Modeling Pipe Networks Dominated by Junctions

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Abstract: The transfer of energy and irreversible energy losses at junctions may dominate flow distribution in pipe networks comprised of connected large-diameter pipes of short lengths. Neglecting the junction effects when calculating network hydraulics is unacceptable and the use of constant loss coefficients to represent these effects in such networks may not be adequate. In the present study, precise relationships based on experimental data are used to represent the junction-loss coefficients. These relations, which depend on flow conditions, are incorporated into a computer network analysis model. Significant deviations in flow rates calculated using constant junction loss coefficients and the semiempirical relationships based on experimental data are obtained using an example network characterized by short pipe sections. It is also shown that although there is always a net energy loss at a junction, there may be significant exchange of specific energy from one flow stream to the other. The use of constant loss coefficients is inadequate to account for this phenomenon.

INTRODUCTION

For many important fluid-transport applications the piping systems are comprised of large-diameter pipes of short lengths with numerous interconnections. For these systems, the losses caused by the various fittings, and particularly the junctions may be quite large, and the method for handling losses at the junctions will have a significant effect on the flow rates and pressures calculated using a hydraulic network model.

To illustrate the problem of concern, the present paper will focus on T-junctions, which are most common. The points covered, however, apply to other types of multipipe junctions.

Most text and reference books list values for loss coefficients or equivalent lengths to account for losses at T-junctions (Bober and Kenyon 1980; Evit and Liu 1987; Hodge 1985; John and Haberman 1988; Vennard and Street 1975). In most cases, two loss coefficients are listed. A lower value (0.2–0.5) is suggested for the loss coefficient for the straight run and a higher value (1.5–2.0) is given for the loss coefficient for the side branch. Even this simplified approach requires clarification, which is seldom presented.

For the case shown in Fig. 1(a) where the flow is dividing the interpretation is straightforward. There is a loss of $K_2(V_2^2/2g)$ in pipe 2 and $K_1(V_1^2/2g)$ in pipe 1 where K_2 is the straight run loss coefficient, K_1 the side branch loss coefficient and V is the velocity of flow. For other situations such as those depicted in Fig. 1(b and c), where the flow is combining, the interpretation is not straightforward. Clarification is made by defining the combined leg as the leg that transports the total flow. The losses are associated with the other two legs and the coefficient depends on whether the flow between the leg to or from the combined leg is straight through or through

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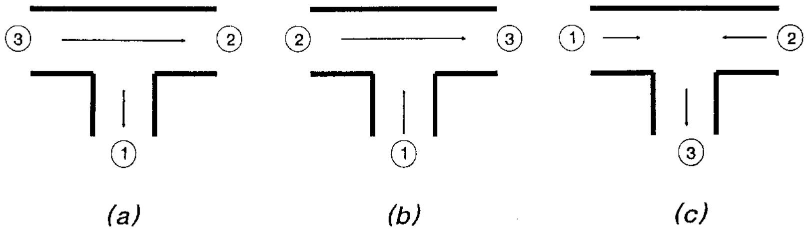

FIG. 1. Typical T-Junction Flow Configurations

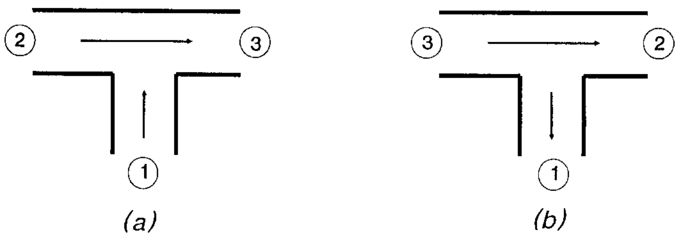

FIG. 2. Combining and Dividing Flows

a 90° turn. Thus for case b, the losses are $K_2(V_2^2/2g)$ and $K_1(V_1^2/2g)$ where K_2 is the straight run loss coefficient and K_1 is the side branch loss coefficient. Using this approach, all situations can be readily categorized. Of course, flow directions must be considered and affect the choice of coefficient and the calculated losses.

Unfortunately the approach described previously greatly oversimplifies the problem and gives results that have little or no correlation to published observations (Idlechik 1986; Miller 1971). Thus, there seems to be limited merit in utilizing junction loss coefficients defined in this manner. In many pipe-network hydraulic-modeling applications the junction losses are of little consequence and the preceding approach, while inadequate, does not greatly affect the results. However, for piping systems comprised of short, large-diameter interconnected pipes, the approach utilized for handling junction losses has a very significant effect on the results and the use of constant coefficients as described previously produces significant errors.

LOSSES AT T-JUNCTIONS

Extensive work has been carried out and published on losses at T-junctions (Idlechik 1986; Miller 1971). To represent the experimental data, loss coefficients are defined as coefficients that multiply the kinetic energy per unit weight (velocity head) in the combined leg to give the difference in specific energy (pressure head + velocity head) between the combined leg and the specific energy of fluid in one of the other two legs. The following definitions are given for these coefficients based on the notation shown in Fig. 2. Pipe 3 is the combined flow leg for both cases and the loss is $K(V_3^2/2g)$ for all situations.

$K_{31}(V_3^2/2g)$ loss $3 \rightarrow 1$ dividing

$K_{32}(V_3^2/2g)$ loss $3 \rightarrow 2$ dividing

$K_{13}(V_3^2/2g)$ loss 1 → 3 combining

$K_{23}(V_3^2/2g)$ loss 2 → 3 combining

The values for these coefficients have been obtained from extensive experiments and vary with both the area ratios and flow ratios. The data is presented in detail in Idlechik (1986) and Miller (1971).

For T-junctions connecting equal area pipes, the various coefficients can be presented as a function of flow ratio on the single plot shown in Fig. 3 (Miller 1971).

To compare these to the constant loss coefficients presented in standard references, the plot must be modified to utilize the velocity head in the applicable noncombined leg. A modified loss coefficient is given by

$$K'_{31} = \frac{K_{31}}{\left(\frac{Q_1}{Q_3}\right)^2} \quad (1)$$

$$K'_{32} = \frac{K_{32}}{\left(\frac{Q_2}{Q_3}\right)^2} \quad (2)$$

where Q_i = flow rate in leg i.

Similar transformations are required for K'_{13} and K'_{23}. The modified coefficients are plotted in Fig. 4. Because of a wide range of variations a log scale is used for this plot. These coefficients can be compared to the constant straight-through and side-branch loss coefficient values often presented in

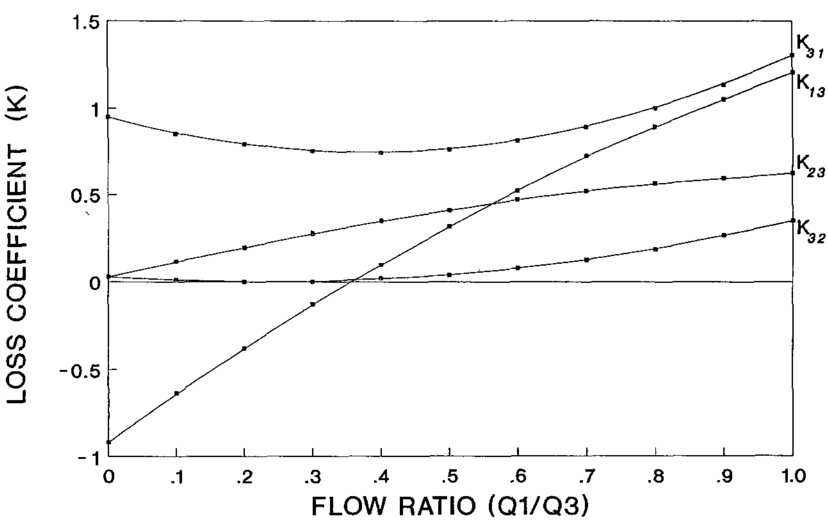

FIG. 3. Loss Coefficients as Function of Flow Ratio

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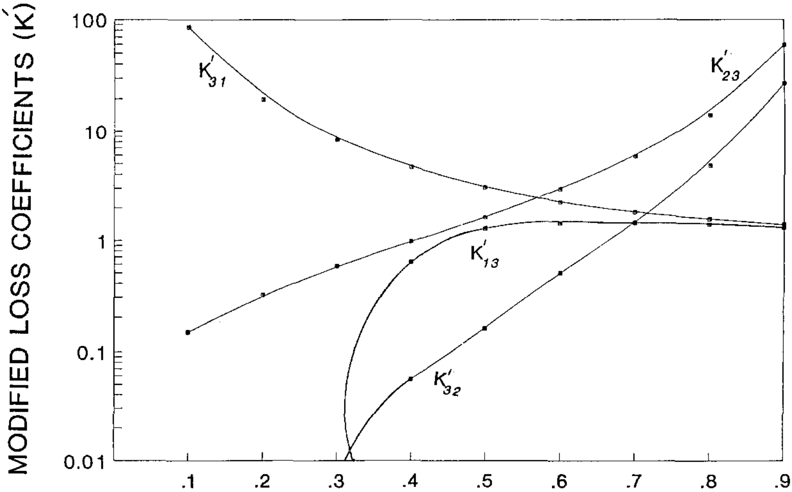

FIG. 4. Modified Coefficients as Function of Flow Ratio

the literature. The following indicates the type of situation represented by the various coefficients:

K'_{31} = straight through (dividing)

K'_{32} = side branch (dividing)

K'_{13} = straight through (combining)

K'_{23} = side branch (combining)

It is clear that the use of constant loss coefficients provides an inadequate representation and has no correlation to the observed results. At low and high flow rates, the coefficients can become very large and negative coefficients are possible.

The inability to represent loss coefficients as constants and the appearance of negative coefficients may require further explanation. In dividing and combining junctions, there may be significant exchange of energy among the streams. For example, when a high-velocity stream combines with a low-velocity stream, a jet pump action takes place that transfers energy from the high-velocity stream to low-velocity stream. The loss coefficient is the number that multiplies the velocity head in the combined leg to obtain the difference between the specific energy (pressure head + velocity head) of the fluid in the combined leg and the leg in question. It might be better called a "specific energy difference coefficient," but the term "loss coefficient" is in common usage and is used in this paper. A negative loss coefficient simply means that the specific energy of the fluid in the particular leg is larger than that in the combined leg (dividing junction) or lower than that in the combined leg (combining junction). Under all circumstances, however, there is a net energy loss at the junction. This is further evidence that the use of conventional constant loss coefficients is not adequate.

APPLICATION TO HYDRAULIC NETWORK MODELING

When conducting hydraulic analysis of a piping system containing junctions, both flow directions and flow rates are unknown. To accommodate precise modeling of junctions, the model must incorporate flow-dependent relations for the loss coefficients and must account for flow directions.

Semiempirical equations developed by Gardel (1957a, b) have been shown to adequately represent the experimental data. These equations are expressed for a general T-junction in terms of the angle of the side branch, area, and flow ratio as follows:

$$K_{13} = -0.92(1-q)^2 - q^2\left[1.2\left(\frac{\cos\theta}{a} - 1\right) + 0.8\left(1 - \frac{1}{a^2}\right) - (1-a)\frac{\cos\theta}{a}\right] + (2-a)q(1-q) \quad (3)$$

$$K_{23} = 0.03(1-q)^2 - q^2\left[1 + 1.62\left(\frac{\cos\theta}{a} - 1\right) - 0.38(1-a)\right] + (2-a)q(1-q) \quad (4)$$

$$K_{31} = 0.95(1-q)^2 + q^2\left(1.3\cot\frac{180-\theta}{2} - 0.3 + \frac{0.4-0.1a}{a^2}\right) + 0.4q(1-q)\left(1 + \frac{1}{a}\right)\cot\frac{180-\theta}{2} \quad (5)$$

$$K_{32} = 0.03(1-q)^2 + 0.35q^2 - 0.2q(1-q) \quad (6)$$

where a = ratio of cross-sectional area of leg 1 to cross-sectional area of leg 3; q = ratio of flow rate in leg 1 to flow rate in leg 3; and Θ = angle between leg 1 and leg 3.

For the simplified case of a 90° T-junction, these expressions can be simplified to the following:

$$K_{13} = -0.92(1-q)^2 - q^2\left[-1.2 + 0.8\left(1 - \frac{1}{a^2}\right)\right] + (2-a)q(1-q) \quad (7)$$

$$K_{23} = 0.03(1-q)^2 - q^2[-0.62 - 0.38(1-a)] + (2-a)q(1-q) \quad (8)$$

$$K_{31} = 0.95(1-q)^2 + q^2\left(1 + \frac{0.4-0.1a}{a^2}\right) + 0.4q(1-q)\left(1 + \frac{1}{a}\right) \quad (9)$$

$$K_{32} = 0.03(1-q)^2 + 0.35q^2 - 0.2q(1-q) \quad (10)$$

Though the Gardel equations are known to represent the actual conditions adequately, they do not accommodate certain possible flow configurations in the 90° T-junctions. They are: (1) Flow entering the junction through the

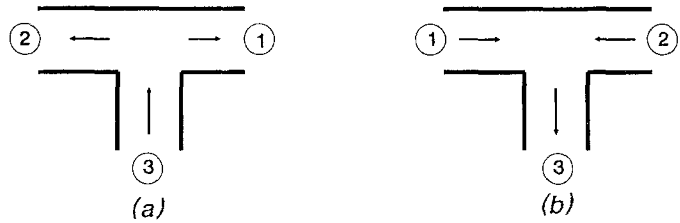

FIG. 5. Flow Entering/Leaving through Side Branch Only

side branch only; and (2) flow leaving the junction through the side branch only, as shown in Fig. 5.

To represent these two flow configurations, equations proposed by Levin (1958) based on his experimental investigations are used in the present study. They are expressed as

$$K_{13} = K_{23} = 1 + \frac{1}{a^2} + \frac{3}{a^2}(q^2 - q) \quad (11)$$

$$K_{31} = 1 + 0.3\frac{q^2}{a^2} \quad (12)$$

$$K_{32} = 1 + 0.3\frac{(1 - q)^2}{a^2} \quad (13)$$

These relations (3)–(13) were incorporated into a computer model for the hydraulic analysis of a pipe network. After each iteration in which flows are calculated, the loss coefficients at all T-junctions are reevaluated using the preceding expressions. The iterations continue until the calculated coefficients and flow rates are essentially unchanged. Because the loss coefficients are reevaluated after each iteration, additional trials are normally required.

EXAMPLE

The example network shown in Fig. 6 was analyzed using these approaches: (1) Case a—T-junction losses were neglected; (2) Case b—constant loss coefficients were utilized for T-junctions; and (3) Case C—Gardel equations were used to calculate T loss coefficients.

Details of the example network are shown in Fig. 6. Length of each pipe element is 3 m. An entrance loss coefficient of 0.5 is used at the reservoir connected to pipe 1. An exit loss coefficient of 1.0 each is used for the other three reservoirs. The example network is characterized by having short pipe sections. Therefore the losses at the T-junction are expected to be very significant.

The calculated flow rates in each pipe are summarized in Table 1. Table 2 shows the loss coefficients that were used for each analysis. For case c, these represent the values calculated using the Gardel (1957a, b) and Levin (1958) equations. Herein, it should be noted that these values are the modified loss coefficients K' based on velocity in the applicable noncombined leg. This facilitates comparison of these loss coefficients to the constant loss coefficients presented in standard references. The results show that the use

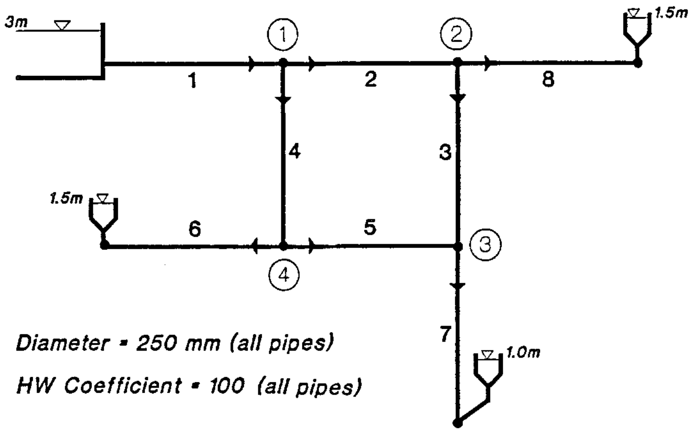

FIG. 6. Example Network

TABLE 1. Network with Insignificant Pipe Friction Losses (Length of Each Pipe = 3 m)

Pipe number (1)	Flow Rates		
	Case a (L/s) (2)	Case b (L/s) (3)	Case c (L/s) (4)
1	269.8	240.1	197.8
2	134.9	149.0	179.7
3	71.3	73.1	32.3
4	134.9	91.2	18.1
5	71.3	48.6	57.1
6	63.7	42.6	−39.0
7	142.6	121.7	89.4
8	63.6	75.8	147.4

TABLE 2. Loss Coefficients for Each Junction

Node number (1)	LOSS COEFFICIENTS				
		Case b		Case c	
	Case a (2)	$K_{13}(K_{31})$ (3)	$K_{23}(K_{32})$ (4)	$K'_{13}(K'_{31})$ (5)	$K'_{23}(K'_{32})$ (6)
1	0	1.8	0.3	112.30	0.00
2	0	1.8	0.3	27.20	−0.04
3	0	1.8	0.3	1.66	4.17
4	0	1.8	1.8	−0.19	0.84

of constant loss coefficients is preferable to neglecting these losses but the errors are very significant.

The calculations were repeated using pipe lengths 10 and 1,000 times greater than the 3 m length used to get the results in Table 1 and the results are noted in Tables 3 and 4. As expected, this shows that the effects of the

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TABLE 3. Network with Less Significant Pipe Friction Losses (Length of Each Pipe = 30 m)

Pipe number (1)	Flow Rates		
	Case a (L/s) (2)	Case b (L/s) (3)	Case c (L/s) (4)
1	116.3	113.0	108.0
2	58.2	60.7	66.7
3	33.8	34.4	31.1
4	58.2	52.3	41.3
5	33.8	30.0	29.8
6	24.4	22.2	11.6
7	67.6	64.4	60.8
8	24.4	26.3	35.6

TABLE 4. Network with Significant Pipe Friction Losses (Length of Each Pipe = 3,000 m)

Pipe number (1)	Flow Rates		
	Case a (L/s) (2)	Case b (L/s) (3)	Case c (L/s) (4)
1	10.28	10.28	10.27
2	5.14	5.14	5.15
3	3.08	3.08	3.07
4	5.14	5.14	5.12
5	3.08	3.07	3.07
6	2.06	2.06	2.05
7	6.15	6.15	6.14
8	2.06	2.07	2.08

approach used to handle the T-junction losses is much less significant for the intermediate length pipes and is insignificant for the long pipes.

The phenomenon of energy exchange between one stream to another is evident from the negative values for some of the loss coefficients in Table 2. Even though there is a gain in energy of the low velocity stream, it may be observed that energy dissipation occurs as expected when the junction as a whole is considered. This important concept is illustrated in the following analysis.

Consider junction 4 of example network as shown in Fig. 7 with the flow directions corresponding to case c. The energy per unit weight may be calculated at a section immediately downstream from this junction on pipe 5. From this value, the energy per unit weight at a section immediately upstream from the junction (pipe 4) may be calculated using the loss coefficient from the following expression.

$$\left(\frac{V_4^2}{2g} + h_4\right) = \left(\frac{V_5^2}{2g} + h_5\right) + K'_{45}\left(\frac{Q_4}{Q_5}\right)^2 \frac{V_5^2}{2g} \quad (14)$$

where h = pressure head; and V = flow velocity. From the network analysis

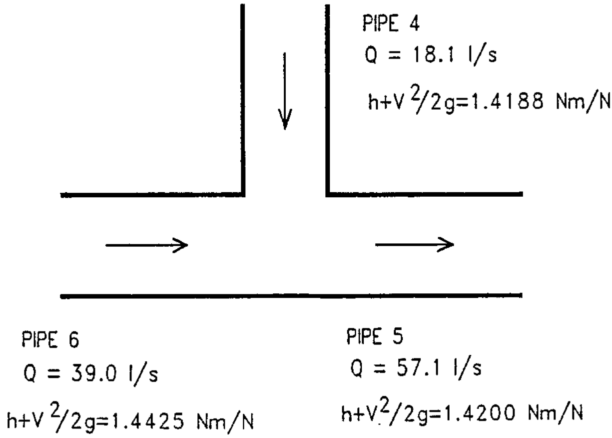

FIG. 7. Junction 4 of Example Network

results the specific energy [energy (N·m) per unit weight (N)] for the fluid exiting junction 4 is 1.42 N·m/N. Substituting all the known values in (14) gives the specific energy for the flow entering from pipe 4 as 1.4188 N·m/N. Similarly for pipe 6, the specific energy is computed as 1.4425 N·m/N. The energy per unit time for the flow entering the junction is computed as 0.08179 N·m/s using the following expression:

$$\rho g Q_4 \left(\frac{V_4^2}{2g} + h_4 \right) + \rho g Q_6 \left(\frac{V_6^2}{2g} + h_6 \right) \quad \text{(15)}$$

where ρ = density of the fluid. The energy per unit time of the fluid leaving the junction is computed as 0.08094 N·m/s using $\rho g Q_5[(V_5^2/2g) + h_5]$. Thus it is demonstrated that a net energy dissipation occurs at the junction despite the fact that a specific energy gain (represented by a negative loss coefficient) occurs for the low velocity stream.

CONCLUSIONS

Pipe network hydraulic analysis using constant loss coefficients for junctions to handle all situations is inadequate but may be preferable to neglecting such losses. Due to energy exchange between the merging streams there is a great variation in the loss coefficients as the flow rates vary. Although there is always a net energy loss at a junction, in some instances there is actually a specific energy gain for one flow stream due to an exchange of energy between merging streams. A negative coefficient is required to represent this situation. For pipe systems comprised of short, large-diameter pipes including pipe junctions, a precise treatment of junction losses is required for accurate hydraulic calculations. For these applications, the Gardel (1957a, b) and Levin (1958) equations, which fit experimental data, may be used to calculate the loss coefficients. If this is not done, then the calculated results are unreliable. There are many examples of piping arrangements in which junction losses are significant and the hydraulic calculations necessary for good design and reliable operation will require precise handling of junction losses. Because of the importance of correctly representing the effects of junctions for their designs, additional experimental data and comparisons of predicted and measured results are needed for a variety of pipe junctions.

For network modeling applications involving longer pipe lines the losses are dominated by pipe friction and the approach used to handle junction losses is not important.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

a = area ratio;
g = acceleration due to gravity;
h = pressure head;
K = loss coefficient;
K' = modified loss coefficient;
Q = discharge;
q = discharge ratio;
V = velocity;
Θ = angle between pipe segments; and
ρ = Density of fluid.