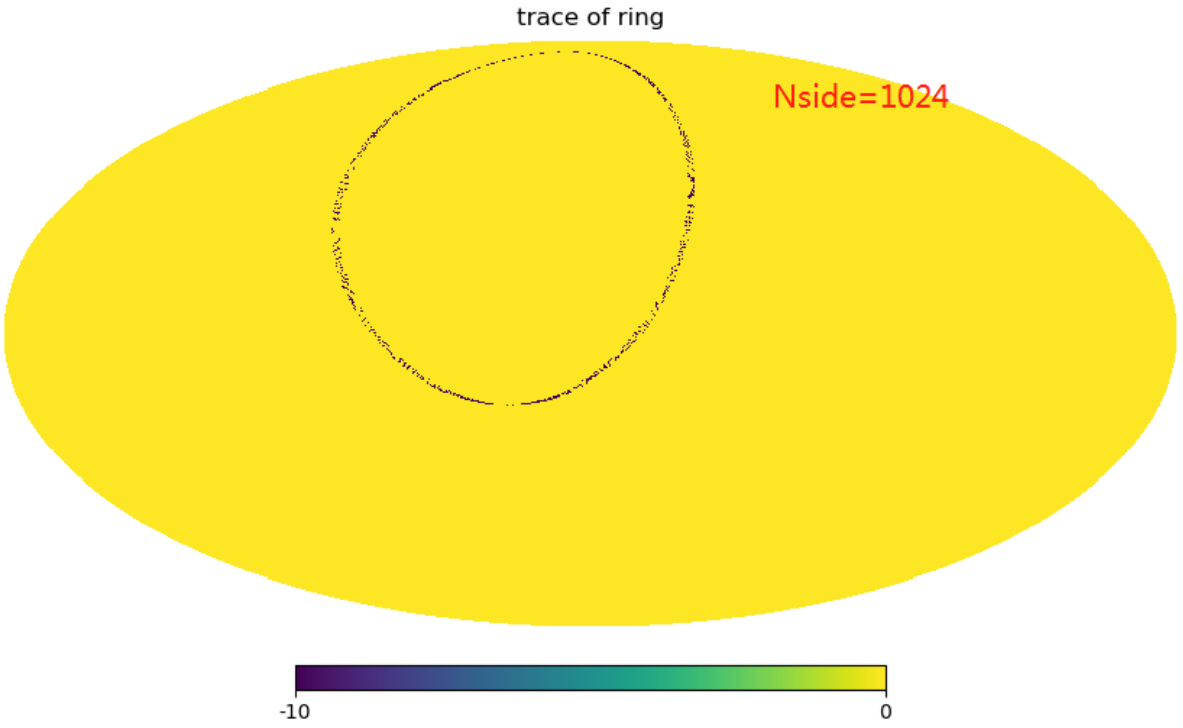
1, ring_scan

The code for circular scan is in `scan_ring.py`, one can get the pixel index of the scan track after running the code by (shown in fig1)

```
python scan_ring.py
```

and save it as `pix_theta_x_deg`.

```
pix = hp.ang2pix(nside=nside, theta=ra, phi=dec, lonlat=True)
np.save('pix_theta_'+str(st.theta)+'_deg', pix)
```



(fig 1, An example of trace of the ring scanning)

2, C_ℓ and Γ_m

2.1 Γ_m from theory

For C_ℓ from theory (e.g., by running CAMB), Γ_m can be given by

$$\begin{aligned}\Gamma_m &= \sum_{\ell=|m|}^{\infty} C_\ell B_\ell^2 \mathcal{P}_{\ell m}^2(\theta_0) \\ &= M_{m\times\ell} C_\ell\end{aligned} \tag{1}$$

where B_ℓ is beam function and $\mathcal{P}_{\ell m}(\theta_0) = (-1)^m \sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos\theta_0)$，$P_\ell^m$ is Legendre polynomial，θ_0 is elevation angle。As $M_{m\times\ell}$ could be a singular matrix，when computing Γ_m from C_ℓ, bin matrix B is introduced:

$$C_\ell \simeq B(MB)^{-1}\Gamma_m. \tag{2}$$

Bin_size for i^{th} element is defined as：

$$b_i = round(a \times b^i), \tag{3}$$

where a and b are two free parameters.

By running `Cl_Gamma_theory.py` , we have fig 2 and fig3

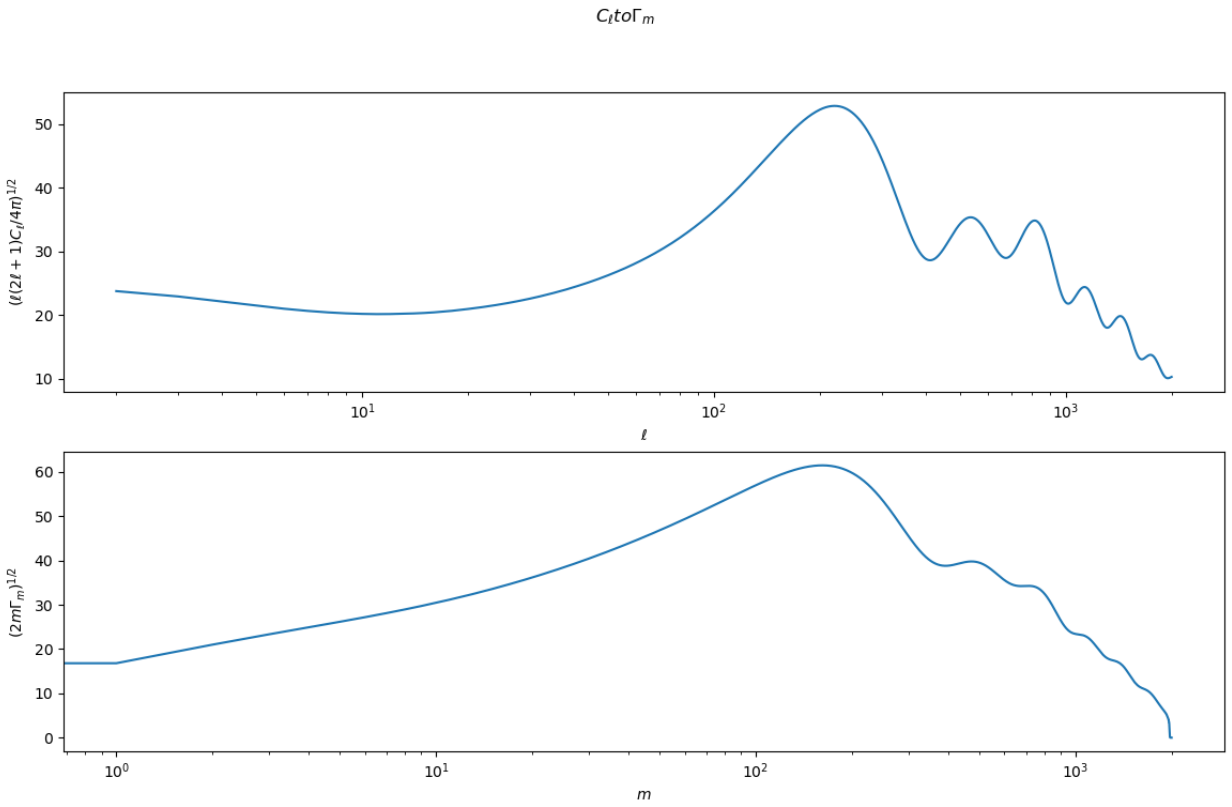


fig2 $\mathbf{\Gamma_m}$ from C_ℓ

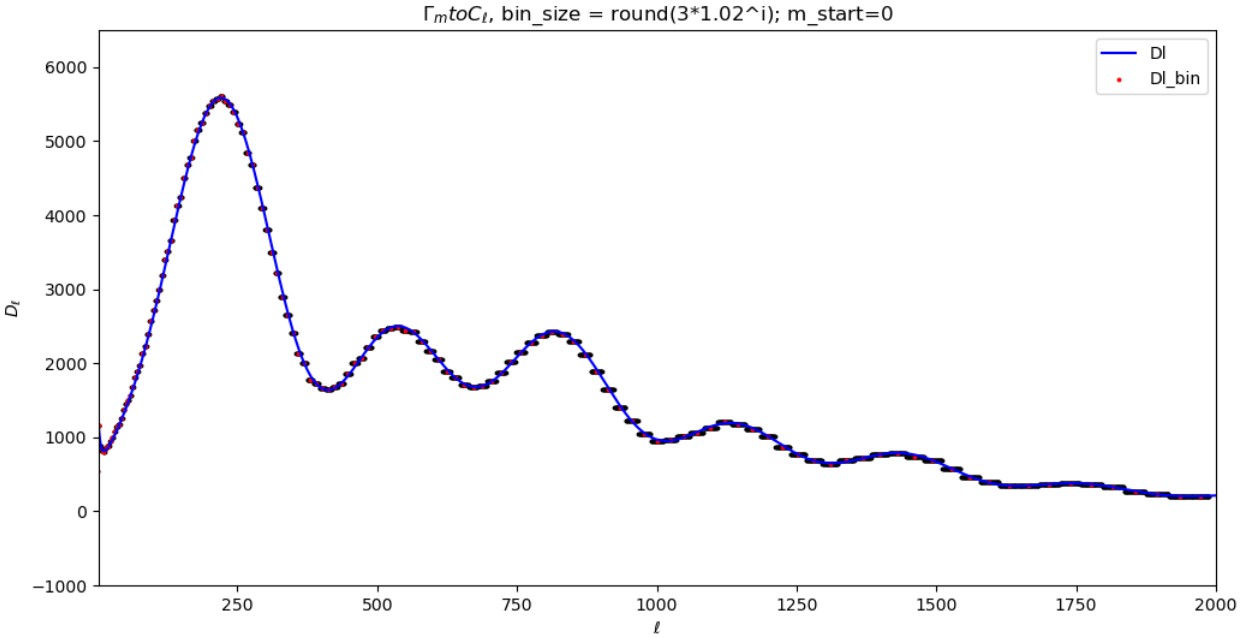


fig3 D_ℓ from CAMB(blue) and by reversing $\mathbf{\Gamma_m}$ (dotted-line, after bin)

2.2 Cosmic variance of $\mathbf{\Gamma}_m$

$$\Delta\mathbf{\Gamma}_m = M_{m\times\ell}\Delta C_\ell = M_{m\times\ell}\sqrt{\frac{2}{2\ell+1}}C_\ell. \tag{4}$$

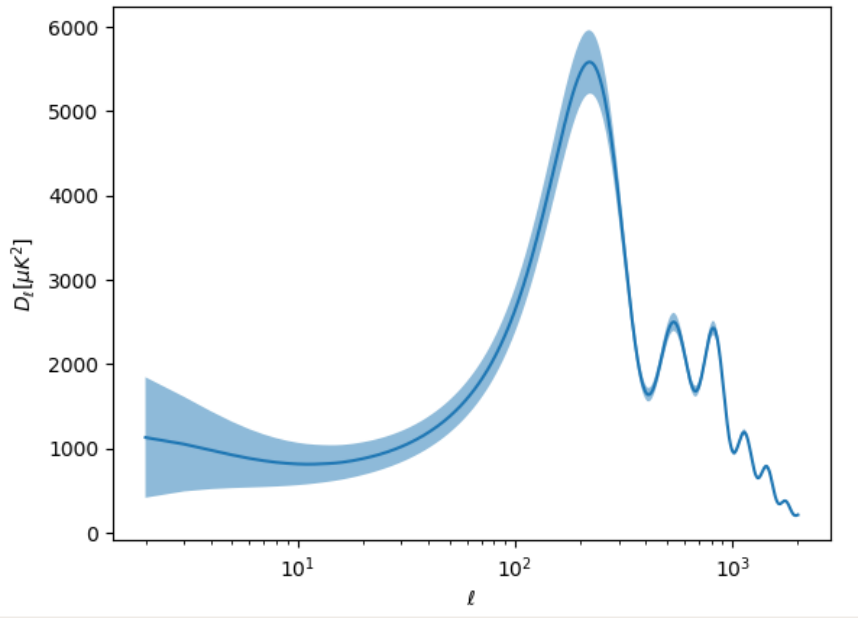


fig4 Cosmic variance of C_ℓ

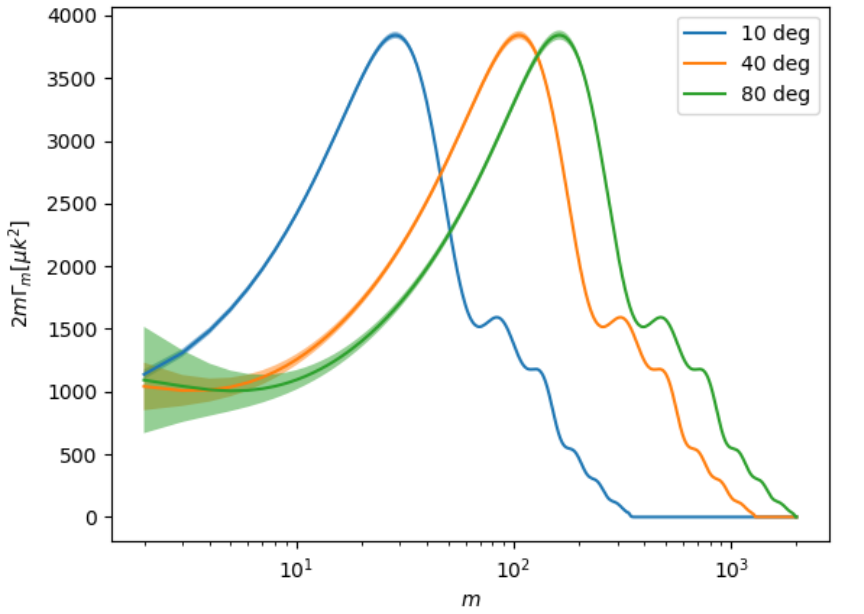


fig5 Cosmic variance of $\mathbf{\Gamma}_m$

2.3 $\mathbf{\Gamma}_m$ from scanning data

There are two ways to get TOD (time-ordered data) for ring scanning.

Way 1: Read data along the latitude. Since `Healpy` sorts pixels from top to bottom when pixelating, a simple way to read the TOD of CMB is along the latitude of the coordinate system (shown in fig6). (It can be considered as coordinate transformation of the scanning track).

After reading rings from multiple simulated CMB maps, $\mathbf{\Gamma}_m$ can be calculated by Fourier transform of TODs and C_ℓ can be calculated from $\mathbf{\Gamma}_m$.

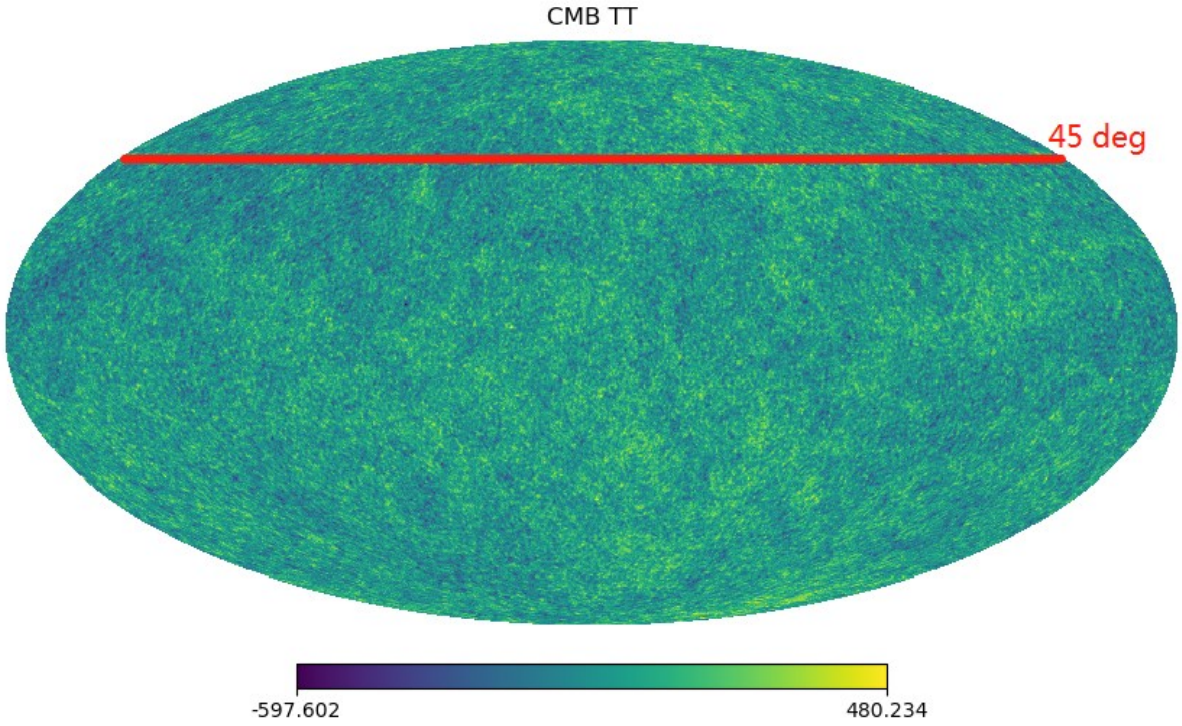


fig6, Read data along the latitude in the map (e.g., 45 deg)

After running `Cl_Gamma_along_latitude_scan.py`, we can get fig 7 and fig 8 (Only 50 rings are considered here, for more precise, one can run more rings)

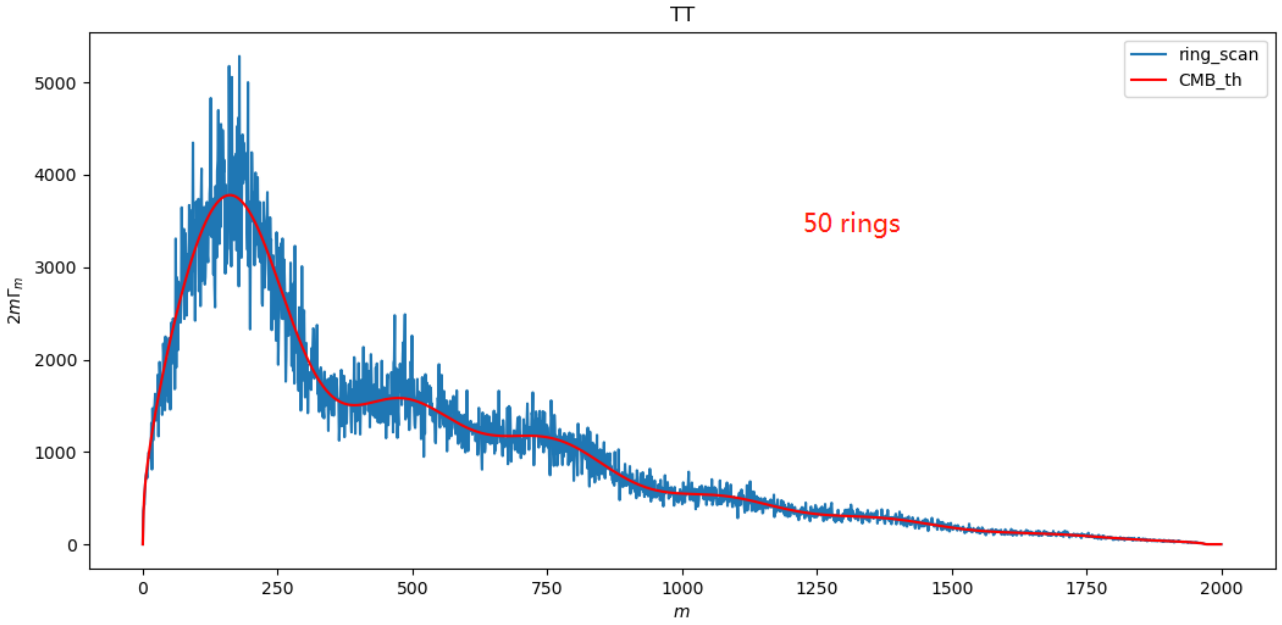


fig 7, $\boldsymbol{\Gamma_m}$ from ring data (blue) and $\boldsymbol{C_\ell}$ (red)

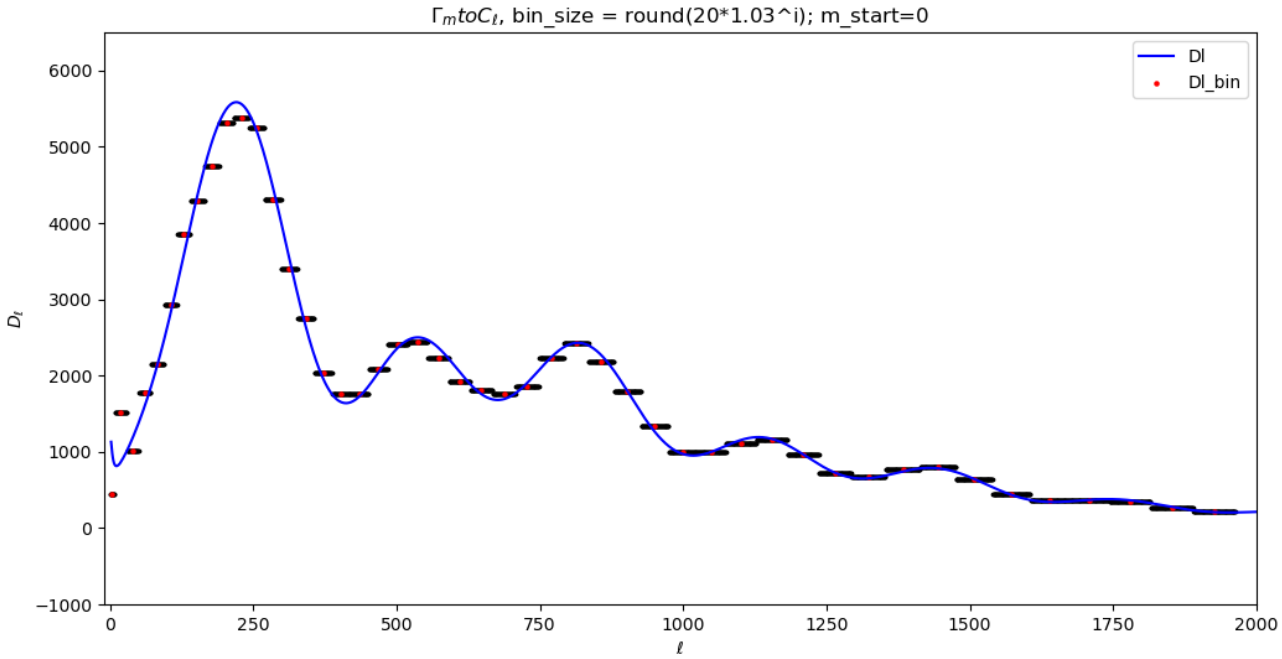
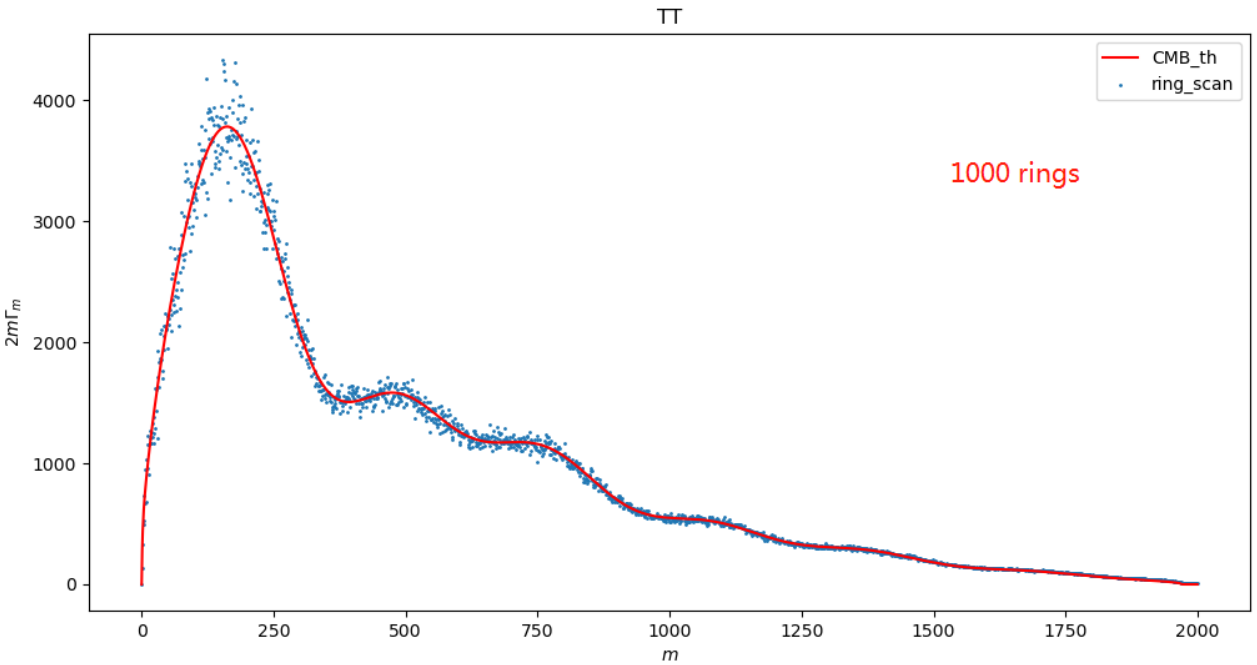
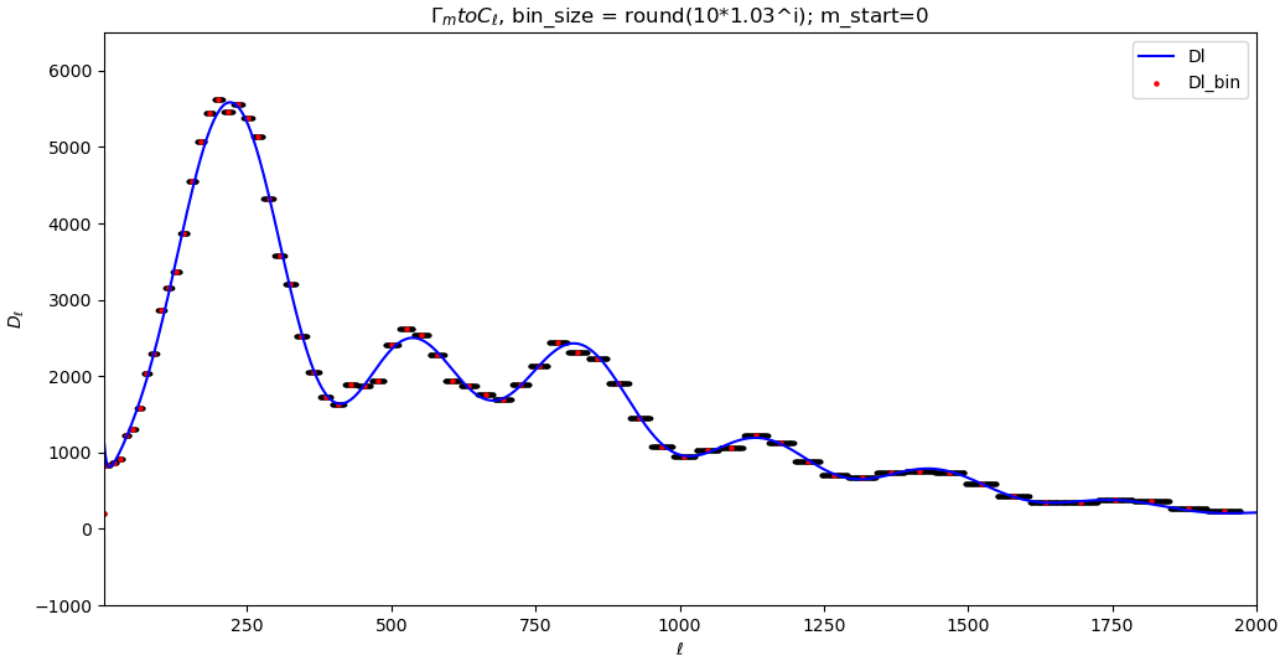


fig 8, $\boldsymbol{D_\ell}$ from CAMB (blue) and $\boldsymbol{\Gamma_m}$ (dotted-line)

Way 2: For real ring scanning: Since there is only one universe, which means we only have one CMB map. To be more realistic, the ring scanning is from `scan strategy` in the horizontal coordinate system (as shown in fig1), and the index of the trace can be obtained by running `scan_ring.py` (In the case, `Nside` need to be higher for accuracy because of the limitation of pixel numbers by `Healpy`, e.g., Nside=4096). After getting the TODs of CMB by scanning map, $\boldsymbol{C_\ell}$ can be calculated from $\boldsymbol{\Gamma_m}$ by running `Cl_Gamma_from_real_scan.py`





3, Appendix

Although the above description is for temperature, the codes also can be used for polarization, but change I to Q and U.