Binomial distribution

Notation:

$$X \sim \mathrm{B}(n,p) \quad \text{with} \quad n \in \mathbb{N} \quad \text{and} \quad 0 \leq p \leq 1$$

The binomial distribution is a discrete distribution with distribution parameters n and p, where n is called the *number of "trials"* and p the *probability of "success"*. Expected value and variance are given by:

$$\mathrm{E}(X) = np \qquad \text{and} \qquad \mathrm{Var}(X) = np(1-p)$$

Probability mass function

The probability mass function (pdf) is given as follows:

$$p(x) = P(X = x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & \text{for} \quad x \in \{0,1,2,\ldots,n\} \\ \\ 0 & \text{otherwise} \end{cases}$$

with $x :=$ Number of "matches" and $\binom{n}{x} :=$ Binomial coefficient.

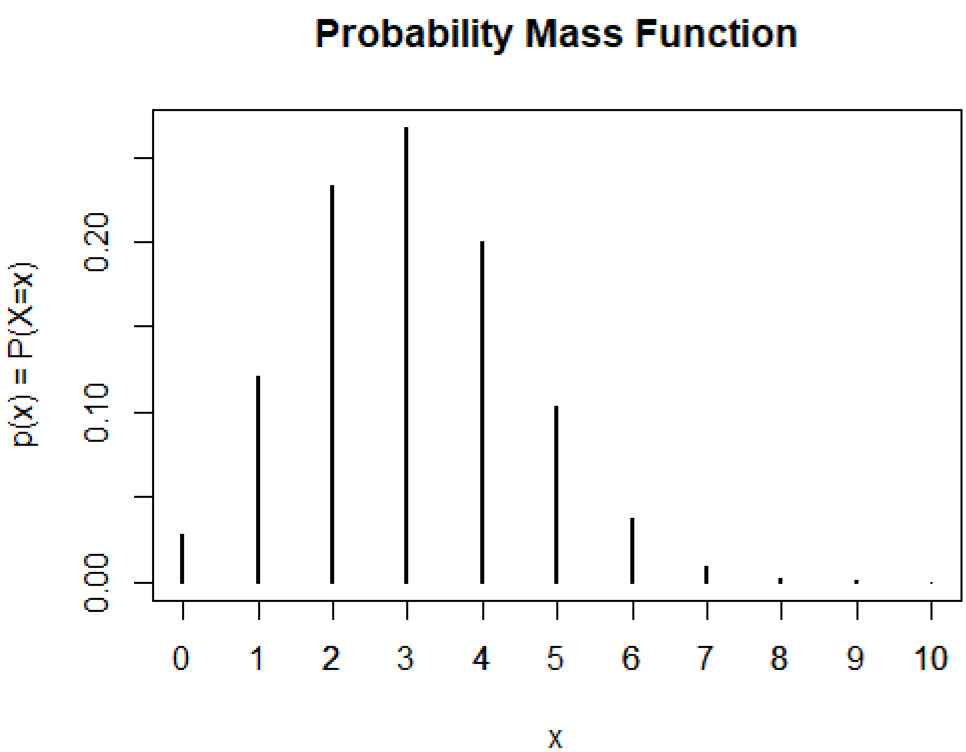


Figure 1: pdf of binomial distribution with n = 10 and p = 0.3

Cumulative distribution function

The cumulative distribution function (cdf) is defined as:

$$F(x) = P(X \leq x) = \sum_{x_i < x} P(X = x_i)$$

The value of the cumulative distribution function specifies the probability that the random variable X is less than or equal to x.

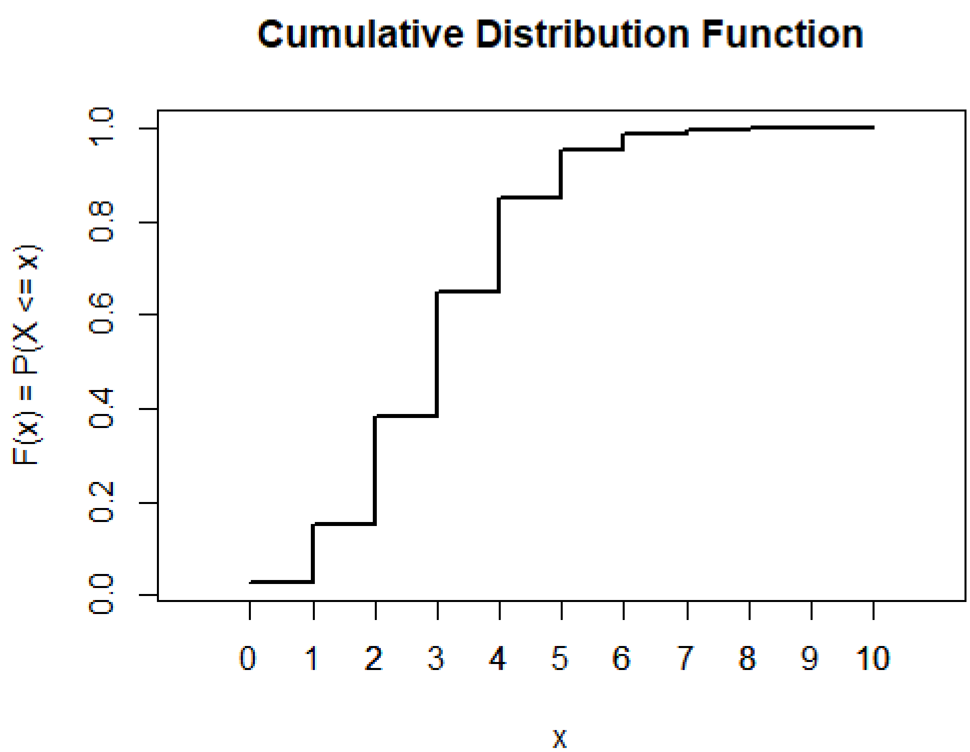
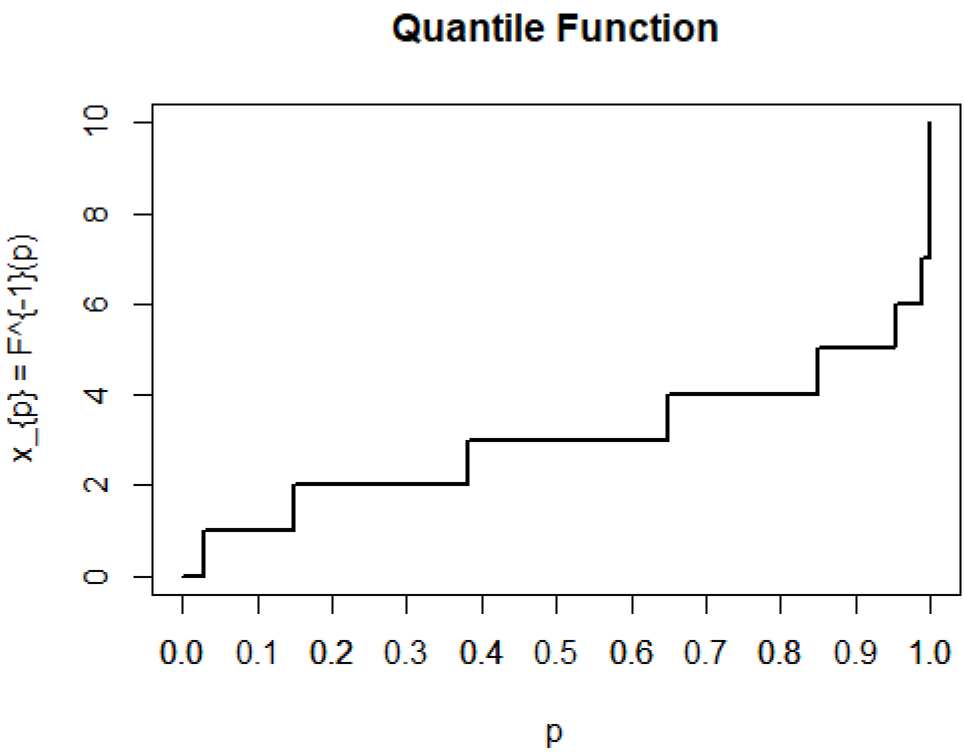


Figure 2: cdf of binomial distribution with n = 10 and p = 0.3

Quantile function

The quantile function returns the value x_p under which is $p\%$ of the probability mass. Formally, the quantile function is the inverse function of the distribution function:

$$x_p = F^{-1}(p) = F^{-1}[P(X \leq x_p)]$$



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Excel commands

Probability function and distribution function of the binomial distribution

- `=BINOM.VERT`(x; n; p; **kumuliert**)
  - x := Number “matches”
  - n := Number “experiments”
  - p := Probability of success
  - kumuliert $= 1$:= Value of the distribution function (probability)
  - kumuliert $= 0$:= Value of the probability function (probability)

Probability range

- `=BINOM.VERT.BEREICH`(n; p; x_1; x_2)
  - n := Number “experiments”
  - p := Probability of success
  - x_1 := Lower limit
  - x_1 := Upper limit

The function `BINOM.VERT.BEREICH` calculates: $P(x_1 \leq X \leq x_2)$

Quantile function of the binomial distribution:

- `=BINOM.INV`(n; p; α)
  - n := Number “experiments”
  - p := Probability of success
  - α := Probability

Note

`BINOM.VERT.BEREICH`(n; p; x_1; x_2) = `BINOM.VERT`(x_2; n; p; WAHR) - `BINOM.VERT`($x_1 - 1$; n; p; WAHR)

because

$$P(x_1 \leq X \leq x_2) = P(X \leq x_2) - P(X < x_1) = P(X \leq x_2) - P(X \leq (x_1 - 1))$$