
Binomial distribution

Notation:

$$X \sim B(n, p) \quad \text{with} \quad n \in \mathbb{N} \quad \text{and} \quad 0 \leq p \leq 1$$

The binomial distribution is a discrete distribution with distribution parameters n and p , where n is called the *number of “trials”* and p the *probability of “success”*. Expected value and variance are given by:

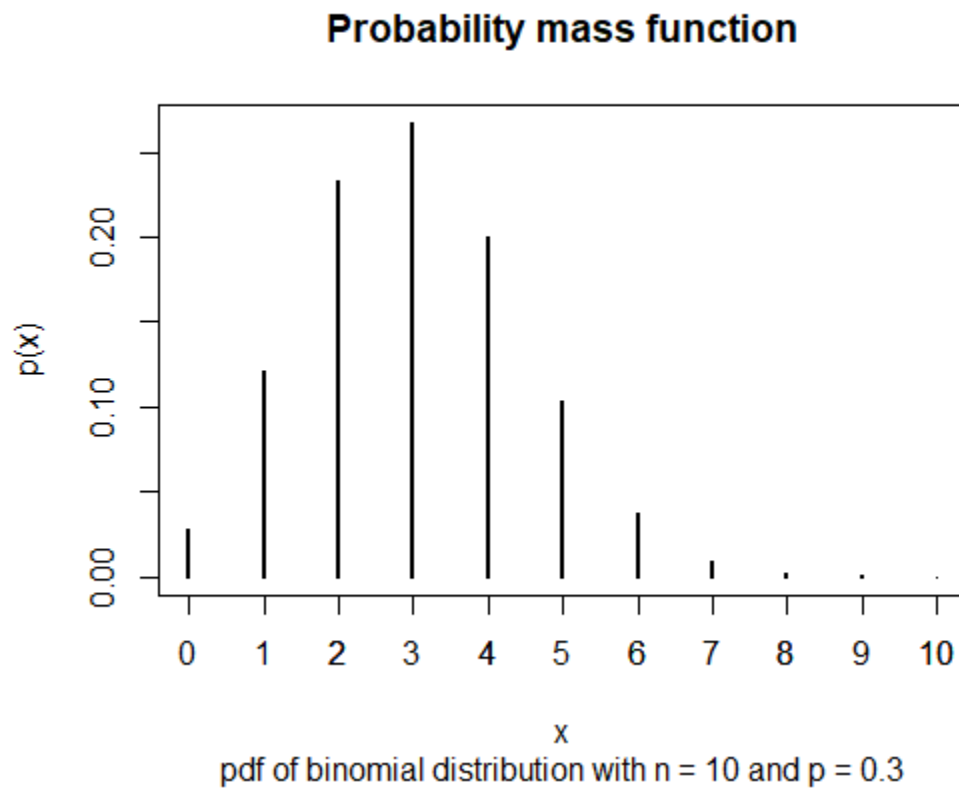
$$E(X) = np \quad \text{and} \quad \text{Var}(X) = np(1 - p)$$

Probability mass function

The probability mass function (pdf) is given as follows:

$$p(x) = P(X = x) = \begin{cases} \binom{n}{x} p^x (1 - p)^{n-x} & \text{for } x \in \{0, 1, 2, \dots, n\} \\ 0 & \text{otherwise} \end{cases}$$

with $x :=$ Number of “matches” and $\binom{n}{x} :=$ Binomial coefficient.

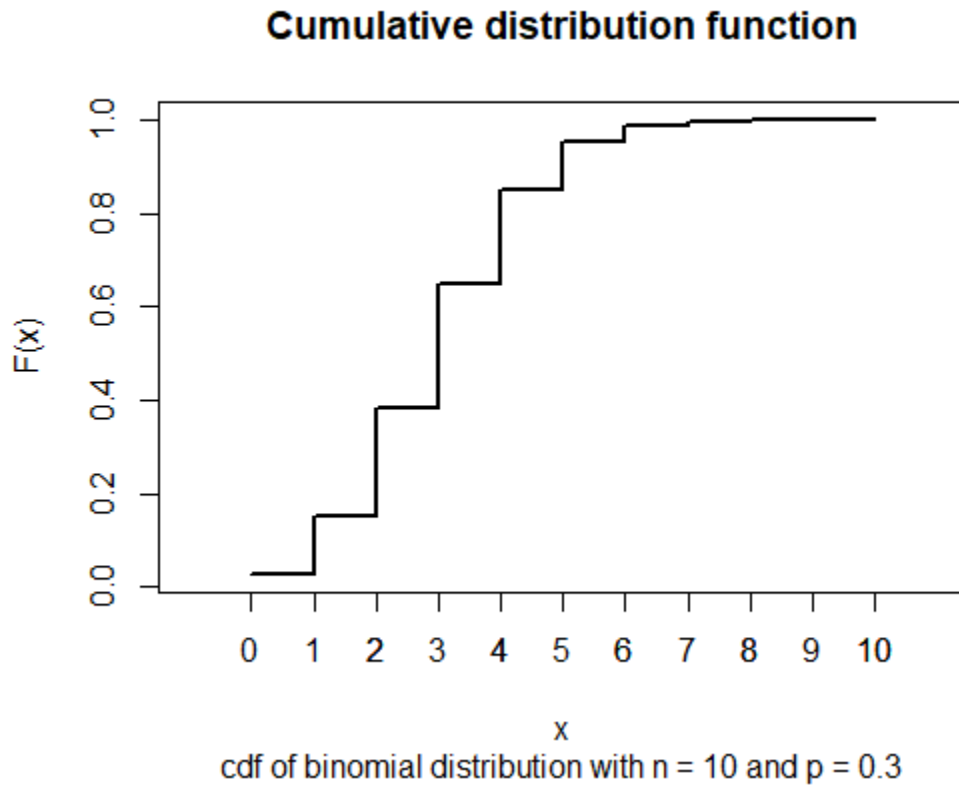


Cumulative distribution function

The cumulative distribution function (cdf) is defined as:

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} P(X = x_i)$$

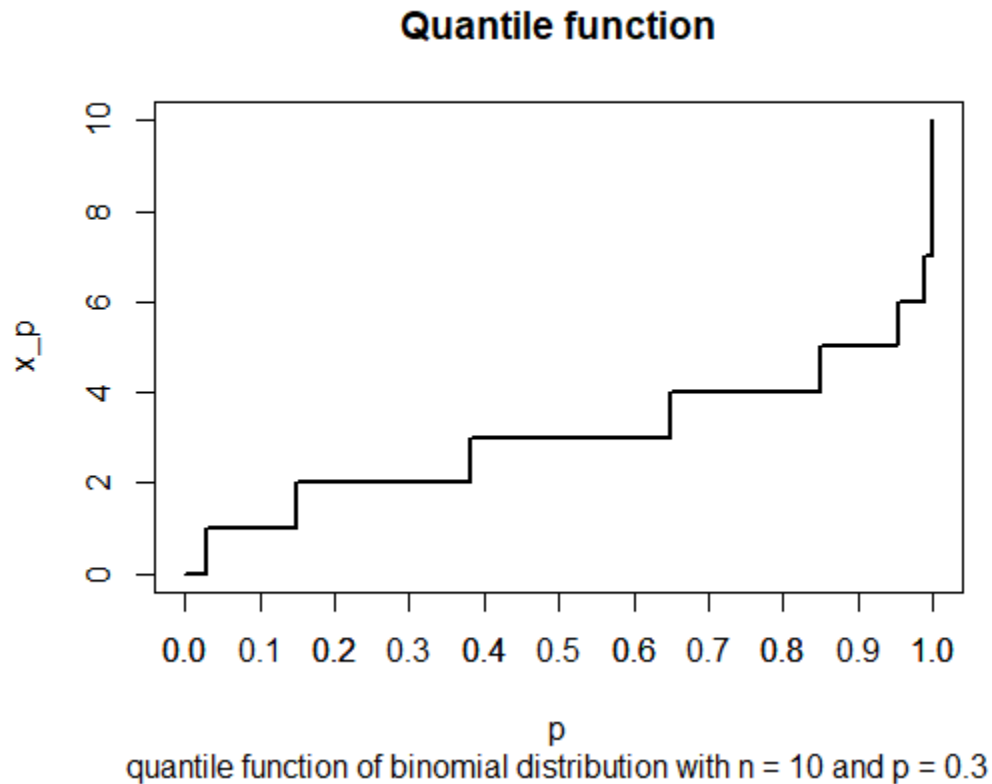
The value of the cumulative distribution function specifies the probability that the random variable X is less than or equal to x .



Quantile function

The quantile function returns the value x_p under which is $p\%$ of the probability mass. Formally, the quantile function is the inverse function of the distribution function:

$$x_p = F^{-1}(p) = F^{-1}[P(X \leq x_p)]$$



Excel commands

Probability function and distribution function of the binomial distribution

- `=BINOM.VERT(x ; n ; p ; kumuliert)`
 - x := Number “matches”
 - n := Number “experiments”
 - p := Probability of success
 - kumuliert = 1 := Value of the distribution function (probability)
 - kumuliert = 0 := Value of the probability function (probability)

Probability range

- `=BINOM.VERT.BEREICH(n ; p ; x_1 ; x_2)`
 - n := Number “experiments”
 - p := Probability of success
 - x_1 := Lower limit
 - x_2 := Upper limit

The function `BINOM.VERT.BEREICH` calculates: $P(x_1 \leq X \leq x_2)$

Quantile function of the binomial distribution:

- `=BINOM.INV(n ; p ; α)`
 - n := Number “experiments”
 - p := Probability of success
 - α := Probability

Note

`BINOM.VERT.BEREICH($n; p; x_1; x_2$) = BINOM.VERT($x_2; n; p; \text{WAHR}$) - BINOM.VERT($x_1 - 1; n; p; \text{WAHR}$)`

because

$$P(x_1 \leq X \leq x_2) = P(X \leq x_2) - P(X < x_1) = P(X \leq x_2) - P(X \leq (x_1 - 1))$$