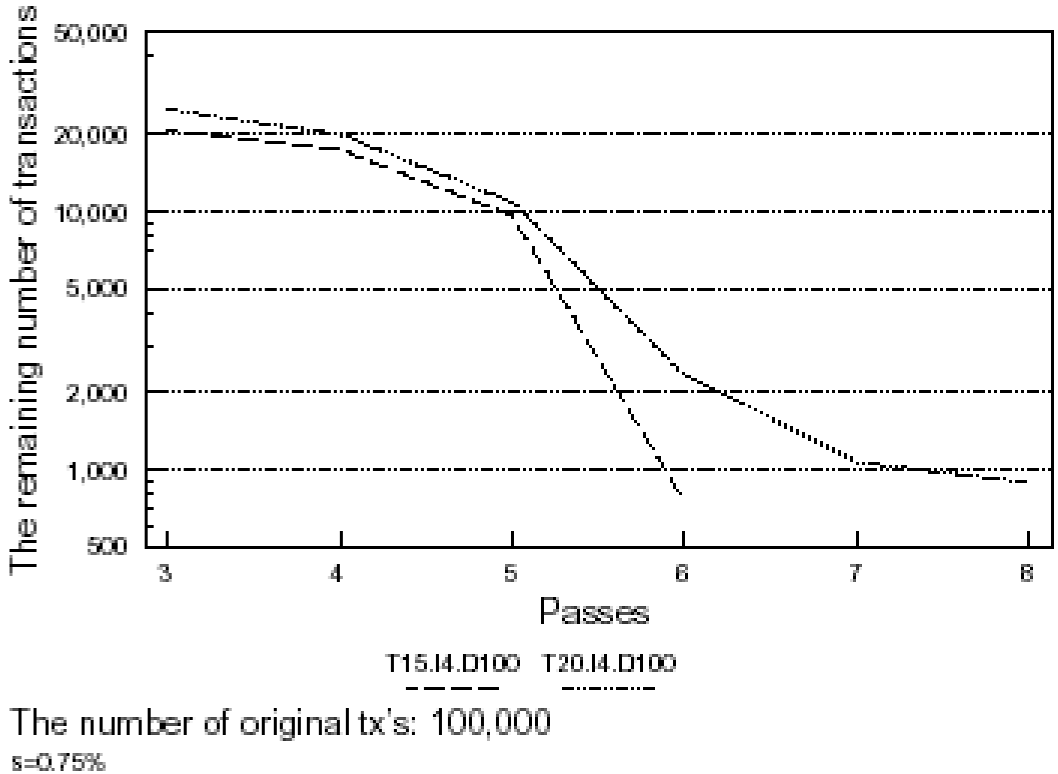
Results from varying hash table sizes (T10.I4.D100)

$\|H_2\|$	524,288	262,144	131,072	65,536	32,768
L_1	559	559	559	559	559
$\|\{H_2 \geq s\}\|$	58	61	75	96	182
C_2	81	120	199	394	1355
L_2	45	45	45	45	45
α	0.0314	0.0320	0.0345	0.0386	0.0545
size of D_3	498KB	500KB	507KB	539KB	603KB
$\|D_3\|$	19,732	19,741	19,755	20,501	21,607
total time	6.44	6.43	6.24	6.77	7.23

Transaction Reduction

- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. In *SIGMOD'95*
- Transaction reduction
  - When scanning transactions to generate frequent k-itemsets, L_k, mark the transaction that contains no k-candidate
  - Remove all the marked transaction
  - The number of transactions drops dramatically

Transaction Reduction

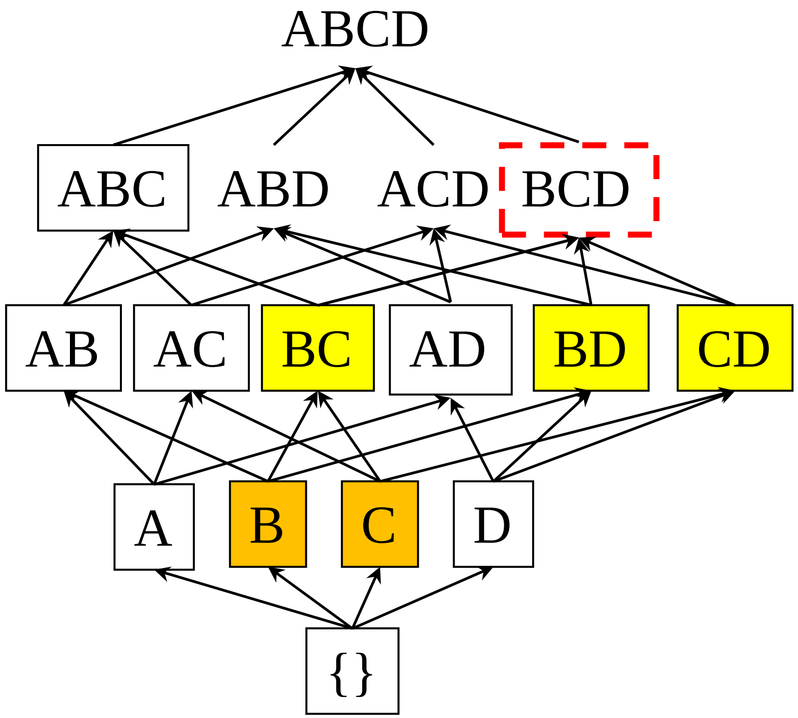


The remaining number of transaction in each pass

DIC: Reduce Number of Scans

- S. Brin, R. Motwani, J. Ullman, and S. Tsur. Dynamic itemset counting and implication rules for market basket data. In *SIGMOD'97*
- *Sergey Brin, founder of Google!*
- Partition database into blocks marked by starting points
- New candidate can be added at any starting point once all its subsets are determined frequent
- Reduce the number of database scans

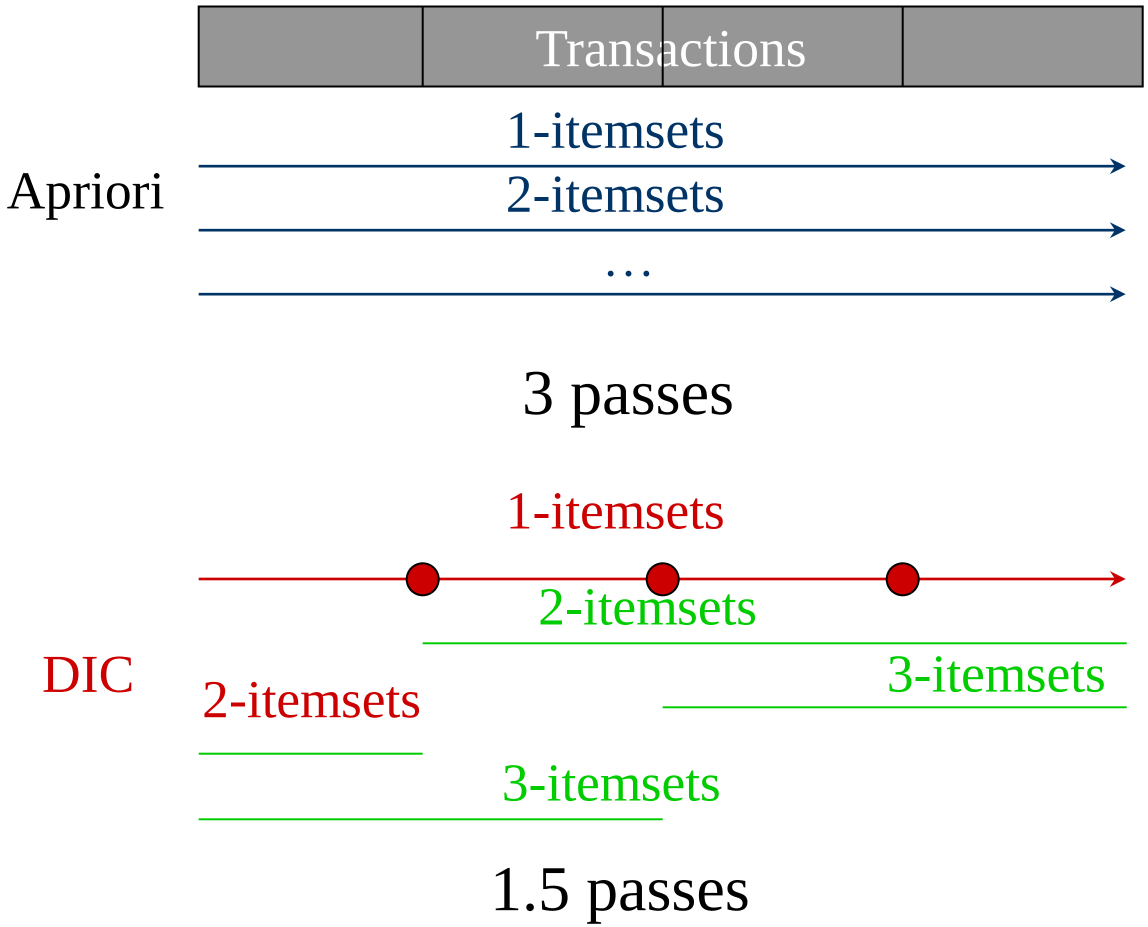
DIC: Reduce Number of Scans



Itemset lattice

- Once both B and C are determined frequent, new candidate BC is added, the counting of BC begins at the next starting point
- Once all length-2 subsets of BCD are determined frequent, new candidate BCD is added, the counting of BCD begins at the next starting point

DIC: Reduce Number of Scans



- Assume 40000 transactions, 4 partitions
- Begin counting 2-itemsets after the first 10000 have been read
- Begin counting 3-itemsets after the first 20000 have been read
- Scan database again, count 2 and 3-itemsets
- After 10000 transactions, finish counting 2-itemsets
- After 20000 transactions, finish counting 3-itemsets

Exercise

2. A database has 9 transactions. Let *min_sup* = 20%. Please present all the frequent itemsets generated by DIC in the first iteration. (Note: partition the data into 3 blocks)

TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

Bottleneck of Frequent-pattern Mining

- Multiple database scans are costly
- Mining long patterns needs many passes of scanning and generates lots of candidates
  - To find frequent itemset $i_1 i_2 \ldots i_{100}$
    - # of scans: 100
    - # of Candidates: $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100}-1 = 1.27*10^{30}$!
- Bottleneck: candidate-generation-and-test
- Can we avoid candidate generation?

Construct FP-tree from a Transaction Database

- Scan DB once, find frequent 1-itemset (single item pattern)
- Sort frequent items in frequency descending order *L*
- Create the root of the tree, labeled with "null"
- Scan DB again, sort each transaction in *L* order, a branch is created for each transaction
  - Increment the count of each node along a common prefix by 1
  - Create nodes for the items following the prefix
- Build a header table, connect each item point in the tree

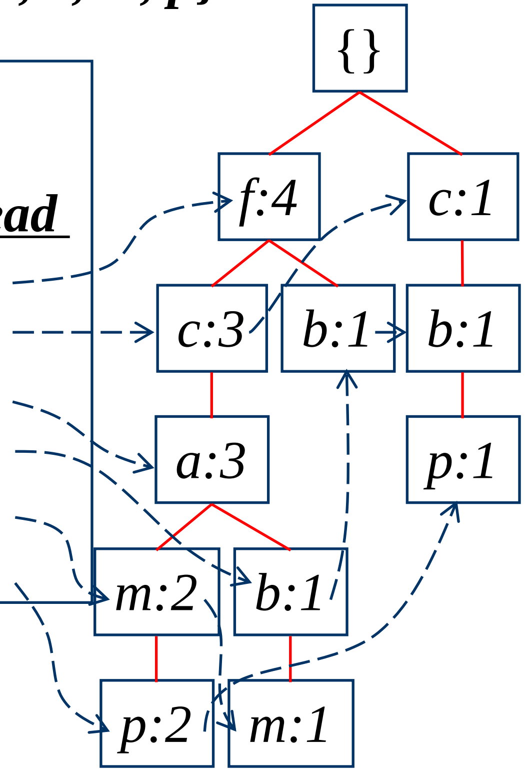
Construct FP-tree from a Transaction Database

TID	*Items bought*	*(ordered) frequent items*
100	{*f, a, c, d, g, i, m, p*}	{*f, c, a, m, p*}
200	{*a, b, c, f, l, m, o*}	{*f, c, a, b, m*}
300	{*b, f, h, j, o, w*}	{*f, b*}
400	{*b, c, k, s, p*}	{*c, b, p*}
500	{*a, f, c, e, l, p, m, n*}	{*f, c, a, m, p*}

***min_support* = 3**

Header Table

Item	*frequency*	*head*
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



F-list = f-c-a-b-m-p

Construct FP-tree from a Transaction Database

- 1. Scan the transaction database *D* once. Collect *F*, the set of frequent items, and their support counts. Sort *F* in support count descending order as *L*, the *list* of frequent items.
- 2. Create the root of an FP-tree, and label it as "null." For each transaction *Trans* in *D* do the following:
  - Select and sort the frequent items in *Trans* according to the order of *L*. Let the sorted frequent item list in the *Trans* be [*p*|*P*], where *p* is the first element and *P* is the remaining list.
  - Call insert_tree ([*p*|*P*], *T*), which is performed as follows. If *T* has a child *N* such that *N.item-name = p.item-name*, then increment *N*'s count by 1; else create a new node *N*, and let its count be 1, its parent link be linked to *T*, and its node-link to the nodes with the same *item-name* via the node-link structure.
  - If *P* is nonempty, call insert_tree(*P, N*) recursively.

Benefits of the FP-tree Structure

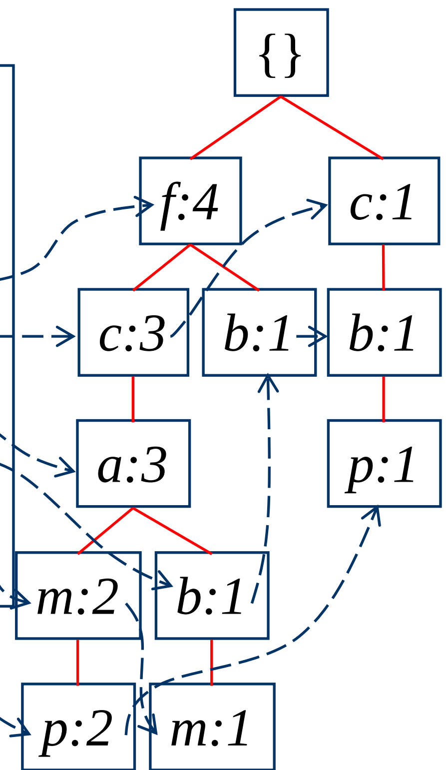
- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info—infrequent items are gone
  - Items in frequency descending order: the more frequently occurring, the more likely to be shared
  - Never be larger than the original database

Construct Conditional Pattern Base

- Start at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item *x*
- Accumulate all of *transformed prefix paths* of item *x* into form *x*'s conditional pattern base

Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



***Conditional* pattern bases**

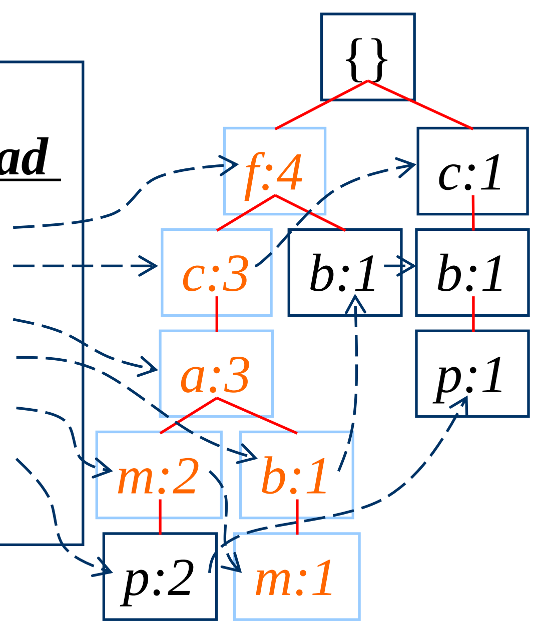
item	***cond. pattern base***
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

From Conditional Pattern-bases to Conditional FP-trees

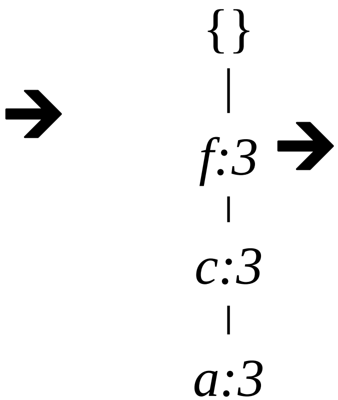
- For each pattern-base
  - Accumulate the count for each item in the base
  - Construct the FP-tree for the frequent items of the pattern base

Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



m-conditional **pattern base:**
fca:2, fcab:1



m-conditional **FP-tree**

All frequent patterns relate to *m*

m,

fm, cm, am,

fcm, fam, cam,

fcam

Recursion: Conditional FP-tree

{}
|
f:3
|
c:3
|
a:3

m-conditional **FP-tree**

Cond. pattern base of “am”: (fc:3)

{}
|
f:3
|
c:3

am-conditional **FP-tree**

Cond. pattern base of “cm”: (f:3)

{}
|
f:3

cm-conditional **FP-tree**

Cond. pattern base of “cam”: (f:3)

{}
|
f:3

cam-conditional **FP-tree**

Mining Frequent Patterns With FP-trees

procedure **FP_growth**(*Tree*, α)

(1) **if** *Tree* contains a single path *P* then

(2) **for each** combination (denoted as β) of the nodes in the path *P*

(3) generate pattern $\beta \cup \alpha$ with *support_count* = *minimum support count of nodes in* β;

(4) **else for each** a_i in the header of *Tree* {

(5) generate pattern $\beta = a_i \cup \alpha$ with *support_count* = a_i.*support_count*;

(6) construct β's conditional pattern base and then β's conditional FP_tree $Tree_\beta$;

(7) **if** $Tree_\beta$ then

(8) call **FP_growth**($Tree_\beta$, β); }

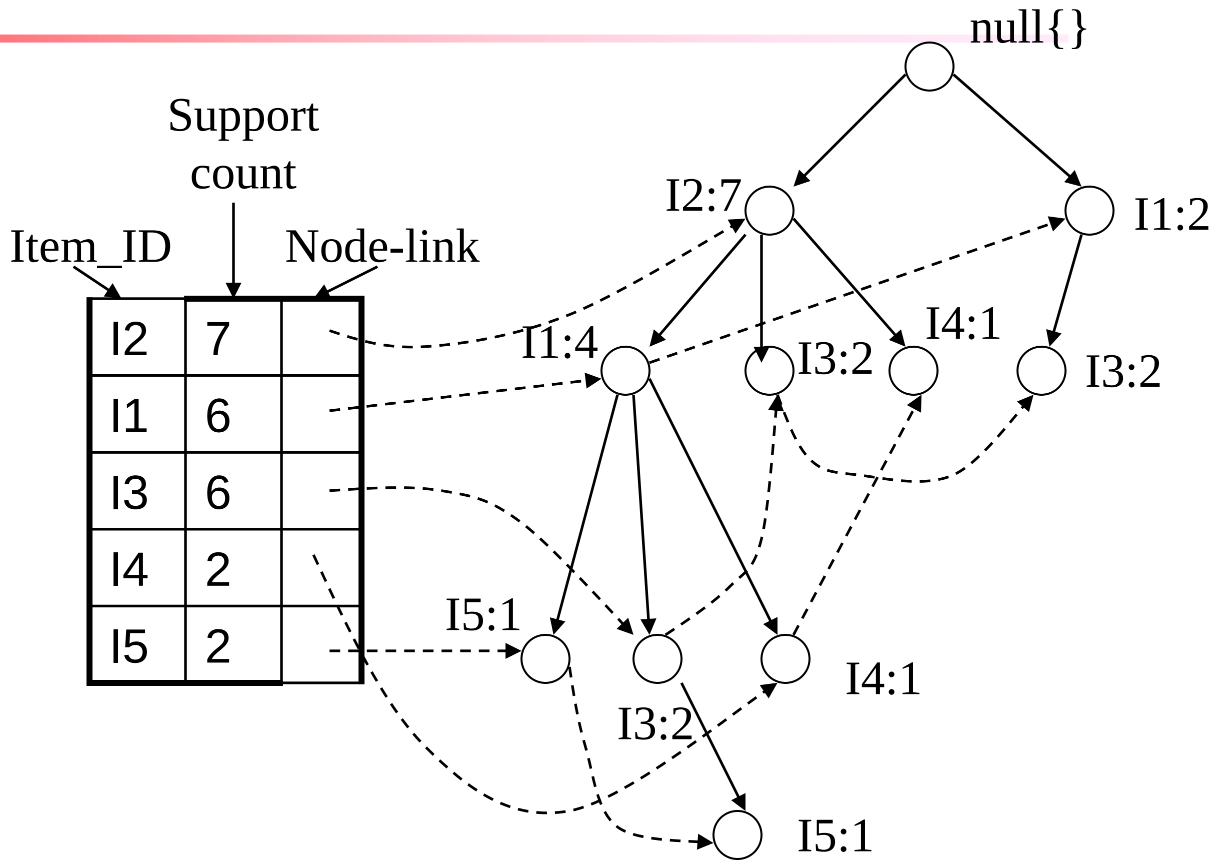
Exercise

3. A database has 9 transactions. Let *min_sup* = 20%. Please construct the FP-tree for the database, the conditional FP-trees, and all the frequent itemsets.

TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

Solution

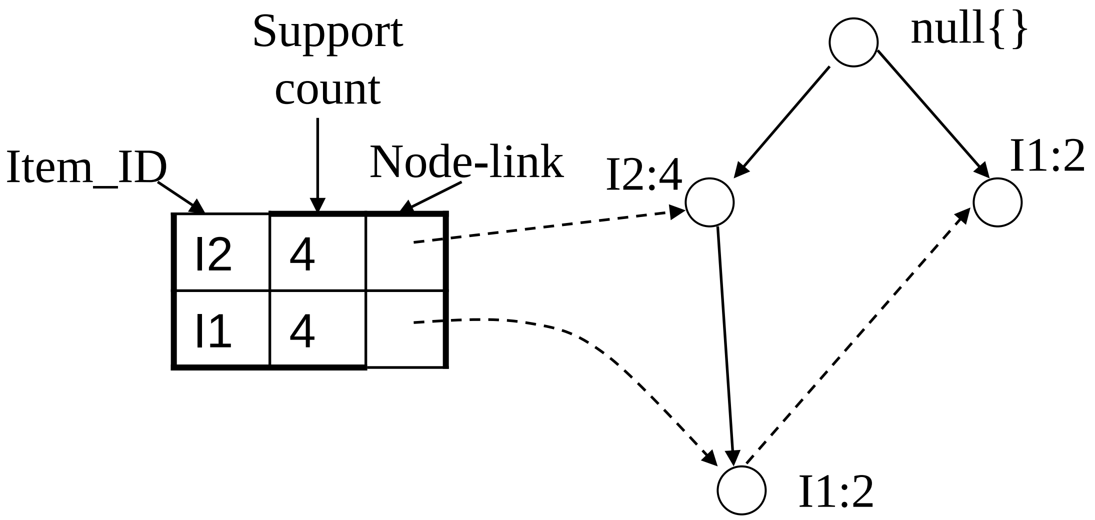
TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3



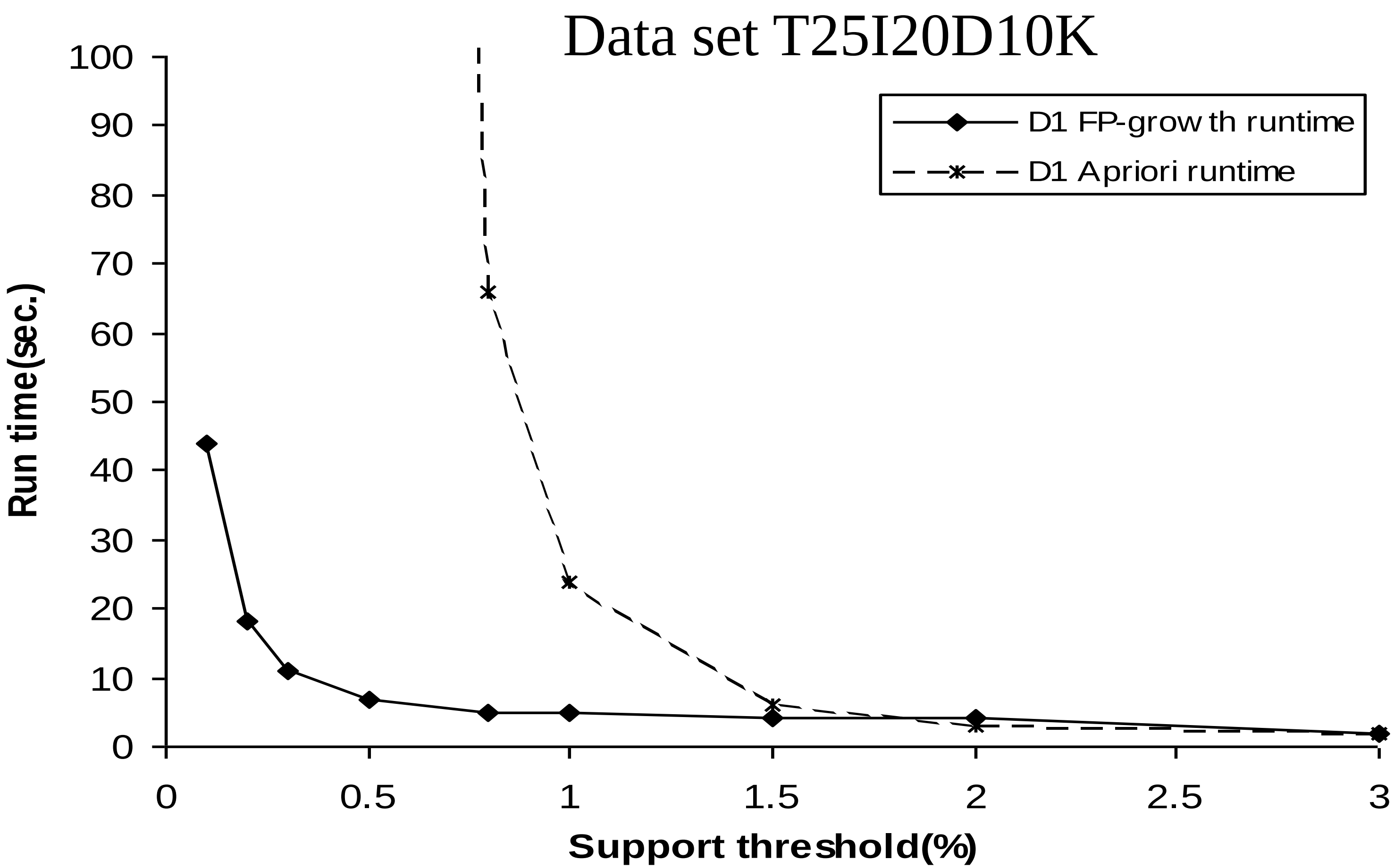
Support =2

Solution

item	*conditional pattern base*	*conditional FP-tree*	*frequent patterns generated*
I5	{{I2,I1: 1}, {I2,I1,I3: 1}}	⟨I2: 2, I1: 2⟩	{I2,I5: 2}, {I1,I5: 2}, {I2,I1,I5: 2}
I4	{{I2,I1: 1}, {I2: 1}}	⟨I2: 2⟩	{I2,I4: 2}
I3	{{I2,I1: 2}, {I2: 2}, {I1: 2}}	⟨I2: 4, I1: 2⟩, ⟨I1: 2⟩	{I2,I3: 4}, {I1,I3: 4}, {I2,I1,I3: 2}
I1	{{I2: 4}}	⟨I2: 4⟩	{I2,I1: 4}



FP-Growth vs. Apriori: Scalability With the Support Threshold



Why Is FP-Growth the Winner?

- Divide-and-conquer:
  - Decompose both the mining task and DB according to the frequent patterns obtained so far
  - Focus searching on smaller databases
- Other factors
  - No candidate generation, no candidate test
  - Compressed database: FP-tree structure
  - Two scans of entire database
  - Basic ops—counting local freq items and building sub FP-tree, no pattern search and matching

Cons

- Building FP-trees
  - A stack of FP-trees
- Redundant information
  - Transaction abcd appears in a-, ab-, abc-, ac-, c-FP-trees

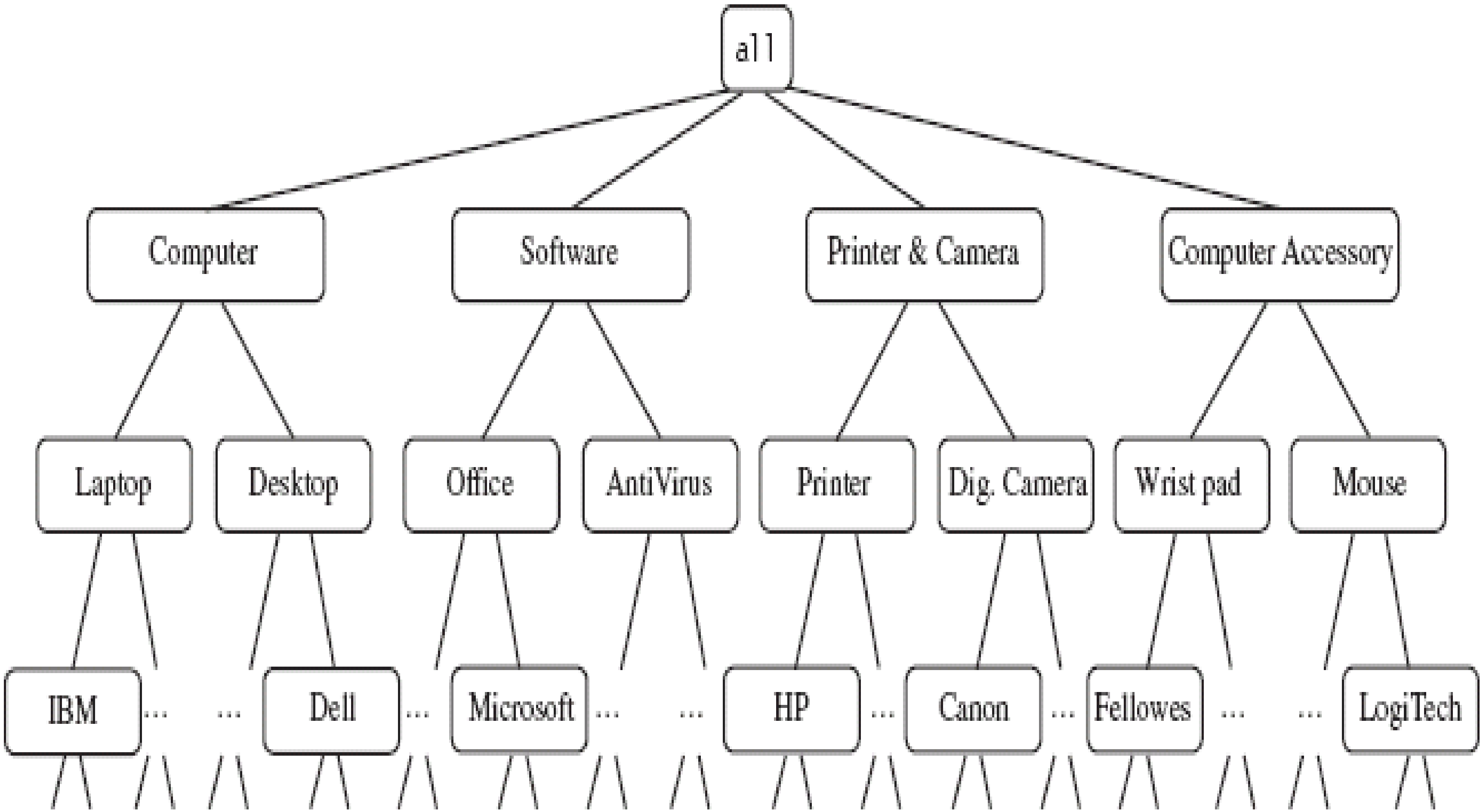
Mining Association Rules in Large Databases

- Basic concepts and a road map
- Mining single-dimensional Boolean association rules
- Mining multilevel association rules
- Mining multidimensional association rules
- Summary

Mining Multiple-Level Association Rules

- Association rules at high concept levels may represent common sense knowledge
- Hard to find association rules at low concept level
  - Items at the lower level usually have lower support, less than min_support threshold
- Mining association rules at multiple levels of abstraction
- Example: sales in AllElectronics store computer sector

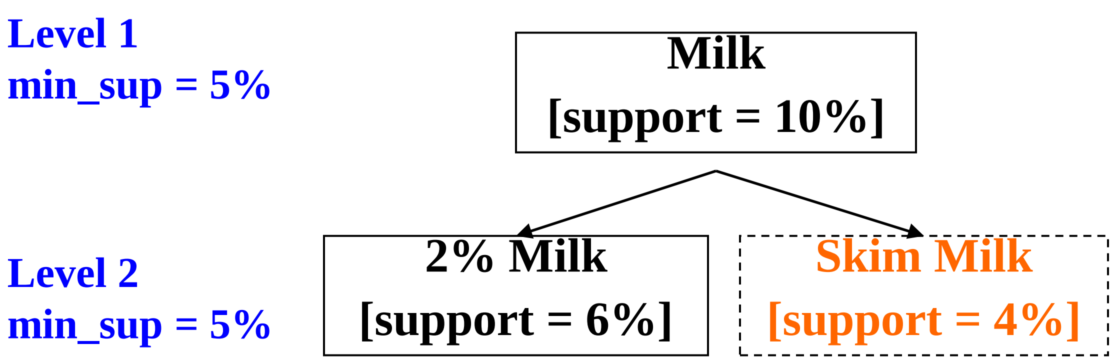
Example


all
Computer
Software
Printer & Camera
Computer Accessory
Laptop
Desktop
Office
AntiVirus
Printer
Dig. Camera
Wrist pad
Mouse
IBM
...
...
Dell
...
Microsoft
...
...
HP
...
Canon
...
Fellowes
...
...
LogiTech

Mining Multiple-Level Association Rules

- Uniform support
  - Top-down, level-wise
  - Use uniform minimum support for each level
  - Perform Apriori at each level
  - Optimization: if an ancestor is infrequent, the search on the descendants can be avoided

uniform support

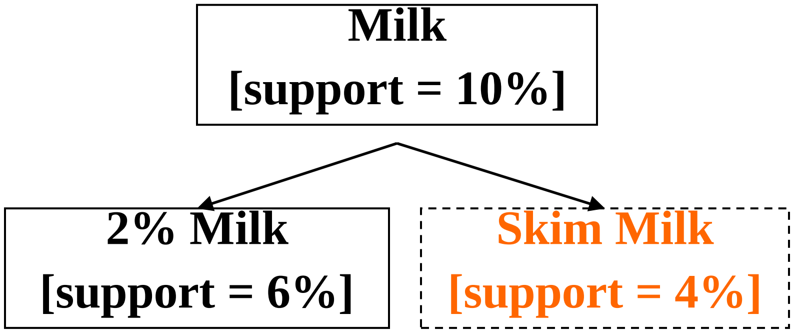


Mining Multiple-Level Association Rules

uniform support

Level 1
min_sup = 5%

Level 2
min_sup = 5%

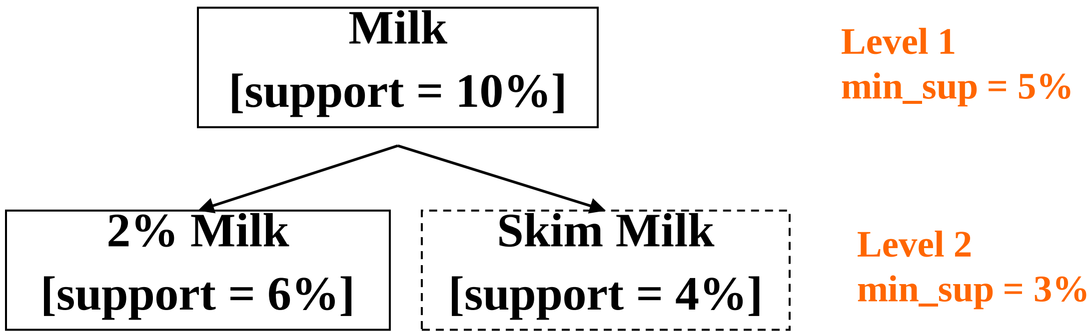


- Drawbacks
  - Miss interesting associations with too high threshold
  - Generate too many uninteresting rules with too low threshold

Mining Multiple-Level Association Rules

- Reduced support
  - Top-down, level-wise
  - Each concept level has its own minimum support threshold
  - The lower level, the smaller threshold
  - Perform Apriori at each level

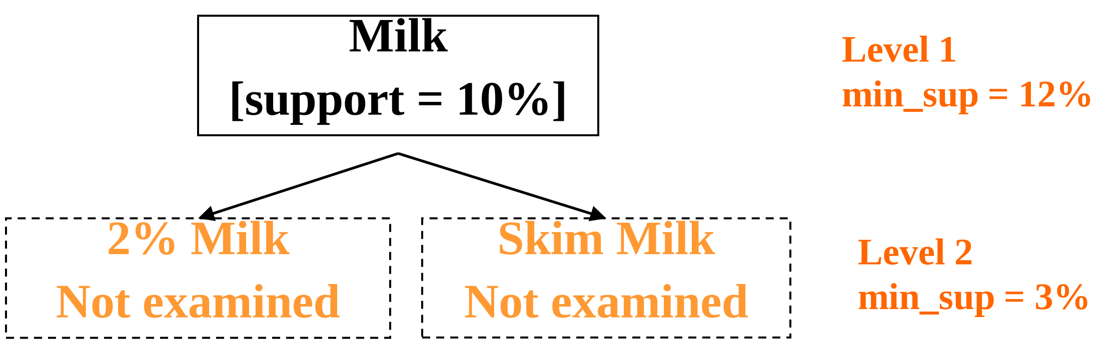
reduced support



Mining Multiple-Level Association Rules

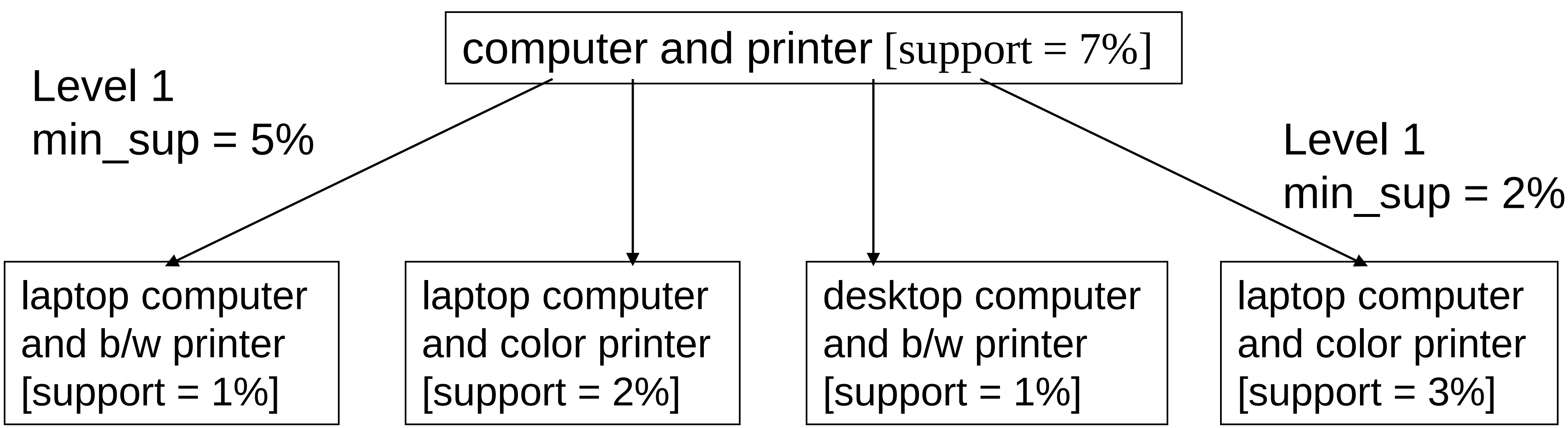
- Reduced support
  - Optimization -- level-cross filtering by single item
    - An item at the *i*th concept level is examined *iff* its parent concept at the (*i*-1)th level is frequent
    - If a concept is infrequent, its descendents are pruned from the database
    - Drawbacks
      - Miss associations at low level items which are frequent based on a reduced min_support, but whose ancestors do not satisfy min_support

reduced support



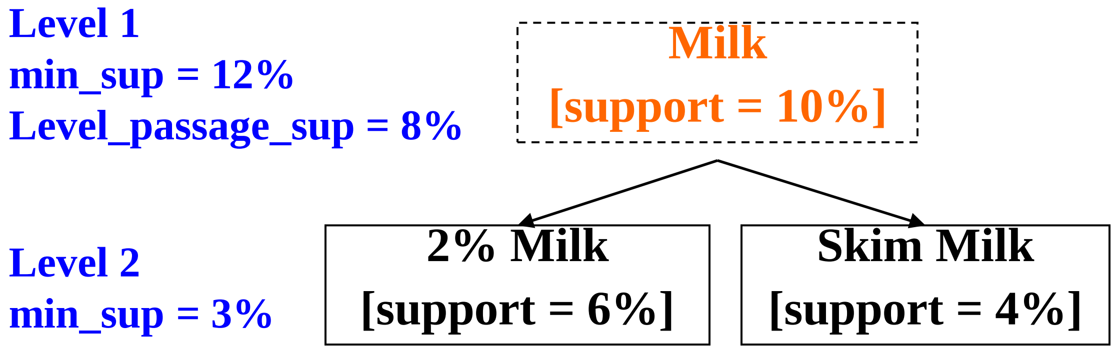
Mining Multiple-Level Association Rules

- Reduced support
  - Optimization -- level-cross filtering by *k*-itemset
    - Only the children of frequent *k*-itemsets are examined
    - Drawback: many valuable patterns may be filtered out



Mining Multiple-Level Association Rules

- Reduced support
  - Optimization -- Controlled level-cross filtering by single item
    - next level min sup < level passage threshold < min sup
    - Allow the children of items that do not satisfy the min_sup to be examined if they satisfy the level passage threshold



Multi-level Association: Redundancy Filtering

- Some rules may be redundant due to "ancestor" relationships between items
- Example
  - milk $\Rightarrow$ wheat bread [support = 8%, confidence = 70%]
  - 2% milk $\Rightarrow$ wheat bread [support = 2%, confidence = 72%]
- We say the first rule is an ancestor of the second rule
- A rule is redundant if its support is close to the "expected" value, based on the rule's ancestor

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Mining Multi-Dimensional Association

- Single-dimensional rules:

  buys(X, “milk”) $\Rightarrow$ buys(X, “bread”)

- Multi-dimensional rules: $\geq$ 2 dimensions or predicates
  - Inter-dimension assoc. rules (*no repeated predicates*)

    age(X,”19-25”) $\wedge$ occupation(X,“student”) $\Rightarrow$ buys(X, “coke”)
  - hybrid-dimension assoc. rules (*repeated predicates*)

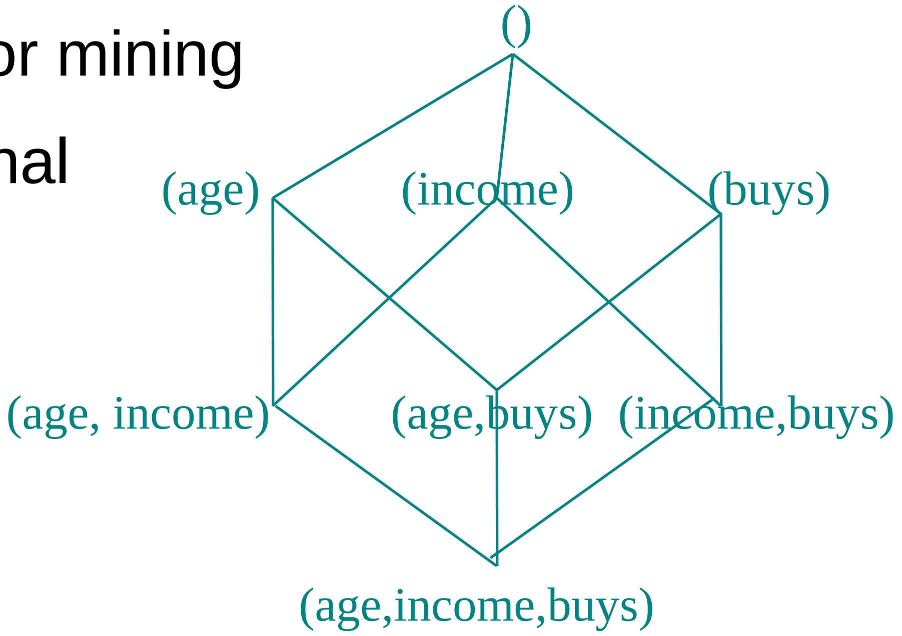
    age(X,”19-25”) $\wedge$ buys(X, “popcorn”) $\Rightarrow$ buys(X, “coke”)
- Categorical Attributes: finite number of possible values, no ordering among values
- Quantitative Attributes: numeric, implicit ordering among values — discretization, clustering approaches

Mining Quantitative Associations

- Techniques can be used to categorize numerical attributes
  - Static discretization based on predefined concept hierarchies
  - Dynamic discretization based on data distribution
  - Clustering: Distance-based association
    - one dimensional clustering then association

Static Discretization of Quantitative Attributes

- Discretized prior to mining using concept hierarchy
- Numeric values are replaced by ranges
- In relational database, finding all frequent *k*-predicate sets will require *k* or *k*+1 table scans
- Data cube is well suited for mining
- The cells of a n-dimensional cuboid correspond to the dimensions
- Mining from data cubes can be much faster



Quantitative Association Rules

- Numeric attributes are *dynamically* discretized
  - Such that the confidence or compactness of the rules mined is maximized
- 2-D quantitative association rules: $A_{quan1} \wedge A_{quan2} \Rightarrow A_{cat}$
- Association rule clustering system (ARCS)
  - Binning: 2-D grid, manageable size
  - Finding frequent predicate sets: scan the database, count the support for each grid cell
  - Clustering the rules: cluster adjacent cells to form a rule

Quantitative Association Rules

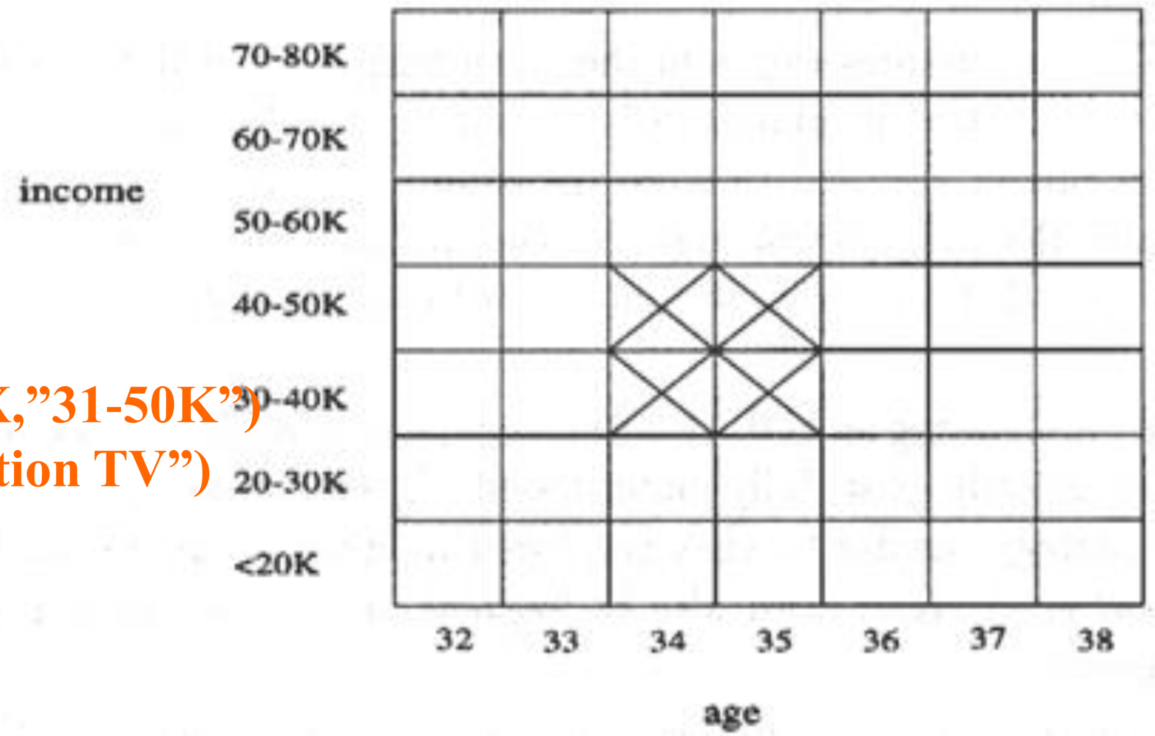
- Example

  age(X,"34") ∧ income(X,"31-40K") ⇒ buys(X,"high resolution TV")

  age(X,"35") ∧ income(X,"31-40K") ⇒ buys(X,"high resolution TV")

  age(X,"34") ∧ income(X,"41-50K") ⇒ buys(X,"high resolution TV")

  age(X,"35") ∧ income(X,"41-50K") ⇒ buys(X,"high resolution TV")



age(X,"34-35") ∧ income(X,"31-50K") ⇒ buys(X,"high resolution TV")

Mining Association Rules in Large Databases

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Summary

- Frequent pattern mining—an important task in data mining
- Scalable frequent pattern mining methods
  - Apriori (Candidate generation & test)
  - Partition, DIC, DHP, etc.
  - Projection-based (FP-growth)
- Mining a variety of rules and interesting patterns