1. Generalized nuclear norm for patches

Reference: [2012 Chartrand] Nonconvex splitting for regularized low-rank sparse decomposition

Consider the minimization problem involving nuclear norm:

where is the patch extraction matrix for the th patch.

* 1. **Selection of**

Let be elementwise generalized Huber function

where

where to make the function continuous.

Define an auxiliary function as:

where the conjugate is given by Legendre-Frenchel transform

has several properties:

1. Proximal mapping
2. Soft thresholding: the minimizer for problem (1) is given by
3. Generalize to nuclear norm

Let

Then

where , are the singular value matrices of and .

The minimizer is given by

where

* 1. **Building Cost function**

Then the original problem is transformed into

Alternating minimization:

where is the surrogate function for

The alternating minimization satisfies:

Giving a monotonic algorithm.

For spectral CT, the function became

which can be solved spectrum by spectrum.

1. SQS for TV norm

Reference: [2018] A separable quadratic surrogate total variation minimization algorithm for accelerating accurate CT reconstruction from few-views and limited-angle data

Consider the total variation penalty:

Define

So

Let

We have Prop. 1:

Second equation is because

Let :

According to Prop. 1, is a valid surrogate of at .

Making further surrogate with convex relaxation:

Then we have

And the equality holds when .

Do the same for the rest 2 dimensions and substitute into and

which is the final surrogate of . It is straightforward to verify that .

The total surrogate is

First order derivative:

And

Second order derivative:

*For implementation:*

First calculate for each pixel and store the array;

Then calculate first and second order derivatives for each pixel.

1. Optimization Algorithm for TV + nuclear norm

The total cost function for spectral CT with be

where

is a matrix.

Minimization w.r.t. nuclear norm:

Then minimizing w.r.t. surrogate of TV and patching for each spectrum independently:

The TV related terms are defined above.

1. SQS for Non-local weighted Gaussian prior

Prior function is defined as:

Use convex splitting:

Equality holds at .

Surrogate:

First order derivative:

Second order derivative:

Derivatives at current position:

For most cases, we have

And the derivatives became:

The loss can be calculated as